**Basic Calculus for Engineers, Scientists and Economists**
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**Lecture – 02**
**Functions – 1**

Welcome to the second lecture. Here we are going to talk about a very familiar thing called Functions. Here everybody has heard about functions, if I have gone through high school I must have heard about function if I was attentive in the math class. Function is essentially a machine which gets one input he gives an output, so just think of a machine like this.

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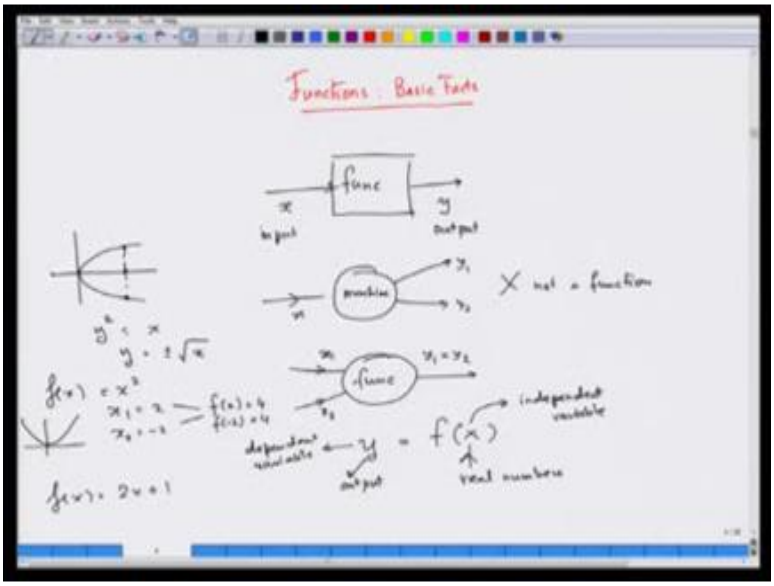


A machine which I called a function, and there is an input which I call x and there is an output which I call y. So here is an input which gives me an output. But remember there are some restrictions in this machine. This machine cannot give you two outputs with one input. So, you take a piece of potato and chop it in to 2 so your 1 potato, this gives you 2 pieces of potato, so that cannot happen.

Cell division, you have a biological cell it divides into two equal identical parts. Such operations are not functions as for mathematic goes. Here, if I have one input x and there is some machine which gives me as a 2 outputs corresponding to this x which is y 1 and y 2, so this is not a function.

Function has the rule one input is being one output gives me one output, but of course one important thing is that a function can have the following behavior I can have 2 different inputs which gives me the same output. This is accepted in a function; because x 1 is giving me one output which is y 1 and x 2 is also giving me one output which is y 1, so at the same time they are not giving me two outputs.

This also has a meaning calls (Refer Time: 02:19) math which we are not going to even talk about here, though economics would need it if we I have time that I will really speak about it a bit. For example, now output is usually written as y and the input that is the function or the machine f is active on x. So, this machine f x on the given input x gives me output y. In mathematics we called it independent variable I have the choice to give my input, and this gives me output, or dependent variable. These also called a dependent variable.

These are some very basic facts you have to know about functions. For example, if you take the function f x equal to x square we are only talking about real numbers here, so all my inputs here are real numbers. For the time mean all my inputs here are real numbers, but it is not that I cannot have any other thing I can have anything as an input, any object as an input, but for us all our examples would come from real numbers. If I take f x equal to x square it is the familiar parabola that you know you have studied in coordinate geometry, this is a function. And you see here for x, x 1 equal to 2 and x 2 equal to minus 2 the function values both f 2 and f minus 2 both of them give me the same number 4.

This function is a function of this particular type. Now the one which we are just said that the two different inputs give me one output. But for example, the function f x is equal to 2 x plus 1, so 1 output gives me 1, 1 input gives me 1 output.

