Basic Calculus for Engineers, Scientists and Economists

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Lecture - 14
Integration – 2

Now coming to sessions on Integration would be essentially simple. You have done high school integration, so we are going to talk about Integration. This is just to recollect or remind you what you have learnt. This is not really to teach you something new.

In the next class we will talk about the integration by parts maybe also little bit about how to handle rational functions. So these are the things you have learnt in high school, but we are just recollecting so that you can keep in mind because you have to start you know doing some integration gradually as you go on.

Then you go out to the real staff the which called Riemann Integration, here you are talking about when a functions which are continuous over the domain and we are essentially trying to talk about integration of that, which is in case of definite integral nothing but we are talking about area under the given curve. But also we have to know what is anti derivative in order to do that because that is what the fun fundamental theorem of calculus shows us. That is what we have learnt in the last class, if I just go back and see what was there.

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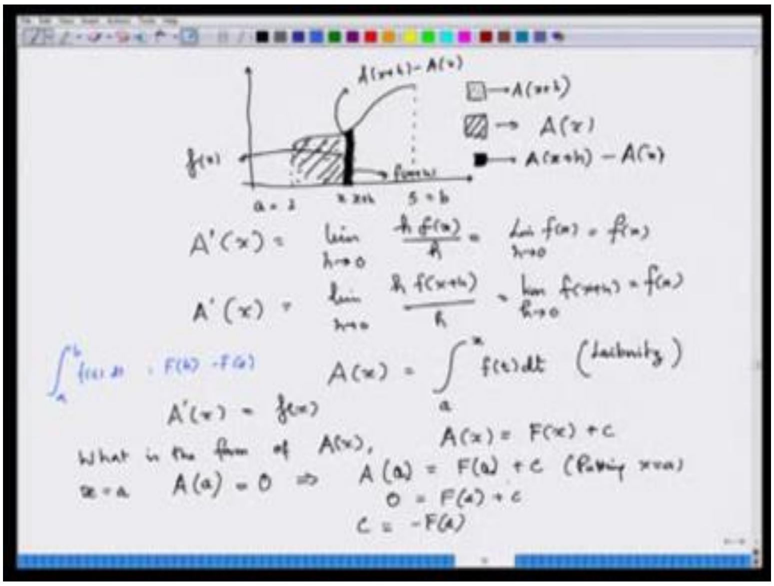

So you have a look at what there and you can immediately see what we have essentially tried to do. This integration this F b minus that integral a to b is F b minus F a. This is something very important that is integral a to b f t d t is F of to b minus F of a. Where, capital F is anti derivative of small f.

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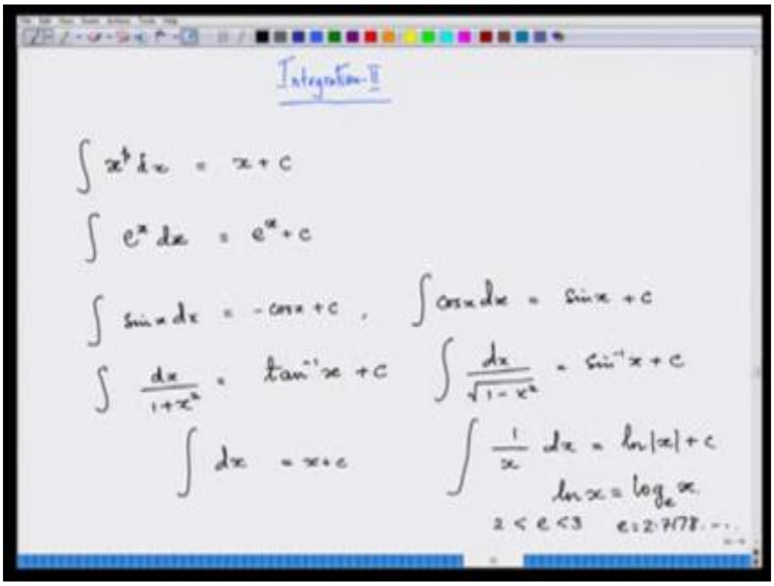

Here, we will just recollect some well known integrals. Let us do something which is very standard. You must be wondering why this sign d x comes from will start explaining that, but let us write down which you have possibly known have not bother much about understanding and work through. I think this is very simple, this is the anti derivative.

