Remote Sensing and GIS
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Lecture – 01
Vectors in plane and space

Welcome to the second part of the promise course on calculus of more than one variable. I am happy that the course on calculus a wonderful variable has been well received. It has had one rerun or two reruns, I guess. Maybe you would have a rerun in the current one then next session of the MOOC courses. As before, I would like to concentrate on the board but keeping in view that there has been a request to be on doing things on the tablet.

But I believe that just been in the tablet is a quite a boring thing. You would be just seeing somebody, then blah, blah, blah, and then something appearing here on the screen. Unless some drama is done on the blackboard marks does not really becomes an interesting stuff. So I decided to make a combination of both the blackboard and this tablet. So I start with my first lecture.

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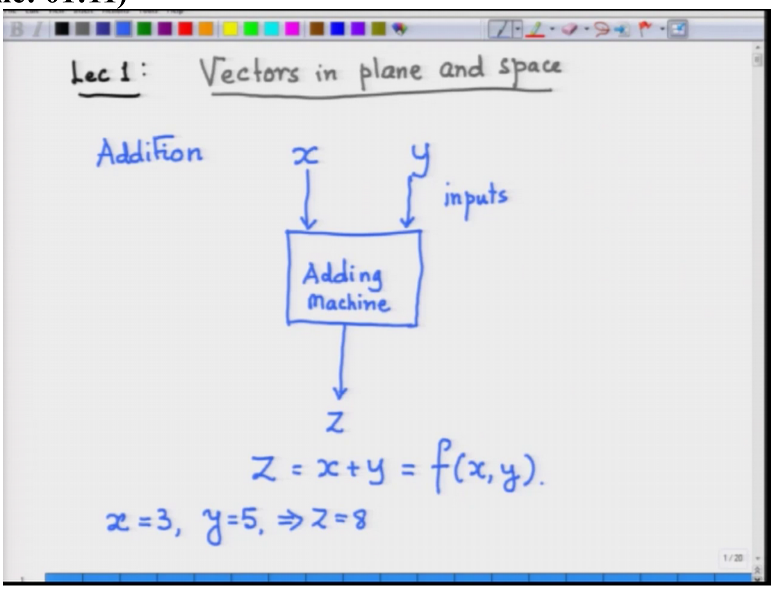

Which is called vectors in plane and space. You most probably already have learned something or geometrical vectors in school. So something like this is a vector in space vector is planning something which I do not have to tell you. But let me first start motivating this whole idea of functions of more than one variables and doing calculus on them. You know, one of the first thing that you learn in school addition.

So you are given two numbers, and you are asked to add. So put it in a more abstract way, what are you doing? You are just doing the following. So I give you a number x, and I give a number y. And there is a kind of machine calls in adding machine. So let me write down this machine is an adding machine. And then if you give me the input x, and if you give me the input y, I give you an output z, these are the inputs.

I give you two numbers, and you add them and you give me a single number of the input. So which I write as z is a call to action x plus y. So when x seven x = 3, and y = 5 z it implies that z = 8, output is 8. So these 8 equal to x + y can be written as f of x, y function of x and y. I want to also mention that this notion of vectors, is intimately linked with the function of more than one variables.

So you observe that here, when I am adding, so you are doing something in school. So before you learn function of 1 variable, here essentially started learning functions of 2 variables. And this is the fun we really do not realize it. And what I want to say is that why the notion of a vector is associated with this function of more than one variables for us.

Let us just Hold on to 2 variables, because you see when you are writing a function of the form z = x + y, you have 2 input variables. Basically what you give me you give me an input variable x and y. This is my input variable and outcomes output. So there is some operation here addition operation, and outcomes output z, which is x + y. So, the input variable is not one real number, but 2 real numbers.

So, what lesson I learned here the first step is that input variable when I add 2 numbers, input variable I can view it like this It consists of 2 real numbers right, it just consists of 2 real numbers. Okay, great. Now how does how do we represent 2 real numbers. We present them as a point on the plane as a Cartesian coordinate that is what you have learned in your basic school.

