Introduction to Probability Theory and its Applications

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Lecture - 6
Discrete Random Variables and Their Distributions

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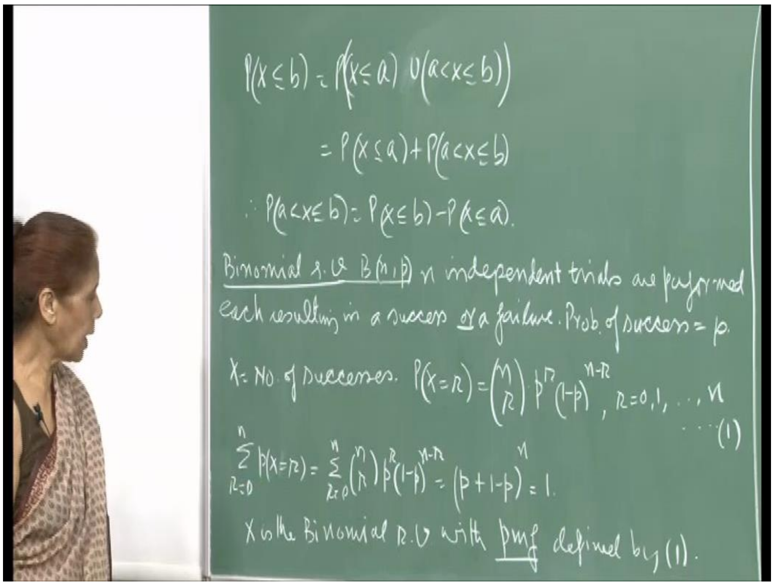

Defining the distribution function, where we said that the this is equal to F b. So, probability x less than or equal to b, now we can immediately write down the formula for the probability of x being in an interval using the distribution function. So, here the idea that we are going to write x between a and b, so am taking script in equality here. And so then you see I am writing this set the event x less than or equal to b as a union of two disjoint events x less than or equal to a, then union x between a and b, so x strictly greater than a less than or equal to b.

So, these are disjoint events and they add up to the union is equal to this event, then because they are disjoint when I write the take the probability, it will be the sum of the two probabilities. So, probability x less than or equal to a plus probability a less than x less than or equal to b and so this probability then can be written as difference of probability x less than or equal to b minus probability x less than or equal to a. And therefore this is F b minus F a, and you can see immediately that, if you want to write equality here, then this event could be x strictly less than a.

So, in that case we will have to write a minus and so on, so therefore since in the discrete case it matters whether equality is there are not, so therefore this is the way I have written it. And then of course, if you want to also have the probability that x is equal to a and I will simply add to it, so plus it will be P a, because x equal to a again is disjoint from this set; and so I can write here plus probability a and so on. So, you can get various forms of the probabilities for x being an interval, you can write it through your distribution function, so this was for lecture 5.

Now, I will start defining, since we have already talked about Bernoulli random variable, which was a very basic random variable, discrete random variable, so will define a discrete random variable which is known as the binomial random variable and then notation is B n comma p. So, here n independent trials are performed each resulting in a success or a failure, so we just say that when you perform a trial, then either the outcome is a success or it is a failure.

Now, probability of success we say is p and that is why the notation B n comma p that means, number of trials is n and the probability of the success is p. And so x the random variable denotes the number of successes, so in n trials you want to know how many successes have occurred, because it is an uncertain event. And the probability of x equal to R is given as n c R p raise to R 1 minus p raise to n minus R, so obviously if you looking for R success in n trials, then they can be any of the R trials out of the total n trials, so n c choose R, I mean n choose R and then since R success so p into R into 1 minus p raise to n minus R.

So, R success and n minus R failures and this is for R varying from 0, 1 to n, so for any value of R you can substitute here and get the corresponding probability that x is equal to that R. And now you can sum up this all the probabilities, so that means for R from 0 to n sigma probability x equal to R, then this will be summation R varying from 0 to n n c R p raise to R 1 minus p raise to n minus R and this you can see is the binomial expansion of p plus 1 minus p raise to n, that is another reason why it is called the binomial random variable.

So, when you write p plus 1 minus p raise to n, this adds up to 1, so therefore this is p m f and of course, so the x is the binomial random variable with probability mass function defined by 1. So, now let me again emphasize this fact that when the random variable is discrete, we describe it is probability mass function that means, when it gives you the function which specifies the probabilities for different values of the random variable x; when that is known as probability mass function.

And when we talk of continuous random variable that means, when the random variable can take all possible values interval, then the function which describes the probabilities of the random variable, that is known as probability density function. So, I will try to keep this thing always in mind, but sometimes may be it may happens that I have random variable is x and instead of saying the probability mass function, I may have used the word probability density function.

But, remember that we make this distinction that for discrete random variables it has to be probability mass function, and for continuous random variables it is the probability density function. So, the notation is B n comma p, so that means binomial and you only need these two parameters, what is the number of trials and what is the probability of success. And so now, you want to verify that this actually is a valid p d f and for which for this verification, I will need to say that all the probabilities must add up to 1, because x can take any of these n plus 1 values.

So, the sum of the probabilities for all these values must add up to 1 and you see that this is nothing but the expansion of p plus 1 minus p raise to n the binomial expansion; and therefore, this number is 1, so 1 raise to n is 1, so that verification is straight forward.

