Introduction to R Software
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Lecture – 08
Matrix Operations

Welcome to the next lecture on the course Introduction to R Software. You may recall that in the last lecture we were discussing some matrix operations and we started with a discussion on how to do the matrix multiplications in R. We had considered one simple example where we have learned how to multiply a matrix by a constant. Now we are going to explore somewhat topics on the matrix manipulation and we will start with the matrix multiplication particularly when we are trying to multiply 2 matrices and then we will try to see some more aspect like as addition, subtraction and some other details about the matrix operation.

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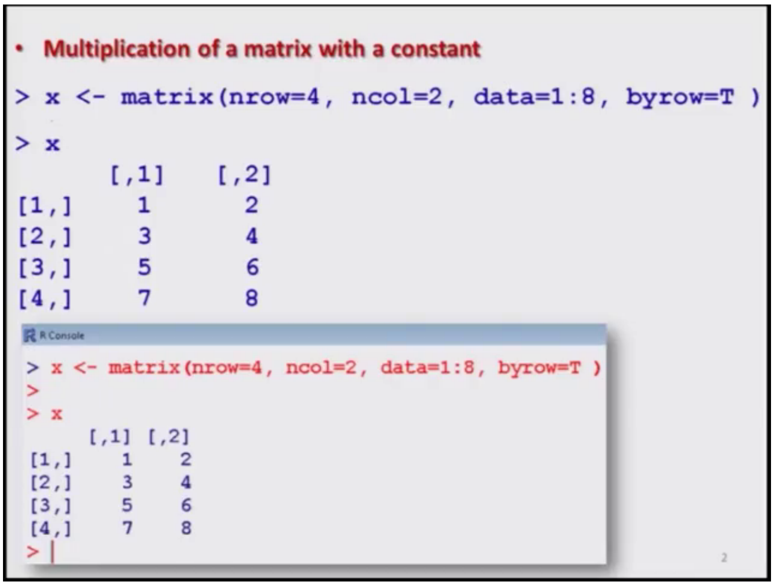

So, just for the sake of your understanding in the last lecture we had created a matrix of order 4 by 2 where the data was arranged in rows and the data was the numbers from 1 to 8. So, the data was arranged in a 1 2 3 4 5 6 7 and 8 and this is again in the screenshot of that matrix operation. So, and this is the matrix output. So, we are again going to work on this type of matrices.

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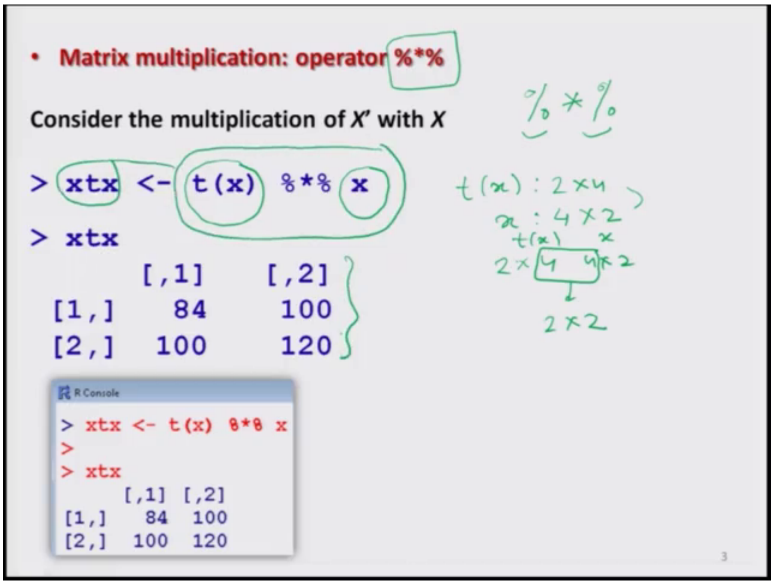

So, now first we consider how to do the multiplication of 2 matrices. The first thing what you have to understand the operator for matrix multiplication is this, which is a percentage sign multiplication and percentage sign. So, the multiplication sign is enclosed inside 2 percentage signs this will indicate that, this is a matrix multiplication. If you do not use this percentage sign then it means that this is a simple multiplication or this is only a multiplication of a scalar with the matrix right. So, let us try to do some elementary operation and then we try to show that how the things are happening.