So, you know a lot of about derivatives. You can do this, but again this particular derivative d x by 1 plus x is square, this is something which is used very very much in actual applications. Of course, I am just writing on the side. There was a famous mathematician who made this comment that when I was a young man all of my friends (Refer Time: 03:16) for luxury, but I only (Refer Time: 03:19) for a knowledge of the calculus. So, knowledge of the calculus is very much under (Refer Time: 03:23) oven scientifically look at a person.

Now, this is something very important tan inverse x and also remember tan inverse x when I told you these are really not functions, but multi functions so said (Refer Time: 03:36). But here we only talking about the principal value please remember this fact. This is also an important integral thing.

These are very much use in engineering. Now, I have two things to write and then I have another thing to write; l n means log natural, l n of x actually also can be written alternatively as log e of x is a natural logarithm at the base is the number e which lies between 3 and 2, and e is equal to 2.7178. This is irrational number by the way. The way approve one can say that there are pi is a irrational number, e is a irrational number means e to the power r for any r rational is a irrational number. There are lot of interesting facts and de-facts associated with calculus which cannot be run in such a basic course. So we will really focus on these two things for the next part of the lecture.

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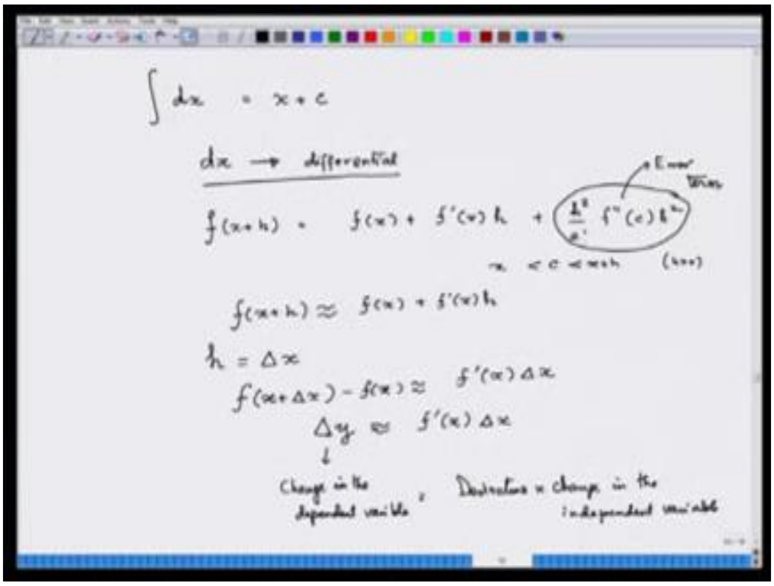

What do you mean by integral d x? And why do you write that this is x plus c. Idea is very simple observe first that what does d x mean. In your mind intuitively you can think that d x is very small, small little parts of x. Basically, you take x some object any have broken it into small parts and each of those parts you have calling d x as we as if we like everything is made of atoms like, if you have all the atoms of solid body and you can pull them inside you get the body itself.

So it is exactly the same idea intuitively or physical it is a intuition that you are dividing x into small, small, small parts and then those things are actually join together to form x. And c is of course the upper the constantive integration which you have say that. How do you physically explain it? That ok fine now you are trying to join because you are broken.

Suppose, you have torn a paper into small pieces a page of a book and now I want to tell you bring them up together by pasting the correct portions, but even if you bring them will be some little part which will be in constant it cannot be exactly the same thing, so it is x plus c. If you can a make it exactly the same thing then c is 0, if you are not the same thing then is c is something. But what d x actually is? Now d x is not the numerator of d d x of f, d x is something calls the deferential. And how did this terminology of deferential originate? You see lot of this thought processes came from with the physical sciences, it came from mechanics and as a result of which lot of this thought processes are very intuitive depending on physical intuition.

For example, here when I am talking about d x what I am actually meaning is the following. Now, if I go by Taylor's expansion of function at x plus h when h is very small then I can write it as f for x, I just want to make of first order expansion. These are the Taylor expansion, Taylor polynomial which we have already studied. Where, c is some quantity language in x plus h and x minus as I am assuming h positive for the time it does not matter put take it negative than it is opposite. So what is the whole issue here? Now, this is something which is an error it is actually an error term.

