Linear Algebra
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Lecture – 12
Row Reduced Echelon Form (RREF)

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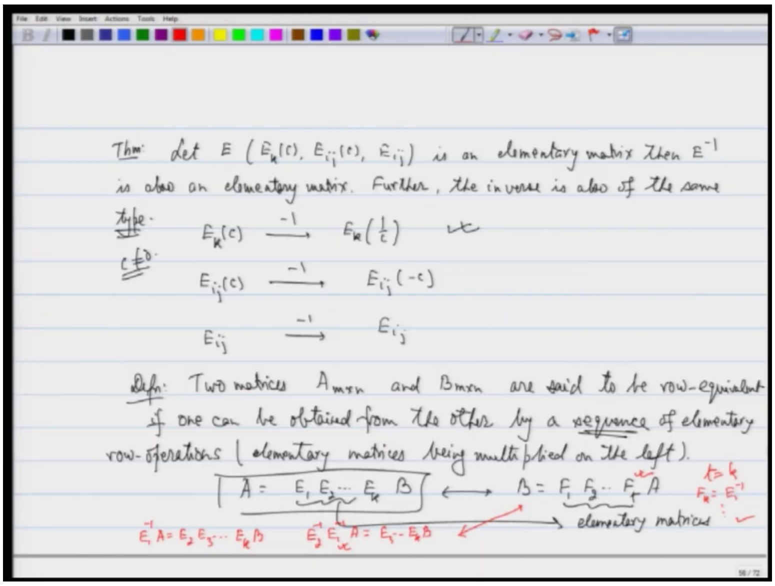

So, having understood elementary matrices, let us sees and we have also seen that they are invertible. So, some results there, what we had seen was that if so, let me write this as a theorem that let. So, I will just write E 4 either say E k of c or E ij of c or E ij, if E is an elementary matrix, then E inverse is also an elementary matrix. Further, the inverse is also of the same type.

So, what do I mean by this of the same type? So, what we are saying is that the inverse of E k of c, if I apply the inverse map it is E k of 1 upon c. If I have the matrix which is E ij of c, here

I assuming that c is not 0 that is a very very important. Then I want to apply the inverse then it is nothing but E ij of minus c and the inverse of E ij is E ij itself.

So, multiplying by c gives us multiplying by 1 upon c replacing the i-th row by i-th row plus c times the j-th row gives the inverse of that as i-th row being replaced by i-th row minus c times the j th row and interchange of i-th and j-th is again interchange of i-th and j-th, fine.

So, some definition now, definition: two matrices A m cross n and B m cross n are said to be row equivalent if one can be obtained from the other by a sequence of elementary operations. So, elementary row operation let me clear about it row operations and what we are saying is by a sequence of elementary matrices being multiplied on the left.

So, what we are saying is I can get A as equal to some elementary matrices E 1 E 2 E k times B is there or which is same thing as saying that I can also write B in terms of some elementary matrices F 1 F 2 F t times B sorry F t times A is there ok. So, these are elementary matrices and these also where elementary matrices, fine and what we know is that they are inverse that invertible.

So, if you want to see you can relate these two also. You can see here that I can have multiply by E 1 inverse here is for example. So, I will get E 1 inverse A as E 2 E 3 E k times B again I can multiply by E 2 inverse. So, I will get E 2 inverse E 1 inverse A will be equal to E 3 till E k of B and therefore, if you want to relate the two, you can see here that look at here F of t will be equal to E 1 of inverse.

So, you can see that t has to be equal to k alright. So, F k will be equal to E 1 inverse and so on. So, I would like you to try that out yourself fine and get a better understanding of things.