So, let us try to create 2 matrices. One matrix I already have which is here x and in the earlier lecture we had created another matrix which was the transpose of x. So, these are the 2 matrix that I already have created and you can recall that the order of the transpose of x was 2 by 4 and the order of your x matrix is 4 by 2. So, whenever I am trying to multiply them together then it becomes here 2 by 4 which is of all the t x and say x here 4 by 2. So, these 2 things are matching. So, my product is well defined and then ultimately I will get an outcome which is of order 2 by 2 matrix right.

So, as soon as I do so, you can see here. So, and I try to multiply the transpose of x and x which are 2 matrices I try to store their values in this variable name x t x and this and the outcome of x t x comes out to be like this. So, now, let us try to do this thing over the R console and let us try to see how the things happen. So, now, let us come to here R you may recall that we already had stored this matrix x and with this we had obtained the transpose of x like this.

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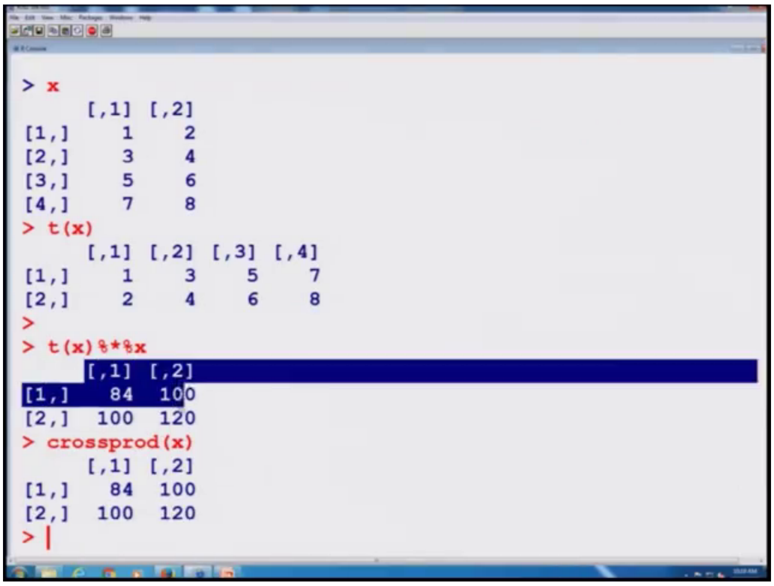

So, now, I am trying to multiply it here transpose of x and multiplied by which is the matrix multiplications. So, say x and you can see here this comes out to be like this right similarly if you try to take any 2 matrices whose orders are well defined for the purpose of matrix multiplication you can get the same outcome and we all know that in mathematics this matrix matrix theory helps us a lot and this matrix manipulations are very important to learn right and transpose is one of the basic fundamental operation.

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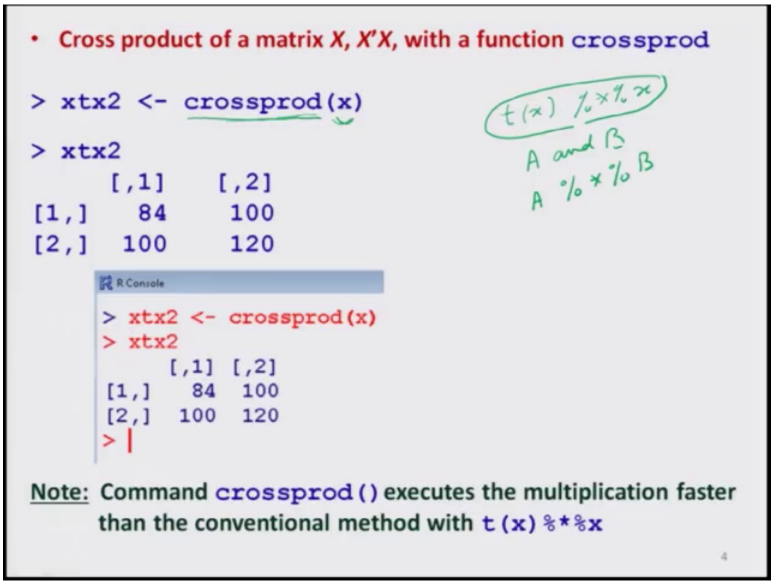

And there is another thing which I would try to show you. Suppose you want to find out this multiplication transpose of x matrix into transpose of x then we already have shown you how to get it done similarly in case if you have any other 2 matrices a and B which you want to multiply suppose and their order condition is satisfied.

