Curves And Surfaces.
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Module-III.
Surfaces-2: First Fundamental Form.
Lecture-11.
Examples Of Surfaces.

Welcome to the 3rd module where we will be actually dealing with the geometry of surfaces but before that let us have some examples in hand. So, I will do 2 typical examples and most of the surfaces are of special cases of these 2 examples. And for our course, we do not need more than this because if you want to develop things in the more general form general surfaces, then we will need more vocabulary, more languages which as I said we will avoid. So, I will write up on this, examples on parameterisation on the screen and I will draw the picture on the board for this lecture.

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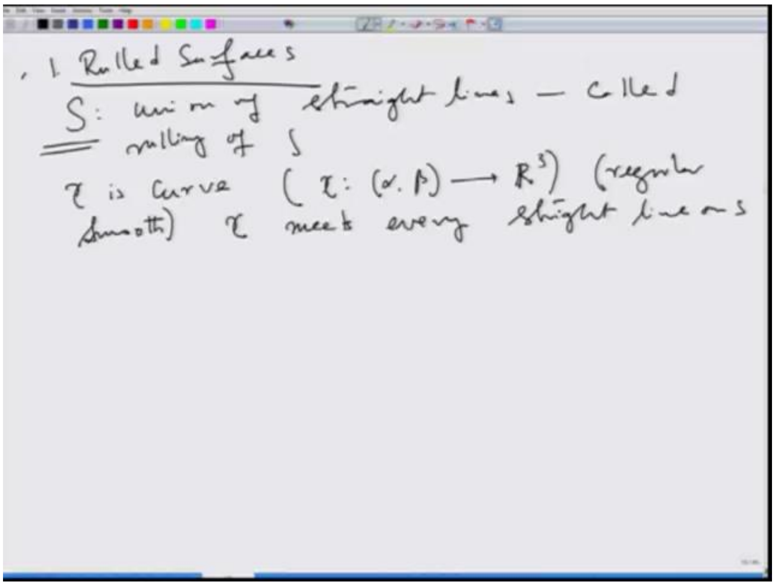

So, 1st class of examples, we will deal with is ruled surfaces. A ruled surface S is just union of straight lines, they are called the ruling of S. It, if I define it like this, it makes no no sense perhaps but it is like this. So, suppose you look at this surface S and gamma is a curve, let us say gamma from Alpha, beta to R3 and its regular, smooth and gamma makes every ruling, that is every straight lines on S, okay. So, what is happening here?

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I mean, the surfaces consists of kind of lines like this. All the lines are there and I have a curve gamma like this. And gamma meets every straight line at some point say Q, surface consist of this straight line, so everything is there, I am drawing it by shade, then any point P, I write delta here, I will explain what it is.

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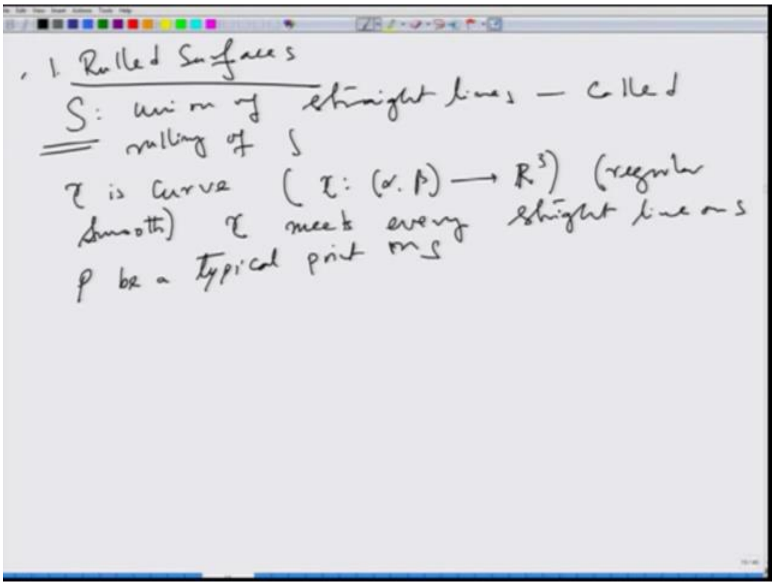

Straight line on S, let P be a typical point on S, what I mean here that gamma is like this,