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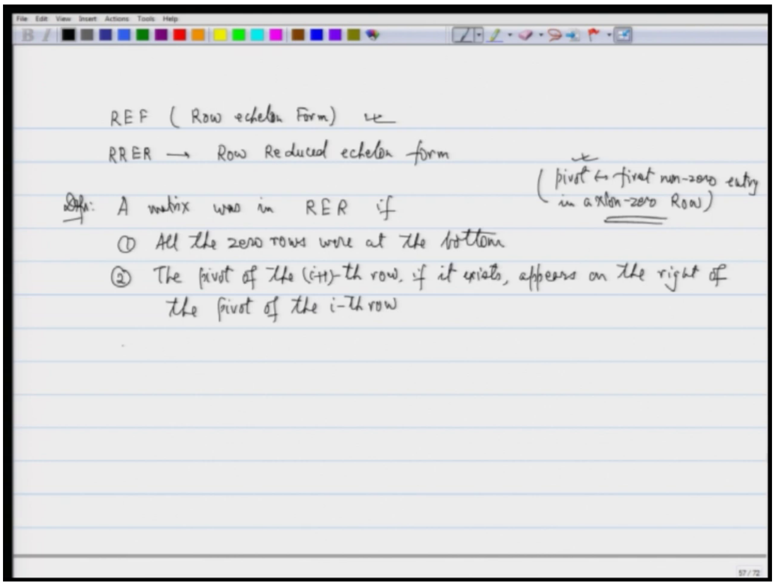

Now, what we would like to understand it what are called row reduced echelon form. So, I want to understand now. So, we are understood what is called REF earlier, what is the row echelon form. Now, we would like to understand what is called RREF. So, the full form of this is Row Reduced Echelon Form. So, let us recall what was the row echelon form.

So, a matrix was in REF if 1 – All the 0 rows; all the 0 rows were at the bottom, alright. 2 – The pivot of the i plus 1-th row if it exists, appears on the right of the pivot of the i-th row, fine and what were the pivot? Pivot was first nonzero row non-zero entry in a non-zero row, fine. That is very very important what is a pivot. Pivot was the first non-zero entry in a non-zero row, alright.

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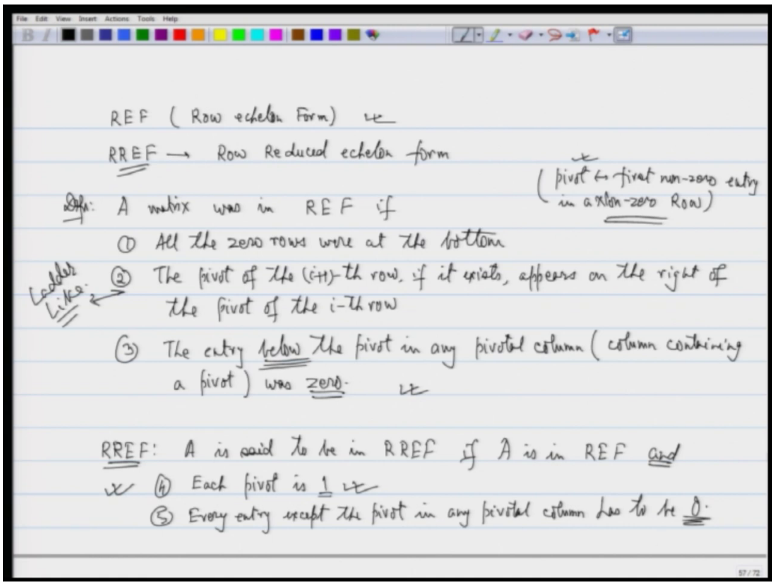

And, the third thing about the REF the row echelon form was the REF was that look at the pivots, the entry below the pivot or entry below the pivot in any pivotal column or bracket, I mean column containing a pivot; column containing a pivot alright, the entry below the pivot in any pivotal column was 0, this was what it was.

So, this part that second part gave us what are called the ladder like or the staircase type and it said that 0 is at the bottom and there pivots and below the pivots everything is 0. So, this was this. Now, sorry here also I wrote wrong I RREF it should have been here.

Now, we would like to look at what is RREF. So, the only difference in RREF and this is that this matrix A is said to be in RREF if A is in REF and there extra condition here. So, I can write it 4 here each pivot is 1. So, there we allowed the pivots to be any number we did not have to divide by any number or multiply by any number to get the pivots at 1, now we are saying that each pivot has to be 1. So, this is one extra thing that we are saying.

