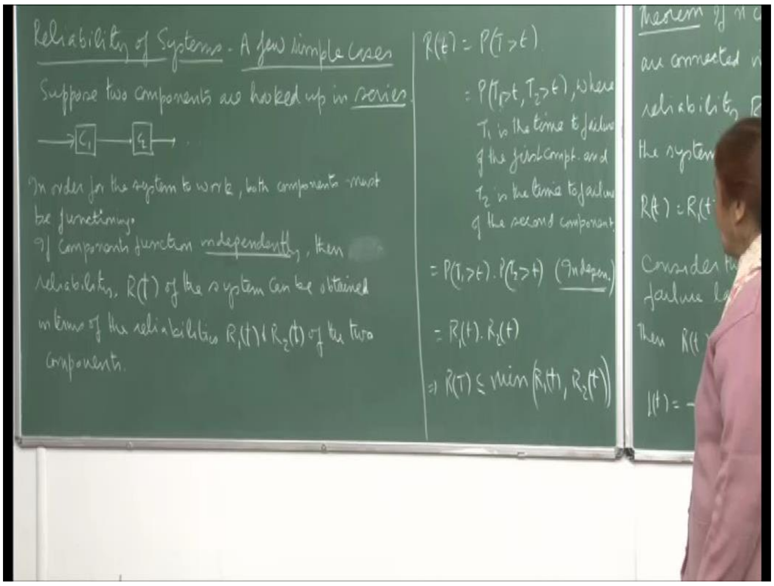
Introduction to Probability Theory and its Applications

Prof. Prabha Sharma
Department of Mathematics & Statistics
Indian Institute of Technology Kanpur

Lecture - 40
Reliability of Systems

Now, having discuss the reliability of a component or a device, where we are assuming that a single piece, single component let me now talk about reliability of systems, where there are more than 1 component. So, here again the treatment will be simple. So, that is why I have stated in the beginning only, the simple cases will be consider, but once you learn the basic technique.

(Refer Slide Time: 00:40)



Then, you can always divide you know break up, complex device into smaller systems. And then you can try to compute the reliability of the whole system. So, let us see we can now, here I have adjusted beginning with this simple case. That 2 components are hooked up in series. So, this how they are right c 1, c 2 there are 2 components.

And there in series, and so for the in order for the system to work, both components must be functioning there in series, may be performing different tasks for the whole device. And, so they both have to function if any of them fails, then the system will fail. So, this is the whole idea they working in series. Now, we also make the assumption, and of course, this is important otherwise; things will get complicated, and we have to learn to methods for handling dependence also. But, right now, we just assume the independents to show you the, how to develop? How to compute the reliability of the system? So, if they are functioning independently then the reliability r t of the system can be obtained, in terms of the reliability r 1 t and r 2 t of the 2 components. So, I am just denoting the reliability of the first component by r 1 and the reliability of the second component.

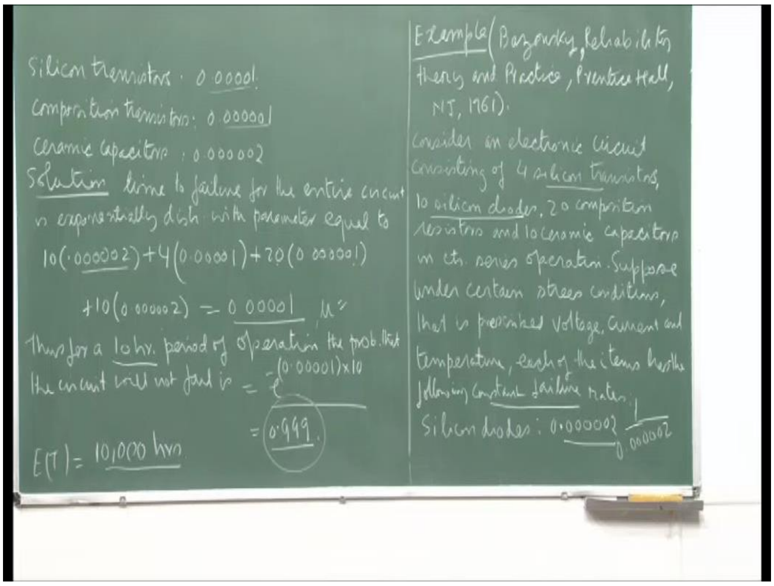
I should probably write just this small t, into value of the... So, therefore, I can compute the reliability of the system in terms of the reliabilities r 1 t and r 2 t of the 2 components. When this is a simple computation we have already done it, so many times. So, here your asking for probability greater than t, and this would be this can what it means says that your t one; that means, t 1 is the life time of the first component, and t 2 is the life time of the second component.

Then we are asking for probability t 1 greater than t, and probability t 2 greater than t. Both must be functioning up to time t if you are saying that the system for the system, the functioning time are the is you know greater than or equal to t. So, now, because of independents this joint probability can be written as, probability t 1 greater than t into probability t 2 greater than t.

This is because, we have assume that the 2 components of function independently. And, so this is r 1 t into r 2 t now, because these are probabilities. So, they are each of the numbers is less than 1. So, therefore, this product would be less than the smaller of the 2. Because, if r 2 t is less than r 1 t then I am multiplying r 2 t by number less than 1. So, the whole product is still less than r 2 t. similarly, if r 1 t is the minimum of the 2 then I am multiplying r 1 t by number which is less than 1, and therefore, the product is again less than r 1 t. So, essentially what we are saying is that the reliability that the function r t, here again, I should use small t. So, r t is less than or equal to minimum of this.

So; that means, the reliability goes down. If you have components hooked up in series, then the reliability the system goes down, because this is less than or equal to minimum of the 2. So, whatever the numbers the 2 numbers here, this will be r t will be smaller than the minimum of the 2 numbers at any time t.

(Refer Slide Time: 04:41)



Now, one can now generalize this result, and to say that if n components functioning independently are connected is series, and if the I th component has reliability r I t. Then, the reliability r t, I have this habit of writing capital t I do not know then reliability r t of the system is given by the product of the individual reliabilities. Because, we are assuming that the components are functioning independent of independently of the other components.

So, therefore, you have this general formula, and you see the movement you have the components hooked up in series, is many components they lower the reliability, because you require all of them to be functioning for the system to function. Now, consider a case when n is equal to 2 and the failure law of the I th component is an exponential. So, then your reliability for the system, when 2 components will be e raise to minus alpha 1 t into e raise to minus alpha 2 t, which is this is e raise to minus alpha 1 plus alpha 2 t. And, your therefore, the p d f for the system for the failure time of the system, would be minus r prime t which is alpha 1 plus alpha 2 e raise to minus alpha 1 plus alpha 2 t. So, this is negative exponential with parameter alpha 1 plus alpha two.