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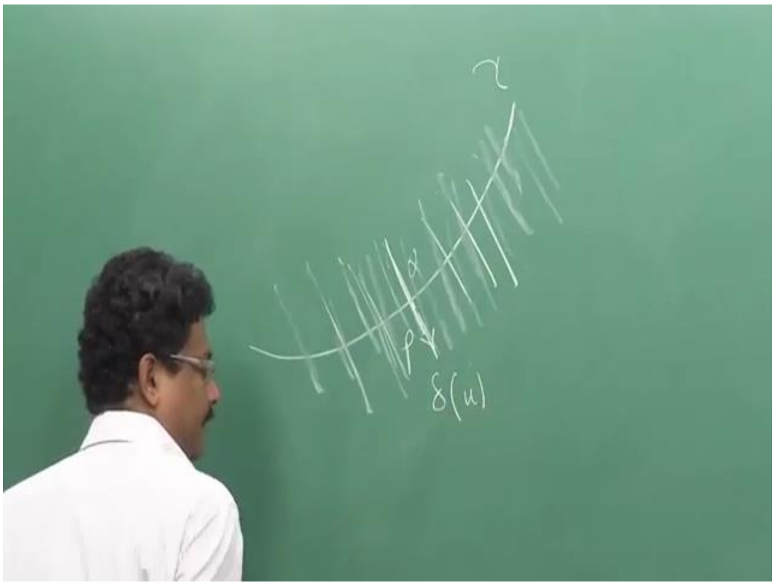

it intersects one of the state lines at Q, this is a typical point which is a direction delta.

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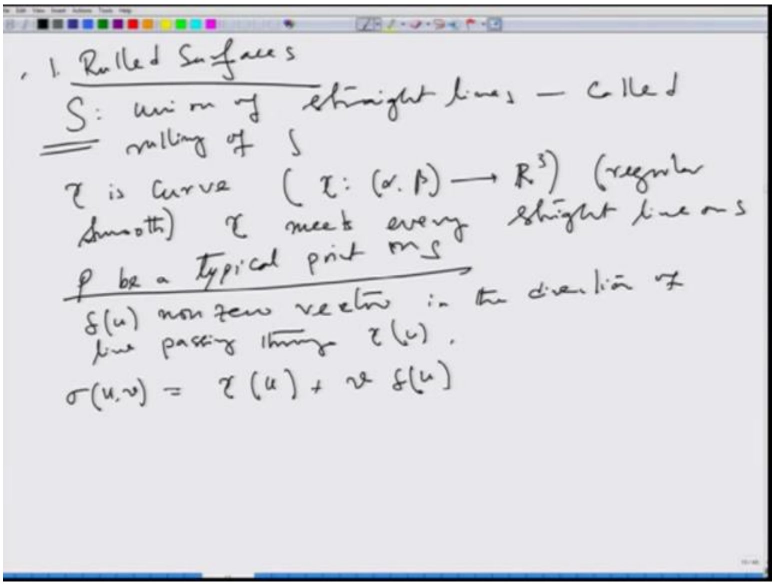

That is delta U is a nonzero vector in the direction of the line passing through gamma U, then a typical point P, if I write that parameterisation, it will be gamma U Sigma U this is gamma U + V + gamma U.

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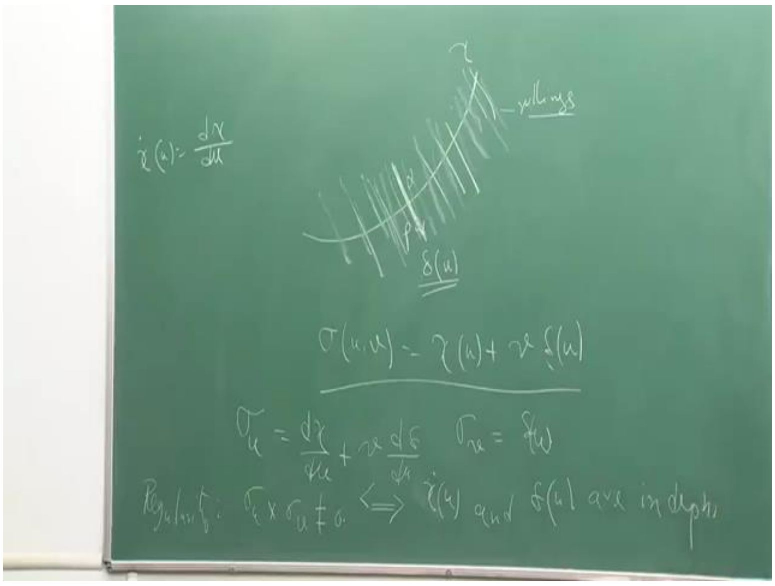

So what is happening here, this is straight line intersects at Q, P is a vector, P is any point on the surface, Delta U is the direction on which P lies, then Sigma U V is gamma U + V Delta U. So, you translate this kind of curve along all the rulings, so these are the rulings of the surface. All these straight lines are rulings and you are translating along every rulings, you get point P and if you translate along the direction gamma U, you will get a point like this, so the thing is that for this it is regular.