And, the 5th what we are saying is look at the 3rd in the previous case the 3rd says that every entry below the pivot has to be 0, alright every entry below the pivot. Now, we want that every entry other than the pivot. So, every entry except the pivot in any pivotal column; in any pivotal column has to be 0.

So, this is a very very important restriction that we have that for RREF we need that the pivots have to be 1 and this has to be there that every other entry in that column has to be 0. So, let us look at some examples to have better clarity of things example.

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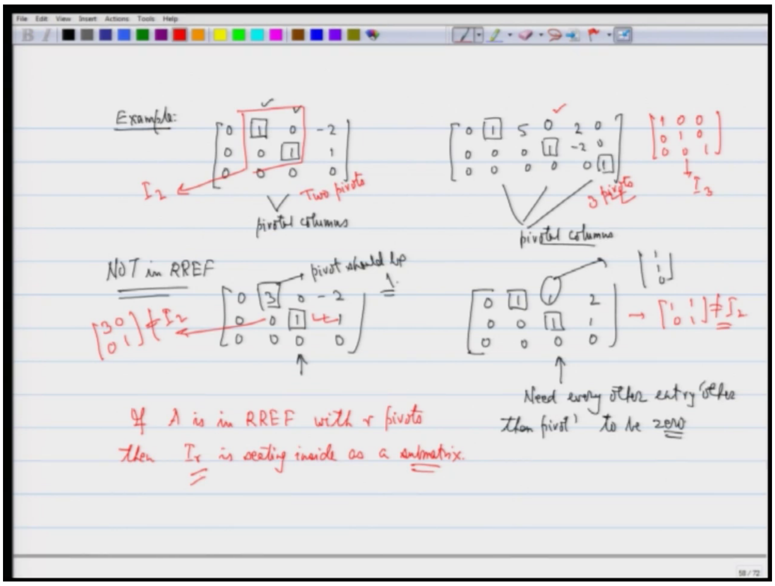

So, let us look at example. So, let us example 1, so 0 1 0 minus 2 0 0 1 1. So, if I look at this matrix the first nonzero entry in the first row is this, the first nonzero entry is this here. So, these are the two which are the pivotal columns, alright. Look at every other entry in this which is 0 here alright, so this is in RREF. Another example, put a 5 here, suppose I have this, alright.

So, here if I look at this is the pivot, this is a pivot, this is a pivot and I have got corresponding things here, fine. So, this is so, pivotal columns are these are the three pivotal columns. Now, what are not in pivots? So, which are not in RREF, see some examples of that. So, you can have your 0 3 0 minus 2 0 0 1 1 0 0 0 0. So, this is not in RREF because this part is fine, here there is a problem the pivot should have been 1; pivot should be 1, alright.

It may happen that I have this matrix here which is 0 1 1 2 0 0 1 1 0 0 0 0, alright. Now, again if I look at this is a pivot, this is a pivot pivots are nice, they are 1, but look at this column this column we need every other entry other than pivot except other than pivot to be 0 alright, which is not the case here alright. So, this is column is 1 1 0 alright this column looks like this which is not allowed, fine.

Now, what I would like you to understand here is that which got very very useful for us that here there are two pivots. So, there are two pivots here. I would like you to see that there is I 2 sitting here. So, there are two pivots, so, there is a I 2 which is sitting here. Here in this example there are 3 pivots and therefore, I have got 1 0 0. Sorry, there was a mistake here. It should have been 0 here because alright. So, this should have been a 0 here, fine.

Since there are 3 pivots it is in RREF so, we need that. So, look at the second one now, it is 0 1 0 and this is 0 0 1, fine. So, there is a I 3 sitting inside this. So, what we are trying to say is that if so, what I am saying is that if A is in RREF with r pivots then Ir is sitting inside as a sub matrix, this is very important, alright.

So, just construct your things you can see that. For example, here even though there are two pivots the corresponding part that I have is 3 0 0 1 which is not I 2 fine. Similarly, here if I look at I have two pivots, but my matrix looks like this which is not I 2 alright. But, if I have got r pivots and my matrix is in RREF, then I will always have alright always have I r coming into play. This is very very important thing.

Now, we will try to understand in the next class that how to get this matrix RREF and what is the importance of this RREF.

Thank you.