Linear Algebra
Prof. Arbind Kumar Lal
Department of Mathematics and Statistics
Indian Institute of Technology, Kanpur

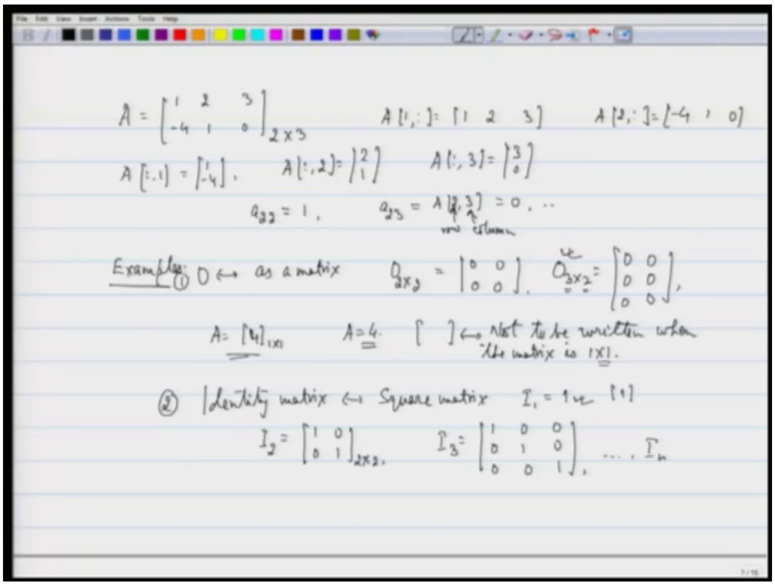
Lecture – 02

Alright. So, in the previous class, recall that we looked at notations and we also had definition of what the matrix is, that it is just an array and the entries are being represented as a ij, then rows are represented as a of something and so on. So, we will recalculate those ideas, alright.

And, we also had this idea that whenever I am solving a system of equations then solving a system of equation also gives some relationship between the vectors that relationship that whether I can write a vector in terms of some other set of vectors get related to what are call linear combination. So, can a vector be written in terms of other vectors or not that will come when we come to vector spaces and so on alright. So, that will take quite a lot of time to understand.

So, now for the time being we will again look at matrices, we will try to look at what are different types of matrices, examples there, then we will also look at what are called transpose of a matrix addition and try to understand matrix multiplication. So, let us look at the understanding of the notations itself

(Refer Slide Time: 01:25)



So, let me take an example. So, I start with a matrix A let me write it as 1 2 3 minus 4 1 0, then in this matrix this matrix has 2 rows and 3 columns. The first row of this matrix is 1 2 3 and the second row of this matrix is minus 4 1 0. Sometimes we may put call commas also because of understanding better understanding.

So, we have the first row and the second row, then there are three columns. So, the first column is 1 minus 4; the second column is 2 comma 1 and the third column is 3 comma 0. The entry a 22 is 1, the entry a 23 or sometimes as I said I may write it as A of 2 3. So, it means second row and third column alright, second row and third column so, which is nothing, but 0 and so on, fine.