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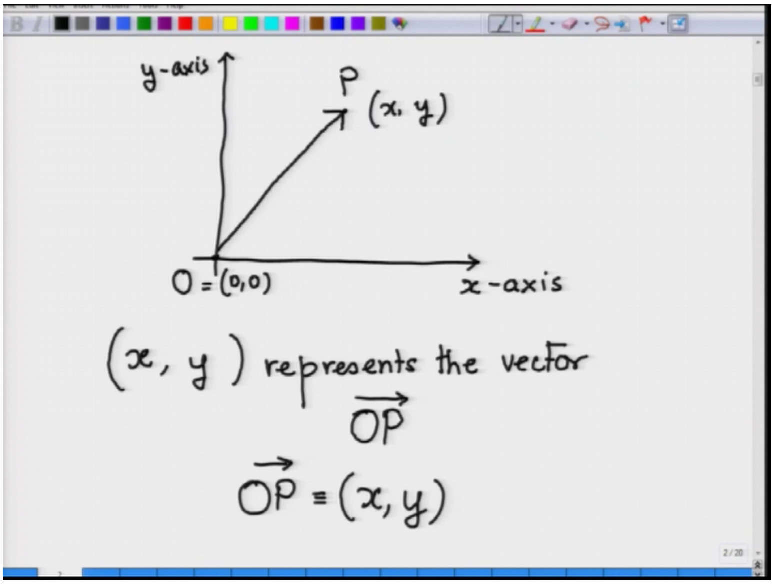

So here I draw drawings do not become clear why straight lines are usually curvy lines as my hand is shaking, which sometimes does I cannot just pull them straight for some strange reason. So this is the x axis, which you know barrier and this is the y axis, the vertical axis. With this point, we know about region 00. So any point x while it can be anywhere, I am just putting it here in the first quadrant does not mean that it is only in the first quadrant.

So, any bond x y, I can view it as a geometrical vector just draw a line segment connecting 00 with x y and at the point x y put a direction. So, this x y call this point x y which has coordinate x y, these x y represents the vector OP x y represents the vector OP. So the tip of the vector is presented by 2 real numbers. So I cannot sometimes right because O Always 0 0 if I write this is Oh sorry,

I not given always origin, always 00. So OP vector can always be written as x y. Great, I think it is good up to here. So the plane is a collection, the plane is a collection of these points, this whole two dimensional plane. And we call all these collections we give a name. We call it the R to space. So R is the space of real numbers on which we did calculus one variables.

So it is just a line the origin and the numbers written on the left and right the negative wants to the left and the positive wants to the right. And now R 2 which you see is the whole plane, the plane of the board actually, it is kind of infinite. We do not imagine it like that. So, a lot of things in this happening in your imagination, which you might find very strange when you want to actually related with physical objects.

Now, that is like a different story will not get into that at this moment. Go. So, let me just write down that 2 dimensional of Cartesian plane is given as a collection of vectors of this form. Where x is in R and y is in R. Now you might ask me why you are calling this OP a vector. So what is a vector you have learned in physics? That vector is a quantity which has which is an object not a quantity really is an object, which has both magnitude and direction. So, let me write down.

So is the quantity having magnitude means a length, length means length from origin, magnitude and direction? So, let me just give an example, look at this quantity OP here. So, OP is in one direction suppose I can draw a vector the same length as OP, but will have a different direction for this can be portrayed a drawing a circle.

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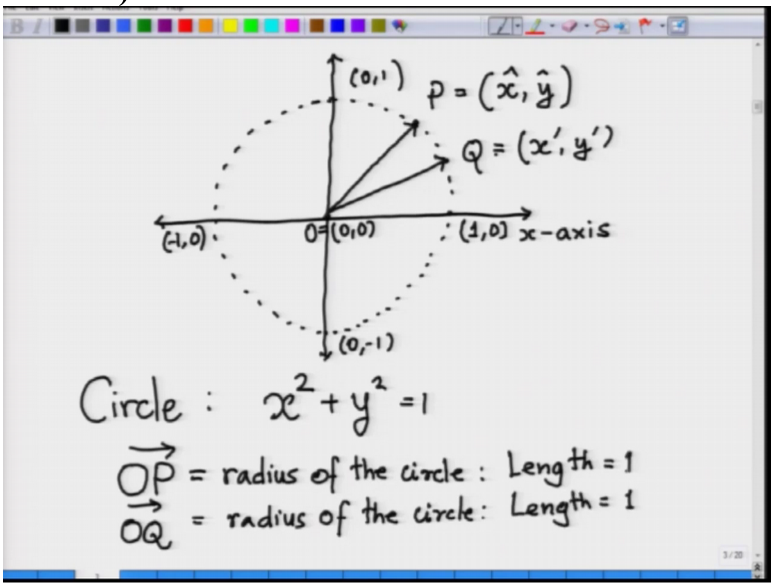

