Curves And Surfaces.
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Module-II.
Surfaces-1: Smooth Surfaces.
Lecture-07.
Transition Maps Of Smooth Surfaces, Smooth Function Between Surfaces, Diffeomorphism.

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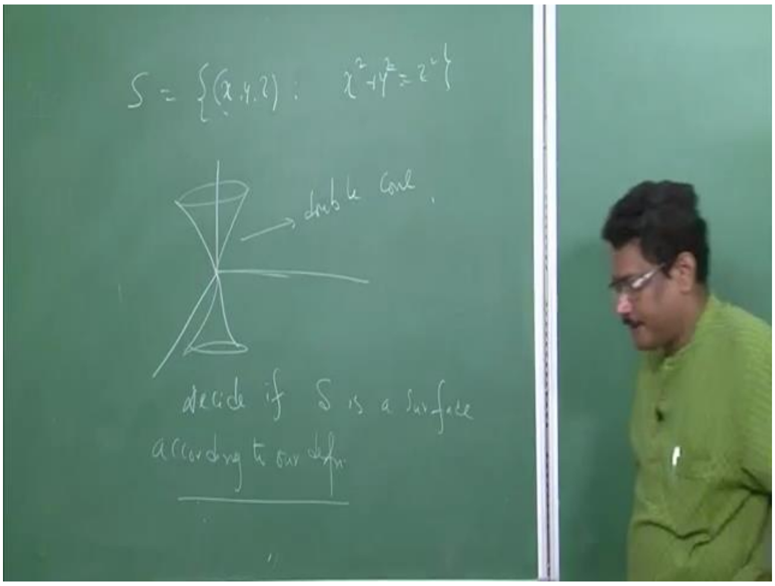

Okay, so last lecture we left a double cone to decide it is a surface or not.

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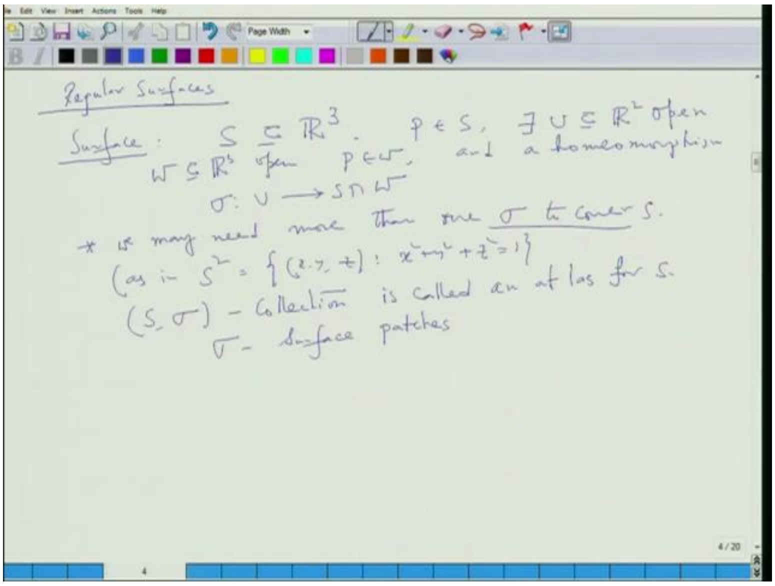

Now, for this particular lecture series will be concerned about not all surfaces but regular surfaces. So, let us make things little bit formal. So, 1st of all we have defined what is a surface surface is subset of R3 and for any point P in a sphere, recalling, there exists U in R2 open, to continue last lecture notes, and W in R3 open P continuing the P is and W and a homeomorphism Sigma from U to S Intersection W. And we may need more than one Sigma to cover S, as in the example of of S2 other sphere.

More complicated surfaces will need more and more Sigma, so this S and collection of Sigma, this collection is called this collection comes with U, W, etc. is called an atlas for S and Sigma is called a surface patches actually, because we need more than one Sigma.