Okay, for regularity condition let us see, so what you need? Gamma U, this is gamma d gamma du + V into d delta du, correct. And gamma V is simply V. Now, sorry, V is gone, Delta U. So, I must have for regularity this vector not equal to 0 and you can easily check that means that this is gamma U d gamma u, d gamma du, let me write as gamma dot U. If this is vector is independent, then of course the cross product will be non zero.

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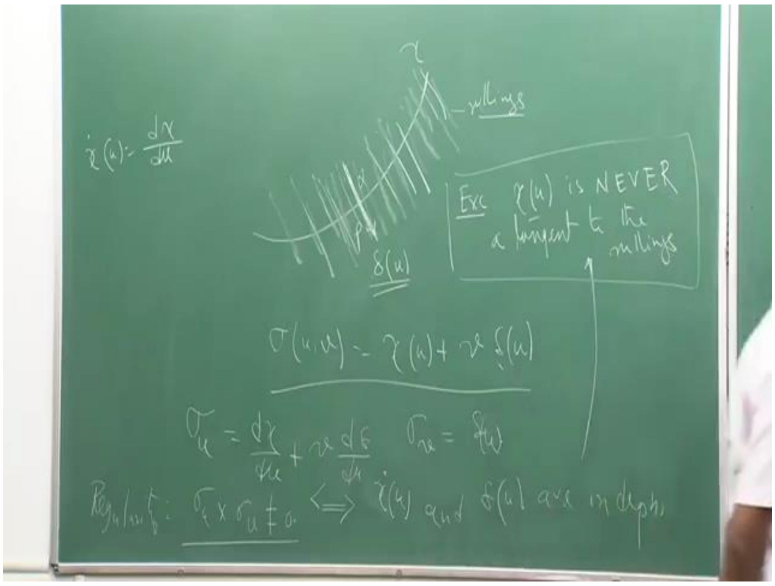

And I put it as a simple exercise, I mean it is contained in the statement, this means and tangent to the rulings means what, ruling such a line. So, there gamma U will never be parallel to any of the rulings. If it is parallel, then I will not have this and I will not have this gamma U and dell U or gamma dot U and dell U are independent.

So, this is the regularity condition and for regularity condition, this is what I need. Please remember this example, this is a very typical example and we can generate lot of examples of surfaces out of this. And this is a very general example, I will show you 2 more special cases of this.

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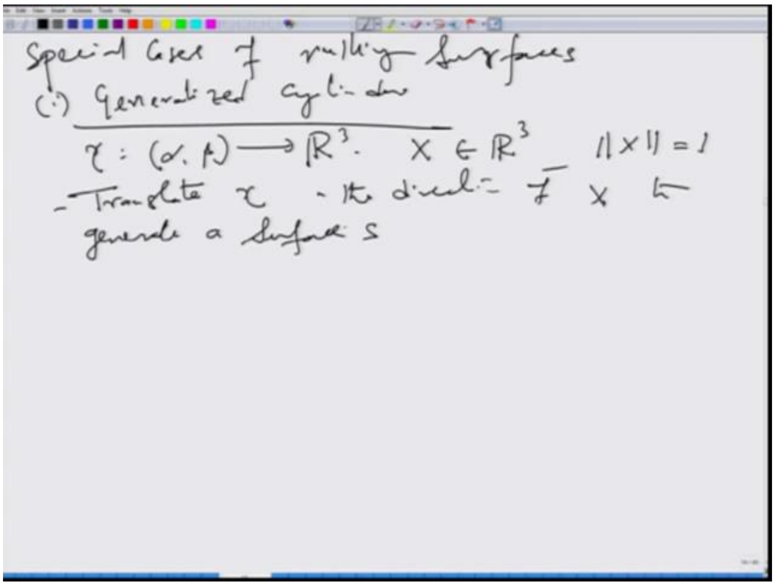

Mainly, so, special cases of ruling surface, 1st one is generalised cylinder. So, again, since ruling surface, it has to be union of straight lines, so my curve gamma is is from some interval Alpha, beta to R3, X is a vector in R3, let us take unit vector and you translate gamma in the direction of X to generate surface S.