So, I will try to write down here a percentage star percentage B there will be no issue, but here I would like to show you that whenever you are trying to find out a particular type of product where you are trying to multiply the transpose of x and x then there is another build in command in R which is called as cross prod which is the cross product of transpose of x and x. So, instead of writing the entire thing I can simply write here cross prod and inside the bracket I will try to write down the matrix whose multiplication I want in a way like transpose of its matrix multiplied by it is multiplied by the matrix.

So, and the advantage of this operation is that that this execute the multiplication faster than doing the basic manipulation by transpose of x and multiplication by x matrix, oh well here if you try to see let us try to first see what happens and then I will try to explain you something more. So, if you try to see here if I try to write down here cross product of here x and now you can see here this is the same thing which we have obtained. Now if I try to say here cross product of x you can see here that this outcome earlier outcome and this outcome is the same right.

Now, before I go further I just told you that in case if you try to use the command cross prod then it is faster and it gives you whether the faster multiplication, but you have seen that there was no difference in the speed as soon as you enter you get the values. But please remember one thing these are very small number of values now the time has come where we are dealing with high dimensional data where the order of the matrices may be several thousands even.

One thing you have to keep in mind that every software has a limitation for multiplication finding inverse and different types of matrix manipulations and even when you are trying to deal with a matrix of say order 20 by 20 or say 100 by 10 then you will see that what is the difference in the speed of execution of a command in R in such a 2 by 2 or 4 by 4 matrices you will not be able to find out the difference in the speed of the operation. And similarly when I told you about some other basic fundamental like is finding or the dimension of a matrix or the number of rows in our metrics or number of columns in a matrix. Well in a matrix of order 2 by 2 4 by 4 or 5 by 5 we can count by hand, but suppose you are given some matrix of say order say this 200 by 200 it is practically very difficult to find out how to count the number of rows and columns. So, in such situation these commands like n row or say n called they help us.

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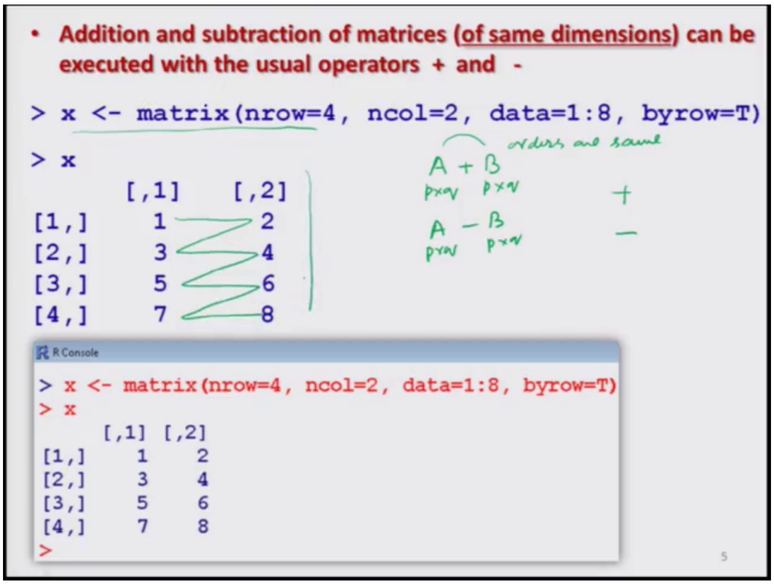

So, now, we move ahead and let us try to come on say another aspect that how we can do the addition and subtraction of matrices. So, suppose I have here 2 matrices A and B when I am trying to say that I want to add this matrices we know from the basic fundamental rule that both the matrices should have the same order. Say if this order is p cross q there are p number of rules and q number of column then we should also have p number of rules and q number of columns. So, the order of both the matrices should be same that you have to keep in mind right this is the basic fundamentals.

Similarly, when you are trying to subtract to 2 matrices in the same operation and the same fundamental continues that the order of the 2 matrices A and B they should be the same right. So, and whenever I am trying to do the addition and subtraction they are operated by simple command like s plus and minus there is no issue there is no additional this operator right. So, let us try to take some simple example and try to see how we can find out the addition and subtraction.

So, I will again continue whether were the same matrix here x because was a 4 by 2 matrix and which the data was entered as 1 2 3 4 5 6 7 and 8 row wise. So, I will try to take here the same matrix and I will try to create another matrix and then I will try to show the addition, subtraction.

