Curves And Surfaces.
Proffessor Sudipta Dutta.
Department Of Mathematics And Statistics, Indian Institute Of Technology Kanpur.
Module-II.
Surfaces-1: Smooth Surfaces.
Lecture-06.
Surfaces And Parametric Surfaces, Examples, Regular Surface And Non-Example Of Regular Surface, Transition Maps.

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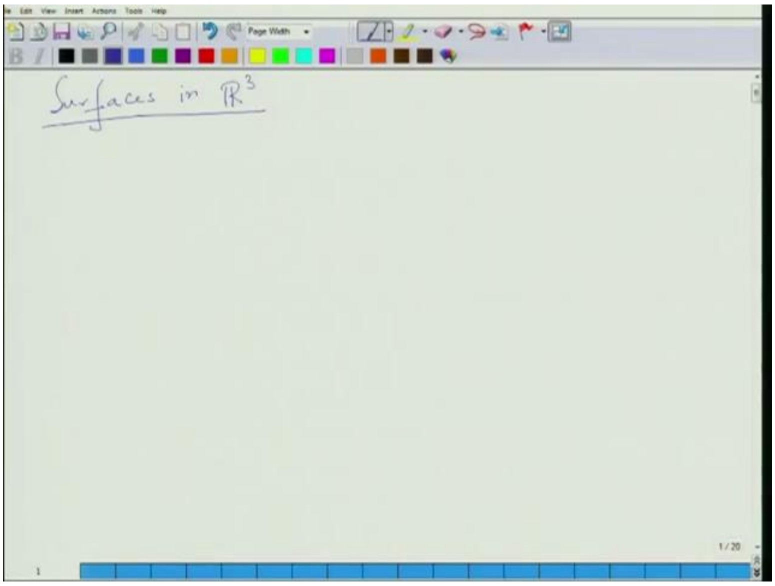

Okay, welcome to the 2nd week of the course on curves and surfaces. So, this we will start with surfaces. And as we have said in the beginning, we will be dealing with the surfaces in R3 only. So, before giving a formal definition of a surface, let us try to see what we understand by in general, what is our notion of a surface.

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Well, 1st of all, we all live in this world and we know this world is a kind of a sphere or ellipsoid and we are kind of here. And we live up we stay, we live around on such a or in general we call it surface.

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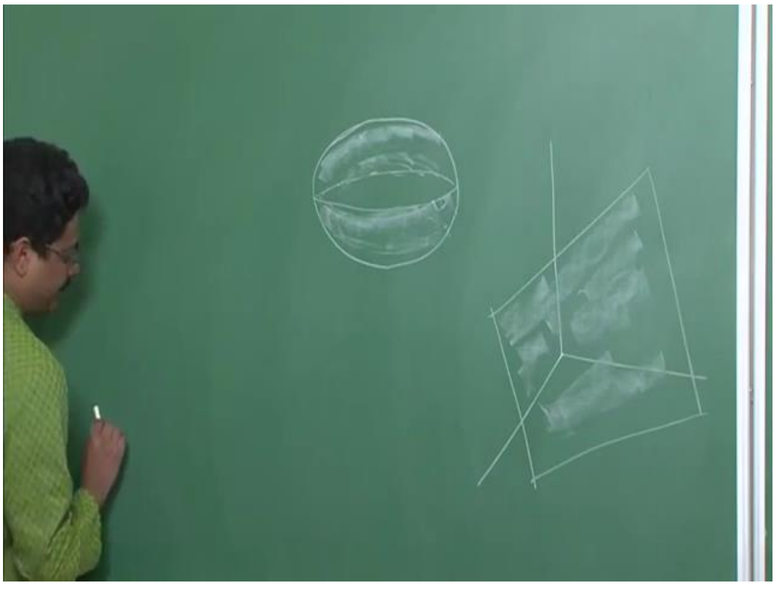

Also, we know our world is three-dimensional and if I take a plane, our intuitive notion will say that such a thing is a surface.