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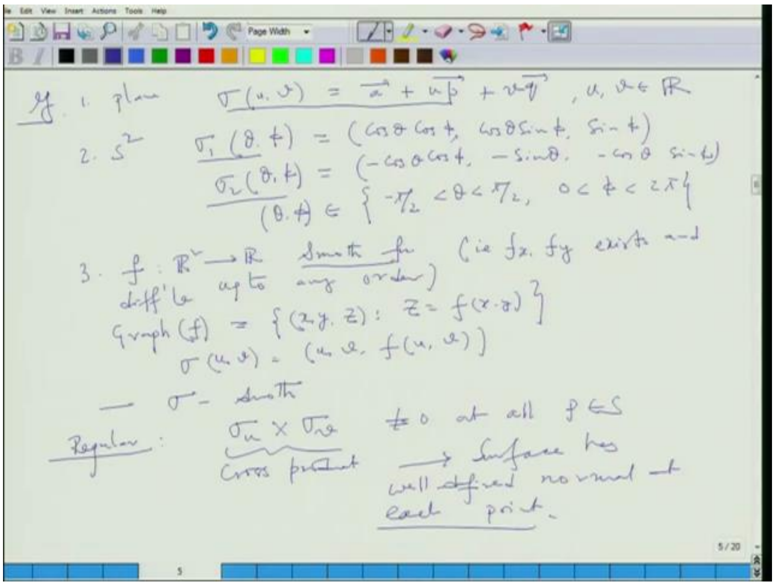

So, examples we have done plane, we can cover it by one surface patch, we have done example of sphere which is covered by 2 surface patches in cartographic sense. If it is not clear how we arrived at it, think over it again, I will do it again, no problem.

More general example is, suppose I have F from R2 to R, a smooth function, what is a smooth function? That is FX, FY exist and differentiable up to any order this is a figure of smooth function and if I look at graph, you remember we did it for the curves also. Now, what is smooth about smooth surface? Here also I have taken smooth function, here this Sigma UV is smooth map, Sigma 1, Sigma 2 is smooth map. So, what I want for smooth surfaces that this, so what is the parameterisation here?

Here the Sigma UV is UV FUV, so in all these examples I want Sigma to be smooth map and regular, by regularity I mean Sigma U cross Sigma V, this is the cross product, this is not equal to 0 at all P in S. But do you need this? This is analogous to what we did in the curves case, you will see very soon in next or next to next lecture. This means that the surface has well-defined normal at each point.

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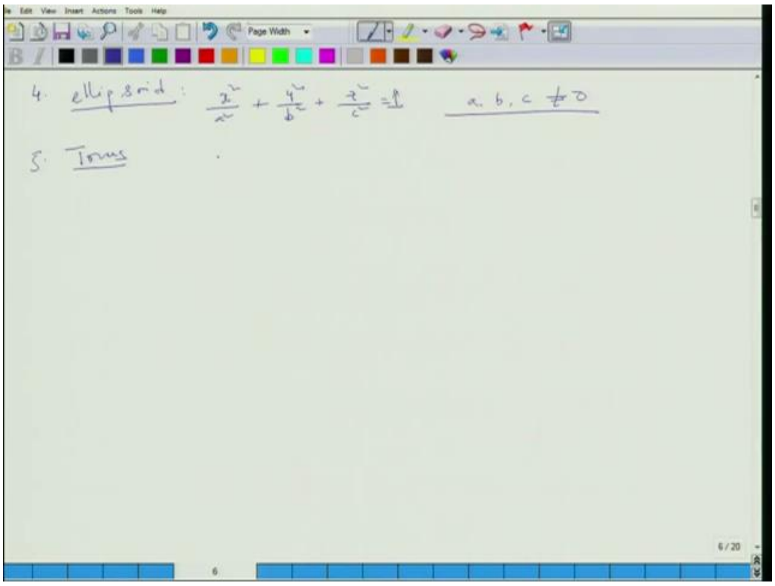

I will give more examples to work you out. And I will be coming back to examples, maybe some of them I will be working out in very details. Equal to1, okay 1 is standard one where A, B, C, all not equal to 0, this is called ellipsoid. If A equal to B equal to C, then I will get a sphere. Then there is torus, how do we generate a torus?