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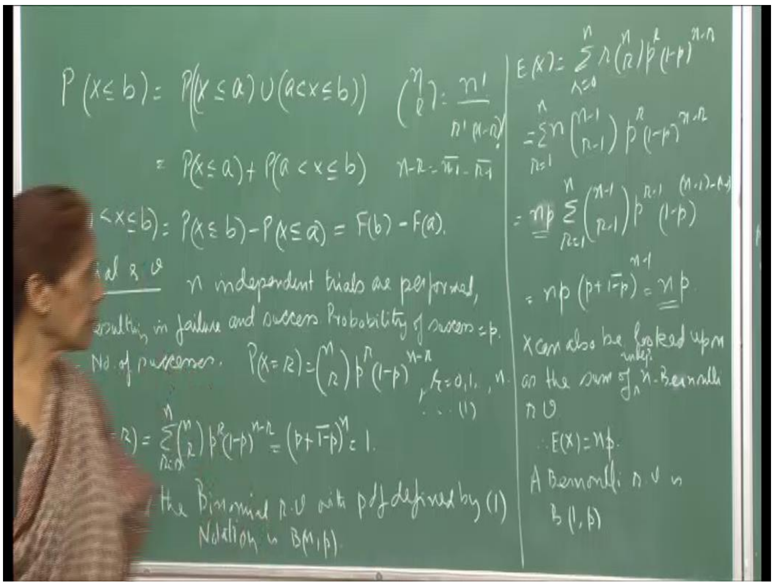

So then I immediately want to compute the expected value of this random variable, so that will be because this x is equal to R, so R into n c R p R 1 minus p raise to n minus R, R varying from 0 to n. So, now here because this is n c R, so you will having the denominator maybe I can use the, so here if I take an outside see the thing is that n c R you can write as n factorial upon R factorial n minus R factorial. So, R if I take out, then this will be R minus 1 factorial, so the R cancels out and n I have taken out from here, so this will become n minus 1 factorial.

So, and then n minus R you can write as n minus 1 minus R minus 1 and therefore, by taking out n here and canceling out the R, I get n minus 1 choose R minus 1; and since you see in this summation R is from 0 to n and when R is 0, this term is the first term is 0, so therefore no contribution. So, therefore, this summation can as well be written as R from 1 to n and this what you have. So, here if I take out p, then I will be left with p R minus 1, so this n p is outside, this n is outside and then you have this.

So, now you can see is again a binomial expansion of p plus 1 minus p raise to n minus 1, because their powers are all n minus 1, so therefore this is 1 and so you are expected value is n p. So, again a straight forward computation we should, because able to figure it out and can also say that a Bernoulli random variable is binomial 1 comma p, it is binomial random variable with parameters 1 and p.

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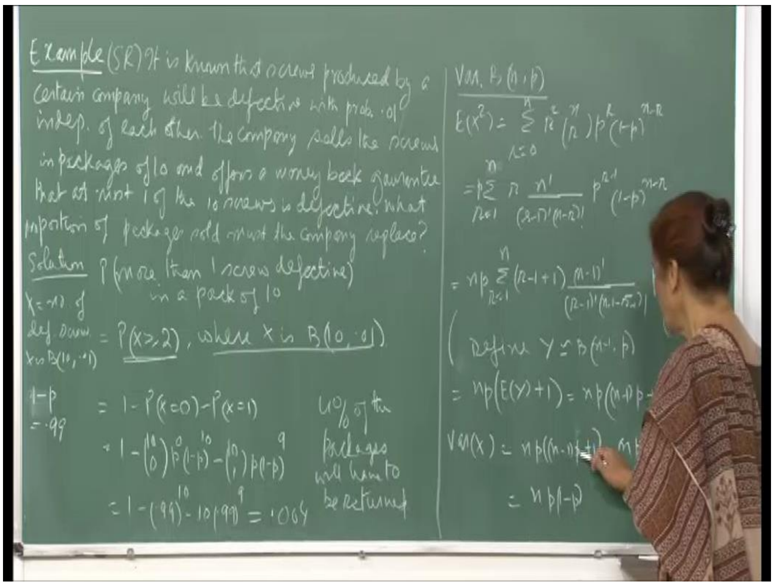

Let us look at this example give taken from Sheldon laws, so it says that it is known that screws produced by certain company will be defective with probability 0.01. So, the p is given here, so here defective we are treating as a success, so the probability of getting a defective screw manufactured is 0.01 and these of course, the defectiveness of the screws is independent of each other. The companies sells these screws in packages of 10 and offers a money back guarantee that at most one of the 10 screws is defective; that means, if more than one a screw is defective in a package of 10, the company will take back the package and refund the money.

So, the question is what proportion of package is sold must the company replace, so that means you wanting to find out the probability of packages of 10 that have more than one screw defective. So, that means, you are wanting to find the if x is the number of, you say that x is the number of number of defective screws, then what is x is binomial 10 0.01. So, because we are treating a defective screw as a success, so therefore in a package of 10 the probability is 0.01, so this is binomial 10 comma 0.01.

And you want to find the probability, let x is greater than or equal to 2, because when x is greater than or equal to 2, the package will be returned back and the money refund it, so here I have said that where x is binomial this. So, therefore, a now this event I can write as compliment, so 1 minus probability x equal to 0 and probability x equal to 1, so these two are acceptable and if x is more than 1, then it is not, so 1 minus, so these two you can see that. Now, x equal to 0 is the probability, so 10 c 0 into p raise to 0 1 minus p raise to 10 and x equal to 1 will be 10 choose 1 p into 1 minus p raise to 9.