So, again we just repeating, what we have already learn that; if you have 2 two exponentials distributions, their corresponding random variables are independent. Then the some would be, I mean no, here we are asking for I over taking the… So, the p d f becomes negative exponential where the parameters get added up. So, this is exponential essentially this is. So, I will written it down here, alpha 1 plus alpha 2 into e raise to minus alpha 1 plus alpha 2 t. So this, what happens if you 2 components and both are exponential both have the failure law exponential failure law, and they functioning independently then for the system, if you want to make the computation for the reliability.

Then it will be parameter added up. So, it will be alpha 1 plus alpha 2 t and e raise to minus this, and the p d f for the time to failure for the system would be exponential negative exponential distribution, with parameters alpha 1 plus alpha 2, this is it. So, now, we can look at some more examples, and then we will look at some more another kind of system, which is when you have components a raised in parallel. I have taken this example, you know from a book which got published in 1961.

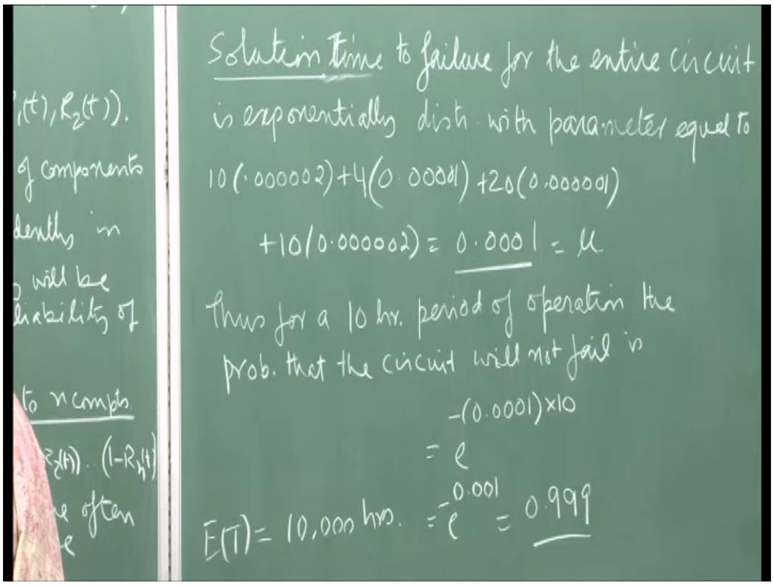
So, this is Beznesky reliability theory and practice prenties hall may be the book is not available now, but I have basically chosen the example. Because, you know these figures, I am not easily available we know we failure wait for a silicon transistor composition transistor and so on. So, just for that reason and I wanted to show you the numbers because you see, what we are saying here is… So, let me read out the problem first. So, you consider an electronic circuit consisting of own silicon transistors 10 silicon diodes, 20 composition position resistors, and 10 ceramic capacitors in continuous series operation. So, they are all hooked up in series, and under certain stress conditions that is prescribed voltage current and temperature. Each of the items has the following constant failure rate so; that means, the failure law is exponential. So, for silicon diodes, it is in hours. So, the your parameters 0.000002; that means, if you convert this into; that means, the mean failure time, mean failure time will be how much? 1 2 3 4 5 6. So, you will you will have to write 1 0.000002.

So, it is a very fairly large number. So, this exactly. So, I thought that since Bezensky has some where got this data from and. So, we can use it and actually the example appears in minus book, which for which the reference I will give you at the end of the lecture. So, any ways. So, the mean failure time is you know, millions of hours will be there right 2 4 6 yes. So, thousands of hours you can see.

Similarly, these are the various numbers. So, the parameters; that means, each has a exponential failure law follows the exponential failure law, and we are assuming that they are hooked up they are they are failure I mean they are functioning independent of each other. So, therefore, just now as we saw that we just add up the parameters to get the distribution for the parameter for the failure law of for the whole system, and that will also be exponential. We just saw, it can be easily shown that of course, I showed it to you for two, but the same thing will easily can be shown for any number of…

So, if you have lot of components many components hooked up in series, each is following an exponential failure law. Then, the when you look up failure law for the whole system then that will be simply again exponential with the parameters added up. So, you have, how many you have, 10 silicon diodes and. So, the parameter is this. So, therefore, 10 times the parameter for a silicon diodes then silicon transistors they are 4 of them 4 silicon transistors of 4 times this parameter, which is 0.00001 plus 20 times we have 20 composition transistors. So, this is 20 times this plus 10 ceramic capacitors 10 times this.

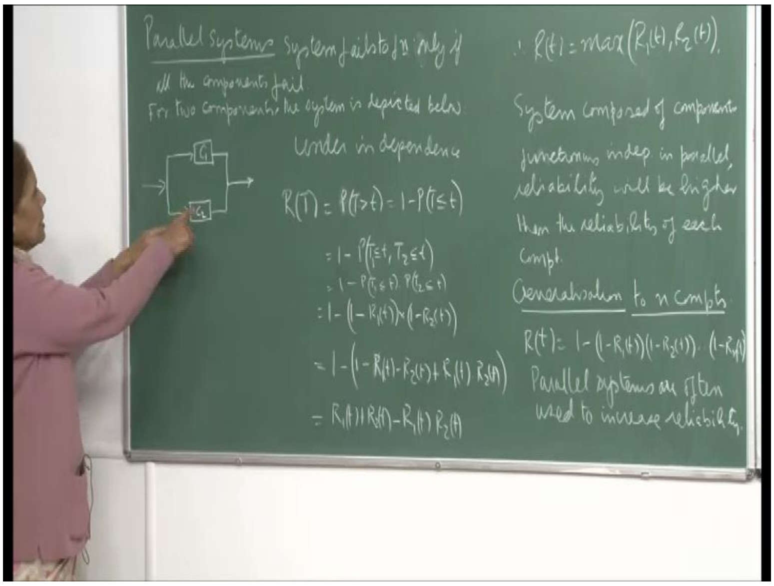
(Refer Slide Time: 11:15)



So, the parameter for the exponentially distributed see the time to failure for the entire circuit is exponentially distributed, and the parameter will be equal to since they are 4 of them. So, 10 into would see the numbers are given to you. So, this adds up to 0.001. So, earlier in the computation I had written 4 zero’s, but actually, when you do the addition multiplication and addition it will come out to be 0.001. So, that is your mu and thus for a 10 our period of operation. The probability that the circuit will not fail will be e raise to minus mu in to 10, so mu ten.

So, the time period whatever the time period the parameter gets multiplied by that for the corresponding parameter during that period. So, e raise to minus 0.0001 into ten, which is e raise to minus 0.001. So, the final answer was given correctly, which is 0.999, and therefore, your e t will be 10000 hours. So, therefore, probability is very higher; obviously, because these diodes and capacitors have life time mean life time in thousands of hours so; obviously, for 10 our period you do not expect another system to fail. So, the probability is very high. So, therefore, we would expect the system to will be the high probability the system will continue to function for 10 hour without any failure this is the idea.