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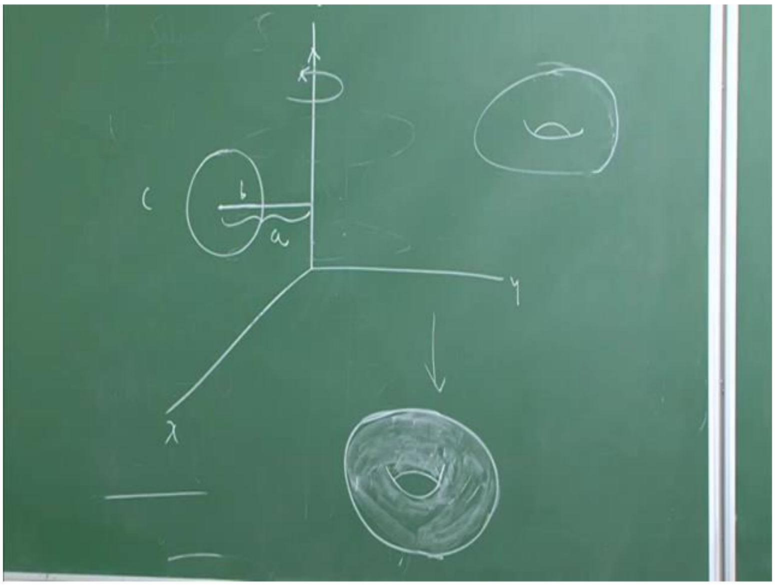

Okay, let me do it here on the board. So, you take a circle, this for sake of convenience, I take the circle on XZ plane. So, circle has radius B this is Z axis, the distance from the centre of the circle to Z axis is A and I rotate this fellow, we will come back to this kind of example more and more, but if you move to any other courses from here, this torus will come back every time. So, this is one of the 1st example while it will come, it it is very basic example of differential geometry and another I will do at the end.

So, what we do here, I take a circle, I have a line which does not intersect the circle and I rotate the circle around that line. I will cycle on the plane, a line in the same plane and I rotate it around Z axis, around the line, for convenience here I have taken Z axis. So, what will be the picture? So a particular point here will move like this. A particular point here will move like this. So, the final picture you will get is something like, this is the standard way of this is one way I mean this is a way people draw torus.

You will see in many math books that let this be a torus, what they mean is that we have a axis exactly this, we have an axis, we are circle in the plane and we are rotating the circle along the axis. So, you should do it from the above.

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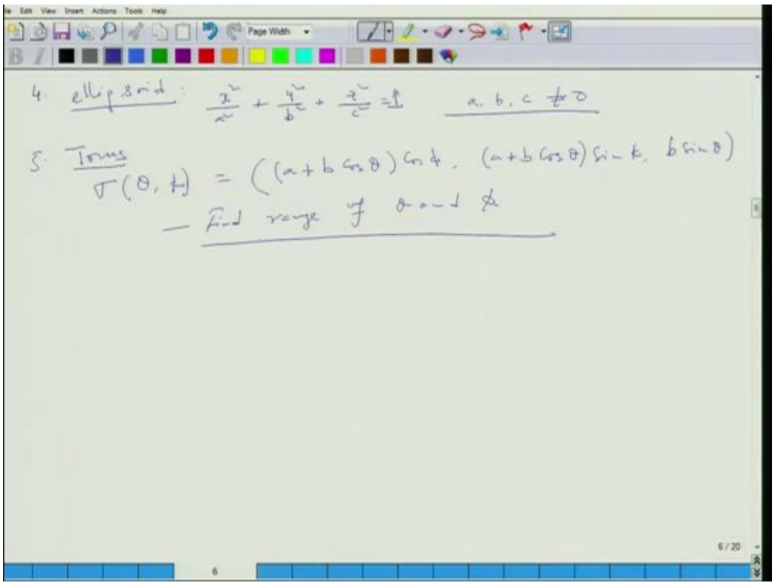

Such a thing is torus and for this particular one, the parameterisation, I mean the surface patch is… So if you cannot do it, we will do it anyways we will do it in next to next lecture but for time being find Theta and Phi. Theta and Phi in terms of, remember, if I have a particular point on a surface, so let me draw the torus here.