And now your 1 minus p is 1 minus p 0.99 and so you write down this number, you can use your calculator to compute it, it comes out to be 0.004. So that means, 0.4 percent of the package is will have to be returned, and this is you can see that this kind of calculation is very helpful to a company, which is trying to budget it is manufacturing of the screws. And anywhere situation you want to know that what is the of course, nobody saying that this exactly 40 percent of the package is will have to be returned, but now the company has an idea as to 4 percent, 4 percent of the package is have to be returned.

So, there is some idea and a guideline for the company to see what can happen. Now, the next thing that we want to compute about the binomial random variable is it is variance, so first we will compute expectation of x square, second order movement. And so that will be given by R square n c R p raise to R 1 minus p raise to n minus R summation is from 0 to n, here again you see for R equal to 0 the first term is 0, so there is no contribution. And I will do the same thing, I will cancel one R with this ((Refer Time: 12:43)) one here and then one R am left with, p I will take outside and then n of course, also we should have written which I have done here.

So, in that case you will because left with R n minus 1 and the same thing, same trick I will do and this is it, n minus R I will write is n minus 1 minus R minus 1, so this is p raise to R minus 1 and this is the whole thing. And the one R that is left here, I will write it as a R minus 1 plus 1, now you see you can break up this sum into two, so R minus 1 times this whole thing, you see now is the expectation of a... So, I define why is a binomial n minus 1 comma p, so why is binomial random variable where n minus 1 trials I have taken place, probability of success is p.

So then you see in this summation I should have written here 1 to n, then R minus 1 into this term, this whole thing summation R from 1 to n will give you the expectation of binomial n minus 1 comma p. So, therefore, that will because n minus 1 into p, so n minus 1 into p and plus 1 and then this summation this again is the say same thing p plus 1 minus p raise n minus 1, so that is equal to 1.

So, therefore, you get n p times E y plus 1 expectation of y plus 1 from here, an expectation of y is n minus 1 p and this is plus 1, so n p into this. So, now, for the variance you have to write n p into n minus 1 p plus 1 minus n p whole square and when you simplify you get this ((Refer Time: 14:30)) and some people also write this as n p q, where q is 1 minus p. So, easy way to remember and you see some more applications of and of course, as the course progresses you will continue to see where all you can use the concept of a binomial random variable.

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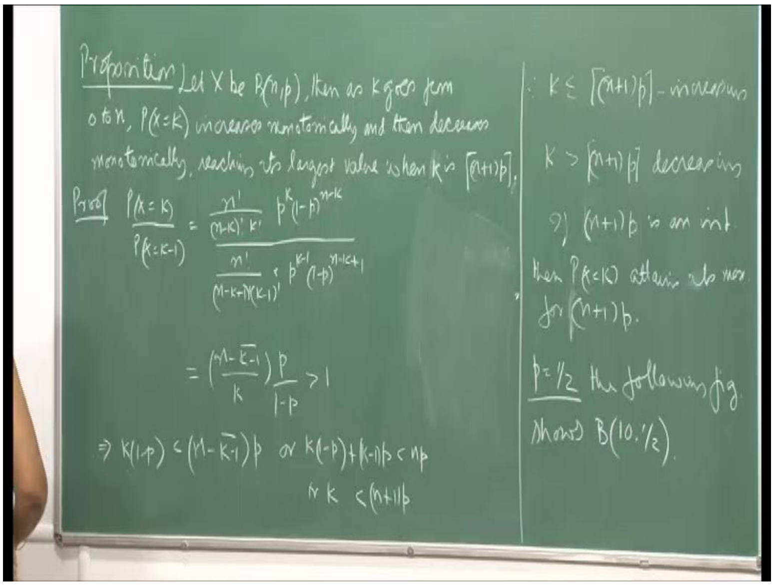

Let us look at this proposition about the random variable binomial n comma p, so then as x goes from 0 to n, this number probability x equal to k increases monotonically and then decreases monotonically reaching it is largest value, when k is n plus you see integer part, largest integer part of n plus 1 into p. See p is a fraction, so n plus 1 p may not be an integer, so when we write brackets like this, it means that the largest integer for example, if you consider 5 by 2, so the largest integer here would be 2, largest integer which is less than n plus 1 into p, so this is this.

Similarly, if you consider what shall I say 9 by 4, here again the largest integer will be 2, because 4 times 2 is 8 and then it is 1 by 4, so this is what mean by that. So, that means, the binomial probabilities they keep on increasing, reach the largest value highest maximum and then starts decreasing this is the idea. And so we want to prove this result, so let us look write down probability x equal to k upon probability x equal to k minus 1; and so if you write down the expressions this is what you get.

So, then things cancel out n factorial n factorial, you have n minus k factorial n minus k plus 1, so therefore, you will be left with n minus k this will come in the numerator, so n minus this is the bracket over k minus 1 upon k p upon 1 minus p. Now, this should be we want know for what values of k the probabilities are increasing, so I want this to be greater than 1. And if you simplify this means that k into 1 minus p should be less than n minus k minus 1 into p or k into 1 minus p plus this is less than n p, so this give you this.

So, the as long as k is less than n plus 1 into p, the probabilities is, because you say this is probability x equal to k divided by probability x equal to k minus 1, so this number is greater than this number as long as k is less than n plus 1 into p. So, that means, and since k is an integer, we will say that this goes on increasing till k reaches the largest integer present in n plus 1 into p, so that is what we say.