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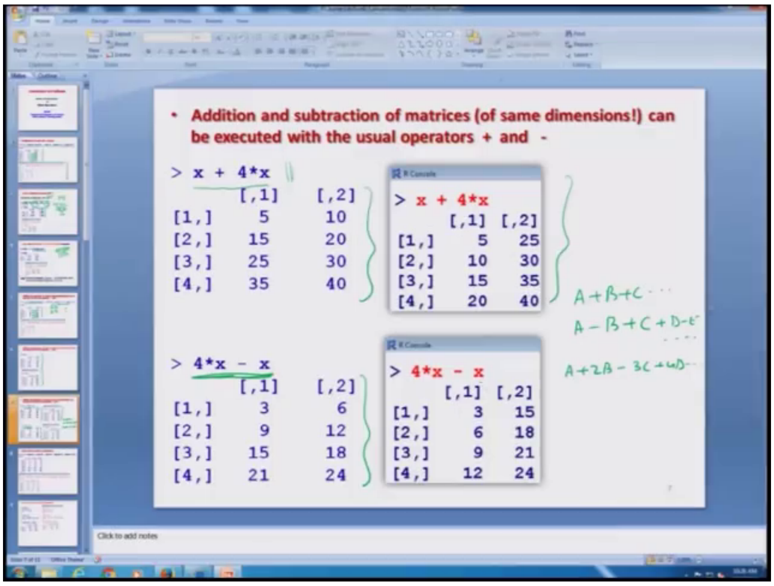

So, I simply try to. So, say create another matrix which we had created earlier in the last lecture just by multiplying the matrix x by 4 a scalar and so we obtain this thing that we had seen in the last lecture. Now suppose if I want to add these 2 matrix x and 4 into x you can see here the command is very very simple, you simply have to write down here x plus 4 star x and you get here the same outcome right. And what really happens in this matrix addition? The corresponding elements are added the element at the address 1 1 is added it forms another matrix having the same address 1 1 and then I try to do the subtraction here the rule is very very simple that I try to suppose I want to subtract a x matrix from 4 star x.

So, I simply have to write down here 4 star x minus x and I get here the same outcome. And the same rule continues for say for example, when we have more than 2 matrices suppose we have 2 2 matrices A plus B plus C and so on the same rule continuous. Even if I have a say combination of addition and subtraction something like a minus B plus C minus D minus E and so on. The same rule applies over here. Similarly if I want to here A plus 2 B minus 3 C plus 4 D and so on the same rule applies over here the only thing is we have to keep in mind that the orders of the matrix should be the same. So, let us try to do it over R console and try to see how do we obtain it.

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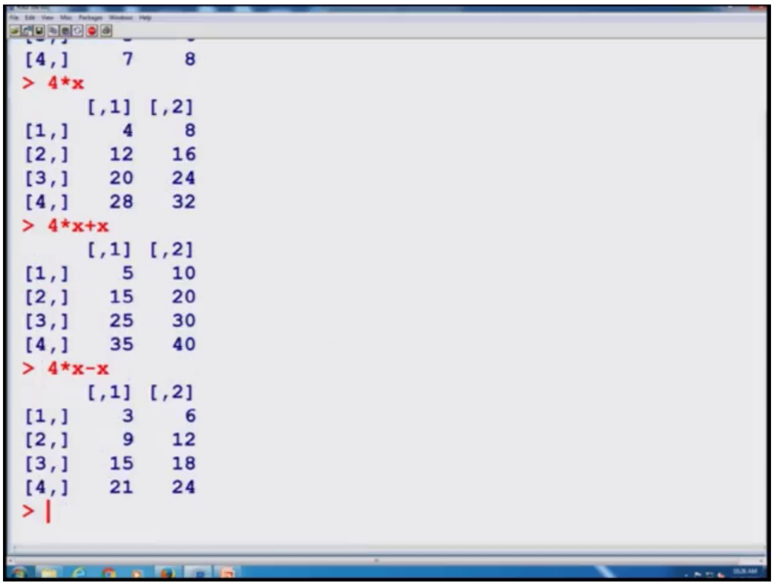

So, you have a seen here this is our here x and I try to obtain here 4 into x and now I want to add 4 x and x. So, 4 star x plus here x you will see here you get here this outcome and similarly if you want to subtract it 4 star x minus here x you get here the same thing you can free over here right. So, you can see here that it is not a very difficult thing for example, if you want to have it here.