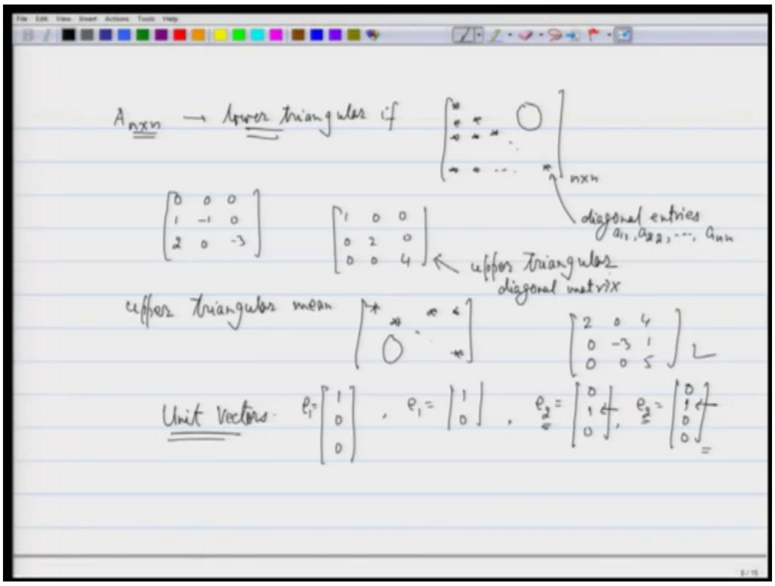
Now, some examples about important matrices. Examples: so, generally we will be writing what are called 0 matrices 0 as a matrix. So, 0 when I write as a matrix I may not write the size of it for example, 0 2 cross 2 will be 0 0 0 0 0 of size 3 cross 2 so, 3 rows here and 2 columns here, so, 0 0 0 3 rows here. So, I will just write 0 I will not write 3 cross 2, 2 cross 3, 1 cross 1 and so on alright.

One thing that I just recall recollect is that whenever I have matrix A which is of size 1 cross 1 alright it could be any integer 4, then I will just write A as 4 itself. It will not be written as inside a bracket. So, this bracket will be lost. So, not to be written when the matrix is 1 cross 1 because it is just a scalar quantity, alright. So, this is the 1st example one of the 0 matrix.

The 2nd example is what is called an identity matrix. So, it is a this is a matrix which is a square matrix. So, if you look at the matrix 0, 0 could have been a rectangular matrix could have been a square matrix also, but identity matrix is always a square matrix. So, if I want to write I of 1.

So, that is 1 cross 1 identity matrix it is just 1 for us or you can also write 1, but we generally write it as 1 itself. If I want to write identity matrix of size 2 cross 2 it will be 1 0 0 1, 2 cross 2; identity matrix of size 3 will be 1 0 0 0 1 0 0 0 1 and so on. And, a matrix of size n identity matrix of size n will be writing as I sub n.

(Refer Slide Time: 05:01)



Some other examples which are what are called now so, matrices again now we are going to look at a square matrices itself. So, if I write n cross n it is a n cross n square matrix. So, a matrix A is called lower triangular if; so, just look at this form here that if I look at. So, these are called.

So, these entries are called so, this matrix is n cross n I wrote it as n cross n these are the elements which are nothing, but the diagonal entries. These are called the diagonal entries. They correspond to the entries a 11, a 22, a nn, alright.

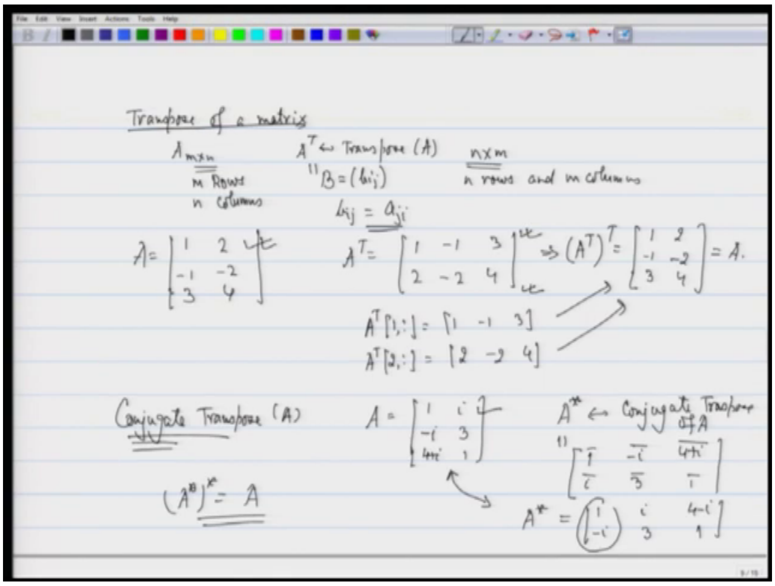
So, if I am looking at the lower triangular matrix these entries are supposed to be anything, but the upper part consist of only 0s. So, for example, if I want to write I could have 0 0 0 1 here minus 1 0 2 0 minus 3 this is a lower triangular matrix. 1 0 0 0 2 0 0 0 4 is also lower triangular. It turns out that this matrix is what is called upper triangular also as well as a diagonal matrix because everything other than the diagonal entries are 0 here.