So, give me some time to draw a circle cell draw circle with center 0 and radius 1. So here I have x axis. So physics tells me vector or quantum objects we just bought magnitude plus direction. Now here this, this vector OP has a magnitude, which is the length of this vector OP, which will come will come to later on in the next lecture, how do we calculate length or all the stuff, but it also the reaction.

Now, I will show you that there are two objects, two vectors, which can have the same length but different direction and hands, they are different vectors. So, two different points on a plane actually represents two different vectors. So that is what is the key idea behind it. So, let me draw a circle like this. So, the circle given by the equation access square plus y squared equal to one circle.

Which has been centred at all the origin and the radius one my drawing is very bad, many of the viewers has much better drawing than me. So I am sure that you would forgive me for my stupid drawings, but I am trying to actually convey some mathematical idea. So this is a circular bond is a circle. So this is 1 0. This is -1 0. This is 0 -1. And this point is 0 1. Not take two different points.

On the circumference of the circle said consider P which is x hat y hat and Q which is x dash y dash. Now observe OP and OQ so what is so you joined with all both OP and OQ. So OP is the radius of the circle. So its length is 1 because this has radius 1 OQ is also radius of the source And hence has the linked one, because these are circle of radius 1 that is what this equation says link one, but look at the direction of OP and OQ you can immediately see.

That the directions are different and so, they cannot really present the same vector. So, that is why a point or input for function of 2 variables is represented by a vector. Now, how can I go and you present vectors in 3 dimensions of course, you know that up to 3 dimensions everything is I can visualize and beyond three dimensions I may not be able to visualize, of course, you cannot visualize then geometry becomes algebra, you can just too many geometry manipulations or view to algebraic manipulations.

So, when you are in 3 dimensions, when you are two dimensions, you are talking what length and breadth when you are in 3 dimensions, you are talking about length breadth and height. So, here I have 3 axis, this is my x axis, y axis and z axis. So this is x axis, y axis and Z axis. So any point in a 3 dimensional plane 3 dimensional space has three components length breadth and height, and has to be represented by 3 numbers.

So here is any point PB the kind of prototype point, and this is having 3 numbers x, y z, so if you drop a perpendicular on the x y plane, this x y plane and then from there it hits the x y plane at some points P dash and from P dash you draw perpendicular on the x axis and then perpendicular on the y axis and this length from the origin O to save Q this is Q dash and Q double dash.

And Q dash OQ has a length XOQ dashes LNY and PP dash this one has a length z this one this this whole thing as a length said so, that is your length, breath and height. So, once you have created a model or with three variables so you can construct this is called Art three which is called the three dimensional Cartesian space. So, let me write down three dimensional.

Now, you might ask me what is a dimension is there a proper definition or dimension Yes, but here when you take on first our intuitive knowledge or into the viewer we know what is the meaning of being in 3 dimension we you ourselves are three dimensional objects sitting in a three dimensional space of this room length better hide. So, these are 3 dimensional Cartesian space Clark Cartesian.

