Introduction to Probability Theory and its Applications

Prof. Prabha Sharma
Department of Mathematics and Statistics
Indian Institute of Technology, Kanpur

Lecture - 4
Random Variables Cumulative Density Functions Expected Value

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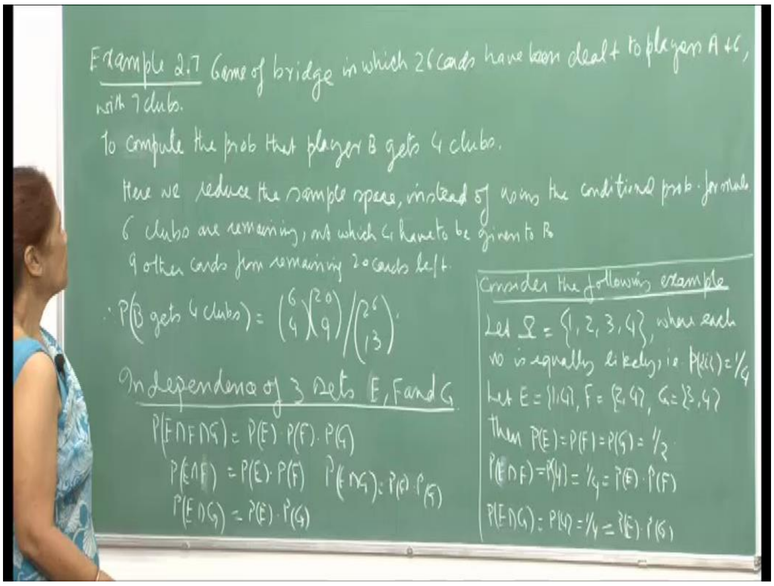

Let me fill up a few gaps in the last lecture. So, for example, this was I had discussed this card game bridge game; there was an example number 2.7, which I was talking about a game of bridge in which 26 cards have been dealt to people to the players a and c. If you remember there were 4 players a b c d. So, 26 cards have been dealt to the players a and c with 7 clubs. So, we wanted to compute the probability that player b gets 4 clubs, right. Now I was trying to say that I was trying to show this as an example of computing the conditional probability that is already 7 clubs have been dealt two players a and c.

So, what is the probability of b getting 4 clubs? But now I wanted to then show you that, and, of course, we solved out the problem; I told you this is the probability, but then it is not possible to explain this through the conditional probability argument. So, therefore, I just want to tell you that there are other ways of computing this probability and that is by reducing the sample space. So, now, instead of you know taking the distribution of 52 cards to the four players; I am just reducing the sample space, because 26 cards are now left, out of which 6 are clubs because 7 have already been distributed to players a and c.

So, then this is now your multinomial distribution. So, what you are saying is that out of 6 clubs, b should get 4, and then from the remaining 20 cards, he should get the other 9 cards. So, that way he gets 13 cards. So, this is the number of ways in which he can get 4 clubs out of 6 and 9 cards out the remaining other 20 cards. So, this is the number of ways in which this can happen, and the total number of ways in which he can get 13 cards out of 26 is 26 choose 13. So, this is the actual probability, and this we can get by reducing the sample space. So, this is another technique which is very helpful and at times you can very easily compute the required probability by doing this, right.

So, you do not have to always go through the actual formula; you can always play around with alternative ideas also, okay. Now remember I asked you to construct examples when we were talking of independence of three sets, right. And in that case, I showed you that there are four conditions which have to be satisfied before you can say that the three sets are independent. And then I asked you to construct a counter example to show that suppose the last three are satisfied; that means pair wise, if we choose any two sets from here, they are independent. They satisfy the condition of independence; that means your probability of e intersection F is p e into p f probability of e intersection g is probability of e into probability of g, and similarly probability of F intersection g is product of the probability p F into p g.

So, you can construct situations where let us say last three conditions are satisfied, but the first one is not satisfied. And so I believe into leave my hand on this example here. Suppose your sample space consists of these four numbers. So, where each number is equally likely; that means the probability of any number coming up as a result of the experiment has probability of 1 by 4, because they are equally likely, and there are four elements in omega your sample space. Now let us choose e to be as consisting of the numbers 1 4; F to be consisting of the numbers 2 comma 4 and g 3 comma 4, alright.