So, effectively when h is very small I can write this as, that is when the distant between x and x plus h is very small I say that the value of the tangent line and the value of the function are very near they are almost the same. (Refer Time: 08:32) of habit of writing h as delta x. So what we can write is, f of x plus delta x minus of f x is almost same as the derivative f dash x into delta x. This was a symbolism or the infinite sought of something call infinite decimal and people where using that, but do not get booked into all these thing.

Let us just look into this very very simply what does it mean. Now as delta x become smaller and smaller these approximate measure become better and better. Now this one this distant between the function value at x plus delta x and the function value is x is often symbolized by physicist as delta y.

Is delta symbol also very important in numerical analysis when the older version of numerical analysis. But an old numerical analysis book in mathematics has this sort of symbolism. But this symbolism use useful, so it tells you this the change in the dependent variable is nothing but the derivative. This is your d d x of f, or this is your d y d x basically into change in the dependent variable. Change in the dependent variable is the derivative into the change in the independent variable. This is called a change in the dependent variable is equal to derivative into change in the independent variable.

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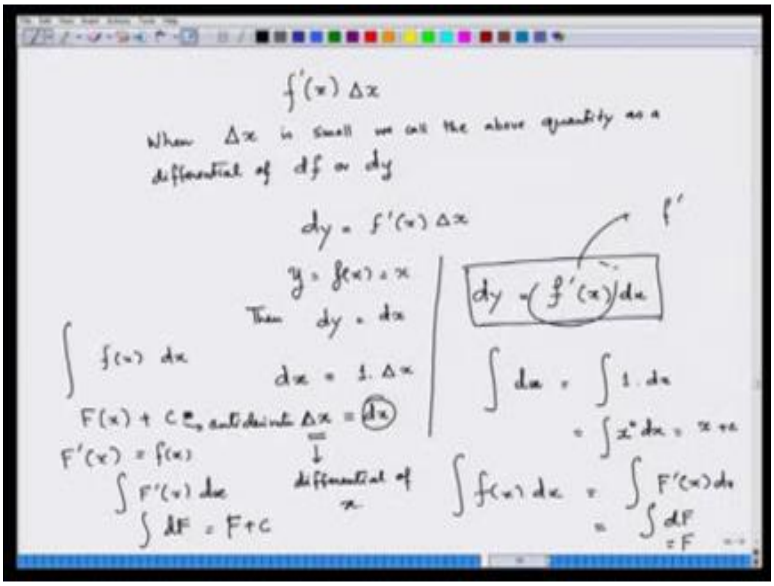

You just look at this quantity, f dash x into delta x. This quantity when delta x is very very small we call the above quantity as a differential y, differential of the function d f or d y whichever one. We write d y is equal to f dash x into delta x. Now, an interesting thing emerges.

Now let me considered the function y is equal to f x is equal to x then, d y is basically minus d x. So what it would mean? So, d y in this case for d x is equal to f dash x is just 1 into delta x. Delta x, whatever be your x it can always be written as d of x. That is why we can say that differential of y. Delta x essentially is then call the differential of x, that is why the differential of y essentially telling the change of y when the change in x is very very small is nothing but the derivative of f at x into the differential of x.

But if you look at the formula, we cannot talk about the integral of just this differential. Essentially, what it is talking about, it is essentially telling that I am talking about this of 1 into d x. Basically, it is talking about a change of y, the f dash x is 1 when it is d x. So, d x is equal to 1 into d x, basically, it is talking about that I find a function whose derivative is f x. So, 1 into d x, so find a sorry, whose a derivative is basically 1 in this case. That is why this symbol of integral of d x it actually means you are integrating of a function, so this function f dash x is the derivative of f of x here, if you look at this point these function f dash x is a derivative of f. So, f when you take the derivative it becomes f dash x.

Similarly, here also this function whatever if I write even integral f x d x. So essentially what, it is integral of F dash x d x, because f is the derivative of capital F. Then this is nothing but del of F and this is actually your F your anti derivative. See everything is link by this very simple formula that that are very small change in y which takes place when there are very small change in x because the function is continuous please remember that. Because every differential function is continuous.