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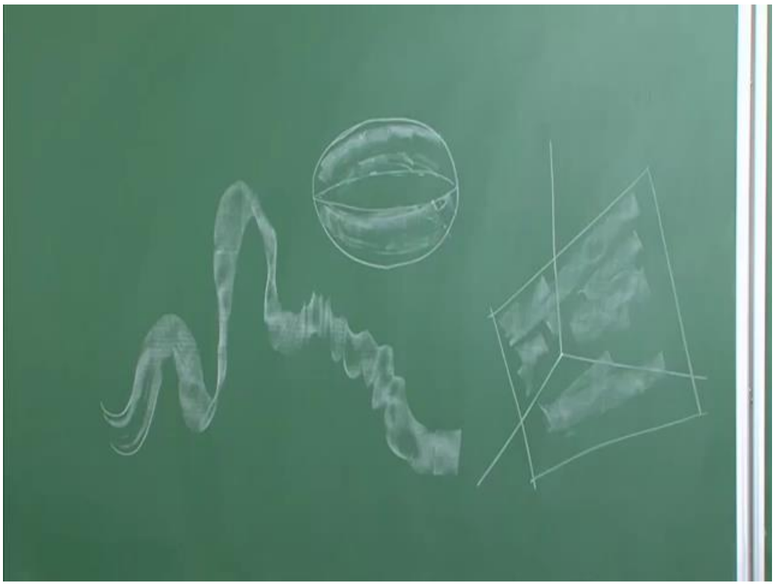

But also, perhaps you are climbing on a mountain then you will encounter a surface like this. So, to come up with a definition, what we do, as we did for the curves, that we call off, we actually talk of so-called parametric surfaces.

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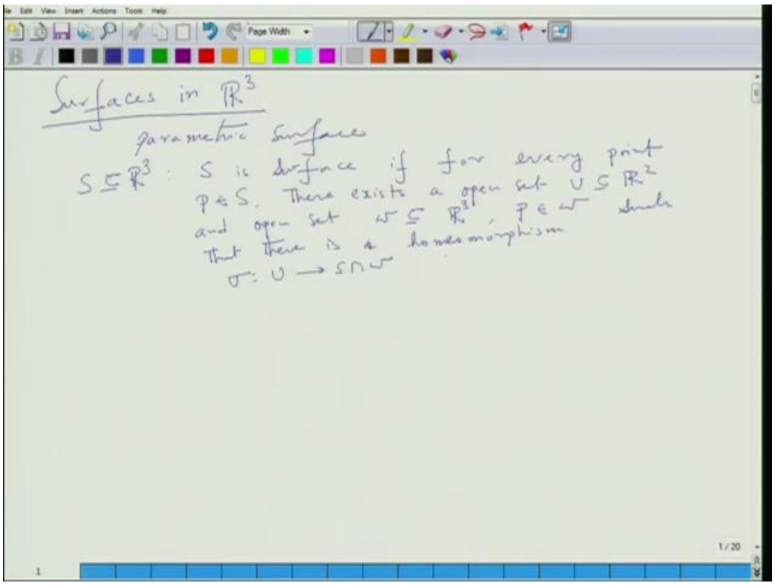

What does it mean by that? That we define our surface, for instance we will see, I will do example today, 3 examples to tell you that we need to define a surface locally, in the sense that, okay…

So, surface in general will be a subset of R3, so I will call it S is a surface if for every point P in S, there exists an open set u in R2 and open set W in R3, P belonging W, such that there exists, there is a homeomorphism Sigma from U to S Intersection W. So, what does it mean?

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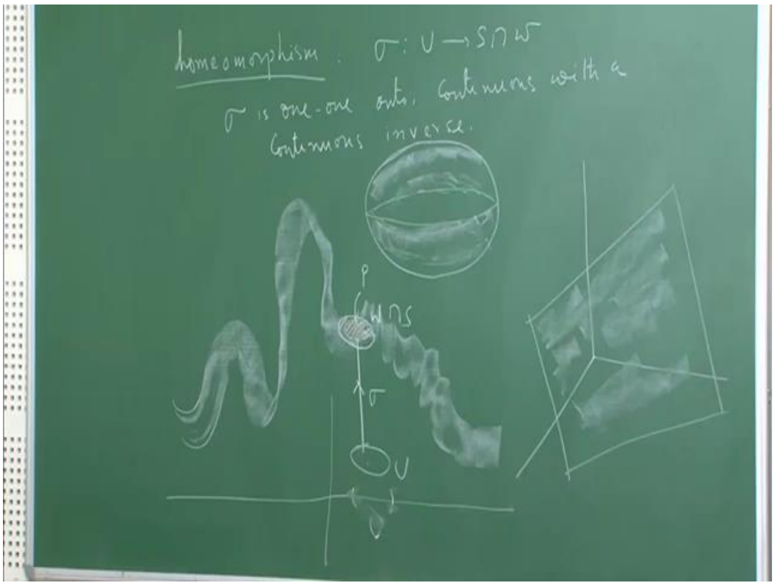