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What is happening here? This is a special case of ruling surface as what I said, what I am doing here is, so let us take a simpler looking curve, where I take this curve and let us take X to be this vector, unit vector. Unit vector differential the direction, so I take this direction. So, I translate this curve gamma along X, so what will happen? Something will go up along parallel to X, things will go down in parallel to X, the curve (())(11:59) and then I will get a surface like this.

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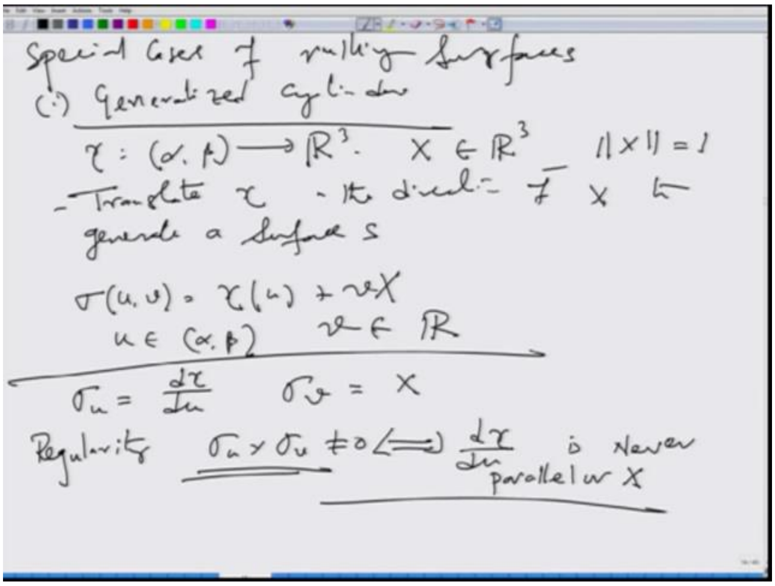

And this will have obvious parameterisation of, that suppose, sorry, I should not put the bracket V dot X, U in Alpha, beta and V in… And let us say for regularity, what is Sigma U, it is d gamma du and Sigma V is just X. So, for regularity, that is Sigma V0 equal to 0, I must have d gamma du is never parallel to X, then I will have a cross product is a nonzero vector. So, this surface is regular provided the condition I wrote, this parameterisation depends, if parameterisation becomes really simple if I take a simpler form.

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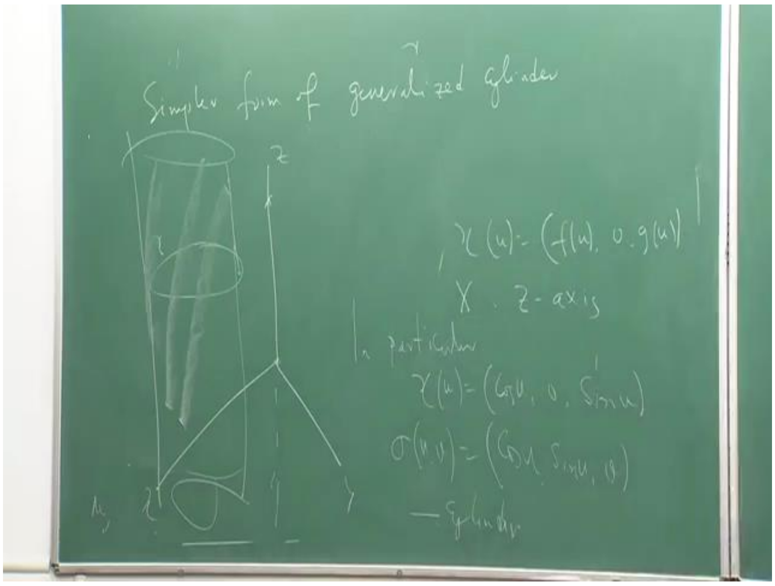

But if I take let us say in the XZ plane, I take gamma, it is a curve in XZ plane and X to be Z axis. Then I transform this, what I get, a figure like this. So, in particular, if I take gamma U to be cos U 0 sin U, then I get Sigma U V is equal to cos U sin U V which is actually cylinder. Therefore this particular type of transformation is called general cylinder.

