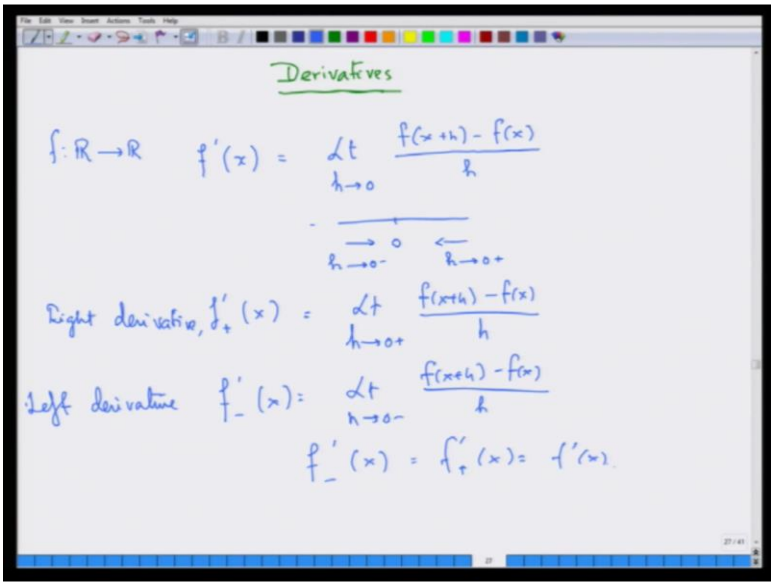
Basic Calculus for Engineers, Scientists and Economists

Prof. Joydeep Dutta
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture – 08
Derivative-1

We are going to talk about Derivatives today.

(Refer Slide Time: 00:16)

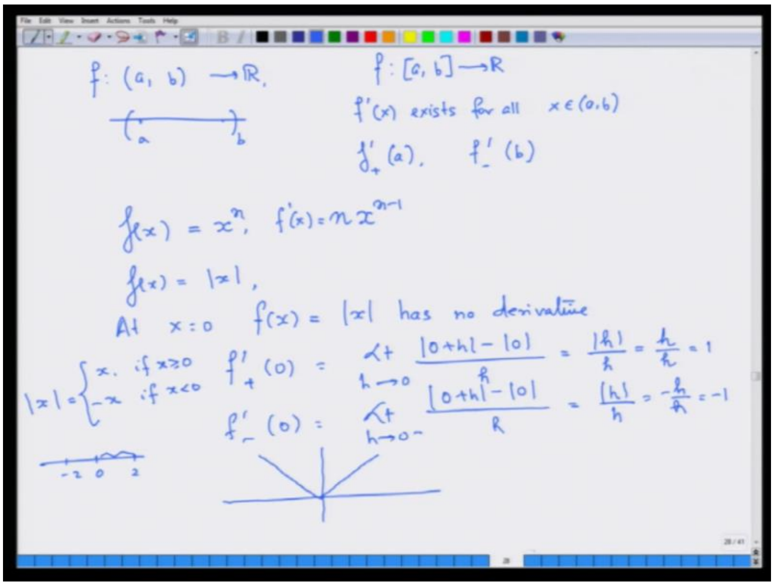


Derivative itself is a limit. It is a limit of ratios. It measures the change of x, change of the function of the f related to the change of the independent variable x. So, f dash of x is a limit as h tends to 0, this is for a general function for what to or so, I take a function from r to r, whose real line to the real line and I am giving the definition of the derivative. Now, you see this is the change in the function, well this is the change in the value of the independent variable x has change to x.

Now, as I told you that h when come to 0 from both this sides, this side or this side. So, you come to these from right side you say, is which is going to 0 plus. If you come from left side you will say which is coming from 0 minus.

Now, what is further important to note is that this limit has to exist and if this limit exist it means it has two components, the right derive right limit and the left limit and both should exist. The right limit of this is called the right derivative; the left limit of this is called the left derivative, which we will write as follows, the right derivative. So, right derivative is a h tends to 0 plus and the left derivative. Now, derivative exists if these two limits exist; left and right limit and they must be equal. The derivative exist only under this condition the left derivative equals the right derivative and which the under common value is the value of the derivative.

(Refer Slide Time: 02:44)



Of course you know of several facts, now what about a function f from open into our a b to r and what about a function f from closed interval a b to r. You observe that when you are taking open interval, the points a and b are not in the interval and hence if you take any point in a and b here or here, however, near they are towards a or b we can still have a both sided limit that are essentially your call the in the quantum of the real line.

In the sense of infinite of the real line these certain things the real line has to, we felt it cannot really be thought and now there are some remark that I want to make when you are talking about this what, when I say the derivative exists I say on for a function from a close interval a 2 b 2 means f dash x exist finitely for all x in the open interval part. Now, when I am talking, what do when I am talking about a derivative at a I am actually then considering the right derivative at a and the left derivative at b. This is very, very fundamental issue, but one has to be very clear in his mind from the very beginning.