In that case, you see if you see the probability of P e p f and p g is 1 by 2 because there are two numbers totally four. So, 2 by 4, again we are using the same concept, because outcomes are all equally likely; therefore, I am using the m by n version of the probability. And so this is half for each of the sets, then probability e intersection F. Now e and F have only four in common. So, that is one singleton, and that probability is 1 by 4 which is the product of p e into p F.

So, similarly, you see that the pair wise independence has been you can easily verify that the three sets are pair wise independent, but when you write e intersection f intersection g that again results in single number 4. And so this probability is 1 by 4, but this is not equal to p e into p F into p g which is 1 by 8, okay. And, therefore, now you get an idea and you should try to construct one example of your own to show that pair wise independence of three sets is not enough to say that the three sets are independent I mean the three events are independent, right. We should be able to construct and as I said that extending this becomes little more cumbersome and so we will leave it here only, right.

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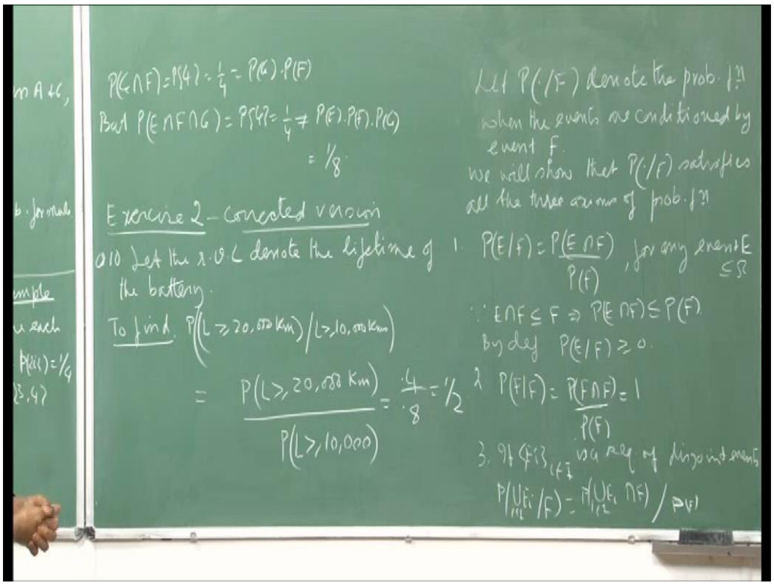

Now exercise two, of course, I have given you the corrected version, okay. So, this is there, and question ten in exercise two, I want to revisit because when I was reading it last time; I said that this needs a little thought. And so let us revisit the problem. So, actually you are given the data that a battery as a lifetime. So, there is a probability that the battery will last for more than or equal to 10000 kilometers that probability is given to be 0.8. And the probability that it will last more than or equal to 20000 kilometers, the probability was given as 0.4 and then that it will more than or equal to 30000 kilometers also a probability is associated with it.

And you were asked to find that if you have bought a battery which is already run for 10000 kilometers what is the probability that it will be running for more than 20000 kilometers; that means its lifetime will be more than 20000 kilometers if it is already run

10000 kilometers. So, this is a simple case of conditional probability computing the probability that the lifetime say if I let l, denote the lifetime of the battery. We are wanting to compute the conditional probability of the event that the lifetime is more than 20000 kilometers given that it is already 10000 kilometers old the battery; it has run that much and so by our formula.

Now when you take the intersection of these two, so obviously, this is a subset of this. And, therefore, the intersection simply becomes l; that means you are wanting the lifetime to be greater than or equal to 20000 kilometer when it is already run for more than 10000 kilometers, right. So, the intersection of these two would be this, because you want the battery to run 10000 kilometers and you want the battery to run 20000 kilometers. Intersection would mean that you want the battery to run for more than 20000 kilometers and so this is divided by the probability of this event which is l greater than or equal to 10000. And, therefore, this is equal to 0.4 by 0.8 which is half. So, this is the solution.