(Refer Slide Time: 12:53)



So, again you may say simple examples, but just to drive from the point that this kind of thinks you consider. Now, other system that we would like to there is a parallel system. So, here the system fails to function only if the of all the components fail. So, you knows the diagrammatically, you can repeat this for 2 component, if you have the component arrange in parallel.

When you it is like this. So, the input comes and then you have either it can go this way or it can go this. So, the system will fail to function only if both of them fail, because this long as 1 them is functioning, they will the things can be the input can the input can be go this if this fails when this will go this way and it will go out this way. So, this still operation will be performing, and if this fail and it will go this way. So, the operation will be still be performed. So, therefore, for the system to fail both of them have to fail. So, that is what to mean; when we say that all the components have to fail, and again and independence.

So, if we are saying that function independent of each other, and that what is expected if you have in parallel then the each component to functions parallels independently of the other. So, then if you want compute the… I do not why I keep writing capital t here. So, this is if you want to compute reliability for the system, when this is probability t greater than t which is 1 minus probability t less than or equal to t. So, there in that case this is what you want that 1 minus probability t 1 less than or equal to t and t 2 less than or equal less than or equal to t. Both of them should not be functioning by 10 t.

So, therefore, because of independence you would write this as the product. And therefore, this becomes, so probability t 1 less than or equal to t is 1 minus r 1 t, because r 1 t is probability t 1 greater than t. So, this will be 1 minus r 1 t into 1 minus r 2 t. Now, when open out multiplied the 2 terms and then you get this. So, this call reduces to r 1 t because, 1 minus 1 cancels out r 1 t plus r 2 t minus r 1 t into r 2 t. So, this is expression for the and of course. So, once we get this expression, which I can because 1 cancels with the minus 1. So, you will be left with r 1 t plus r 2 t minus r 1 t r 2 t.

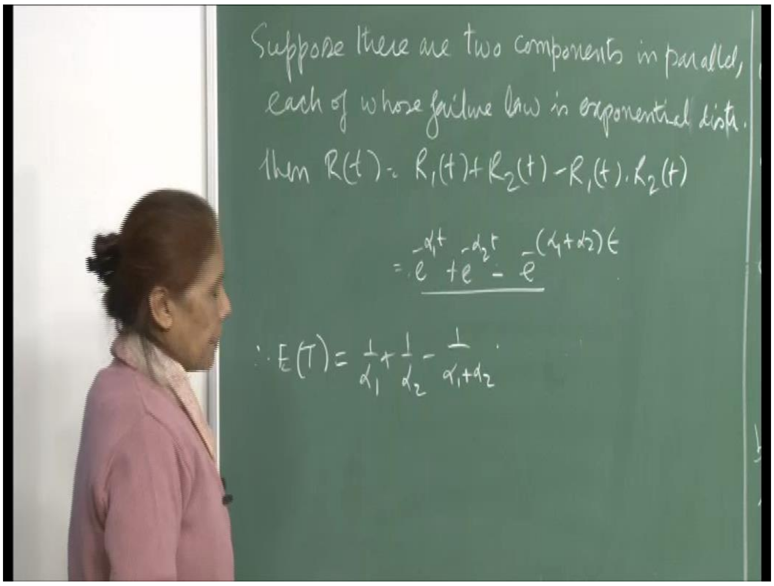
Now, you see that this is this is the reliability for the 2 systems. Because, this is r 1 t into r 2 t n where assuming that 2 two systems function independently. So, therefore, you see that this is a number, which is less than r 1 t and r 1 t both. So, considered see for example, r 1 t is larger than r 2 t then this whole number is bigger than r 1 t. So, if r 1 t is maximum of r 2 and r 1 then this number this whole number because, r 2 minus r 1 t r 2 t something non negative and therefore, r 1 t plus something non negative.

This is going to come out to be and hence I can immediately conclude. So, that is what I have written that since r 1 t r 2 t is less than or equal to both r 1 t and comma r 2 t. Therefore, it follows that your reliability, when you have you know 2 components working in parallel. See, I have shown you that the 2 system have working in parallel. So, in that case the reliability of system because is greater than or equal to max of the reliability of the 2 components. So, that immediately shows that systems compose of component functioning independently, in parallel that reliability will be higher than the reliability of the of each of that components, that are in parallel. So, parallel components often used to increase reliability.

If you want to now, generalized to n components, which are functioning in parallel then this will be r t into 1 minus of 1 minus r 1 t into 1 minus r 2 t into 1 minus r n t. So, the same principle will be used, and you can show that. So, this is the important thing that when 2 so; that means, here we are considering the case when 2 components are working in parallel, and system has to fail only when both of them fail. Because, all components have to fail, in that case the reliability of the system will be greater than or equal to reliability of both the components. So, therefore working in parallel having in components in parallel and having components in series. So, this is the basic the way you make up the devices, and then I can say you can decompose them, and you know into smaller this thing.

Where you can consider component arrange in parallel, when components arrange in series, then put them together. So, I will try to show you some more examples of you know system, you know system, of components arrange in different orders

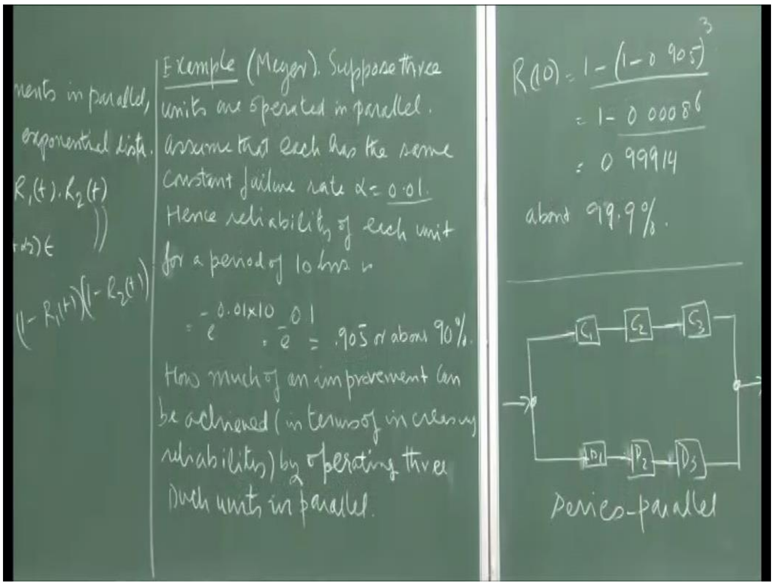
(Refer Slide Time: 18:29)



So, let see take the consider the example, where 2 component are in parallel, and each of whose failure law is exponential distribution. And of course, we are assuming that first component has parameter alpha 1 the other 1 is alpha 2 then the reliability of the system when since they are in parallel. So, reliability is given by formula.