So, lower triangular was everything above the diagonal is 0; upper triangular means means the matrix will look like. So, look at the diagonal entries, they could be anything, but below the diagonal everything is 0. Here these entries could be anything as such. So, an example will be a upper triangular matrix could be 2 0 4 0 minus 3 1 0 0 5. And, this could be of any size, but the important thing here is that it has to be a square matrix.

Sometimes we talk of triangular form and things like that in which case when you use the word triangular form it basically means that we have a matrix which is not a square matrix or maybe a square matrix, but it looks like upper triangular or lower triangular, alright. So, let me first look at what are called unit vectors.

So, example unit vectors. So, for us the unit vectors are going to be so, all our vectors are going to be column vectors. For example, e 1 in 3 dimension will be this; in 2 dimension it will be 1 0. Similarly, if I write e 2, e 2 will mean that the second entry is nonzero and the other entries are 0. It could be also be of size bigger size 4 size 4 or any n as such, fine. So, 2 means the second entry the second entry is 1 and the rest of the entries are 0, fine.

(Refer Slide Time: 08:41)



Now, the next thing that we need to understand is what is called transpose of a matrix. So, given a matrix A m cross n, we will write the transpose as this. So, this will be the transpose of A and this matrix has size n cross m. So, look at here, here it was m rows and n columns; here it is the other way around it has n rows and m columns fine and the entries they got reversed.

So, this is nothing, but so, the entries of this matrix if I write. So, if I write this as B which is b ij then the matrix notation we write b ij as a ji, that is the indexes get changed, the row becomes columns and columns becomes rows. So, example A is say 1 2 minus 1 minus 2 3 4 suppose this is my matrix A, then A transpose will be this row the first row becomes the first column, the second row becomes the second column and the third row becomes the third column.

So, I would like you to understand here and see that if I want to look at A transpose transpose, fine so, I am going to take the transpose of this. Since I am going to look at the transpose of this, the rows of this matrix are going to become the columns. So, the rows of this matrix the first row was so, first row of this matrix is 1 minus 1 3. Here when I go it will become the first column.

So, I will get 1 minus 1 3. Similarly, if I look at the second row of this matrix this is 2 minus 2 4. So, now, here it will become the second column. So, it will become 2 minus 2 4 and I would like you to see that this matrix is same as A, fine. So, A transpose transpose is itself. This is what we are saying, fine.

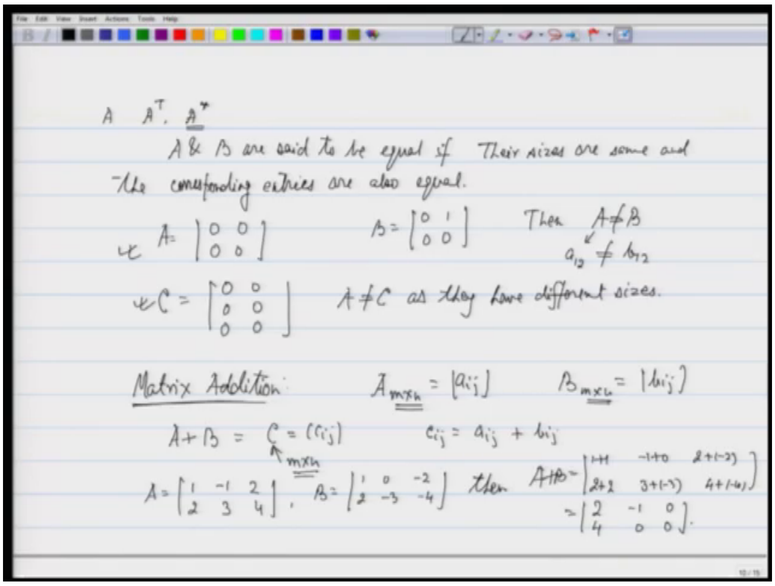
We also have the notion of what is called conjugate transpose. So, when we talk of conjugate transpose we generally look at a matrix A which has complex entries, fine. So, for example, if I have matrix A alright and it has entry say 1 i minus i 3 4 plus i and 1, suppose I have this then we write a star to say that it is the conjugate transpose of A and this turns out to be equal to again we interchange the rows and columns, but we also take the conjugate. So, 1 i is there will take the conjugate of that fine.