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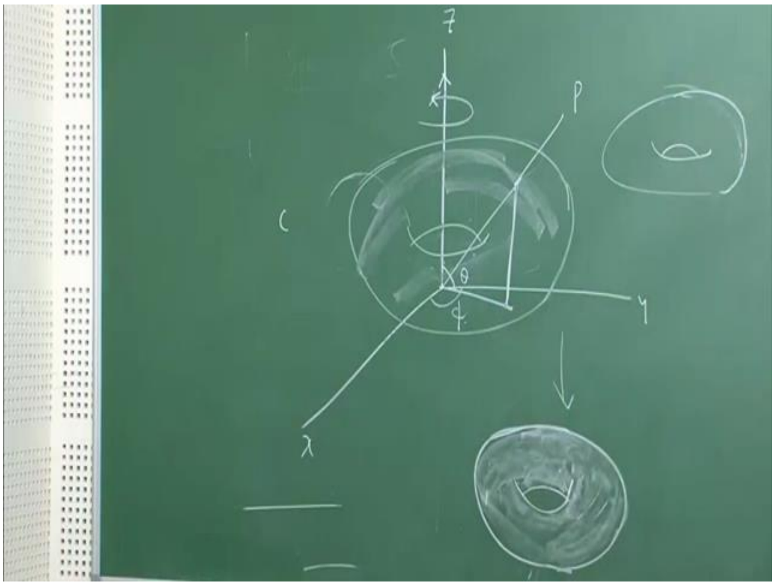

This is a surface, so a particular point here P, project it on the XY plane, theta is this angle and pie is the angle between X axis and Z axis, okay.

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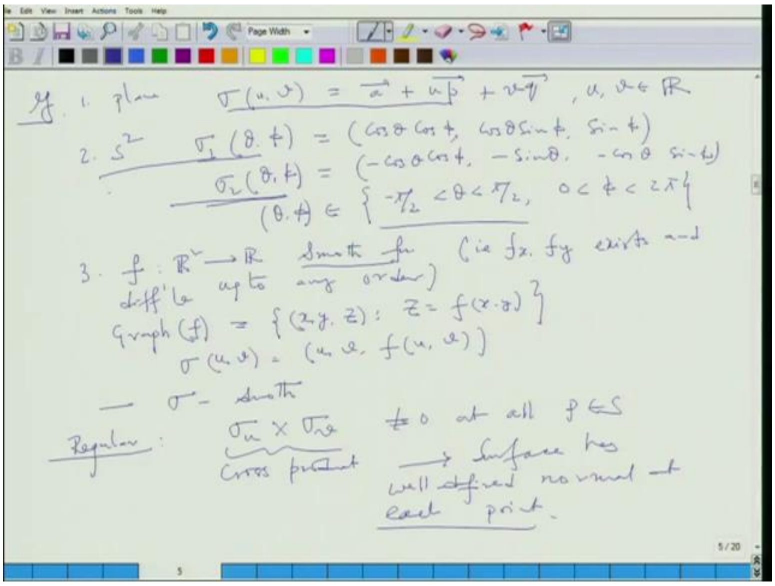

Now, as we have seen in the case of sphere that we need more than one surface patches to cover the entire sphere. For example, recall S2, okay, I can, yeah, here it is, I need more than one surface patches.