So, that means, k less than or equal to this integer part in this number, probabilities are increasing and when k is greater than this, because the any quality will get reversed, so then they are decreasing. And if n plus 1 into p is an integer then of course, that will because the maximum value otherwise, because here it is strictly less and this is less than or equal to... So, k will the probability x equal to k will attain it is maximum value for n 1 into p that means, when this is an integer, otherwise it will attain it is maximum value for the largest integer present in n plus 1 into p. So, I will show you the diagram now for a particular this thing, the bar chart for a binomial distribution, so the here will look at the bar chart when the random variable is parameters are 10 comma half that means, your p is half and the number of trials is 10.

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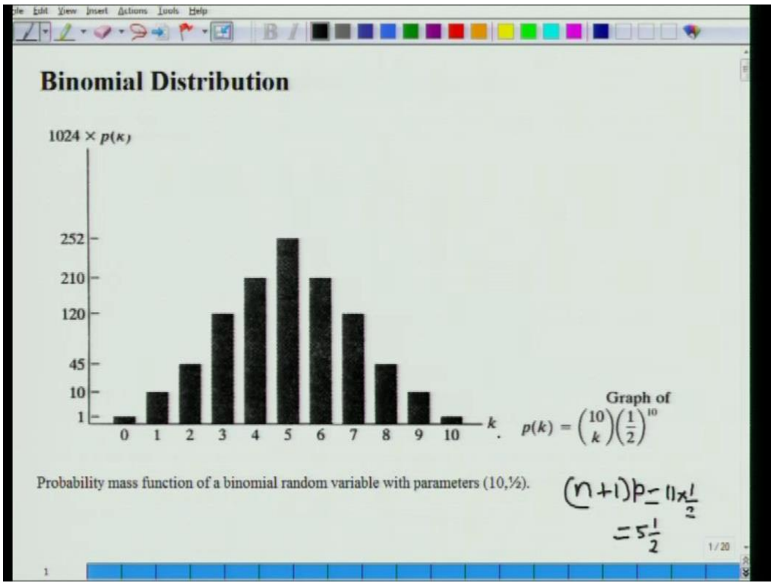

So, you see here if you look at this thing 1 by two, so that means for x equal to 0, then when k is equal to 0, the probability p 0 is 1 by 2 raise to 10 and that is depicted by this small bar here and then x equal to 1 it starts to increase and then at 5 it is maximum. So, if you look at, see we said that, so what is your n plus 1 into p this is equal to 11 into 1 by 2, so this is 5 1 by 2, so the integer part is 5. So, as just now we looked at the we proved this result, that in this case the maximum will value will be attained for the largest integer present in n 1 plus n plus 1 into p which is 5.

So, therefore, you see the largest bar is corresponding to k equal to 5 and then again the values start decreasing and you also see that in this case, because p is half, therefore the graph is symmetric about the value k equal to 5. So, just want to explain that this is 1 0 2 4 into p raise to k, because the horizontal axis is k and here just to otherwise numbers would have been, because probabilities are less than 1. So, therefore, just to make them whole numbers we multiplied everything by 1 0 2 4, which is 2 rise to 10, if we multiply all the probability, then they become integers.

So, this is what your a binomial bar chart will look like, for the particular value and p is equal to half and I have shown you also that how it will continue to the probabilities will continue to increase, reach a maximum for the integer part of n plus 1 into p and then start decreasing.

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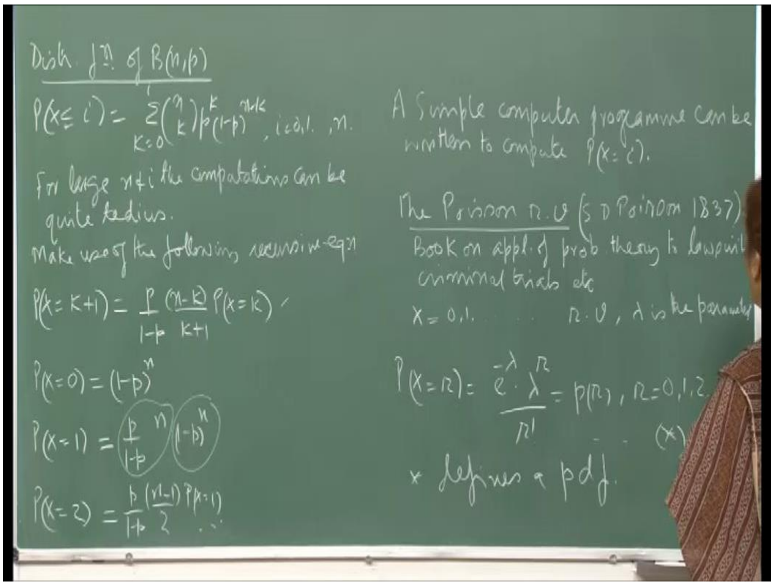

Now, let us look at the distribution function of the binomial random variable that means, you want to compute probability x less than or equal to i and here, you see the expression would be you have to sum up all these probabilities from 0 to i for the individual probabilities at the random variable takes. Now, these numbers can be very very large, in fact if your n is moderator large and you want to compute even for n equal to 20 and if you want to compute for, let us say i equal to 16, then you will have add up 17 terms here and this can become very tedious.

So, there is a very simple and nice formula which you can recursive formula, because we obtained this here in this expression I have this here, so just place k by k plus 1, then k minus 1 becomes k, so in that from that formula you get this recursion. So, which says that, if you have obtained probability x equal to k, then you can obtain probability x equal to k plus 1 by this simple formula, recursion formula. And you can see that to start off when x is equal to 0 when x takes a value 0, then this will be 1 minus p raise to n.