His name is coming after the famous French mathematician for any they caught they were invented to coordinate geometry. So, how do I represent this? Any point is a triplet of numbers x y z, where x is element of R y is element of R and z is element of R no can there we are four dimensional space of course, the modern theory of special relativity depends on four dimensional space. So, special relativity

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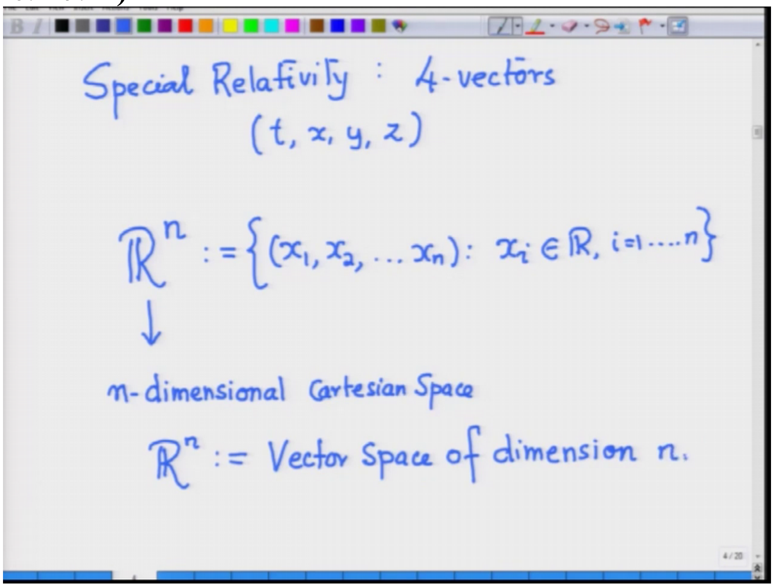

For example, uses just called four vectors. So, where the first vector is time and the rest are the space vectors. So, that gives us makes a four dimensional space and four dimensional system of space time. So, special relativity what is the time vector and x y z or the space vectors. So, the geometrical view of physical space is much better really realized.

Because time as Einstein showed that time is not as Salute neither is space and everything is relative and hence, it is much better visualize if we club up time and space together, but anyway, we are always all in our normal physical spaces three dimensional and when we add on time, we get a four dimensional space. So, this is natural to have spaces of more than three dimensions.

So in general mathematicians speak about an interdimensional space and the model is called R n. It is called a card international Cartesian space. So any vector in n dimensional Cartesian space will call 2 dimension you have to 2 elements, 3 dimension you have three elements and in any mention you must have any elements. So, you have x 1 x 2 x n says that each x i is a real number.

Where I is equal to 1 2 and that is our model. So such spaces are in which consists of vectors is R n. So, R n is often categorized as a kind of space with contents vectors and sometimes it is called a vector space vector space of dimension n now, in geometrical vectors that you have learned, how do you add two vectors, you know when you when you do your geometry in two dimensions.

You know, how do you add two vectors? I am just removing this part. So, you can easily have an idea.

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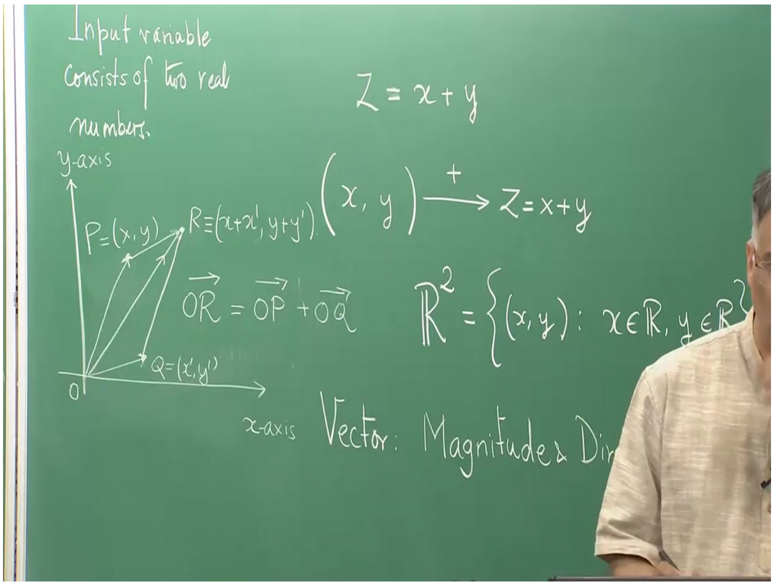

So, what do you do you add two vectors, how do you add two vectors you know in the in the standard school high school vector geometry what do you do is that okay you have 2 vectors. One is so, this is OP this one is P with the coordinate x y another is say, for OP is OQ. So, given a x dash y dash how do you add it? How do you add these two vectors? So, you draw a line from P parallel to OQ and so I draw a line parallel to OQ.