So, I just thought that if you have thought about the problem may be some of you have already done it but then this is the right answer, okay. We have talked of condition probability but I just thought that should formulize this in some more in the sense that see we are defining now given an event a, then we are computing the conditional probability of events conditioned on f, right. So, want to show that this is the function that you have defined here that conditional probability function satisfies the three axioms of probability and not too difficult to compute to show, because the first axiom requires that this for any event e, the conditional probability with respect to F must be within 0 and 1, alright; that is the first axiom,

So, here if you take probability e condition on F, then by definition this is this. Now you see that e intersection F is a subset of F, alright; this is a subset of f. And, therefore, we have already shown remember after giving the axioms, we proved some propositions and there I showed that if subset is subset of another subset, then the probability would be. So, that means here probability e intersection F will be less than or equal to probability F, alright. This is because this is a smaller event in the sense that it is a subset of this; therefore, the probability of this is less than or equal to this; this we could easily show, right.

And by definition, of course, probability e condition F is nonnegative, alright. This is nonnegative; this is nonnegative. So, this is nonnegative. So, therefore, the first axiom is satisfied, because p intersection F is less than p f. So, I divide by this; this is less than 1, and this is nonnegative. So, first axiom is satisfied. Now second axiom said that your probability omega should be one so here for us because we are conditioning on f. So, the corresponding axiom would be that probability, okay; this should be again the same thing. I have to say that omega condition on F, so what will this be? This, and, therefore, now when you take omega intersection F, this will be probability F this is equal to 1, right, okay. So, therefore, probability f divided by probability f is 1. So, probability for mega condition on f is 1. So, the second axiom is satisfied and the third one requires e I intersection f divided by p f.

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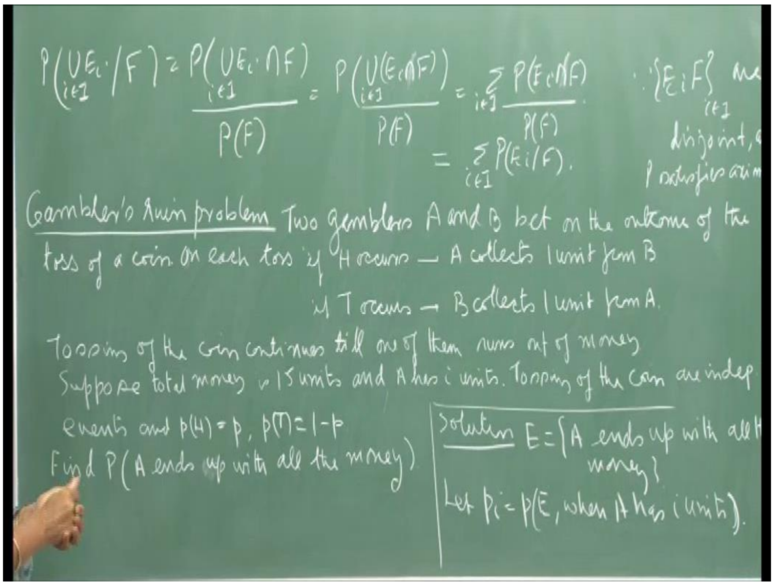

Now this I can write as see here this is this, then because distributive law holds, this can be written as union e I intersection F, I belonging to I this thing. Now because e I intersection F are disjoint, since, e I are disjoint, right. I have taken sequence of disjoint events e I is here. So, e I intersection F r disjoint and probability p, the original probability function satisfy axioms three, alright. And therefore, this can be written as summation of probabilities e I intersection F divided by p F, alright, which is equal to. So, this can be written as summation probability e I condition F, alright, I belonging to I. So, the third axiom is also satisfied.

Therefore, the conditional probability function is also a probability function; that is it satisfies the three axioms, okay. Now again I just want to continue working because conditional probability function is an important concept. So, again little elaborated example that I would take gamblers. Oh, here fine. So, this problem would be coming after I define for you the conditional probability. Now I want to discuss the Gambler's Ruin problem which is interesting example of use of conditional probability. So, let us see there are two gamblers a and b, and they bet on the outcome of the toss of a coin, alright.