Now, you are all experienced about derivatives, you have learned it in calculus. You all know that this simply means where n is a real number. So, if I take the derivative of this you know how to calculate it through first principles, but I want to tell you that the sudden must interesting use of the derivative which will soon come, but let me tell you something, for example, if you want to compute a derivative of x is equal to mode x does it have a derivative everywhere absolute value x, answer is no; so will show that at x equal to 0, f x is equal to mode of x has no derivative.

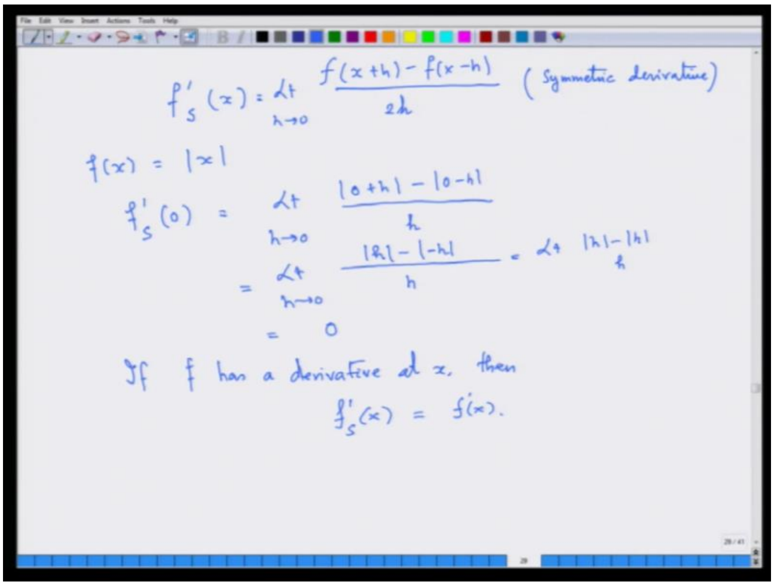
So, when I am talking about a derivative of a function over the whole domain or the whole or makes meaning that each at every each and every point the derivative exist. So, these things has to be understood when these are very simple things. Now, how do I know that the derivative we have taken at x equal to 0 does not exist? This is by checking the right derivative and the left derivative. So, for this function the right derivative is given in this way. At x equal to 0, my x is now 0. So, I am replacing x with 0, 0 plus h minus 0 by h, now this mode of h y h, because h is positive. So, h mode of h is h. So, h by h is one left derivative let us calculate.

Now, because if since h is now will strictly less than 0 mode of h is minus h. So, minus 1 is if those who have forgotten what is mode of x, let me tell you mode of x is equal to x if x is greater than or equal to 0 is equal to minus x, if x is strictly less than 0 then if x is a negative one, if you only get its positive plot that is meaning of modular x that is the mode x simply means the distance of a number from 0. If you have 0 you to have 2 and minus 2 the distance of if you are measuring centimeters each block 1 centimeter then distance of 0 to 2 is 2 centimeter and distance from 0 to minus 2 is 2 centimeter.

Every function need not have a derivative and if you look at f x is equal to mode x you look at its diagram f x is absolute value of x then you see these continuous function because you can draw the graph of this function without lifting your pen from the paper. So, again I want to go back to Renova Mendeleev's about statement that you would know calculus much better, if you know at the very outside at every continuous function did not have derivative of course, they have functions which are continuous at every point and not differentiable at any point this a monster functions by Weierstrass which we are not going to make any discussion.

There is an interesting idea called the symmetric derivative this is just for I am talking about I am really not going to deal with it much.

(Refer Slide Time: 08:13)



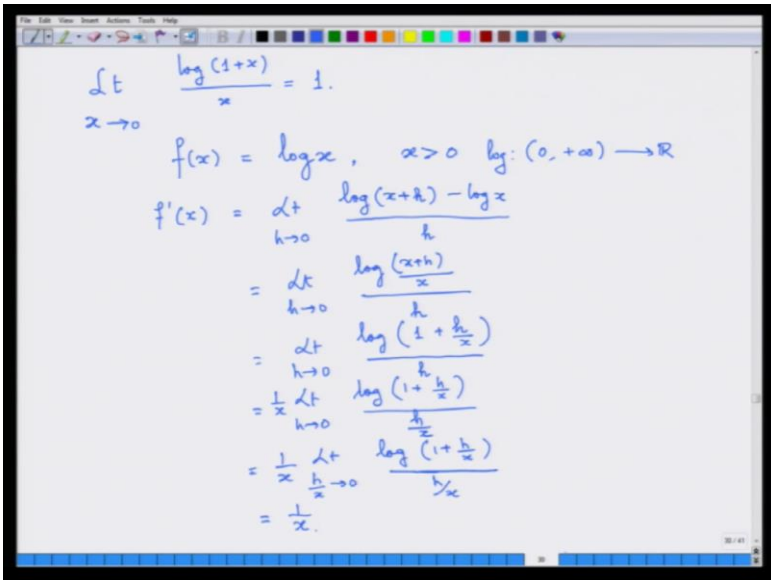
It is called the symmetric derivative. In symmetric derivative at x is given as follows limit h tends to 0 f of x plus h minus f of x minus h. So, I am looking at a two n points both movement this side and that side divided by twice of h. Now, the funny part is that these called a symmetric derivative, now the funny part is that this derivative can exists even the derivative itself do not exist, for example, let us now consider f of x is equal to mode of x and let us try to calculate its symmetric derivative. It is limit h tends to 0 that is 0 plus mode h minus 0 minus mode h by h. So, you can have two parts, right.