Basically I from QI draw a line parallel to OP and then from here I draw a line parallel to OQ and the meet at the points R. So the new vector we joins O and R is called Addition of the two vectors or an OQ. So OR is viewed as an addition of OP and OQ. Fully we have OP plus OQ. So, what is the coordinates which you presents this vector.

If you are slightly if you take a little bit of time out of your busy schedule of today's life or you leave your mobile phone for quite a while, you will immediately show that this coordinate is nothing but x plus x dash will be the x coordinate and y plus y dash would be y coordinate. So, any such space vector space of dimension n has two major operations that are important.

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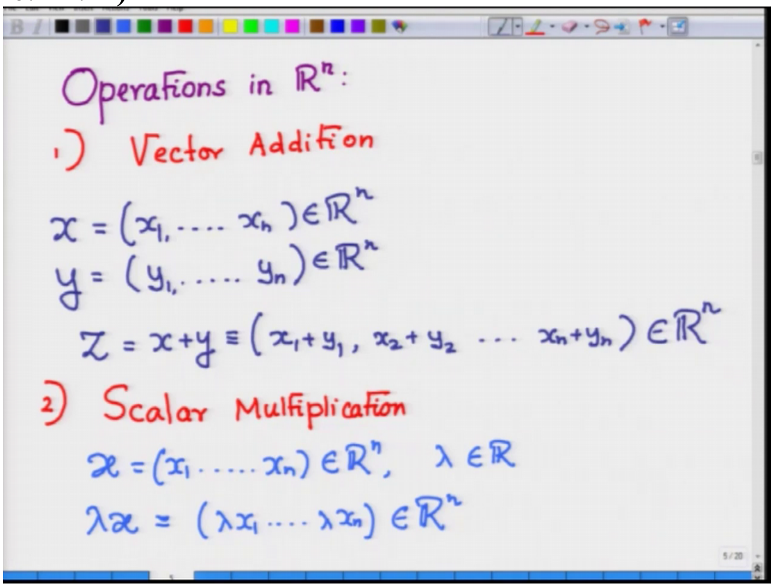

Operations in R n so there are two operations in Rn. So what are those operations? Number one is called vector addition. So how do I add two vectors you know ultimately the result and vector will be represented by a vector who is a squad x coordinate with the sum of the x coordinates and the y coordinate will be the sum of the y coordinate the inclusion that you have in two dimensions come up.

As I told you in the introductory lecture, the difference essentially is when in order not to one once The slight difference in our in our three was one zero some idea or three, you can never have your fair idea what are three, the rest is algebra vector edition. So what does vector edition mean? So you have two vectors x, x 1 x 2 x n in R n and y equal to y 1 on y 2 y n. So these are the coordinates in R n.

And then the new result and vector OG the addition of these two vectors has a coordinate. So, you add the first coordinate of x with the first coordinate of y So it a x 1 plus y 1 then the second coordinate is x 2 y 2 to an y n you the same institution what we did here, we added the first coordinate with the first coordinate of P with the first coordinate of Q and coordinate of QP with the second coordinate of Q.

So you add the two x axis, the first position and identify positions, the same thing we have done here, you just define this whole thing. There is another operation which is called scalar multiplication. Basically, you cannot pull a vector or single vector. So here is a vector line, you can pull it like a string, and even also string it down like a string. So these called scalar multiplication scalar here for us would always be a real number.

So what is scalar multiplication what do we do with this killer multiplication what we do is that you take an x x x 1 x 2 x n and in R n and you take a lambda element OR then lambda times x is same as a vector lambda of x one lambda of x n. So direction of these vector new vector lambda x is same as the direction of the vector x while the direction of the vector x plus y is like or as a different direction from both OP and OQ.

So, x and y has different directions from it as a different direction from x and y. But when you do a scalar multiplication lambda acts as a same direction but the length only changes and this is also in R n okay. So, these are something which is typically absolutely. So, I have to also write for here you can understand that is also in R n, because these are a top of a numbers.