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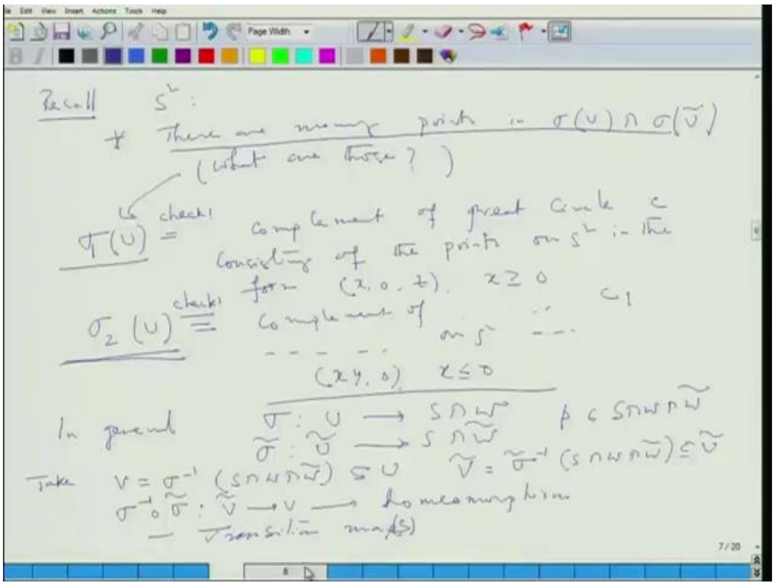

And here there are many points in, so I put an exercise there, what are those? So, what are those points which are in the intersection but you will see there are many points. Okay, let me answer this question here. What is Sigma U? Sigma 1U? Sigma 1U, this will be, so check, this is compliment of great circle C consisting of the points on S2 in the form, okay. And Sigma 2U, check again, compliment of…

Great circle C1 consisting of points on S2 in the form XY0 X greater than less than equal to 0. So, there are many points in the intersection. So, in general if I have email to surface patches Sigma U from a surface for a surface and Sigma tilde from another U tilde and P is a point which belongs to S Intersection W intersection W tilde, then I can take V to be Sigma inverse which is a subset of U and V tilde Sigma tilde inverse subset of U tilde. And then I have map Sigma inverse compose Sigma tilde from V tilde to view V which will be an homeomorphism again.

Why it is homeomorphism, think about it, otherwise ask it in the forum. Everything is homeomorphism here, so this map into homeomorphism and this map is called the transition map. Actually it will be transition maps because a surface usually comes with many surface patches and it can be, I will use the word reparameterised with another surface patches, so there will be many transition maps. So, given an atlas, I will have given 2 atlases, it is regular smooth surfaces, so I will have transition maps.

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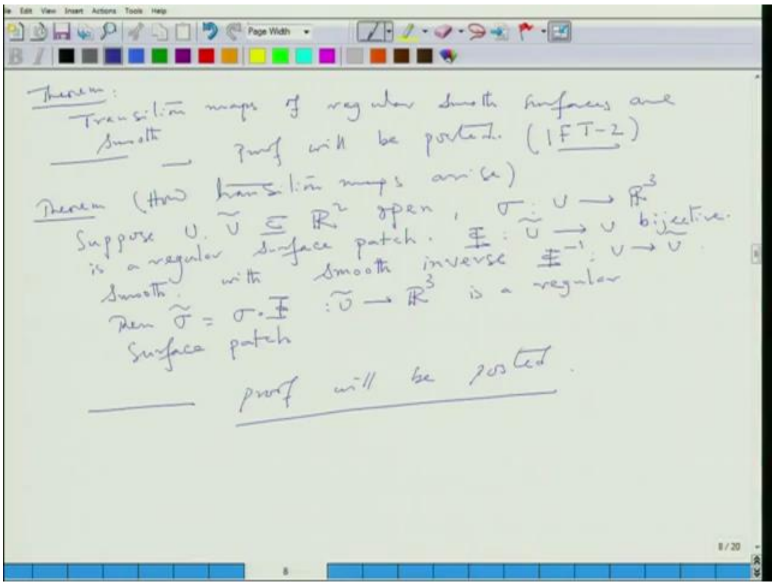

And what I need here that in the theorem here, since we are talking about regular smooth surfaces, that transition maps of regular smooth surfaces are smooth. This is the 1st theorem I wrote down for surface, this week's lecture, I will not give the proof, I say the proof will be posted in the forum because this uses so-called IFT 2, which is inverse function theorem, which is an application of implicit functions in which I talked about in the introduction of the course.