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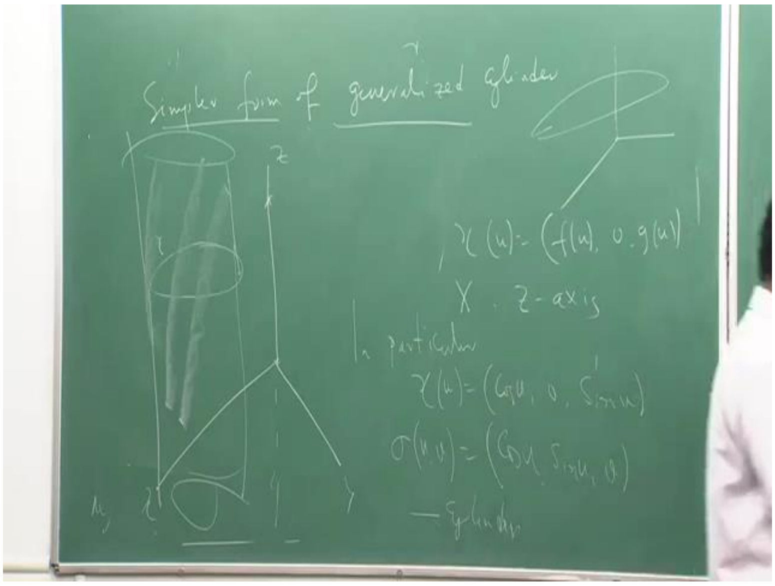

But on the other hand if you take any curve for that matter, by transformation you can put it in the Z XZ plane by rigid motion and you can take the line of rotation, line of translation to be Z axis and so all the surfaces like this will be homeomorphic generalised cylinder, provided your gamma is smooth and your axises satisfy the regularity condition. Remember this, I will not repeat it. This may be read important for your exam.

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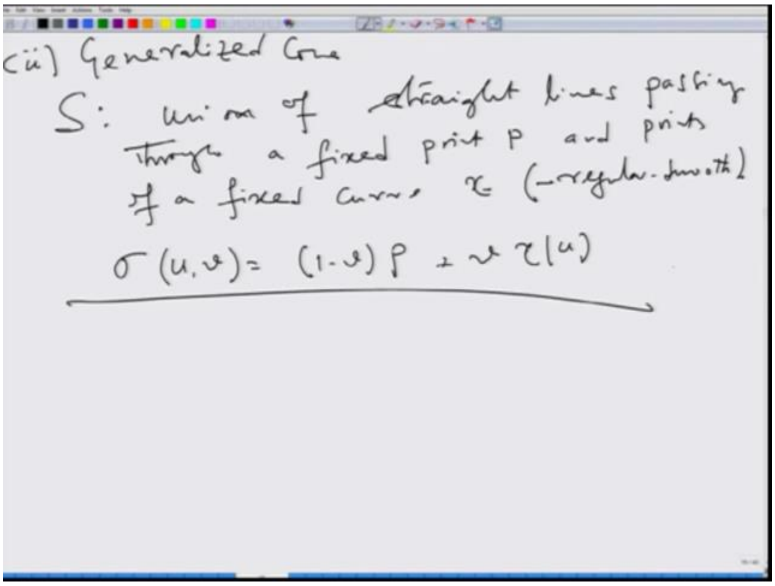

I have that example of ruling surface is generalised cone. We have done this example before. So, this is a surface S union of straight lines passing through a fixed point P and points of a fixed curve gamma, where gamma is regular curve okay, regular smooth curve. And parameterisation is usually is very easy to realise, 1 - V P + V gamma U. What is happening here? I will draw the picture on the board.

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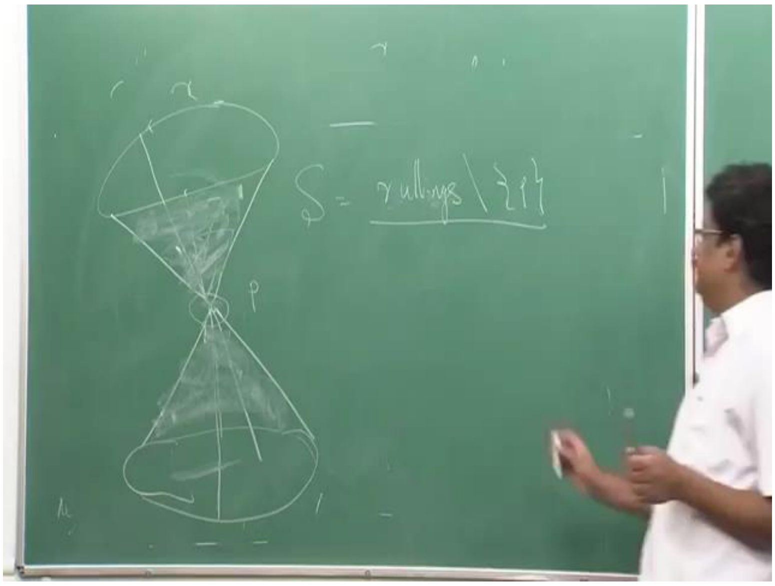