So, without telling anything you should also be able to know that this is a normal. Now, these are some properties I if I keep on telling you all the properties will bore you either you go to the text which is mentioned, which I can show you that this still available you can go to them and go to Amazon and buy it basic multi variable calculus. It is an excellent, excellent book. I do not think there is anything which is better than this.

I have not found anyone which is written in a simple, nice way for everyone to understand. I guess the audience here is not just about mathematicians it is all consists of physicists, chemists, engineers and mechanical engineers, electrical engineers, economist and statisticians and any maybe even from biology. So I am trying to make it look as much as simple as possible there, but there are certain things about the properties you can do certain things one and for example.

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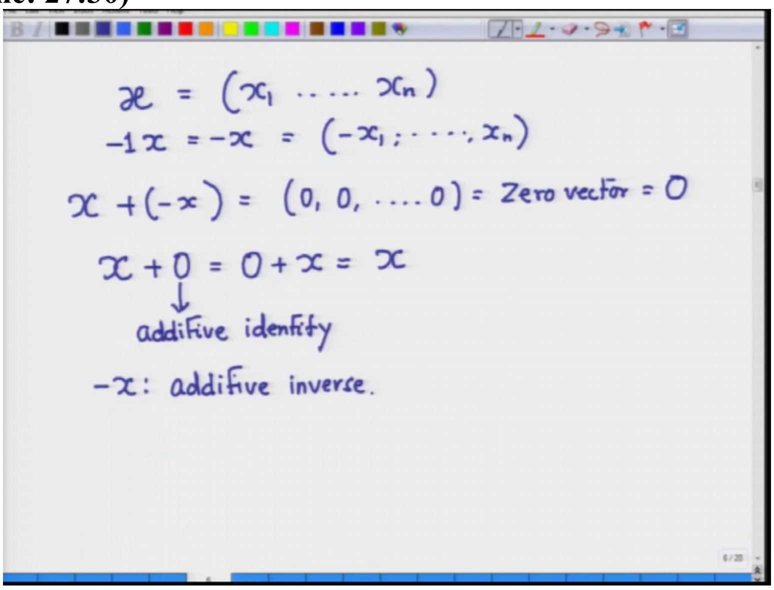

So, if you have an x as a vector x 1 x 2 x n now what I do, I also, I take a vector and I multiply 1 with x, which I write as minus of x and this is the vector whose coordinates are now suppose they add these two vectors x plus minus x, what do I get? I get 0 0 0 it is a zero vector. Zero vector which we can also right as a big zero. Another interesting fact is that if you now add 0 is any vector.

With addition is community advisory, right, x + 0 = 0 + x does not matter or x plus y and y plus x are the same. This will make x remain as x. So sometimes by taking a little advanced mathematical language borrowing it from group theory in algebra, we call this as an additive identity. So this is slightly more erudite language you may choose to use it you may not choose to use it minus x by the way, is called an additive inverse.

Now, what is more important is how can I represent any numbers in 3 dimensional space. Can I represent any number any vector in 3 dimensional space in terms of some basic vectors in terms of some standard symbol vectors can I always do that.

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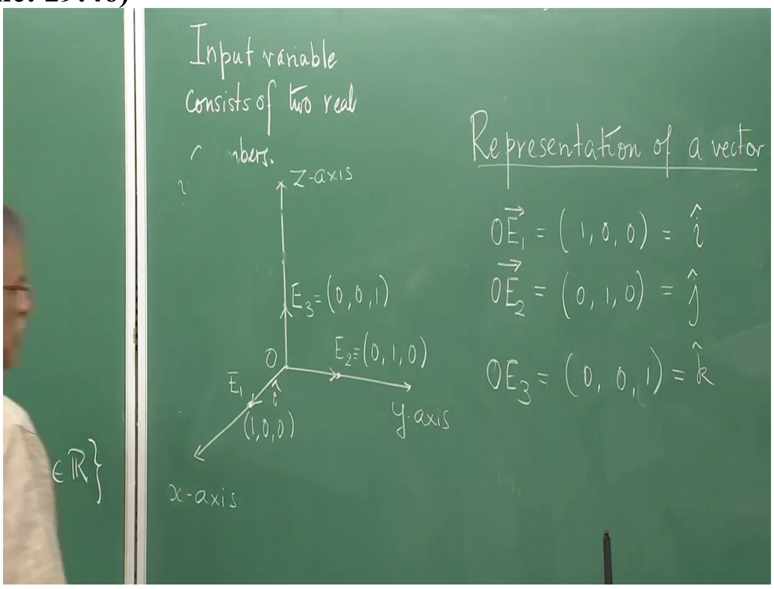