That okay, I may have a surface any rough surface like that, in general terms. But I am viewing surface from my, I am standing in a plane, usually R2. And I am viewing it from the plane, so any point here P, well, this W may go out and I will have a U on the plane, not on the real line, on the plane, such that I will map Sigma which maps U to W Intersection S, that is precisely this portion homeomorphically.

And what is a homeomorphism? I hope all of you know what is homeomorphism, homeomorphism between 2 open sets means Sigma is one-one, onto, continuous with a continuous inverse. Such a thing, we will be calling a surface. It defines ever notion little bit, in the sense that okay, if I am in a plane or on a sphere, why you have to define it locally, what is the reason for that? We will soon see, let us start with some example. Let us take the simple example plane.

You will see that for a plane I do not need to define it locally but when I move to sphere, there will be actual problems, to, you can intuitively understand that you cannot put the sphere, you cannot flatten the sphere on the plane. So, okay, let us talk about some examples.

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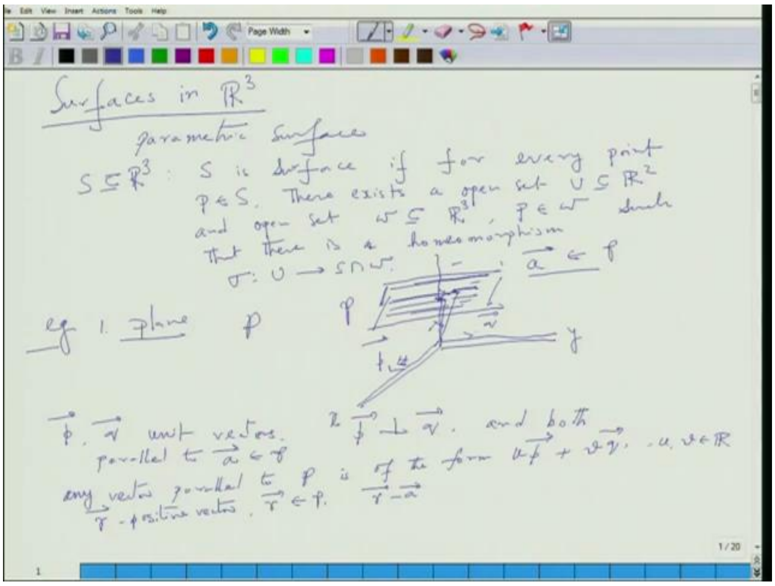

Plane. What about a plane? So, let us take a plane P. So, I draw up the coordinate axes, we have a plane here, for reasons I will just take it mostly in place one.

And I take a vector here, let us say A in the plane P, so this is my plane P. Now, how do I describe this plane P mathematically? Okay, I take a vector A here, in the plane P and I take 2 parallel vectors along the axis P and Q. So, and I take unit vector P, Q, unit vectors perpendicular to each other and both parallel to the vector A I have taken in the plane P. Simplest example I have taken Z plane and A is a vector unit vector around Z or any vector around the Z axis and P and Q you can take unit vector around X axis and Y axis.

Now, you observe any vector parallel to P is, what will happen to that? Okay, P and Q are parallel vectors, I have already taken a plane and A is better on the plane. So, it will be of the form some U P scalar U + V Q and U and V belong to R. So, if r is unit vector or any positive vector, r is in U P, then r - A, A is also on the, so, here is r, r is another vector here, so r - A, so this should be parallel.