So, limit h tends to 0 even divided into two parts, the h positive h negative and left limit, left limit, left limit, right limit and do it, but that is not really require thus this is mode h minus mode of minus h. So, mode of minus h is same as mode h, basically this is 0 because this same as limit mode h minus mode h. If h is positive this is negative, this will again have minus and become mode h. So, we can do the same thing if h is positive say this is this is negative, there will be minus in front of it and it will become h.

It will become h minus h you can try it out with h negative separately if you are not convince with the fact that mode minus h is minus h because the distance of h from 0 and distance of minus h from 0. We got the same we just have to understand that the absolute value of a given number is nothing, but it distance from the 0 and distance from the origin 0. So, this is 0.

The symmetric derivative of this function exist even though symmetric derivative is not when the actual derivative does not exist. In fact, if f has a derivative then the symmetric derivative is equal to the derivative that is which are the points. Let us now look into the uses.

(Refer Slide Time: 11:19)

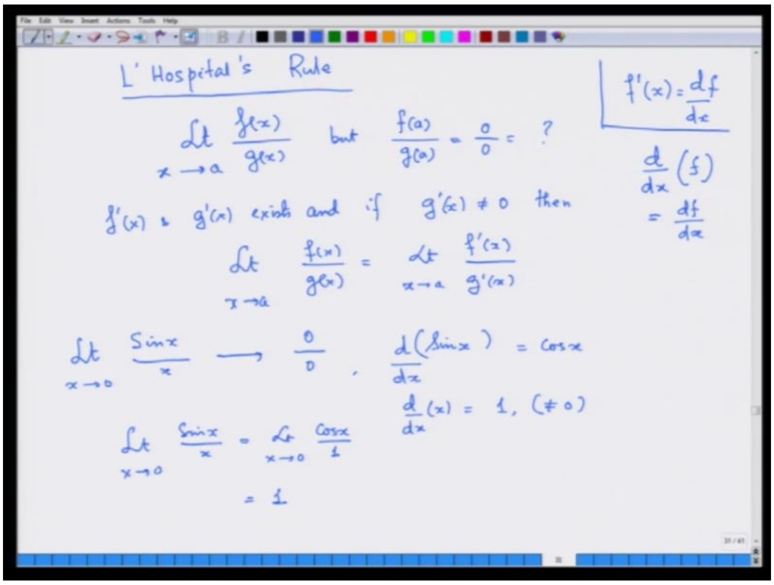


For example, yesterday we were talking about a limit of this form and will see how useful it becomes, now let us try to look at the function f of x is equal to log of x of course, x has to be greater than 0 that is a domain. The domain of this function, basically this function f log x this log function it actually defines from 0. So, you know already that the domain of this function is 0 to infinite. Now, we want compute this from the first principles at when want to compute this limit. So, this is same as limit h tends to 0. So, I write again.

So, what I will get is h log of 1 plus h by x see my x is now fixed x is a fixed number and x cannot be 0 x has to be bigger than 0. Now, what I will do is here my I have one limit h by x. So, I will divide, multiply both sides top and bottom by one by x. So, I will have limit h tends to 0 one by x multiplied at the top which can be brought out where it just a constant it is not depending on it h by x.

Now, observe that on when h is going to 0 and x is a fixed positive number h by x also goes to 0. So, this is same as 1 by x limit h by x going to 0 log of 1 plus h by x by h by x and that is known to be 1. So, ultimately we have 1 by x, this is a very, very useful way to obtain the limit you see how we are made a use of this limit, but now you will see we have never given a proof to how do I actually get this limit though we have used it.

(Refer Slide Time: 14:29)



Here, interesting way to prove it is, to use what is called the L' Hospital's Rule. In India we will call it L’ hospital rule, but if you go by the French pronunciation it is called L' Hospital's Rule, he was taught by Jon Bernoulli and possibly the result was given by Jon

Bernoulli to L' Hospital's Rule was a Richman Sonin, he later on wrote a book on the calculus.