So, we have a curve gamma out here and I have a point P, okay. So, I am looking at all and I look at a point on the gamma, I look at all the straight lines that passes through P and some points on gamma. So, I will have a surface like this and we have seen that this looks like a double cone but double cone is not a surface, that we have done last week. So, I have to remove this point P. So, the surface S is that ruling - the point P, I have to remove the point P, otherwise I will not get a regular surface because we have seen it that if I have this double cone type picture, then there is a problem. So, I have to remove the point P and then I will get what is called a generalised cone known as double cone.

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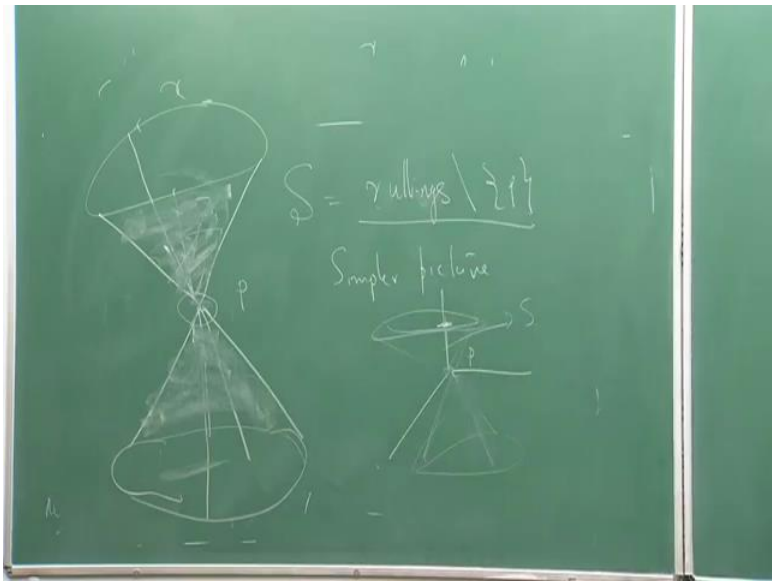

And again if you see I mean this picture can be made very very simpler in the sense that… you take P to be the origin and some point on the Z axis, let us say Z equal to1 and you take the curve which is passing through this, then you will have a cone like this, you have to remove the origin, this is your S and I will leave you to calculate Sigma U or Sigma V to see some

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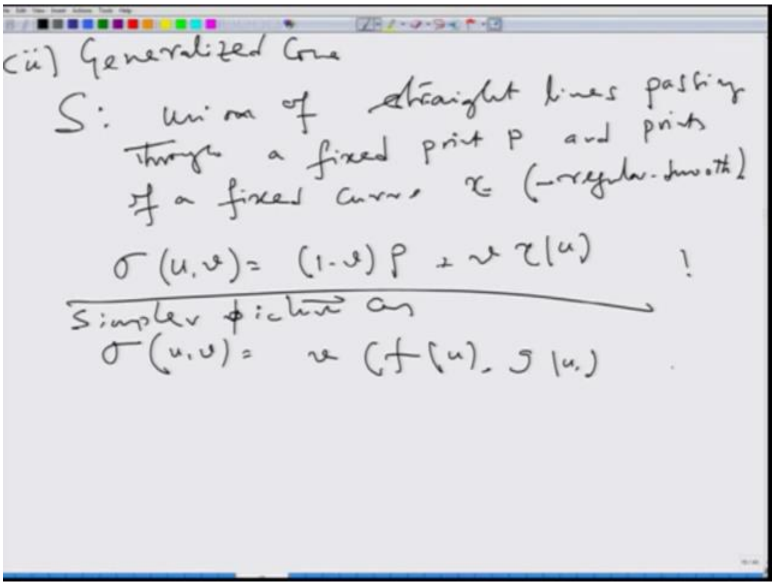

I mean, in this case, in the simpler picture case Sigma U V is actually V FU GU, let us say Z equal to, here we have Z equal to Z0, that will be similar parameterisation.