So, once you have compute this, then p x equal to 1 will be from this formula p upon 1 minus p k is equal to 0, so this is n and p x equal to 0. So, therefore, this will because p x equal to 1 will be p upon 1 minus p n 1 minus p raise to n, so that means you if you have already computed this number, then you have to simply multiply the probability x equal to 0 by this number to get probability x equal to 1. And then similarly probability x equal to 2 would be this number multiplied by probability x equal to 1 which we have already computed here.

So, a nice simple recursive formula that you have obtained and you can write this compute the program feed the values and then it will go on computing. That means, you have to just feed the value n and p and then it will compute the successive probabilities for you; so this is the computational part, because the wise things can become very tedious. Now, another special kind of discrete random variable is the Poisson random variable and this is named after the mathematician S D Poisson, who defined this random variable in 1837.

And he in fact, wrote a book which was application of probability theory to law suits, criminal trials etcetera, so you see the kind of applications that the Poisson random variable has. And at that time in 1837 he wrote this book, where he apply the theory Poisson random variable to predicting things about law suits and criminal trials etcetera. So, x is a random variable which takes values 0, 1, 2 up to infinity, lambda is the parameter and lambda has to be positive.

Then we define the probability that x is equal to R by this number, E raise to minus lambda lambda raise to R upon R factorial, defines a probability mass function of the random variable x and this is the Poisson p d f. And now you want to make sure that this defines the p d f that means, you have to show that summation probability x equal to R is 1; so very simple calculation will immediately give you the verification that this is indeed a p d f.

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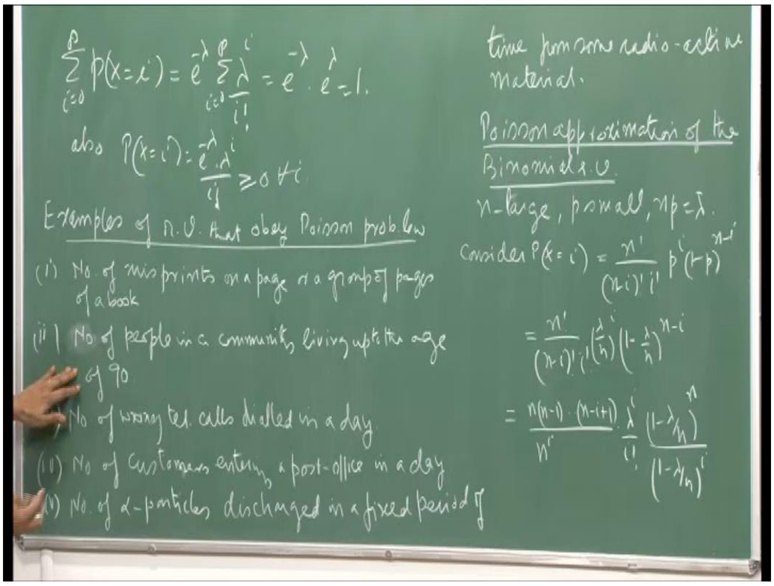

So, to show that the probability mass function define for Poisson random variable is valid p m f, we add up all the probabilities here which is equal to this, but you see lambda raise to i upon i factorial i varying from 0 to infinity is nothing but the expansion of E rise to lambda. So, therefore, E minus lambda into E raise to lambda gives you 1, gives you E raise to 0 which is 1, so we have check the validity and of course, each of the probabilities defined are also non-negative.

So, therefore, what we have defined is the probabilities for the Poisson random variable forms a valid p m f, probability mass function. Examples of random variable that obey Poisson probability law, some of the situations am just writing down to get give you better feeling for the random variable. And this is for example, number of misprints on a page or a group of pages of a book, which we believe is a random phenomena.

Number of people in a community living up to the age of 90, which again is a random phenomena, because how long one is for how long one lives is not certain event. Then number of wrong telephone calls dialed in a day, a wrong numbers what I mean is wrong telephone calls dialed in a day, this could be at particular exchange or you mean take a particular number and then count a number of times the wrong telephone calls come.

Number of customers entering a post office in a day, then number of alpha particles discharged in a fixed period of time from some radioactive material. Now, you see that here all the situations, there is some sort of discreteness and that is why we are saying that these situations can be modeled by a Poisson random variable. You can just get the feeling, because you cannot have two numbers dialed simultaneously, there as to be a gap; then number of misprints of course, will be current discretely told some gap of time and so on.

Now, I want to show you the relationship and as we go on the discrete random variable that I defined, I want to show relationships between among these discrete random variables. So, Poisson approximation of the binomial random variable and this is when n is large, so for n large and p small, then you expect that n p would be a very, a moderately small number, n p and so we defined that is lambda. So in other words, what you are saying is that a p small and n large and then this n p number sort of approach is reasonably small number equal to lambda.

Now, let us look at the binomial probability for x equal to i, when x equal to i when x is equal to i, then this is the probability defined, then let me start writing, so here if I take this, then p is lambda by n. So, I will make that substitution here lambda by n raise to i 1 minus lambda by n raise to n minus I, then if you cancel out the n minus i factorial part here, you will be left with n into n minus 1 n minus i plus 1. And then this n raise to i, I am writing here and i factorial is underneath, because you see am trying to converse to the Poisson probability. So, lambda raise to i upon i factorial and this one 1 minus lambda n raise to n and then divided by 1 minus lambda n raise to i. Now, this n 1 n cancels and I am left with n raise to i minus 1 and you will have i minus 1 terms here, so I take n inside and divide.