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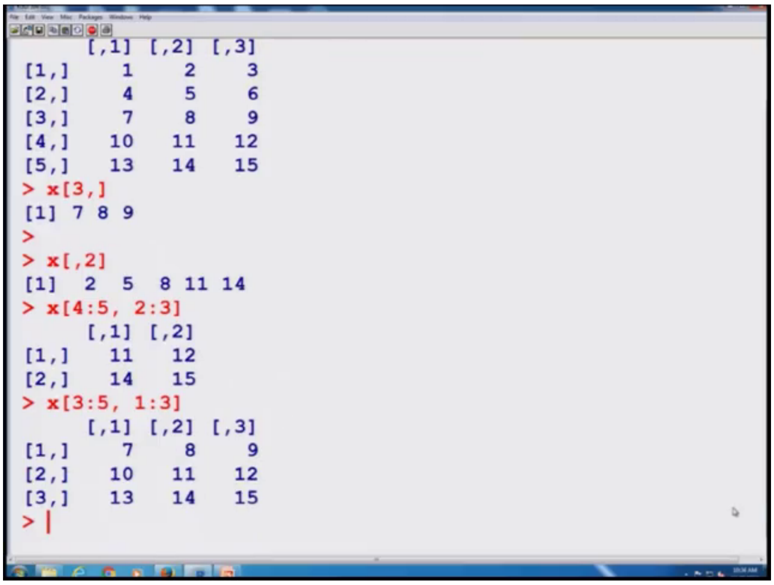

Suppose we takes another example where I want to have x plus 2 star x minus 3 star x plus 4 star x and so on you can see here you get the same thing over here it is not difficult at all it is just like any other subtraction and multiplication right. So, let us try to continue further and once again I will request you that you try to do some more examples and try to practice it.

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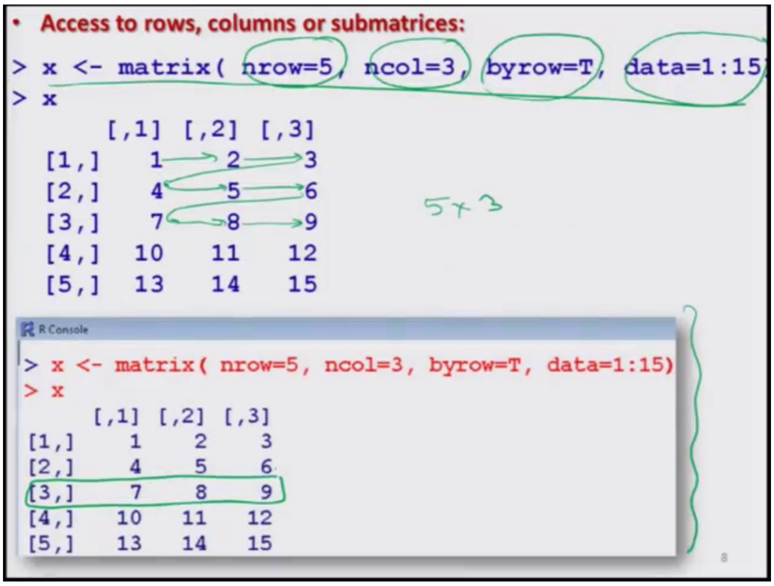

Once we have created a matrix there are situations where I would like to access a particular element of the matrix a particular row of the matrix or a particular column of the matrix. So, now, I am trying to show you how we can do it. So, I am trying to create now here another matrix because I am sure that you all of you are getting now bored with the same matrix x having the numbers 1 to 8. So, I try to create here say here another matrix here x which has 5 number of rows, 3 number of columns and the data values are the number from 1 to 151, 1 2 3 4 up to 15 and these values are arranged by rows. So, you can see here that now this is going to be here a 5 by 3 matrix whose values are 1 to 15 arranged in rows from the 1 then 2 then 3 and then come here 4 5 and then here 6 and then come to here 7 8 and then here 9 and so on. So, once you try to do it over R you get here this type of a screenshot.

So, this outcome can be obtained now very easily and now you can see here that suppose I want to obtain here suppose third row of this matrix. Now, how to do it here?