We just obtained r 1 t plus r 2 t minus r 1 t into r 2 t because they have functioning independent of each other and therefore, this will be your reliability function. And then the e t the expected time to failure for the system would be because, you know when you take the expectation will be integrating each of them separately, this t into d t this t into d t integral of 0 to infinity t into this, each them is exponential distribution. So, it will be 1 upon alpha 1 plus 1 upon alpha 2 minus 1 upon alpha 1 plus alpha 2. And you can now, since you have the old machinery with, you do all any competition that you want to do once you know the functional form of the, you know of the reliability function. You can make these computations.

(Refer Slide Time: 19:44)



Now, again just to drive of the point that, parallel arrangements of components definitely increases the reliability of the system. And I have take this example from Meyer’s book which again is very old one, but very good 1 and. So, the thing is that yes. So, any way I have just given reference I will give you reference, but the book may not be easily available does not matter. Suppose, 3 unit are operated in parallel, assume that each has the same constant failure rate alpha equal to 0.01. So, there are identical components. So, all have the exponential failure law, follow the exponential failure law with parameter 0.01.

Hence reliability of each unit for a period of 10 hours is, e raise to minus 0.01 into 10, which e raise to minus 0.1, which is 0.905 or about 90 percent. So, if each component is functioning by itself then the reliability is in 10 hour period that it will not fail is, you know 90 percent. Now, how much of an improvement can be achieved, in terms of increasing reliability of the system by operating 3 such unit in parallel. So, by our formula this is of course, I am not looking at the expanded form.

This is 1 minus 1, 1 minus p probability of t less than t raise to 3. So, that is what will it will be. This is what you see that I got after opening up the brackets, but if you do not do it see this whole thing you wrote as, 1 minus 1 minus 1 minus r 1 t into 1 minus r 2 to t. So, this was the formula which by which then we opened up into 1 1 got cancelled and so on. So, I am just and since they are identical. So, it will be the same function. So, it will be 1 minus r 1 t square in our case it will be q. And, so this is 1 minus 0.905 it because argue of this came out to be 0.905. So, 1 minus of that raise to 3, and that cancel to be this.

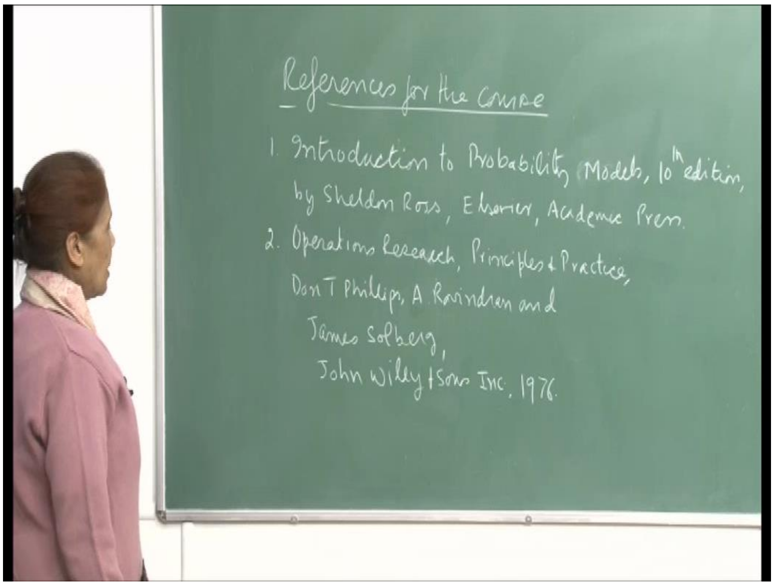
Therefore this is equal to 0.99914 and therefore, reliability has g 1 up to 99.9 percent. So, the numbers drive on the point, and that is why it is important, that you individually if you just had 1 component in the system. Then, for the probability its operating for 10 hours would be only 90 percent, you expect 90 percent of the time it will be function still at the end of the 10 hours period. But, if you have 3 in parallel then you will almost be showed that the device with 3 parallel components, will still be functioning at the end of 10 hour period right.

So, this is the idea when therefore, the reliability can improved upon,, but I had said that it has to be verses reliability verses cost reliability verses volume of the device and so on. So, you cannot just go on having components in parallel now, see the thing is that as our saying that I discuss the very basic arrangements for you, basic systems. And then you can have things like you know series parallel. So, you have these 3 components here in series, these 3 components in series, and then parallel. So, there is no problem because, you can complete the reliability for this, and for this, and then you know how to compute the reliability for the parallel because, when they 2 are parallel.

So, we just have to you know iteratively do this arrangement we compute this we shall be the product, and this will be the product, and then it will be you know 1 minus of. So, what we have been doing, so for n. Similarly, if you have arrangement paralle,l and series then these are parallel. So, you can compute the reliability of this you can compute the reliability of this. So, this will be form your r 1, and this form you r 2, and then you are doing it in series. So, that is what I meant that all complex devices can be broken up into you, whether either they are series parallel, parallel series and so on. And then you want to put them together.

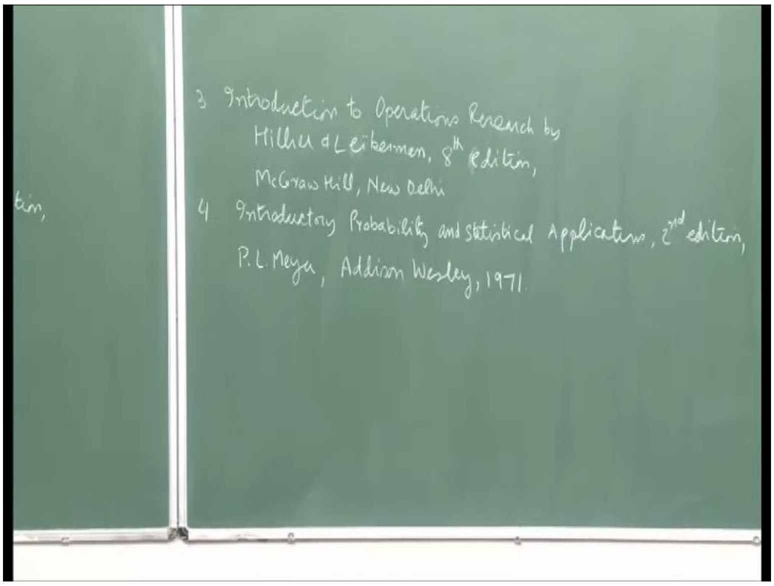
So, most of time you should come up whether reasonable functional form for the reliability of the of the whole device. And, you thing just and of course, I will discussing if you problem like this in the exercise, which will follow. So, that exercise on problem related to whatever we have discussed about reliability theory.

(Refer Slide Time: 24:45)



So, let me give you the reference that I have the books that I have been referring to all along in this course, and yes I agree that. So, of them are out of print,, but in any case the idea is that even though like this is a 76 edition and that 1 is 71 edition of course, ((Refer Time: 25:05)) 8 th edition have just come, and I think even the 9 th 1 may be ready.