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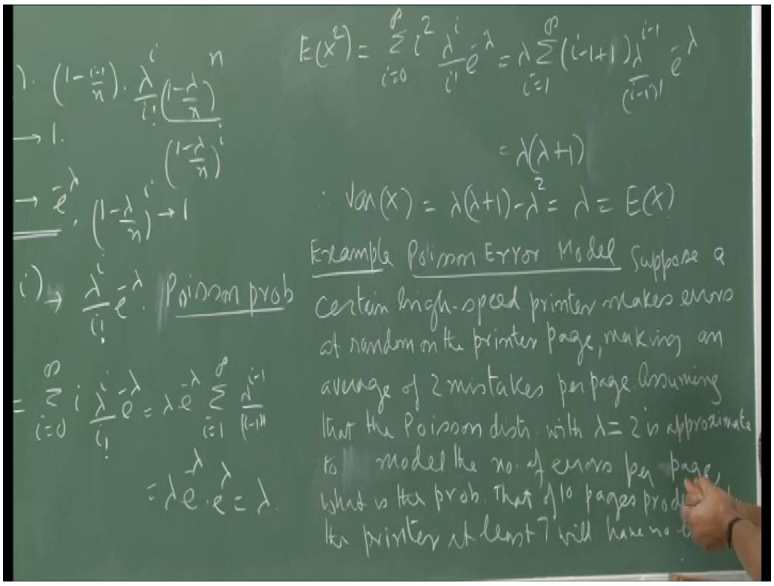

So, therefore, each of this term becomes 1 minus 1 by n 1 minus i minus 1 by n, so the this n raise to i gets absorbed here, lambda i raise to i factorial 1 minus lambda n raise to n and this. Now, as n goes to infinity becomes very large, then 1 minus lambda by n raise to n will approach E raise to minus lambda, so I hope you are all familiar with this limit. And then 1 minus lambda by n raise to i, because as n becomes large lambda is fix, so this number is becomes smaller and smaller and so this will approach 1, for n sufficiently large.

And then and all these numbers again as n goes to infinity are becomes very large, each of these numbers approach 1, so the product is 1, so therefore this is gone and this raise goes to E raise to minus lambda. So, the whole thing approximates to lambda raise to i upon i factorial E raise to minus lambda, which is the Poisson probability. So, essentially I should say that this approaches this and this is your, so these things are actually inter linked. And therefore, for large n your binomial probability is the same as the Poisson probability that is one result.

Now, they computing the expected part expectation of a Poisson random variable again straight forward, sigma 0 to infinity i lambda i E raise to minus lambda upon i factorial. So, here I do the same trick as I did for the binomial, so lambda E raise to minus lambda you take outside, then your i becomes, this summation becomes 1 to infinity lambda i minus 1 upon i minus 1 factorial, which is again the expansion of E raise to lambda.

So, lambda E raise to minus lambda into E raise to lambda it gives you lambda, so whatever the parameter of the Poisson distribution that is also it is mean, so this is one result. And now variance also straight forward again the same thing I will do as, I did for the binomial expectation of x square we find out, then again the i cancels lambda you can take outside and the i that is left you write it as i minus 1 plus i and this thing. And just do the same argument as we did for the binomial, you will see that i minus 1, this will give you the again a Poisson this thing expectation of a Poisson with lambda parameter.

So, this will be lambda plus 1, because this should be plus 1, i minus 1 plus 1, so 1 because these are again Poisson probability, so will add up to 1, so this is what you have; and therefore, variance will be given as lambda into lambda plus 1 minus lambda square, which is the expectation. So, this is a particular situation where your expectation and the variance are the same, and both are equal to the parameter of the Poisson distribution.

Now, best way to give you feeling about particular random variable is always through examples, so here let us look at the Poisson error model and as I said this is same the first one there, that is your modeling the number of printing mistakes done by let us say high speed printer. So, here it says that certain high speed printer makes errors at random on the printer page, on the making an average of two mistakes per page. So, on the average two mistakes are made per page, assuming that the Poisson distribution with lambda equal to 2 is approximate to model, the number of is appropriate actually I should say, I should say appropriate.

Let me correct the word appropriate, appropriate to model the number of errors per page, what is the probability that obtained page is produced by the printer at least 7 will have no error. So now, you want to probability that when the 10 pages are printed, 7 of them are without any errors.

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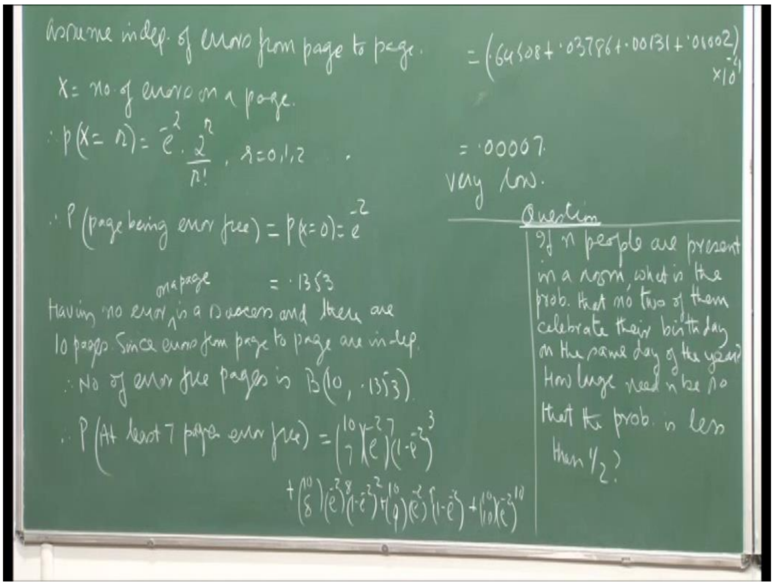

So, we assume independent of error from page to page that means, number of errors that occur on one page is independent of the number of errors that occur on the second page and so on. Now, let x be the number of errors on a page, then as we have said that we will model this pi the Poisson random variable and so probability x equal to R will be E raise to minus 2 into 2 raise to R upon R factorial, where R can vary from 0, 1 to anything.