But it is very very fundamental thing to remember that we twist this function a little, say we look at this function not a function or this curve I would say y square is equal to x. That is these a function? The answer is no, because y gives me plus minus route over x. So, corresponding to a given x - i f, two values of y you see that is where it cuts the curve. A function, here as you see what I have done I am try to draw certain graphs here.

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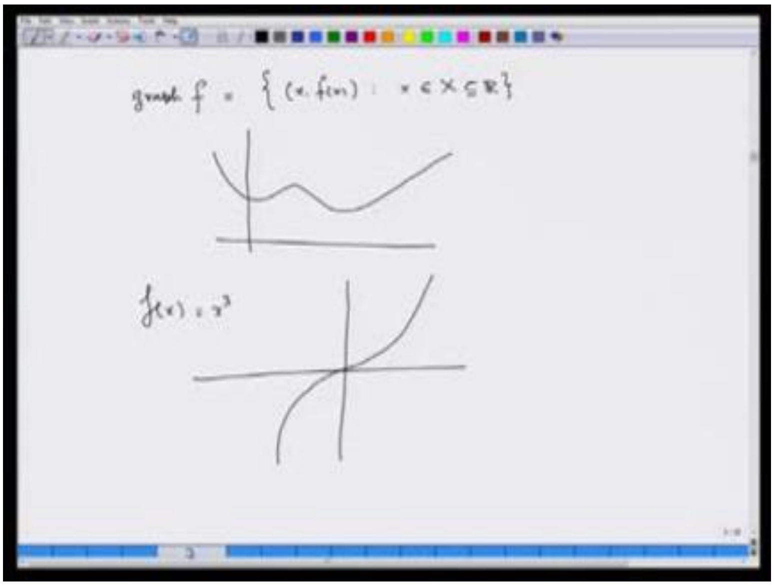


So concept that is intimately linked with a function this is the notion of a graph. When we are talking about functions about real number we are talking about the graph over function. It consists of two real numbers x and the function value f x y basically. So, x is somewhere I would write to some set x which is the subset of the set R, I am assuming that you know set theory. So, because I have already mention that these are slightly a course for the first year level not for plus two level students.

Graph over function - you can think something like this, this looks like a graph of some function of course, I am enable to right down exactly what function it is for example, if you say f x equal to x cube now to (Refer Time: 05:57) precise let me go back to the high school definition of our function which you already have some idea about.

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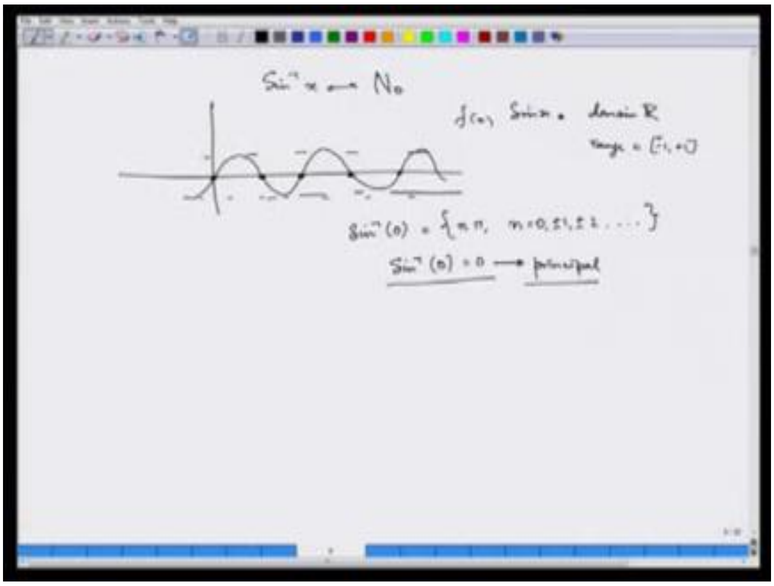

In your high school they teach you the following definition of our function they said let us take two sets which are non empty x and y. And by a function one means, so this is no x and this is my y and by a function we mean a rule which takes a one point x in x to only to one point y in y not to two points or more points. So, f takes this x to this y. So, y is often as we have written as y equal to f x.