Now if on each toss if h occurs that means if the head shows, then a collects one unit from b and if tail occurs; that means if t occurs, then b collects one unit from a. So, this is all a simple game. You toss a coin and then if the head shows, a gets the money, and if tail shows, b gets the money from a, okay. Now tossing of the coin continues till one of them runs out of money. So, that is the Gambler's Ruin problem, right. So, one of them will end with all the money and the other one will have no money left. So, suppose total money is 15 units, I am just taking a number 15.

You will see that through a solution, it does not matter; whatever the amount of money that they start with and has I units. So, that means b will have 15 minus I units with him, okay. Now tossing of the coin are independent events, and we are assuming that the coin is biased. So, therefore, probability h is p and probability is 1 minus p. We will also consider the case when p is half, okay. Now if we want to find the probability that a ends up with all the money and we have assumed that at the starting point, a had I units of money with him, alright. So, let e be the event that a ends up with all the money and let p i denote the probability because we are staring the game when a had I units. So, I am just denoting it by p I which is the probability of e ending up with all the money and a had i units to start off, right.

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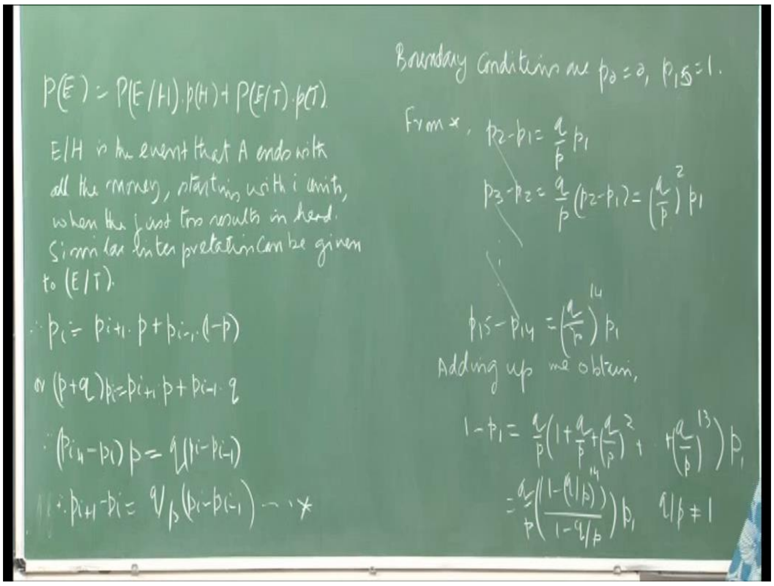

Now we have been using this technique very often, because when you are tossing a coin, your sample space is consisting of just h comma t, right; two points on the sample space. And so you can write p e in terms of conditional probability p given h; that means now this we are saying that we are starting the game. So, the first toss of the coin either results in h or in t. So, this is conditional of e; that means this is the probability of e this is the event that a ends up with all the money starting with I units when the first toss results in head. So, this is that event, right.

So, that into p h plus this is again the similar interpretation that a ends up with all the money when the first toss of the coin shows a tail, alright. So, this is this into p t. Now when the first toss of the coin gives you head, then a collects one unit from b. So, therefore, the money at the end of this after the first toss, a ends up with money I plus 1, alright, and if the first toss shows a tail, then a will have to give one unit of money to b, and therefore, he will have I minus 1. So, therefore, as I said in the beginning, I am denoting by this p I because a had I units of money when the game started.

So, therefore, this equation can be rewritten as p I is equal to p I plus 1 because this will be then the probability that now a has I plus 1 units, and you want to compute the probability that a ends up with all the money, alright, and then this into p; probability of getting a head, then here a will lose one unit of money to b. So, it will be p I minus 1 into 1 minus p. So, 1 minus p we can also write as q. Now since p plus q is 1, I can multiply this by p plus q and write this equation in this way, alright. And then rearranging the terms that means see p I plus 1 minus p p i I bring here, and p I minus q I take to this side.

So, I get this equation from this equation, and that gives me that p I plus 1 minus p I is equal to q by p into p I minus p i minus 1. So, I get a recursive relationship between this p I's, okay. And boundary conditions are that if p had no money; obviously, the probability of his ending with any money is 0, because, he will not be able to play the game, alright. So, p 0 is 0 and p 15 because the moment he has won the game and so the probability is 1. So, p 15 is 1, okay, and p 0 is 0, alright. So, therefore, now we use this recursion and we start with I equal to 1. So, when I is equal to 1, this will give me p 2 minus p 1 is equal to q upon p; p 1 minus p 0. So, p 0 is 0.