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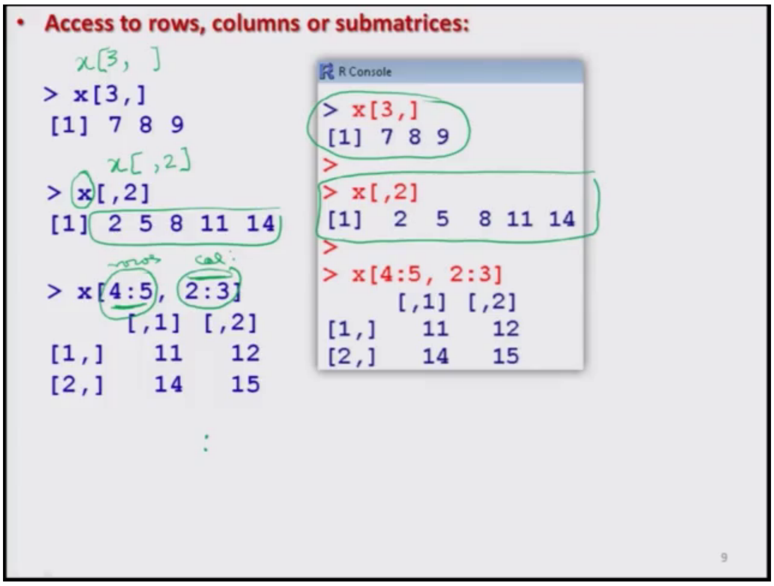

With this I am trying to do it here I simply have to write down here the name of the metrics and inside the square brackets. Please remember this thing this is very important now I am going to use the square bracket inside the bracket you have to rise the third row and that is all comma and then blank. Once you try to do so here you will get the outcome as the third row you can see here.

Similarly, in case if you want to have access to the second column second column here is the containing the values here 2 5 8 11 14 suppose I want to extract only this column from the entire matrix then I have to give a similar command here the name of the variable here x and then I have to write down the square brackets and then I have to give here blank space do not write anything and then comma and then here 2.

So, this will give me an access to the second column and as soon as you type it you get this value over here or this is the screenshot you can see over here right, and suppose you want to have a subsection of a matrix. Suppose just for the sake of understanding I am trying to take here 4th and 5th rows and second and third column the symbol the meaning of this symbol colon is simply from and 2.

So, this is trying to say here from 4 to 5 and this is trying to see here from 2 to 3 this is about the rows and this is about the columns. So, I am trying to say here that I want to extract here a sub matrix which is containing the 4th and 5th rows and second and third columns. So, once you try to do it here you can see here in this matrix that these are this element 11 12 14 and 15 or you can also clearly see over here this is the this matrix. So, you can see here that this is governed by this 4 and 5 and this is the 1 by this 2 and 3. This is how we can obtain a particular element, particular row, particular column or even a particular sub matrix.

Let us try to do it over the R console and try to see how it happens right. So, first I try to create here this matrix here well I am trying to save my time. So, if you try to see here I have this here matrix the same matrix over here and now I try to obtain here the same thing the third row. So, you can see here the third row is containing the values 7 8 and 9 here and this is the thing what I have obtained here right suppose I want to have here information about the second column. So, I have to write down here x the variable name matrix name blank then comma and then here 2 and then bracket closed. So, I get here the same thing which is here 2 5 8 11 14 which is here and then I get it over here and similarly if I want to have a particular sub section. So, I can write down here like this and you can see here that 11 12 14 15 this is a subsection of here and say here right.

Similarly, if you want to have here something here more for example, if I want to have a 3 to 5 and say here 1 2 3 and. So, on this type of sub matrix that can also be obtained over here and you can see here this is a particular sub matrix of the given matrix right ok.

So, once again I would request you to please try to do some more practice with these things.

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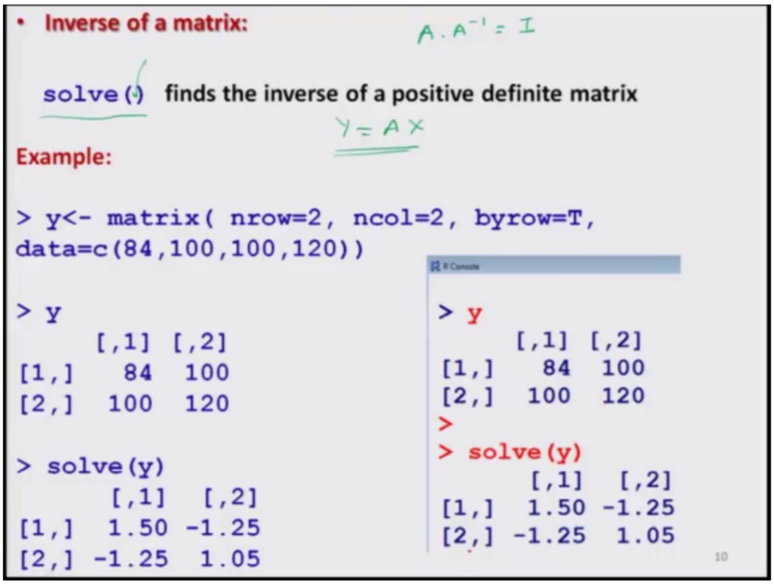