And that is what I am going to show and that is what we are going to discuss which is called the representation of vectors. And I am going to use a notation which physicists would like an engineers all also like mathematics mathematicians who most some linear algebra maybe why some luck. Might be disappointed, but does not matter what I write is also can be read very easily in the way modern mathematicians, right representation of actors.

So, we will just bother about 3 dimensional vectors first. I will leave you with a question, or how do you represent n dimensional of vectors. So in 3 dimensions which we already have there. So here is a 3 dimension. Again, again I am drawing, so there is little time which flows off as you draw it. Now underline this. So, in this axis, I will first draw 3 vectors.

So the point where I have the coordinate 1 0 0. I will call these vector I will call this point E 1. And this origin and this point E 2 which will have 0 1 0 vector, and the point E 3 on the z axis, maybe not that we are going too far, E 3 which will I will call it 0 0 1 vector. So, the 3 vectors OE 1 OE2 OE 3 represents a point 1 0 0, 0 1 0 and 0 0 1 respectively there these vectors are representing those coordinates.

Now, this vector OE 1 given by 1 0 0 is called the i vector by physicists. So this is what physicists would call i vector. And OE 2 0 1 0 is called a j vector by a physicist. And OE 3 0 0 1 is called the k vector by physicist. Now I come back to the drawing board and let me draw.

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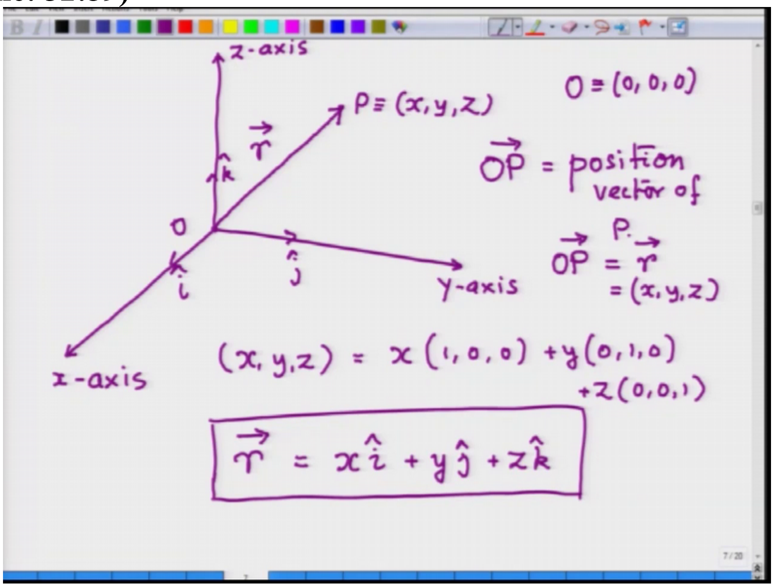

So I now have the x axis, the y axis and the z axis. Let us consider any number of P. So in any not number I am so sorry any point p, which is represented by x y z the 3 coordinates. So I connect them and complete the vector OP were always the origin with 0 0 0. So the origin is always 0 0 0 am just to remind you, I am writing it on the side so that you do not forget when you receive the video. And here you have the i vector. Here we have the j vector. Here you have the k vector.

Physicists would always like to call OP a position vector the position vector of a point that is a vector OP is talking about is giving you the position of the point being in space. So OP, as many physicists would call as a position vector, position vector of P. Now, many physicists would like to write this position vector as r vector. OP vector is equal to r vector. How do I represent r vector?

Now you go by simple mathematics, so now, if you look at it very carefully, I can write this vector x y z in the following way. So, x is a real number take x as a real number multiplied with 1 0 0, the scalar and multiply scalar multiply y with the vector 0 1 0 and scalar multiply the vector 0 0 1 with z and add them up that would give you these vector x y z you can just try it out it is very simple. Now, once you have done it, you know that this is representing the r vector.