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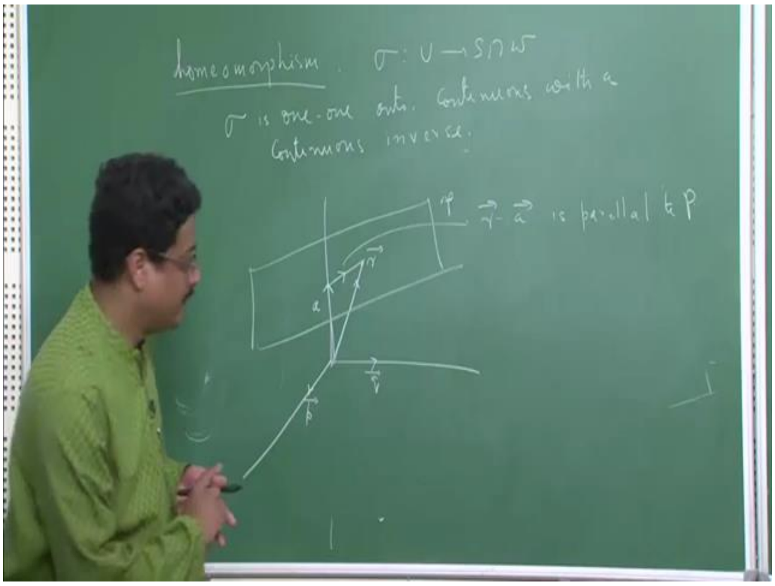

If I draw it very clearly here, it will look like, I have taken this vector to be A, this is my P, this is my Q and I take any vector r on the plane P, then r - A which is, this is r, so this is r - A, this is parallel to P. Correct.

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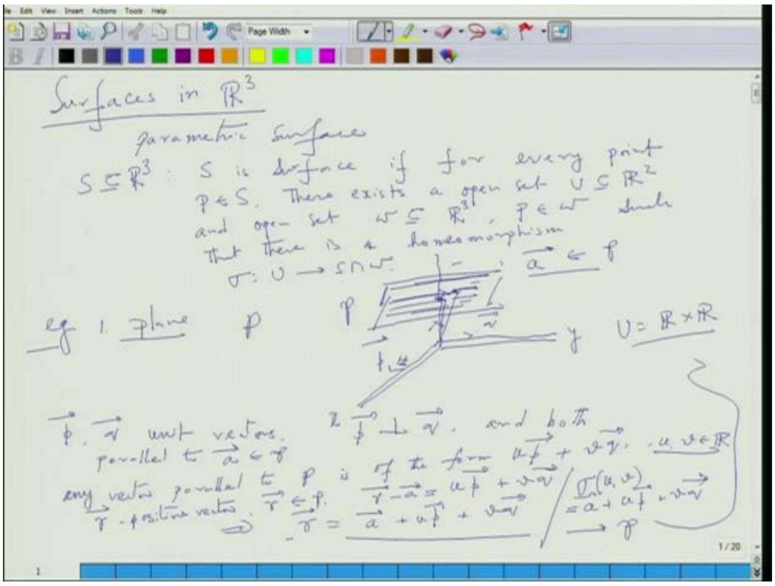

So, r - A will be of the form U from the above observation V + Q, which implies r will be A + U P + V Q. So r is a general vector in the plane I have described as r equal to A + U P + V Q. So, fixing any P and Q, I can describe my Sigma now, Sigma U V is equal to A + U P + V Q, so this will describe my plane completely. And my U is, what is U here? U is r cos r which is an open set in R2. Okay, so any, every point I can describe by one particular U. One open set of does… But let us move to the example of a sphere. And it is better explained on the board.