There is another important aspect in matrix operation to find out the inverse of a matrix the inverse of a matrix is defined in such a way such that if I have here a matrix here A and if I try to find out here its inverse a inverse then their matrix multiplication will give me an identity matrix I right. So, how to find out the inverse of a matrix in matrix operation? There is a simple command in R to find out the inverse right, this command is here solve s o l v e and inside the bracket you have to give the name of the matrix whose inverse you want to find. Why this is actually solved? Those who are familiar with the metric theory and some linear equation we always try to find out we always try to solve the equation something like y a x and from there after solving this type of equation they are trying to obtain the inverse of a matrix and that is why this is called as solve command in R all right. So, let us try to take some example and with let us try to see here how you can obtain here.

So, I have simply taken here a matrix here for example, here I have created here. So, another matrix remember one thing whenever you are trying to find out the inverse of a matrix the matrix has to be a square matrix and a positive definite matrix right. Earlier I had created a matrix which was of order 4 by 2 I cannot find out the unique inverse of that matrix well there are something like generalized inverse those things can be found, but here we are not going to that detail discussion.

So, now I try to create here a matrix here say here y.

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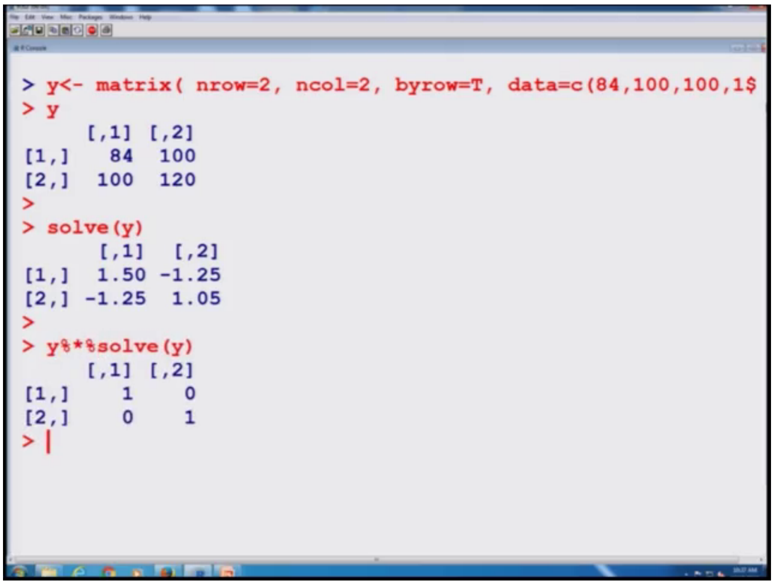

And you can see here that this is here y here is like this and. So, this is essentially a 2 by 2 matrix and I want to find out its inverse. So, I will say here simply solve y and this will give us the inverse of this matrix and if you want to really verify whether this is really an inverse or not. So, I will say here I. So, let us try to multiply y and solve y together. So, y my matrix multiplication solve and let us try to see I get here an identity matrix of order one by one. So, you can be sure that solve is trying to give us the inverse of the matrix.

So, now, we let us come back to our slides and the last command which I would like to share with you is another important command in matrix theory to find out the Eigen values right.