And that is nothing but the derivative into the differential of x. This essentially means the x to the power 0 into d d x. So, is x to the power 0 plus 1 by 1, so is exactly forms into that formula. What is the function whose derivative is this? Basically, you are actually using this formula. Again, this is something I want to repeat. So, you are looking at integral f x into d x, now you have the function f of x plus c which is your f so such that f of x take the function f of x plus c and now you have that f dash of x is equal to f x. These are anti derivative of f.

Now, what happens here? This is the very very important thing. Now I can put instead of F x I can put F dash x, and this is nothing but this formula integral of d f. An integral of d f means again breaking the parts and bringing it in, so it is again f plus c. That is all about it and that is the whole game, so please remember that. Again you can also use this formula. It again the link between the differential y and differential x that is giving you this simple formula, this is the formula that you do in anti derivative, it is again all link with the Taylor’s theorem.

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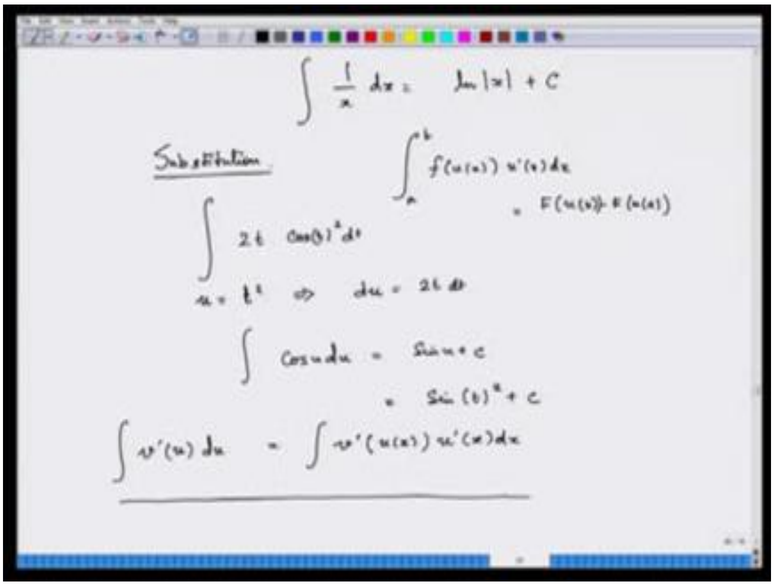

And Taylor’s theorem has is route in the Binomial theorem. I would like you to thing why do we write 1 by x d x is equal to l n mode x plus. See I am sure many of you really would not realise it if I just ask you. I will request my t s to put an explanation to this as a p d f file in your forum so that you get some understanding of it. We have very less time now three minutes to go, so what we are going to talk about is something calls Substitution.

For example, use some other function and make your function looks simpler. For example, what is the first method that you learn in integration apart from calculating this anti derivative is Substitution, and that means the following. I will just give you an example.

So see you want to integrate 2 t cos t square d t. Now what you want to do? You want to simplify this into a form where you can immediately apply the anti derivative. Suppose, I take u is equal to t square, this would immediately imply by the differential of u is nothing but the derivative of t the differential y is equal to f of x so u is equal to f of t; where, derivative of t which is 2 t into the differential of t so you see here two t d t. So what was t square here, t square was u. This becomes integral cos u d u which is nothing but sin u plus c. Now you put back u is equal to t square. So it will give you back sin t square plus c.

What is the general process of this whole thing? I will just come back to the this slide where I have made a mistake I had put an h square here which I have now removed the error term I had made a mistake is such I am just it is h square by 2 into f double dash c, there is no h square here I just made a mistake while writing. This is the basic substitution method. Basically what you do? Suppose, you have integral of v dash of u, but you are integrating in terms of u. So, v is the function of u and you are taking is derivative that can be written as v dash u of x into u dash x d x.

Basically, you are replacing u is equal to some function of x u of x and then this u is equal to u dash x, but u d x is equal to u dash x. So, d u is equal to u dash x d x. So, u you are writing as a function of x. So, v is the function of u and you want to integrate v dash u of d u, but now you because u is a function of x you are replacing u as a function of x and now you putting the whole integration in terms of function of x. So, u has the function of x and d of u the differential of u has we have just learn is the derivative of u into differential of x and this is exactly what you get.