And we said that, since the average number of errors made by the printer on a page is 2, so am taking 2 as the parameter of the Poisson random variable that am using to model this particular situation, so this is what is given to you. Now, if you want the probability that a page is error free that means, there are no errors, so probability of x equal to 0 is equal to E raise minus 2. Now, the thing is that you want to find out the probability that at least 7 pages are error free, so you see we modeled this situation by a Poisson random variable to find out the probability of a page being error free.

So, that I got as 0.1353, which is equal to E raise to minus 2, but now there is a next step and see that is why example is very interesting, because you see is now you will make use of the binomial random variable. Because, having no errors on a page is the success, let us treat that as a success and since we have said that and we are looking at a 10 pages, so I will consider scanning every page as a trial of the experiment, and if there is no error on the page then that will be a success.

So, this is ideal for a binomial random variable, this situation is ideal, because since pages are the errors from page to page are the independent, therefore I will treat this as 10 independent trials. And if a page turns out to be error free and that is a success and what is the probability of a page being error free that is 0.1353 that means, the probability of a success our p is 0.1353 and the number of trials is 10, so this is a binomial random variable.

And so probability that at least 7 pages are error free, I require that 7, 8, 9, 10 at least 7, so the error free pages can be 7, 8, 9 or 10 and the probabilities will be 10 choose 7 E raise to minus 2 raise to 7 into 1 minus E raise to minus 2 raise to 3. Similarly, for 10 choose 8, we will write the expression E raise to minus 2 raise to 8 and so on, then 10 choose 9 E raise to minus 2 raise to n into 1 minus E raise to minus 2 plus 10 choose 10, which is E minus 2 10.

And even as I have saying that here also the computations have to be done by using a calculator or a computer, so I write down the numbers this is into 10 raise to minus 4 and so the probability is 0.0007, so which is very low. So, therefore, what you will say is the probability that 7 pages out of 10 will be error free is a very very low probability, has very low probability, so chance of this happening is very small. Now, again I will try to take as many examples are possible, so make the concepts clear, so first this question am taking, because that will lead to a Poisson approximation which I want to show you.

((Refer Time: 37:56)) So, first consider the situation here that, n people are present in a room what is the probability that no two of them celebrate a birthday on the same day of the year. So, this is the first question, what is the probability that no two people will be having a same birthday, celebrate it on the same day and then we want to ask the question how large did n be. So, that the probability is less than half that means, how many people should be there in a room, so that the probability of any two of them having the same birthday is less than half. We want to first look at this question and then I will take you to the how I approximate this situation and give you the through Poisson and then again show you the connection between the two.

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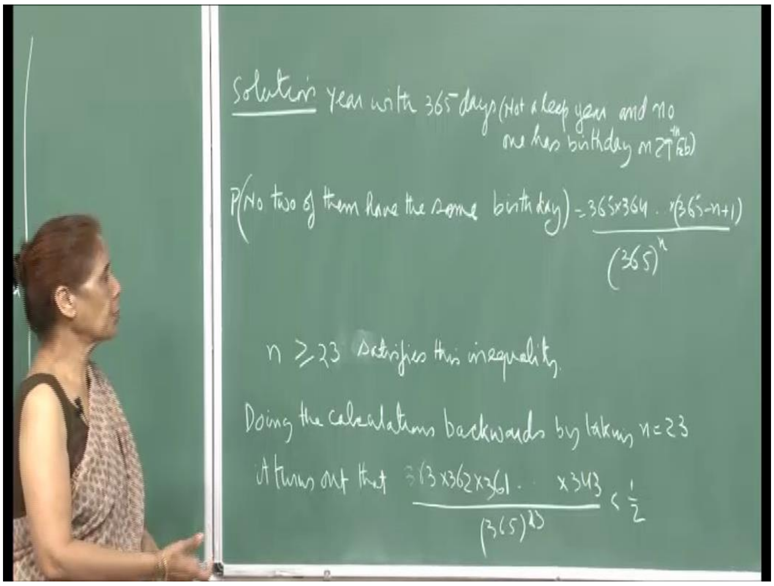

So, the birthday problem as we saw that, if no two of them have the same birthday then of course, we will write it as 360... see the first person can have a birthday on any of the 365 days and we are of course, ruling out the leap year, so nobody has the birthday on the 29th February. So, this is 365 upon 365, then the second person can have in the group, group of n people the second person can have is his or her birthday on any of the remaining 364 days and so 364 upon 365 and so on.

So, this will go on up to 365 minus n plus 1 upon 365, so this gives you the probability that no two of them have the same birthday and we want the number n such that, this probability is less than half. So, the probability that no pair, no two people in the group have birthday on the same day, we want that probability to be less than half and we want what should be the smallest number of such people. So, that this probability is less than half and so it turns out that n greater than or equal to 23, satisfies this inequality.