(Refer Slide Time: 25:11)



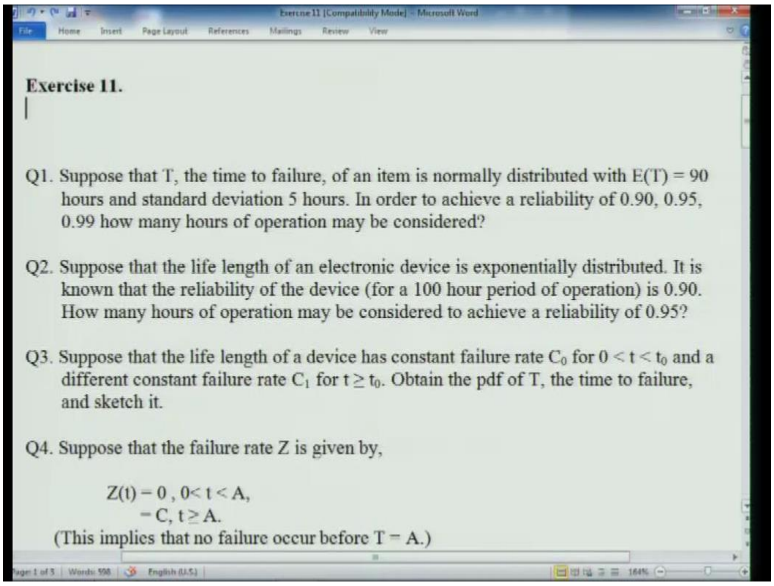
So, any way the thing is to get the basic material I have to refer to these old books, but certainly substitute will be there and now lot of materials is available on the net. So, you just have to type the word that you would want, whatever subject matter you want and lot of thing come out. So, therefore,,, but in any case I just want to refer to this books, because I have use material from these books. So, the first 1 is the introduction to probability models 10^{th} edition by Sheldon Ross this is Elserser academic press.

This I think is that 2010 addition, and may be this 1 will keep coming with new addition. So, therefore, no problem getting a copy of this book, operation we search principle and practice Don t Phillips, A Ravindren and James Solberg this is 76 th book, but some treatment or some topic have been treated very well. So, I have use a lot of example are fall from here and I have whenever I use figures, I have refer to that part also.

So, this is 76 th book then introduction to operation search by hillier and lei berman eighth edition McGraw hill. I do not remember the for this particular year, this is being come out with new addition. So, therefore, no problem this is McGraw hill. So, cheap edition I should be available easily, and 4 th 1 that I have used is introductory probability and statistical applications, second addition pl Meyer this is classical book, and very neatly and simply, the material has been presented.

So, reliability theory portion, I have use this book and this is the addition Wesley 1971 book. So, in any case this is, what I of my source is, where and now, you can as I told you Google search for any subject matter that you need, and that I hope you get interested enough topic to read more.

(Refer Slide Time: 27:17)



Exercise 11.

Q1. Suppose that T, the time to failure, of an item is normally distributed with E(T) = 90 hours and standard deviation 5 hours. In order to achieve a reliability of 0.90, 0.95, 0.99 how many hours of operation may be considered?

Q2. Suppose that the life length of an electronic device is exponentially distributed. It is known that the reliability of the device (for a 100 hour period of operation) is 0.90. How many hours of operation may be considered to achieve a reliability of 0.95?

Q3. Suppose that the life length of a device has constant failure rate C_0 for $0 < t < t_0$ and a different constant failure rate C_1 for $t \geq t_0$. Obtain the pdf of T, the time to failure, and sketch it.

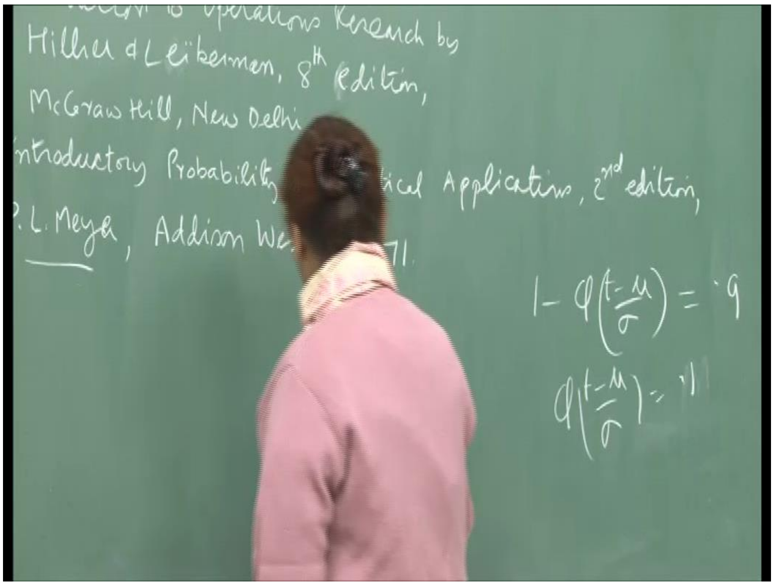
Q4. Suppose that the failure rate Z is given by,

$$Z(t) = 0,\ 0 < t < A,$$
$$= C,\ t \geq A.$$

(This implies that no failure occur before T = A.)

Now, let me just discuss the last exercise 11 with you, which is on based on probability theory. Let us just look at question 1, suppose; that t the time to failure of an item is normally distributed with e t as 90 that is mu hour and standard deviation 5 hours. In order to achieve reliability of 0.9, 0.95 and 0.99, how many hours operations may be considered?

(Refer Slide Time: 27:55)

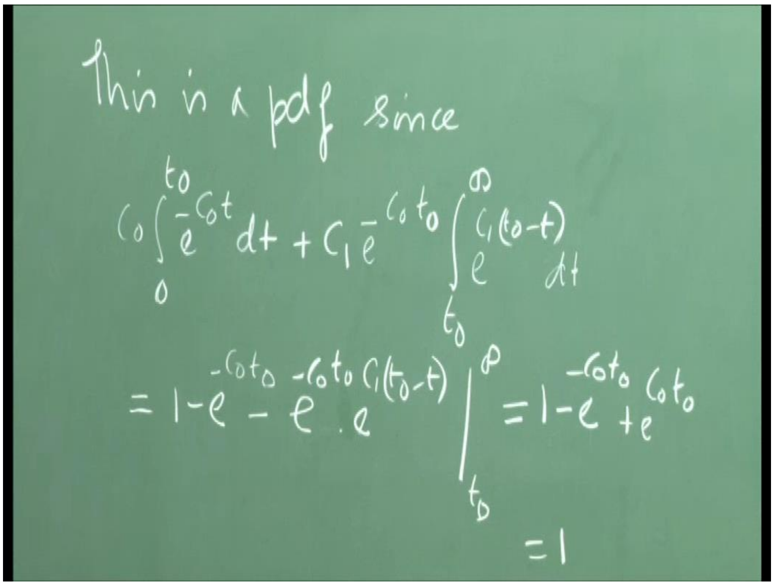


So, now you know the reliability function this is 1 minus 5 t minus mu upon sigma. So, this would be if for example, you put it equal to 0.9 and you know you do not mu is it mu is given to you sigma is also given to you now, perform the normal tables you are looking for pi t minus mu, I have d 1 problem will give like this is equal to 0.01. 0.1,1 minus 0.9 is 0.1. So, then give mu 1 sigma you will look up the tables and for corresponding 0.1 what is the value here and they corresponding value of t will be available.