So, I can talk about arbitrary sets, functions can be defined between two arbitrary sets it need not be even defined just between to real numbers. So, this is what you learn in your high school. This set x is often called the domain and this set y is called the codomain. There is another concept called range of the function, range of f or in short ran f. It consists of all elements in the codomain, it consists of all y in this set y for which there would exist an x in x such that y is function of x - y is equal to f x. So, this means that I am trying to say that there meant all elements of y need not be an output corresponding to a given x that is something very very important.

So, there is a different between codomain and range. So, essentially what we are trying to say range is the subset of the codomain. Range of f also called the image of x under f, sometimes if the people use a terminology as f of capital X, I will give an example how these are two different - for example, I will take again the standard function f x equal to x square. Here the domain means, when I am talking about the domain means I have to operate this function f on each and every element of the domain this is the fundamental importance that each and every element of the domain has to be acted on by f.

So, if I take f x equal to x square I can act I can square any real number. So, domain which I now write in a short form dom f is R, but domain of f is or the codomain of f is which I right now as codom; the codom of f is also R. But the range of f is not all because it only maps gives me non negative number it is the set R plus, R plus is a set of all x as that x is greater than equal to 0, see it is set of all x in R; x all real numbers are, so x here is in R and these x's must be greater than equal to 0. So, it is just one side of the number language you have an idea about.

We are just here, so my range is a subset of the codomain for example, now a take the function f x is equal to logarithm of x - natural log of x. Where the base is the number e then the domain of f is R plus plus that is the set of all x belonging to the real line, that is all real numbers which are positive. Of course, you know the log you cannot define log for a negative number or 0. Now what is the codomain of f? The codomain of f is R. Codomain of this log function, codomain of log is R. So, what is the range of the log function? The range of the log function is also R.

Codomain now has become equal to the range, when codomain of f becomes equal to the range of f we say that the function is onto or subjective we say that f is onto or subjective. If we are speaking about some other sort of behavior of functions and important behavior is one to one relationship that is, if you have two functions such that if you have a functions such that f of x 1 is equal to f of x 2 then it should imply that x 1 must be equal to x 2. It cannot be that two different numbers has given me the same function, the same function value.

It can happen for example, in f x equal to x square we saw minus 2 and plus 2. So, effects equal to x square it does not satisfy this definition. If something satisfy this definition is called injective or into - into function. So, a function is bijective or one to one correspondence, function is bijective we have used the word bijective in the last lecture if function is bijective if f is both subjective and injective.

Now, we come to a very very important notion of the inverse of a function because this question is naturally you might ask why you are talking about inverse. So, what the question of inverse is a very natural concept - inverse of a function. What does it do? It does the following - Inverse means for example, I take the function y is 2 x plus 1. So, here I have expressed y in terms of x.

So, what happen if I swap it means I make y as the independent variable and x as the dependent variable, then can I that take such function of y. So, here y is the function of x my question is can I (Refer Time: 13:14) takes as some g of y. In this case the answer is g s because I can write the whole think as the x is equal to y minus 1 by 2 which is my g of y, right.

But, it might not always be possible. So, inverse of a function which we denote as f inverse some times, so g is here called f inverse - f inverse of y; so f inverse of y - where y belongs to the domain. What is f inverse of y? If you take any y and then you ask the question can I find an x in x such that y is equal to f of that x. So, there can be more than one x for which y is equal to f of x, so if I take the function f x is equal to x square and if I take f inverse of 4 then i f this set would consist of 2. For example, if I have f x equal to x square and then I ask you what is f inverse of 4 then this set consist of 2 elements - plus 2 and minus 2.