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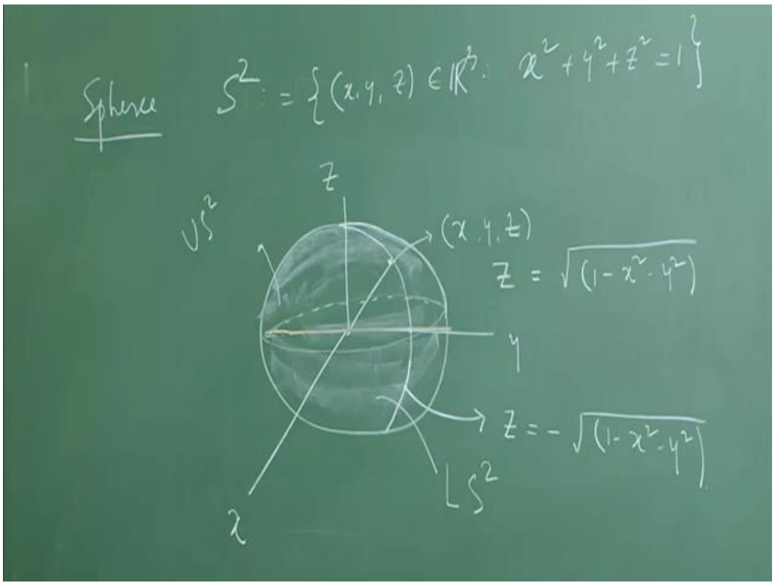

So, next example is sphere. Let us take unit sphere, which we will be denoting by S2 which has all points X, Y, Z in R3 such that X square + Y square + Z Square equal to 1. So, if I draw it, it will be like… This part is not visible, if when you see it as a globe, so this is a sphere. Okay. So, now you see any point here on the sphere X, Y, Z, if I look at the this part, whatever given at this part, that is, I this coloured line, this part we call upper hemisphere and this part we call lower hemisphere. Just think of our world, I mean our globe.

Any point here Z X, Y, Z is related by motion is related by relation, correct. Because X square + Y square + Z Square equal to1, whereas if I extend it at any point here, here Z is -… so what is happening here?

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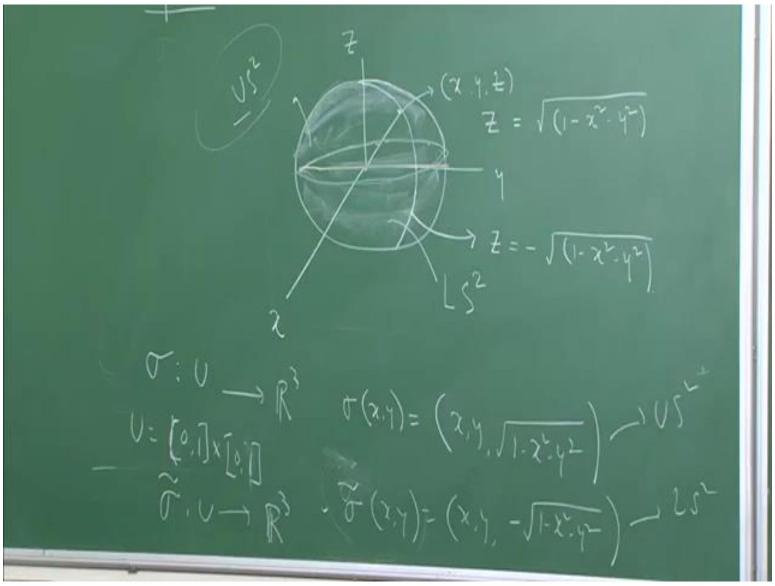

According to the definition, if I take Sigma from U where my U is 01 cos 01 to R3 Sigma X Y equal to X Y 1 - X square - Y square, then this will cover the upper hemisphere of the sphere. And similarly I take Sigma tilde from same U to R3, this will cover the, so this will cover upper hemisphere, this will cover lower hemisphere.

But the points on this great circle, that is equivalent that will be covered by both. And we should keep it in mind that these are covered by both but if you can see that these both of them are homeomorphism restricted to… so if I take open set which is restricted to upper hemisphere only, Sigma is homeomorphism, on the lower hemisphere, Sigma tilde homeomorphism but this homeomorphism has intersections. Sigma and Sigma tilde are not disjoint. This will come, this will prove a very crucial point in our discussion ahead.

That I covered this H2 by 2 homeomorphisms Sigma and Sigma tilde and they have intersections. Actually you learned in school that you do not do this thing, you do not describe the globe in this way, there is another way to describe the globe, that is by longitude and latitude. It is cartographic description. That I mean suppose we are in a particular place in a world, we say okay, it is 72.21° North and 58.3132° East, so that is the way we describe places in the world and they are saying… You need to describe that.