So, similarly when you put 0.95, 0.99 you can accordingly get the value of t. So, how many hour of operation may be considered? So, you can answer for all the 3 value of the reliability level. Question 2, suppose, that the life length of an electronic device is exponentially distributed, it known that the reliability of the device for a 100 hour period of operation is 0.9, how many hours of operation may be considered to achieve a reliability of 0.95? So, first the first data that is given to you.

You will compute the parameter for the exponentially for the exponential failure law of and then once you get the alpha then you can compute the time. We have corresponding to the reliability level of 0.95. Question 3, suppose, that the life length of a device has constant failure rate c0 for 0 less than t less than t 0, and a different constant failure rate c 1 for t greater than or equal to t 0. Obtain the p d f of t the time to failure, and sketch it. This is sketching part I will level to you, but see here all that is saying is that you have. So, up to t 0 you have 1 failure law, and then after t 0 you have another failure law. So, therefore, but at the point t 0 the 2 you must meet right. So, therefore, you what you will say is that c 0. So, you see the required density will be c e raise to minus c 0 c as long as t is between 0, and t 0because, this is the failure law. So, the rate of failure is c 0 and is exponential failure law. So, therefore, for t less than or equal to t 0; that means, laying between 0 and t 0 you will write this. And, for t greater than t 0 it will be c 1 e raise to minus c 0 t 0 plus.

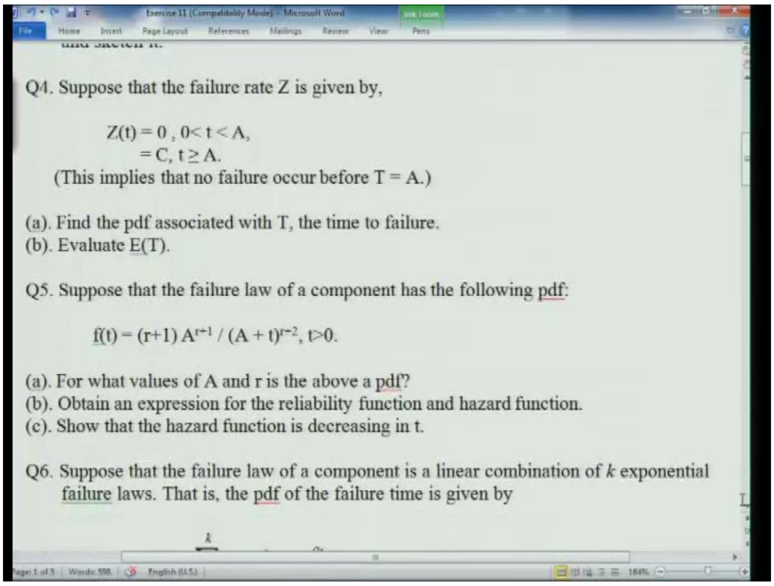
(Refer Slide Time: 30:09)



So, this is e raise to I should, and then you see c 1 t 0 minus t. So, since t is greater than t 0, when you write it as t minus t 0 then it will be minus here anyway. So, this is now this will be the probability density function. So, to show that f t is a p d f, we have to show that integral from 0 to infinity, would be equal to 1 and then particular. So, the first part we will integrate from 0 to t 0 and the second part of the function f t we will integrate from t 0 to infinity.

And therefore, the calculation shows, that the integral comes out to be equal to 1. So, this is the required p d f and for question 3. And then you can try to sketch it. Suppose, failure rate z is given by. So, now, 4 is special case of 3. So, here your time between 0 and a the failure rate is 0 and for t greater than a it is c. So, it is constant. So, again it is the same thing as 3 except that now, c 0 is 0 and for c 1 is c fine and . So, therefore, you can because, we have obtain the form for the failure law in 3's now just substituting this special values you can compute you can compute the…

(Refer Slide Time: 32:07)

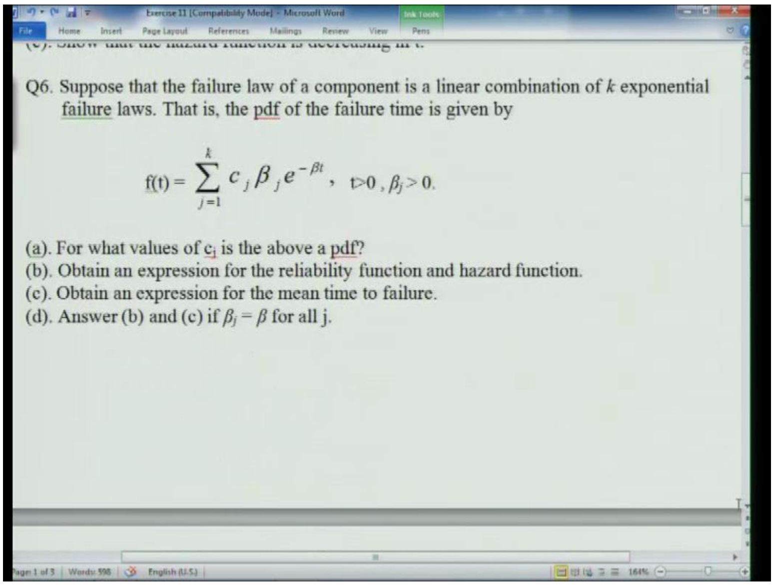


Let me just see. So, here you have to find the p d f associated with t the time to failure. So, that you can find out because, I have already obtained for you. And now, put in the value of c 0 c and of course, your t 0 is a, and evaluate e t. So, then you also a evaluate by actual integration. Now, let us look at question 5, suppose, that the failure law of a component has the following p d f. So, this is f t is r plus 1 into a r plus 1 divided by a plus t raise r plus 2 t greater than 0. So, for what values of a and r is the above a p d f we can look at it I can look at it…

Suppose, that the failure law of component has the following question 5, suppose that failure law of a component has the following p d f, f t is r plus 1 into e raise to r plus 1 divided by a plus t raise to r plus 2 t greater than 0. So, it should not be difficult because, all we have to do is hence we have to say that for what value of a and r is the above a p d f and I think if my this thing is right. Then it really does not matter it is a p d f for all a and r. Because, you see simply we write the integral as a plus t raise to minus r minus 2 and then integrate, from 0 to infinity. And then you just have I think the value of a will cancel out and r. So, you will get the integral as equal to one without specifying any values for a and r anyway.