Similarly, you have minus i, you have to take conjugate of that 3 4 plus i conjugate 1. So, as such you are supposed to conjugate of everything, but since they are real numbers so, the conjugate is itself. 1, conjugate of i is minus i, conjugate of minus i is i, conjugate of 3 is 3 itself, conjugate of 4 plus i is 4 minus i, this. So, this is what are our so, given A this is our A conjugate or A star, fine.

So, again I would like you to check that A star A star will be A itself. Just verify it because again if I look at 1 and minus i this is a column vector, so, this will become a row vector and I have to take a conjugate of that. So, conjugate of minus i is i, I will get back this and so on. So, try that out yourself, I am sure you can understand it.

So, now I would like you to understand what is called matrix addition.

(Refer Slide Time: 13:21)



So, here I just looked at certain things which was conjugate. So, given A I could compute A transpose and I could compute A star, alright. I forgot to mention one thing which I should have said in the very beginning after the definition of matrix that two matrices A and B; A and B are said to be equal if their sizes are same and the corresponding entries are also equal.

So, for example, what I am trying to say is that if I have matrix A which looks like 0 0 0 0 and a matrix B which is 0 1 0 0 then A is not equal to B, because if you look at the entry here which is a 12, a 12 is not equal to b 12. Similarly, if I write a matrix C as 0 0 0 0 0 0 then A is not equal to C as they have different sizes.

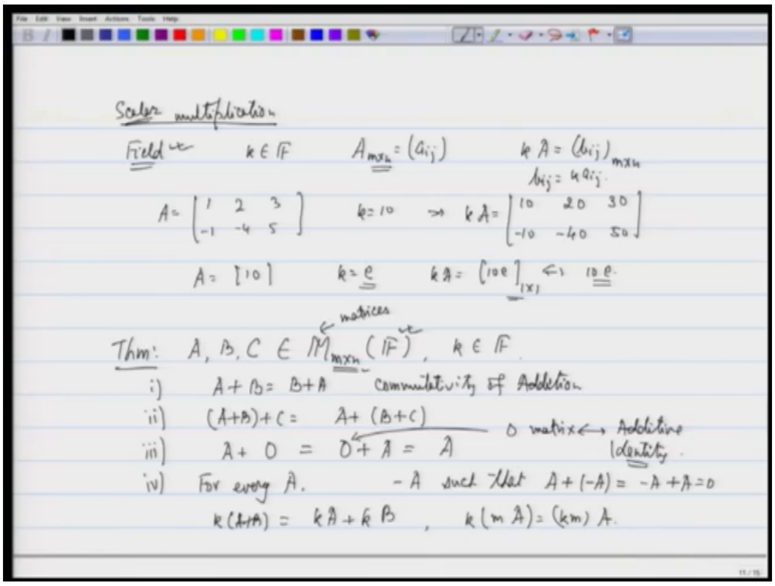
So, even though both the matrix are 0 matrices, but their sizes are different and hence they are not the same. So, you have to be careful when you say that two matrices are equal it means that their entries as well as their sizes they have certain properties. The properties are they need to have the same size – m rows m rows, n columns n columns and the entry a ij should be equal to b ij, that is very important.