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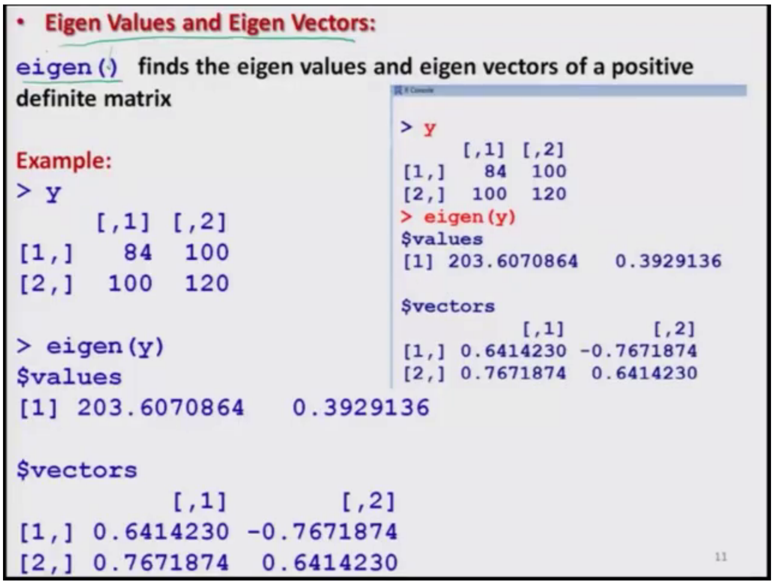

Here I am not going to discuss how you can find out the eigen values in matrix only, but these are very simple things and they are the part of any elementary course on matrix theory.

So, suppose you want to find out the eigenvalues and eigenvectors of a matrix then the simple command here is eigen e i g e n and inside the bracket you have to write down the name of the matrix of which you want to find out the eigen values. Sometimes this eigen values and eigen vectors are also called as characteristic roots and characteristic vectors. So, quickest to the same thing all right. So, here I will try to do the same thing I try to create here the same matrix which I had used earlier in finding out the inverse of a matrix and let us try to see it over the R console that that how the things happen right.

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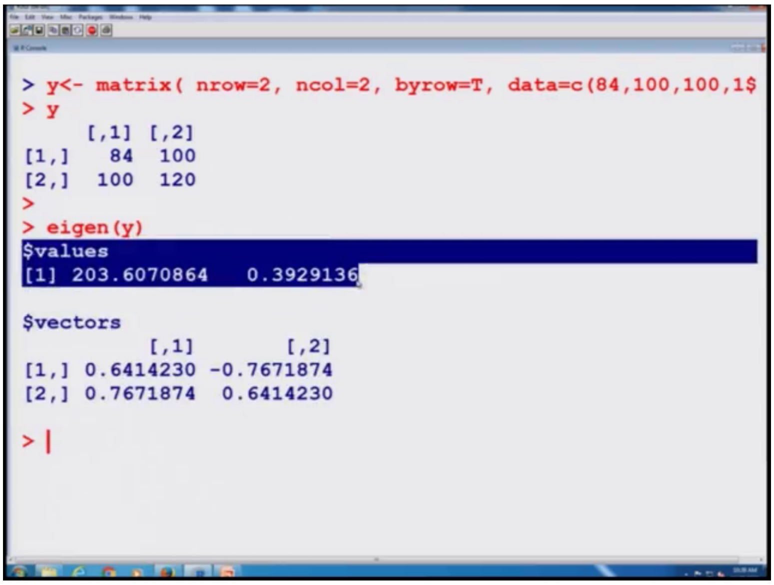

So, if you see here we had created this matrix here y which is here like this one and. So, now, this is my here 2 by 2 matrix and yeah means I have checked it earlier that it is n it is a positive definite matrix right. So, now, if I want to find out its eigen value I simply have to type eigen and inside the bracket say y. So, you can see here that these are the eigen values which are obtained and these are the 2 eigen vectors corresponding to each of the 2 eigen values. So, you can see here that these commands are not actually difficult to operate and you can also see here that I have obtained these things and then here I am trying to give you these outcomes and here is the screen shot of finding out the eigen values.

So, now I would like to stop here and I will also conclude with the discussion on different types of matrix operation, but definitely I would say that this is not and this is only your starting. There are so many matrix operations available chronicle product inner product add mud product and different types of this inverse like a generalized inverse mood controls inverse right all those things are there, but again as I said earlier my objective is not to teach you here the matrix theory my objective is simply to help you that how you can do this matrix operations in R which are related to your areas. So, some engineers might have a different type of requirement then those who are dealing statistic or say it mathematics, but these are the basic fundamentals by which you can do your programming you can write down a function or even you can do a Monte Carlo simulation.

So, now, I will stop here, but now you have to start and you can go as far as you want depending on your needs and requirement. So, we will see you in the next lecture till then goodbye.