So, this r has been represented by this coordinate x y z and this is our direction of ijk vector this character. So you can write what physicists always write that r vector is x into i vector y into j vector and z into k vector. And this way of writing has a lot of advantage, especially when doing applications in natural sciences. And I am an engineering and that is what will you will really use in many, many our applications.

We are not going to go beyond this today. So, just to end it, I am giving you an example. Now, consider how do I present a line for example, in 3 dimension, is an way I can represent a line, vector and write down. It is the question in three dimension. You see how we can do it using vectors.

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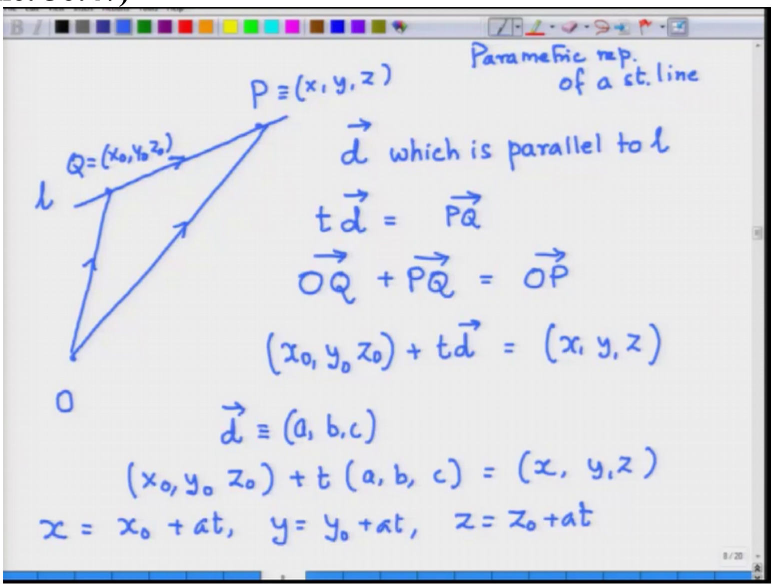

So, take a line l and which passes through a point Q am not an always origin here. I am not trying to access which hurts coordinate x 0 y 0 z 0 and another is P which is a called a running coordinates any at point P which is lying on this line l. So from O you draw this position vector of Q OQ and the position vector of P OP. Now called tis vector PQ. And let us consider a d vector.

And in this direction of l, this parallel to l, there is a vector d consider vector d d vector, which is parallel to l. And so, it does is essentially along the same direction of PQ. So, d is say given us some real number t times PQ vector or PQ can be written as t into d the right you will soon realize that okay I have basically over some taken over to d and you have scale it up scale that or I can take a vector like this a bigger one and I have scale it down or scale it up.

I will get it could be in this direction d I can scale it in the negative way putting the negative number and I can get it make it exactly equal to PQ basically what do you do. You take the length of do you take the length of PQ you take the ratio and see what is t. PQ by d should be T. Now, if you look at the vector P, you will immediately know from your basic geometry vector geometry that OQ plus PQ is exactly equal to OP vector.

Now, what is OQ? OQ is presented by the coordinates x 0 y 0 and z 0 and PQ is t into d and OP is x y z now t Now, suppose d is a position vector of a point whose coordinates are a, b and c. So, a, b and c are basically lying on line parallel to the line l then what would happen I can write this equation now becomes x 0, y 0 z 0 plus t times a b c is equal to x y and z.

So, what do you have you have that x is nothing but x 0 plus a t y is nothing but y 0 plus a t and z is nothing but z 0 plus a t. So, this is exactly the equation of a straight line lying in the space lying in 3 dimensional space in terms of the parameter to it is called a parametric representation of a straight line. So, what we have learned now is parametric representation of a straight line.

And I am using shortcut in this. What is up, world shortcut is much more important. So, friends, I think you have enjoyed this talk, I hope so you will decide. And we, I hope this mixture of the board and this makes much more sense. And so let us send it up here today and the next lecture will be on lens of vector and in our products dot products. Okay, thank you very much and good night.