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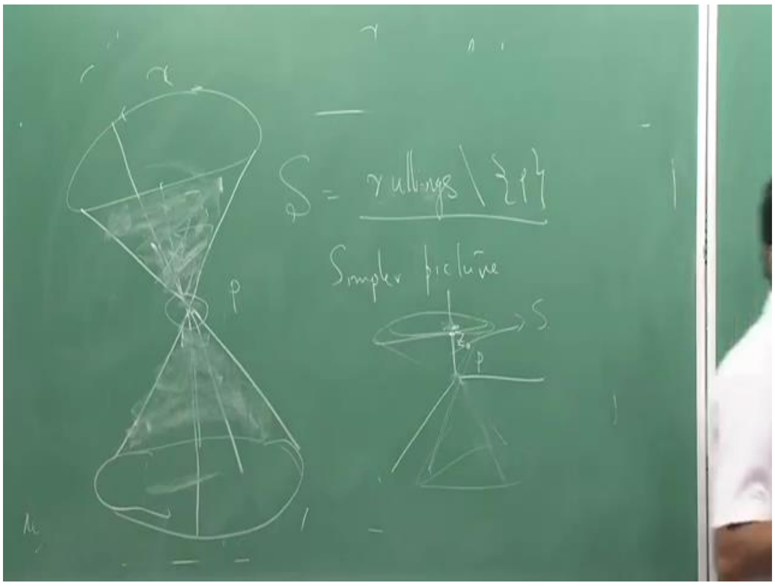

So, this is about the ruling surface which gives double cone and also gives generalised cylinder.

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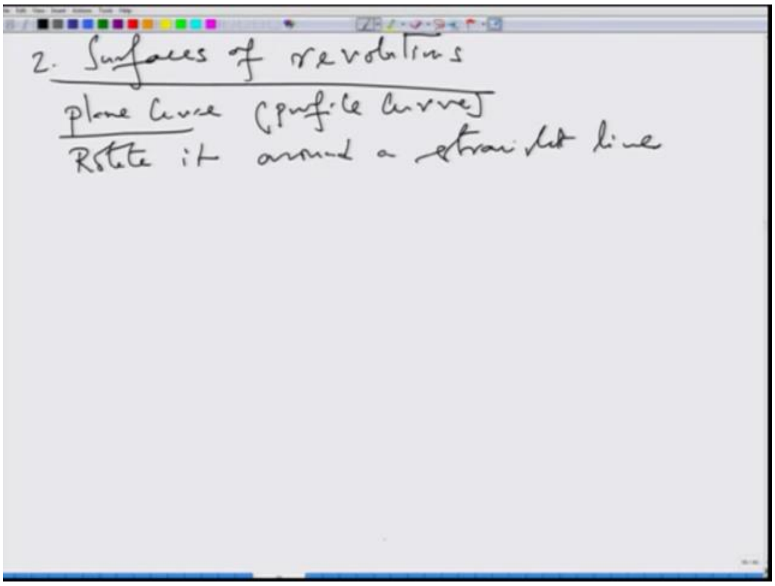

Next example of surface is general example of surfaces, what I will do is surfaces of regulation. And this is the most important for applications. So, what we are doing here? So, you will take a plane curve, it is called the profile curve and then rotate it around a straight line. This is a general surface, right. What will happen here? I will take the most simplest example.

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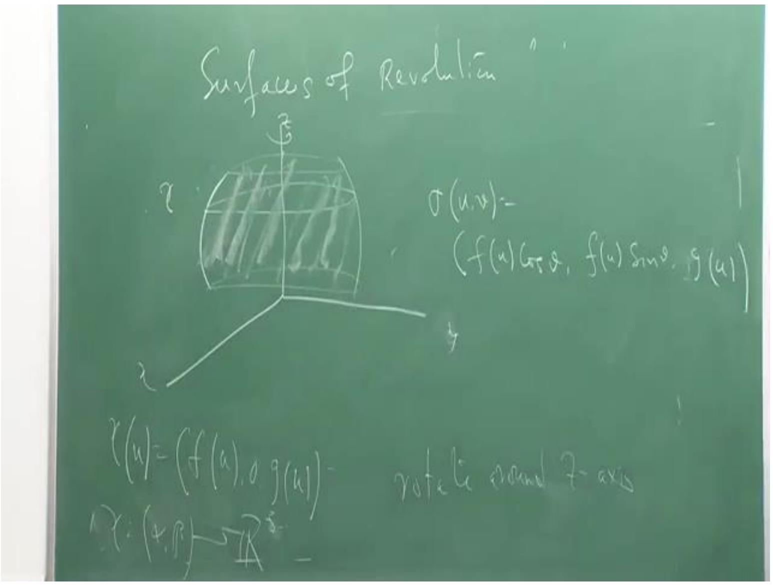

Let us say I have a curve gamma on the XZ plane okay. And then you rotate it around Z axis, what we have, a typical point here will rotate like this and come here, so the surface I will have is this and it is very naturally I will have the parameterisation as… Correct.