So, now we come to matrix addition. So, given a matrix A of size m cross n entries are a ij, another matrix B of size say m cross n b ij. Important thing here, that the sizes of the two matrices A and B are same. A and B both have m rows and n columns, fine. Only when their sizes are same we can talk of addition or the sum of two matrices.

So, you can define the matrix A plus B as a matrix C whose entries are c ij. This matrix C is again of the same size as the size of m or B of A or B. So, again a matrix of size m cross n and the ij entry c ij is nothing, but a ij plus b ij. So, for example, if I take A as 1 minus 1 2 2 3 4, B as 1 0 minus 2 2 minus 3 minus 4, then A plus B is nothing, but. So, 1 plus 1, minus 1 plus 0, 2 plus minus 2, 2 plus 2, 3 plus minus 3, 4 plus minus 4 which is same as 2 4 minus 1 0 0 0, alright.

So, this is what the matrix addition is that we are supposed to add what is called component wise addition that the a 11 is to be added to b 11, a 12 is to be added to b 12 and so on till a mn is going to be added to b mn, alright. So, important thing they need to have the same size.

(Refer Slide Time: 17:38)



Then we have the notion of what is called a scalar multiplication, alright. So, mistake in the spelling, the scalar multiplication. So, when we talk of a scalar basically a scalars are nothing, but the elements of a field. So, recall what was field. The field was some set in which we had addition, subtraction, multiplication and division by your nonzero element.

So, a scalars are elements of these fields. So, take any k belonging to F and a matrix A of size m cross n which is a ij, then when I talk of k times a then k times A is a matrix which is b ij say b. Again, if the size of the matrix A was m cross n the size of this matrix is also m cross n and b ij is nothing, but k times a ij. So, for example, if my A is 1 2 3 minus 1 minus 4 5 and k is say 10. This will imply that k times A will be equal to 10 20 30 minus 10 minus 40 50. So, you just multiply every element by that number, fine.

Another example if you want. So, if I have A as say mine just 10 itself and k is e the exponential e, then k times A is nothing, but 10 times e. So, either as I said since it is a 1 cross 1 matrix, you can write it as inside the bracket or you may just write it as 10 times e whatever way you want, fine.

Now, there is a theorem which relates addition with scalar multiplication. So, let me write that theorem. So, here I have got matrices A, B and C, they are all of the same size say m cross n and over some F. So, this m cross n means all matrices. So, capital M for matrices; again, you see that I have got two vertical lines and one extra vertical line. So, that indicates that I am looking at a set, but a set of matrices which have m rows n columns and the scalars are going to come from the set F, fine.

So, I have these and I also have a k scalar which is coming from F itself. Then the following holds: i – you can check A plus B is same as B plus A, commutativity of addition; ii – associativity holds. So, A plus B plus C is same as A plus B plus C, that I can add them in whatever order I want, alright.

iii rd – for every matrix A whatever A I am got I have a 0 matrix. So, that A plus 0 is 0 plus A and it gives me A. So, I have the 0 vector for me. So, here this 0 0 matrix behaves as additive identity. So, identity basically means which does not change. So, with respect to addition if I add A to 0 A does not change a remains as it is.

iv th – for every A we have a matrix B, we generally write it as minus A such that A plus minus A is same as minus A plus A which is 0. I think I should not write B here because you may get confused with a B that is given above. So, given A I have A minus A such that A plus minus A is minus A plus A and so on, fine. And, now the scalar multiplication k times A plus B is k times A plus k times B and k times m of A is k m times A alright.

So, these are the properties that are true of vectors also if you remember the sum of two vectors x and y was x plus y and we know that x plus y was same as y plus x. I could add the vectors x, y and z that is there was notion of associativity there also, for every minus x which was a negative direction which gave me 0 the vector x plus 0 was x itself and I could increase the size of any vector or reduce the size by multiplying by reciprocal of some number and things like that.

So, that is all I have written here and nothing else. So, I would like to end this class here itself. In the next class, we look at what are called matrix multiplication.

Thank you.