Now, what is a L' Hospital's Rule? L' Hospital's Rules essentially deals with trying to find limits of this form now suppose f and g are both nice continuous functions you do functions got, but suppose f of a by g of a is in the from 0 by 0 then of course, you cannot say I will just take limit f x by limit g x tends not possible because its 0 by 0. So, your divisibility rule that you have about limits completely breaks down here right. What should be done now handle such limits. So, that this interesting rule its say that it is if f dash x and g dash x exists and if when if the functions are differentiable.

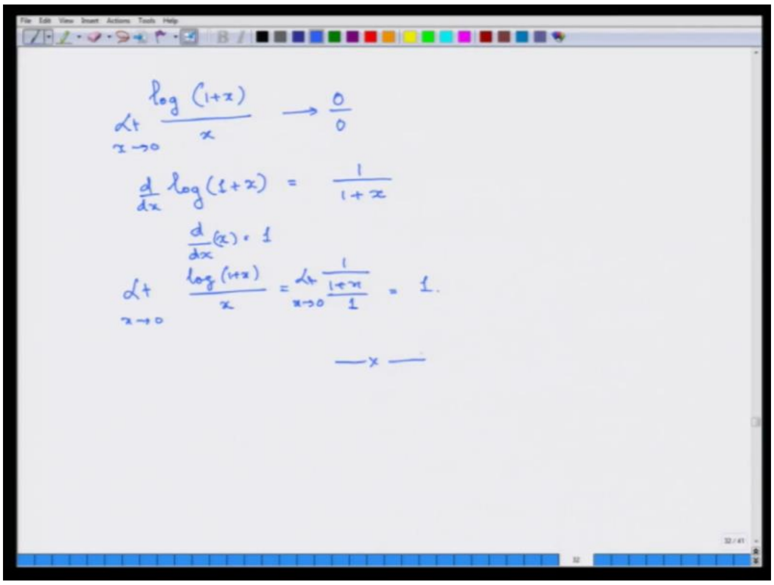
Of course, we have to assume needs differentiable at every x and if g dash x is not equal to 0 then limit of x tends to a f x by g x is equal to limit of f dash x by g dash x and when I am talking about f dot x g dot x exist means I am talking about that the derivative exist at all x. So, from the domain of the function and then if g dash x is not equal to 0 on that domain for every x then this limit is same as this limit what could be a possible application of this let us see.

Let us look at the following limit again, if I put x equal to 0 that is sin 0 y 0 if this is again in the 0 y 0 form, but I know the derivative of sin exist sin of x if I take the derivative which we write at as like this the derivative is also a just for your recalling because you know it in high school the derivative of if I take also written as f dash x, this was a notation of Newton.

While Olier give the following notation some people also credited, but Olier is really the person who gave this made this notation popular where we call it d d x of the function f never usually we say d f d x or d f by d x. It is not a way d is not a ratio d f is not something and d s is not something its d d x is an operator which is operating on f.

So, you have to remember you should always call d d x of f that is the right thing which we are basically we adding just the short end to right if very fast is not d f d x is is not a ratio of two numbers. So, d d x of sin x is cosine of x right and when cosine of x is not equal cosine of x is there. It exists and d d x of x is 1 and that is not equal to 0, which means limit of x tends to 0 sin x by x is same as limit of x tends to 0 cos x by 1 and as x tends to 0 cos 0 is continuous function and that is what you get.

(Refer Slide Time: 19:08)



Then let us look as a last example for today is the logarithmic is the one which we are just used to compute the derivative of log x. Now, if I put x equal to 0 it will become log 1. So, log is 0, it will again in a 0 by 0 form. So, let me did the derivative of log x now I am a see some sort of a little bit of circular thing of course, there is a geometrical proof this you can do a proof this, but we are not going to do a proof of this of course, these limits came into existence when we were this people when they have first got to this limits they actually did it by computation did it by experimentation.

Remember, mathematics is also an experimental subject, but thing which I tried to tell you in the last lecture and very specifically so is calculus. Now, if you take the derivative of this, it is 1 by 1 plus x and you know if x is strictly greater than 0 these also known 0 this is the very well defined thing and derivative of x. So, d d x of log x is this and d d x of x x is 1. Basically, now again by applying the L' Hospital's Rule or L’ Hospital, if you want then we have limit x tends to 0 log of 1 plus x by x as 1 by one plus x limit x tends to 0 by 1. Now, these are quantum function where x is strictly greater than 0 an x is tends into 0 and that gives you 1.

With this we end today's talk and tomorrow will tell you some more important properties of the derivative, how to differentiate sum of 2 functions, the product 2 functions, the cosine 2 functions, the ratio of 2 functions and some of their more interesting properties. So, with this we end out of today.

Thank you very much.