So, now you can do this and then you can obtain and expression for the reliability function and hazard function. And show that the hazard function is decreasing in t. So, I let you do this problem.

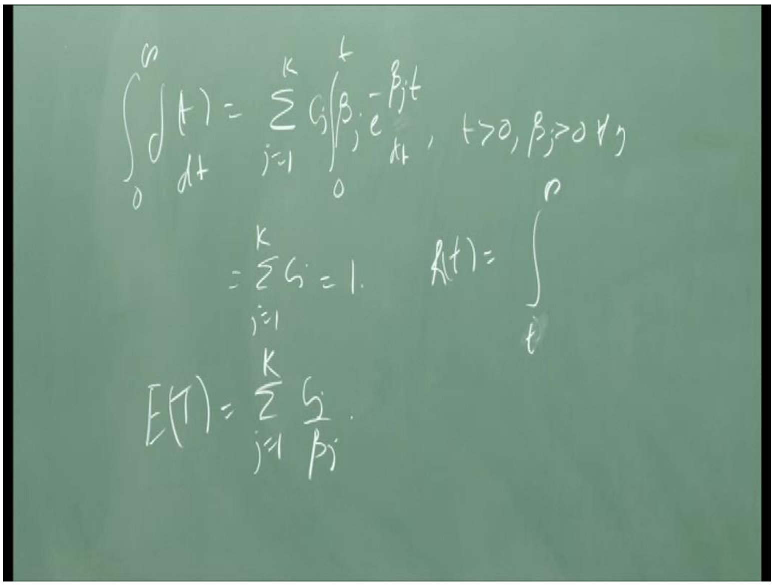
(Refer Slide Time: 34:09)



Now, question 6 suppose, that the failure law of a component is a linear combination of k exponential failure laws, that is the p d f of the failure time is given by f t is equal to sigma j varying from 1 to k c j beta j e raise to minus beta t, t n beta j from all j is positive. So, for what values of c j is the above p d f. So, now, when you integrate because, this is finite sum.

So, I can take the integral inside. And, so when you integrate beta j integration integral of 0 to infinity beta e raise to minus beta, it should be there is missing, let me write it down I think you f t should be...

(Refer Slide Time: 34:58)



This happen the typing errors take tripping. So, this is j being from 1 to k and this is c j beta j, e raise to minus beta j t. This is for t greater than 0 and beta j greater than 0 for all j. So, this is beta j make the correction. So, when you integrate this from 0 to infinity d t will be integrating all separate 1 this is 0 to t d t. So, therefore, now this is equal to 1.

So, therefore, this whole thing will add up to sigma c j, j going from 1 to k. So, this is the condition that all and of course, c j have to be non negative, and then you say that sigma c j while, actually it say linear combination. So, I will just to be on safe side say that c j be non negative and they add up 1. So, this becomes convex combination in that case, of all these different exponential law, and so this is also again and p d f this will be a p d f. Then, obtain and expression for reliability function and hazard function, obtain a function expression for mean time to failure.

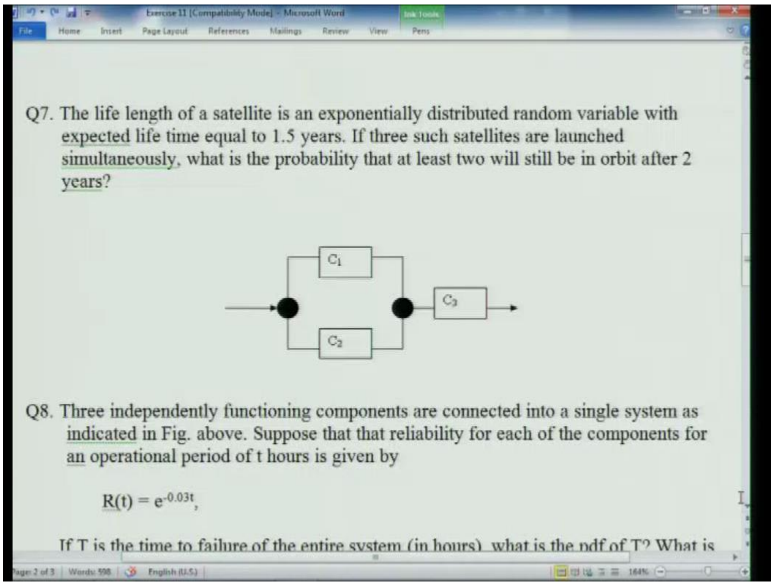
See here again because it the summation. So, you will have to integrate if you have to compute this integral, the same principle you will use for the reliability thing you will have to show r t, when you do t to infinity. So, you again then integrate separately and. So, it will be a convex combination of all the separate reliability functions, r 1, r 2, r 3. So, it will c 1 r 1 plus c 2 r 2 plus c k r k. So, straight forward this is not right and then answers b and c beta j equal to beta for all j. And of course, obtain an expression for the mean time to failure. So, the mean to failure would be, see for each of them 1 by beta j.

So, it will be summation. So, mean the failure; that means, your e t will be summation c j beta j, j going from 1 to k.

Now, let us go to next problem, then the question 7 expected life time this 3 by 2 years. So, which means that lambda is 2 by 3 again, exponential failure law. And, probability that it is still functioning after 2 years will be e raise to minus. So, it will be e raise lambda t, which is minus 4 by 3 after 2 years. So, this will be e raise minus 4 by 3 now, you want to probability the 2 still functioning after 2 years. So, 2 still functioning after 2 years at least so; that means, you may have after 2 years either 2 of them are functioning or 3 them.

So, when 2 of them are functioning it will be 3 c 2 e raise to minus 4 by 3 into 2. Because, 2 of them are functioning, and 1 of them is not functioning. So, it will be 1 minus e raise to minus 4 by 3 and all you have all 3 of them functioning. So, it will be e raise minus 4 by 3 into 3; that means, here this is. So, actually it will be e raise to minus 12 by 3. So, this will be required probability.

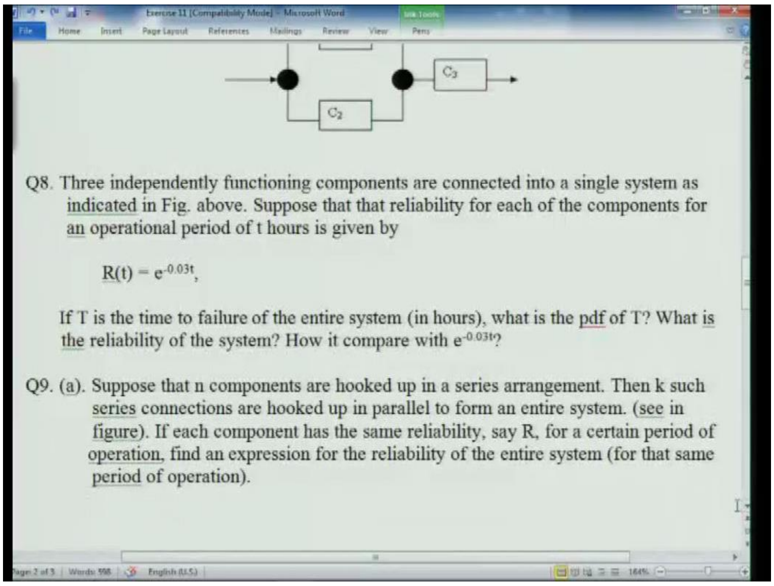
(Refer Slide Time: 38:42)



Now, this figure refers to problem 8, I think 3 independently functioning components are connected into a single system, as indicated in figure above. So, 2 are parallel and then 1 series, suppose that reliability for each of the components for an operational period of t hours is given by, e raise minus 0.03 t.