Now and of course, then I started calculating backwards, so for example, for n equal to 23, for n equal to 23, 365 minus n plus 1 is 343, so then I started computing 343 upon 365, 344 upon 365 and so on. And then just 363 upon 365 when you come up to this point, then the probability just turns less than half and so the next number is 364 by 365 which is also less than 1, so the probability will remain less than half. And of course, the last number is 365 upon 365 which is 1, so therefore and in fact, if I took n to be less than 23 that specify took n to be 22, then this will not happen.

Because, then you will be starting with 344 and then when you calculate backwards it will not turn less than half, so this is just enough and that means, if you have more than 23 people in the group, then the probability will be still less than half, so this is the idea. Now, here calculating it otherwise become difficult, because what is happening is that you are in the group if a and b have the same birthday, then it is possible that b has a same birthday with c and so this thing is transitive.

And so it is not very easy, I mean one cannot daily assuming dependence to calculate the actual probability, but we will see that some approximation is possible and so we will see that. But, so this is interesting to see that, even group of people having 23, I mean group having 23 persons the probability that no two people have the same birthday is less than half.

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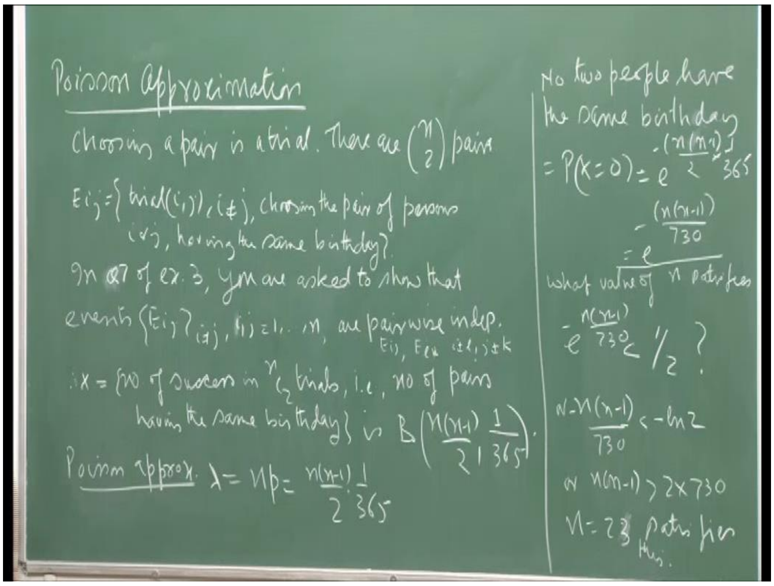

So, let me continue with the Poisson approximation of what we discussed the common birthday problem we discussed, so now suppose choosing a pair is a trial, choosing a pair of people present in the room, then there are n c 2 pairs that I can, different n c 2 pairs that I can choose. Now, consider E i j as a trial i j i naught equal to j such that, and of course, what I mean by this is that am choosing the i'th in the j'th person here, i and j are different having the same birthday.

So, E i j would be considered a the event, when I choose the pair i j it is the success, if they have the same birthday. Now, in question 7 of the exercise 3, which I will be discussing at the end of having gone through all different types of discrete random variables. So, the exercise 3 question 7 am asking you to show me that these pairs the events E i j are pair wise independent. So, of course, choosing of each pair is independent and then I want you to show that the pairs E i j that means, this set of events E i j i naught equal to j.

That means, n into n minus 1 by 2, such events they are all pair wise independent, which means that i j and E l k this is important, I must show i j and E l k, where i is not equal to l and j is not equal to k. So, between different sets of pairs when you choose, different sets of events I will, so that is what I should say that when you pick up two such events from here, where i is not equal to l and j not equal to k, then any two such pairs are independent events, this you should do as an exercise at the end of this chapter.

Now, x is the number of successes in n c 2 trials having the same birthday, so let me now start looking at the Poisson approximation, so number of successes in n c 2 trials having the same birthday. And this am calling as and so as I discussed with you is a binomial n into n minus 1 by 2 comma 1 upon 365 remember, I showed you that pair of people having the same birthday, the probability is 1 upon 365 and the number of trials that I have is n into n minus 1 by 2, so this is a binomial random variable.

If I do the Poisson approximation, then we said that the corresponding Poisson parameter would be n p and so that will be n into n minus 1 by 2 into 1 upon 365. So, this is my n p and now you want to compute the probability that no two people have the same birthday that means, you want to find out probability x equal to 0, which is E raise to minus n into minus n into 1 by 2 into 1 upon 365, so which is this number. So, the next part of the question was what value of n that is twice this the inequality, that this is less than half, no two people having the same birthday that probability should be less than half.

So, we should take the logarithm of both the sides, then this is minus n into n minus 1 upon 730 l n of E is 1, this is less than minus l n 2, so minus minus cancels inequality reverses. So, therefore, n into n minus 1 is greater than 2 into 730 and you can see that n equal to 23 will satisfy this equation. And therefore, this also is validated, the answer that I got earlier by other arguments, now I have been able to validated by... So, therefore, this is what, so these examples that am trying to show you that, how you model and sometimes your modeling can be such that, the getting the answer can be very comber some, but sometimes when you model it in the right way then you can get the answer.

So, here the computation was quicker, then because remember there I was solving the, I was trying to say 365 into 364 up to 365 minus n plus 1 divided by 365 raise to n, this should because less than half. And so I was wanting to compute an n and you could see that, this will require trying out trial and error of lot of values of n before you get the answer and we got the answer I showed you that the answer would be 23. But now, if you model the situation through Poisson random, approximation of binomial through Poisson, then you get the answer in a much simpler way.