So, f inverse is really not a function because for a single element here 4, I have two outputs plus 2 and minus 2. One input giving me two outputs. So, f inverse need not be a function in general. When if inverse is the function, we said that the function is invertible like the one we have just done and then we can say that the function has an inverse. So, f is invertible, when f inverse is a function. When f inverse is a function? A very important result to remember is that a function is invertible if an only if it is bijective. This is absolutely fundamental to once understanding.

Now, what can we do with functions? We will give you some more examples to play on with, finding inverse of logarithmic function and there are certain cases where you can write down the inverse exist because the function is the bijection, but you cannot write it down explicitly. We are now going to talk about some more properties of functions called increasing functions from of real numbers.

In our study we would be largely considered the functions defines from R to R or functions defined on a closed interval a b or an opened interval a b does not mind. So, functions of these forms. These are the type of functions we would be interested in our study we now study of calculus these are the type of things which will come repeatedly.

We will talk about limits of this functions, continuity of this functions integration differentiation all sort of stuffs what which you have heard about. But now it is very important to us to note two important types of functions which play a fundamental role in the study of calculus these are call increasing functions and decreasing functions.

So, what sort of function is called increasing? So, if I take x 1, so I take a function from real number to real number R to R and the first one, this one. And if I take x 1 bigger then x 2, if 1 whenever this happens if I have f of x 1 strictly bigger than f of x 2 I say that the function is increasing there is also a concept called non decreasing, which says that if x 1 is strictly bigger then x 2 the function value at x 1 can be strictly bigger then f x 2 like this one or can remain the same.

Similarly, you can talk about decreasing functions. For example, in that case we are whenever x 1 is strictly bigger then x 2 you have f of x 1 strictly less then f x 2. In the similar spirit 2, in the non decreasing function is as notion of our non increasing function. We will see that how derivative is a role in understanding of these type of functions. So, whenever x 1 is strictly bigger then x 2 it will imply that f of x 1 must be less than or equal to f of x 2.

You can immediately in you are mind imagine and draw graphs like this, functions like this which is increasing - functions like this which is decreasing. So, if you take f x is equal to logarithm of x then that function is obviously in increasing function or if you take its inverse f x is equal to e to the power x.

That sort of function is if you come from this side is an increasing function. This is the graph of e to the power x, e to the y equal to e to the power x and this is the graph of log x, logarithm of x. So, these types of functions are very very important and these play a fundamental role in calculus and that is exactly what we are going to talk about.

Now, another important thing that I want you to note at this stage at the very end is that you have to also keep in you are mind that, when you use the notion of function. Never ever by mistake think about inverse functions like for examples sin inverse functions; people use sin inverse functions, sin inverse x this is standard in calculer in our trigonometry. Sin inverse x function, is it sin inverse x we use it as if it is a function, but is it really a function - the answer is no. For example, if you draw the graph of sin x what would happen. Sin x is a function whose domain is R and whose range by the way is minus 1 plus 1 or the values where we written minus 1 and plus 1.

For such a function what is the meaning of sin inverse x because you see at this point the sign value is 0, at this point the sin value is 0, at this point the sin value is 0, at this point the sin value is 0. So, sin inverse 0 are called set n pi, but when you write sin inverse 0. In many cases we just write it to be 0 means we are using one of the values of this set of numbers. So, it has to be very important that with inverse econometric function never think of them really as functions. Inverse econometric functions are not functions you are just using one of the values of this functions which is in this case 0 as call the principle value.

This has to be very kept in mind because trigonometric functions are very very important because Fourier analyzes of Fourier series that you learn about has a major role to play in engineering sciences. So, it is very important that you keep in mind that, inverse trigonometric functions which can never be a function they are always so called said valued maps which are not functions.

But when we write sin inverse x equal to y we are essentially talking about only one of these values and a value which is chosen, the first value or the base value which is call the principle value. The inverse econometric functions are not functions. So, these are very very important example. Our thing which is used as a function which looks like as if it is being used as the function, but it is not really a function.

We will talk about sequences in our next talk, in our next lecture. I hope that you are having a more or less basic idea what was going on.

Thank you very much.