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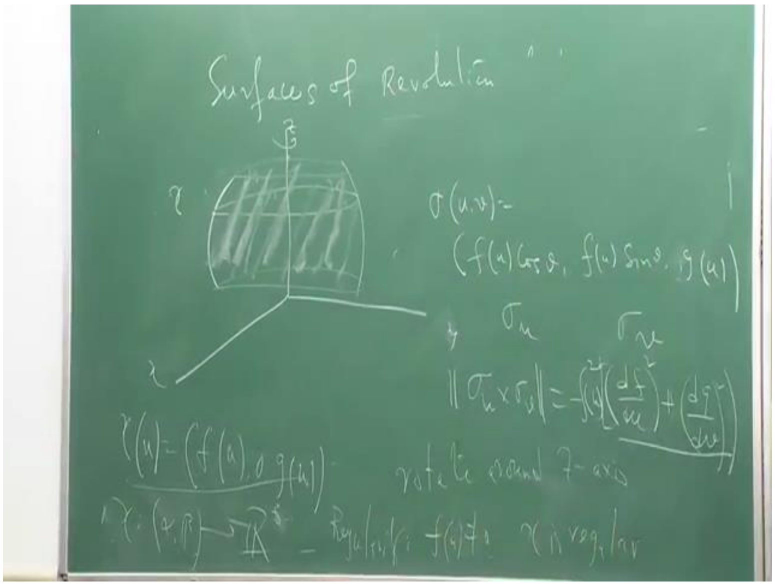

You can easily verify this as a regular surface, that is not a problem, you can calculate Sigma U calculate Sigma V and then you see Sigma U cross Sigma V, this is nothing but, it comes out to be how much X square ds du square + dg du dv square, so to be regular surface, I must have X square U.

So, regularity condition is, FU0 equal to 0 and this fellow need not equal to 0, this is not equal to 0 means gamma is regular. This is the tangent vector at gamma because gamma is this fellow. But this surface actually gives rise to lot and lots of examples. And we will see some of the complications here, in this week and also next week that how to deal with such a surface and when you do the geometry, there will be certain forms associated to surface and certain curve which are called Geodesic s associated to surfaces and for this kind of surfaces different kind of gamma or different gamma.

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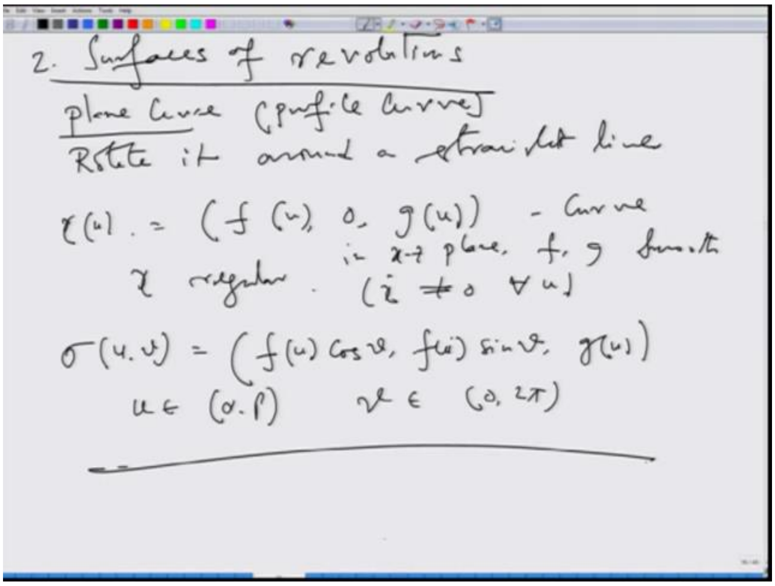

Calculation of Geodesic s is calculus forms may become really difficult but this is a very general form and means we will see many applications of it, so I just tried the parametric form for your record. So, gamma U is a curve FU0 GU curve in XZ plane, F and G smooth, gamma regular, that is, that is gamma dot not equal to 0 for all U. And then this surface I am looking at is FU cos V FU sin V GU, U belongs to Alpha, beta and V in 0 to 2 pie. This is a surface of regulation, so remember it, we will have many of this in next 9 lectures to discuss about it, thank you.