So, now by now, we have discussed all this. So, therefore, you have 3 these 2 has parallel. So, then you compute the reliability of these component, and then this together with this component c 3 in series. So, now, you do it, and they all identically distributed the failure law for the 3 components. So, therefore, you first compute these 2 in parallel, and then it will be, so which will be… I will give you the formula for this and then that into c 3. So, that will be you know multiplication.

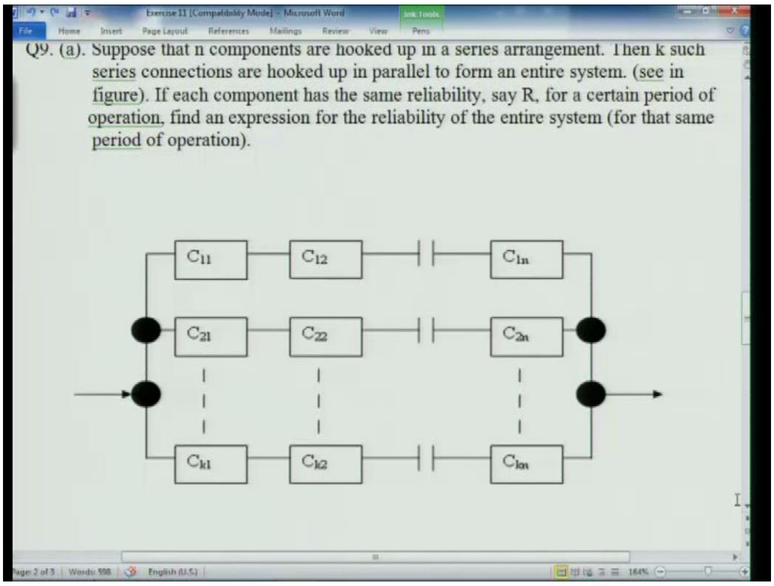
(Refer Slide Time: 39:41)



So, you can do that and let us say what are else is ask. So, if t is the time to failure of the entire system, what is the p d f of t? That you can find out well, you will first find out the reliability function or you can try to find the p d f directly, what is the reliability of the system? How it compares with e raise to 0.03 t? So; obviously, I think your guess should be that it will definitely improve the reliability because, they are 2 components in parallel. So, definite even though see, the thing is that the reliability of this component consisting of c 1 and c 2 will go up, but c 3 will have the same.

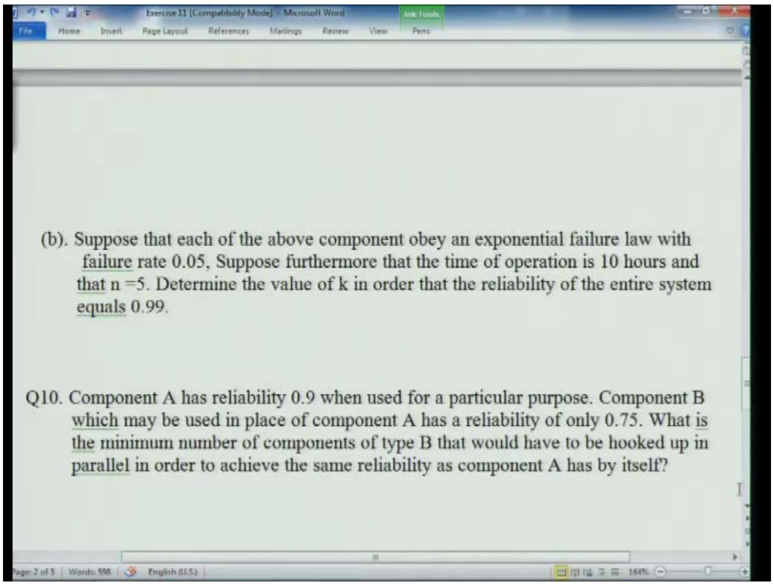
And since when the take the combination in series, when you hope then up in series then your reliability is the minimum of the 2. So, therefore, you cannot say much it will almost with the same. In fact, it will not be the better e raise to 0 or minus 0.03 t. In fact, it will not be better than e raise minus 0.03 t.

(Refer Slide Time: 40:53)



Now, I have given you a big system here. So, this is suppose, that n component are hooked up in a series arrangement. Then k such series connections are hooked up in parallel to form an entire system. So, the figure below is given, if each component has the same reliability say r, for a certain period of operation, find an expression for the reliability of the entire system for that same period of operation. So, you should enjoy doing a just sit down patiently write down the first for these series connection, you write the reliability function for this and these it will be the same. And then you do the parallel thing. So, their k in parallel, and you have how many n in series. So, just patiently sit down, and you can write down expression for the reliability function for the whole system, should be able to do it.

(Refer Slide Time: 41:40)



Now, I am saying the suppose, that each of the above component obey an exponential failure law, with failure rate 0.05. Suppose, further more that the time of operation is 10 hours and that n is 5. So, now, I have give the numbers and determine the value of k in order that the reliability of the entire system equals 0.99. So, you will enjoy do this problem because, case not give to you, but then when you write down the reliability function, it will be a function of k.

And then you put value for 10 hours. Your t 10 hours then you substitute that in the expression, and put the whole thing equal to 0.99 then you will get the value of k. So, interesting problem, I am sure you will enjoy by doing it now a component has reliability 0.9 when use for a particular purpose, component B which may be used in place of component A has a reliability of only 0.75. So, A has reliability of 0.9 in for a certain period, and component B which may be used in place of component A has reliability only 0.75. So, what is the minimum number of component type b that would have be hooked up in parallel? So, now, you can write down the reliability function suppose, k you can take k number of B component and then hook then up in parallel.

You know the reliability function, and then you want say that its reliability should be equal to 0.9. So, that also again you can use in the formulas that have give you for computing the reliability, when they are hooked up in parallel. Then when you 0.75

reliability of 1 component, then how many should be there. So, that reliability goes up to 0.9. So, these are the kind of question which I am sure you will, that is it.

So, I think this is the last lecture, and effort has been made to equinity with probability theory, basic probability theory. I would say, and then its applications and I have try to trough exercises, try give you good in site into the subject, and I hope you enjoy doing these exercises.