And perhaps I believed another theorem here, that how transition maps arise, this is a converse to the above theorem, that suppose U and U tilde in R2 are open sets and Sigma from U to R3 is a regular surface patch. I have map Phi from U tilde to U which is bijective, that is one-one, onto, smooth, smooth inverse Phi inverse from U to U tilde, then Sigma tilde Phi compose Phi from U tilde R2, sorry I should have R3 no… This is regular surface patch.

This proof will be also posted. I do not want to bother you with the proof in this course but will just give you the idea. So, these 2 theorems actually leaves you to something. Leaves you to what? Let me summarise it, that when you are looking at a surface, you are looking it locally. When you are looking it locally, then I can consider this local portion where I am concerned with because we will be finally looking at the surface very locally because we look at the surface from a plane and we can see some point and around.

So, instead of saying that, okay, we have an atlas, but I am not moving through the atlas, I am fixing on its, I am standing on its particular open set U which maps to a particular patch of the surface and for me that is a surface. And what I can proof for this patch, I can roof for the entire patch, provided my transition maps are smooth, that you are standing somewhere else and we have intersections and we look at the same points, you are looking at some other open sets, I am looking at some other open sets, we have intersection in the surface.

What you were to infer of the surface, they should be same. What these 2 theorems actually tell you that okay as long as my surface is regular and smooth, I should be defining the properties of surface in a way that it should be invariant over under what I see and what you see. So, any property of a surface to be defined, which are invariant under transition maps.

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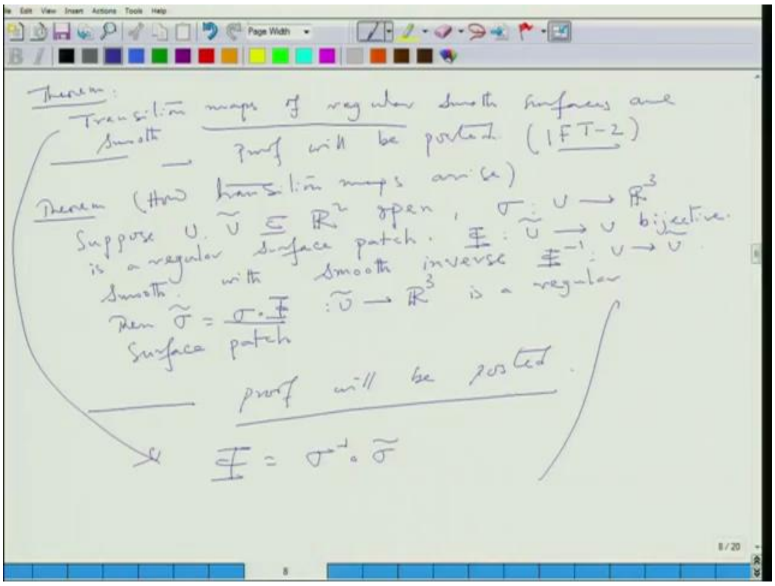

For instance, here as you see, what are the transition maps here? Transition maps here, what is the Phi here, Phi is actually Sigma inverse compose Sigma tilde as we done in the last page. And conversely any such Phi, here also you can see, Phi is a Sigma inverse compose Sigma tilde, so this is a**,** these 2 theorems guarantees that the properties of surface as well as the transition maps are smooth, I can define invariant of where I look at, if the surfaces are living in R3 and I am looking for R2.

So, this is the viewpoint I will we will take for the rest of the course and next time I will give an example, I will start with an example to define one property of surface called as, how do you define maps between surface, smooth maps between smooth surfaces. Okay, so that is it for this lecture. So, wait for the next lecture, thank you.