Let me give that cartographic descriptions of sphere. The reason I am doing it that you will see that 1st of all we cannot cover sphere by one single homeomorphism and secondly when you cover by different homeomorphism, we will have intersections.

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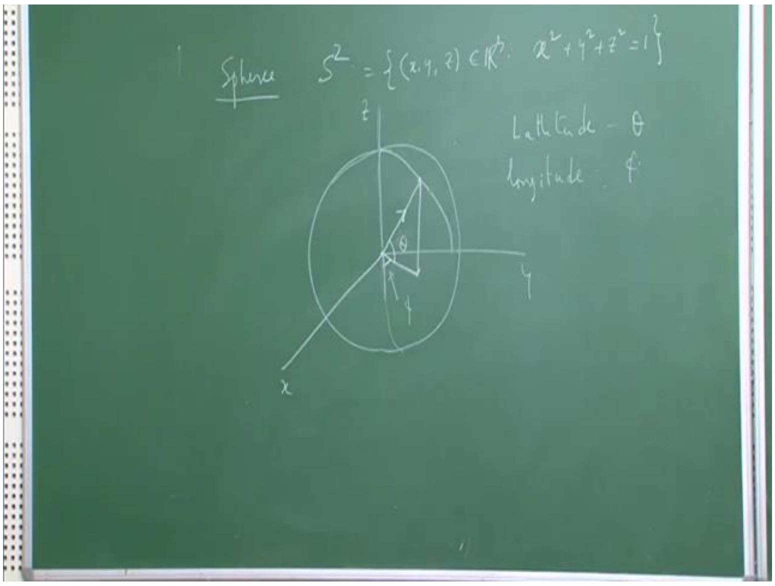

So, here is a sphere again. And I am at a place somewhere here. What is the Latitude? Latitude is described by this angle Theta, right. Z and Y, no, not clearly, this is Theta, sorry, this is the place and then I project it on the XY plane, I join this, so Theta is this angle, this is Theta and longitude is the angle between X and Z axis, that is this angle Phi.

And we have a make sure you align the great circle passing through like this and according to we move east or west, we call it the longitude east this way longitude Phi East, Phi ° East and if I am on the upper hemisphere, which will be latitude theta degree north. And how do you cover it, what is the range of theta?

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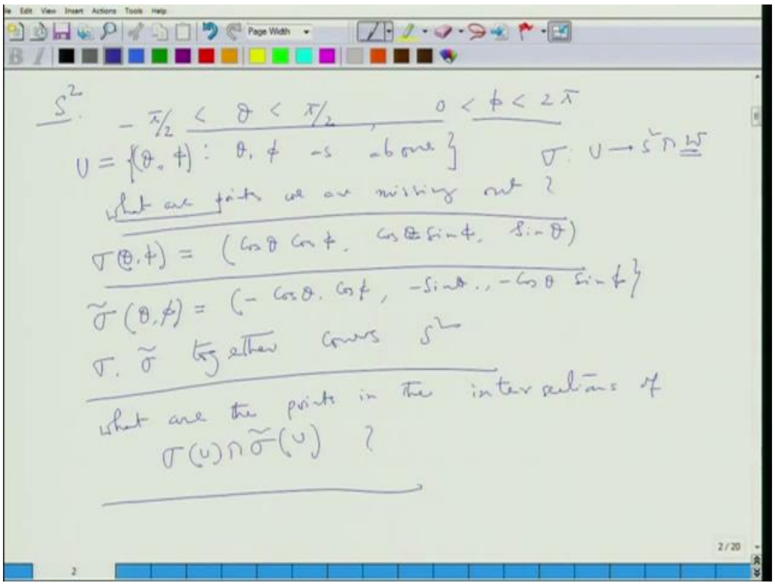

So cover S2 the sphere, you take theta, okay, sorry, I should make it open. And Phi dt 0 to 2 pie. So, U is theta Phi, theta Phi as above. But then you see there is a problem that I am not covering the entire sphere, because my definition of surface is that I should make it Sigma from U to S 2 Intersection W, I should choose my W and U should be open set, but make it open, I am missing out certain points in this sphere.