Now, some more examples - if you want to even a look at what is the meaning of this in terms of substitution, that is f of u x u dash x d x this is the substitutional basically with the same substitutional law, this because u dash into d x is basically d u so f of u b minus f of u a. So, you may not remember this algebra etcetera because you know substitutional is something you would very much use for example.

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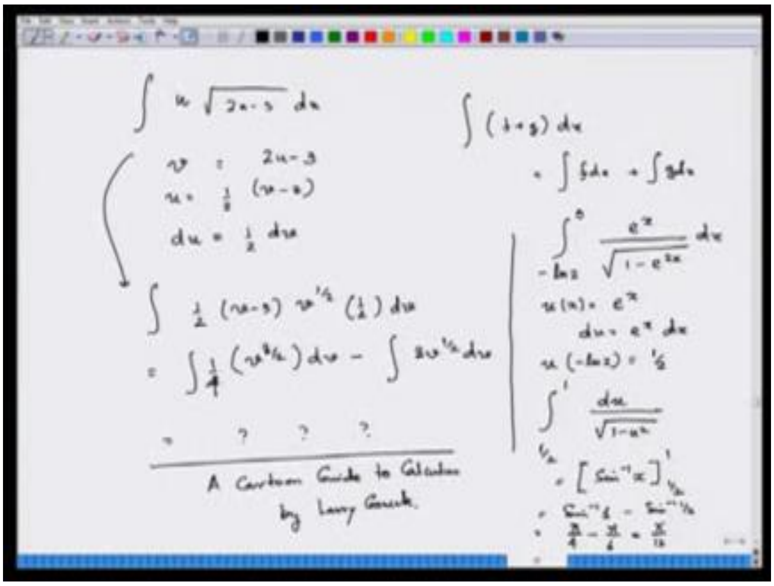

Substitutional something you would very much use for example, if you take integral u into route 2 u minus 3 d u. So what should you use? What should be the function that you should substitute? So what should be the relation between v and u, because v was a function of u. So, v I should choose here as 2 u minus 3, so u would you equal to half of v minus 3. Basically, d u would be equal to half of d v. So you will replace d u with half of d v, you will replace this with v and u with that. This integral will now become half of v minus 3 v half, half of d v.

Now it is very simple because now it is just integral of half of, you know then integral of f plus g integral of f plus g of d x is same as is addition of the integrals when my addition integrating the sum of two functions. These are very simple known formulas so I am not getting repeating such things at this slightly higher level. Here, you know what to do. So, half of will become one forth, so it will become v to the power 3 by 2 d v minus integral 3 v to the power half d v and you know how to write down the answer which I leave to you.

Following, for the case where you really want to use substitutional method and do definite integral, let us take an example. For example, minus log base e log 2 to 0 e to the power x route over 1 minus e to the power 2 x d x, and let us see what happens here. And

I will tell you which book I am telling all this from just as I finish it. Let us try u x is equal e to the power x.

If you find this then d u is e to the power x d x very good. So, what does this whole thing becomes? When x is minus log of 2, so basically what does it become? It becomes u of minus log of 2, because now you have to also change these two points. Because you have to understand the following fact that now I am changing over from x to u so now this n point has to be also in terms of u, so this is nothing but half and e to the power 0 is 1.

Basically this integral will become half to 1 d of u root over 1 minus u square, and we know what it is. It is sin inverse x which is the anti derivative plus c is not requiring you could because you cancel out the c from both the cases. And by fundamental theorem calculus now this is half and 1. So what it is? You will have will have sin inverse 1 minus sin inverse half. So, sin inverse 1 is pi by 45 degrees and sin inverse of is pi by 60 degrees. And so this is nothing but pi by 12, and that is a neat simple answer

And tomorrow will talk about integration by parts. As given these little examples you must be thinking that I am joking is from a book called I have already mentioned, a Cartoon Guide to Calculus. I am not spoken about the differential from this book of course, that is what just I said, but the examples that I have taken is essentially from the Cartoon Guide to Calculus and this little two three examples that I have taken.

And it is serious book though you might thing that is the cartoon so it will be just a joke. But I would suggest every engineer, every economists, any scientist should keep this book on their desk, they will enjoy calculus and they will start loving mathematics and they will start loving calculus. Available on Flipkart or Amazon which we use more than book stores now a days. So do not worry (Refer Time: 26:01) is not a joker he is a (Refer Time: 26:03) mathematician.

Thank you very much.