What point I am missing out? So, what are points we are missing out? You decide it, I will give the answers in the next lecture or next to next lecture, but for the time being we just think about yourself, what are the points you are missing out and what is my Sigma theta Phi, that you can easily see, this will be cos Theta cos Phi cos theta sin Phi sin theta, I think you have learnt in your +2 courses, that this is the usual parameterisation of the sphere. But as you are missing out certain points, actually I need another (set) Sigma to cover the entire sphere which is - cos theta cos Phi - sin theta and - cos Theta sin Phi.

Sigma and Sigma tilde together covers S2 and there will be many points in that section, so what are the points in that section?

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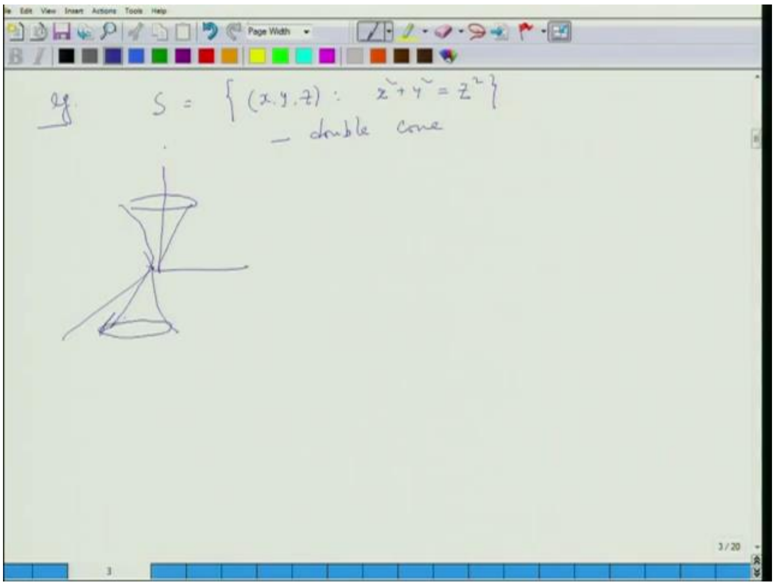

So, this is my 2nd example and next I want to end this lecture with another example with a surface, I do not know it is surface or not. So, this is called a double cone. This looks like well, how it will look like? If I draw like this, then this will be, I draw better on the board, let me draw it on the board.

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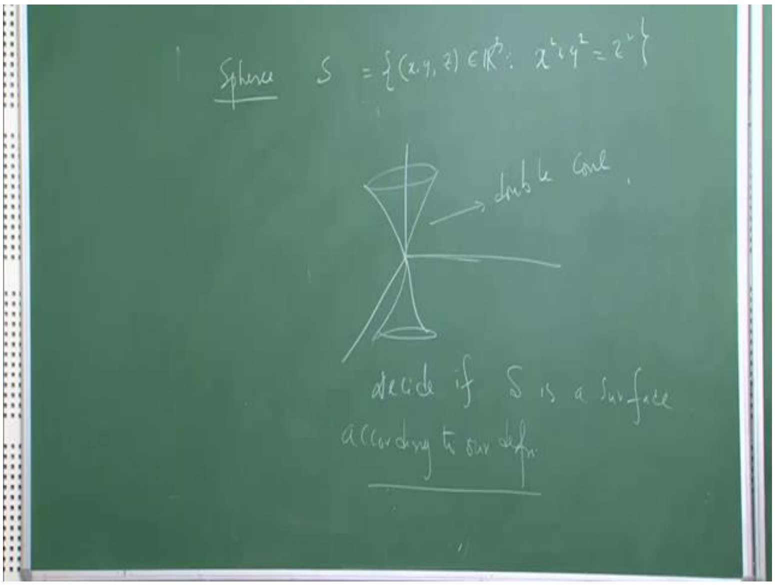

So, decide if S is a surface according to our definition. So, this lecture of introduction to very basic examples of surfaces plane, sphere and double cone, I do not know it is a surface or not. Next time onwards be will try to make it more formal. Okay. That is it.