And therefore, I get this equation, then when I put i equal to 2 in this recursion, I will get p 3 minus p 2 equal to q by p, p 2 minus p 1, but p 2 minus p 1 from here is q by p p 1. So, it will be q by p whole square p 1. And so this way you can go on writing the differences. So, p 15 minus p 14 will therefore become q upon p raise to 14, because whatever the number here and that is 1 less than this. So, the power of q by p is 14 into p 1, alright. Adding up, we obtain; so now I add up all these equations, then you see these things will cancel out in pairs, alright, and you will be left with p 15 minus p 1, but p 15 is 1.

So, this is 1 minus p 1 is equal to and on this side q by p you can take outside. So, it will be 1 plus q by p plus q by p whole square and so on up to p raise to 13 into p 1, alright. And now this is a geometric series; I can write down the sum. So, this will be 1 minus q by p raise to 14 upon 1 minus q by p. Now this is a finite geometric progression. So, therefore, it does not matter. The only thing I need is that I can do this if q by p is not equal to 1, alright, because, otherwise, I will be dividing by 0.

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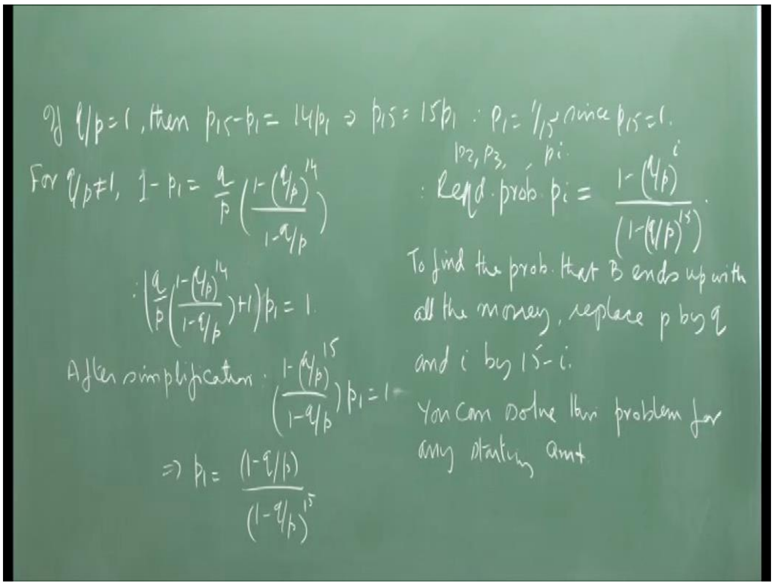

So, if q by p is 1, then p 15 minus p 1 you immediately get is 14 p 1; in the last equation you substitute q by p is 1 everywhere. And so from here, you will get p 15 is 15 p 1, alright, because p 1 gets added here. And, therefore, p 1 is 1 by 15, and now you can easily compute your p 2, p 3 and p I, okay; by going backwards you can compute your this thing, fine. So, we consider the case when q by p is not 1; in that case, what we got from the last slide is 1 minus p 1 is q by p 1 minus q by p raise to 14 upon 1 minus q by p using the geometric progression series sum.

So, if you take p 1 to this side, then you get this. And now you just simplify, and after simplification, you will get that 1 minus q by p raise to 15 upon 1 minus q by p into p 1 is 1. So, p 1 is 1 minus q by p upon 1 minus q by p raise to 15, alright. And so the required probability again from your recursion equations, you will immediately get that required probability p i; that means when a started with I units of money would be 1 minus q by p raise to I upon this, because that sum will be up to q by p raise to I minus 1 and so you will get this. So, you substitute for p 1, so 1 minus q by p will cancel out, and you will get this, alright.

So, now if you want to find the probability that b ends up with all the money replace p by q and I by because now for b the p is q and the money that b starts with is 15 minus I. So, when you make these two replacements, you will again get the formula for using the probability that b ends up with all the money, alright, okay.

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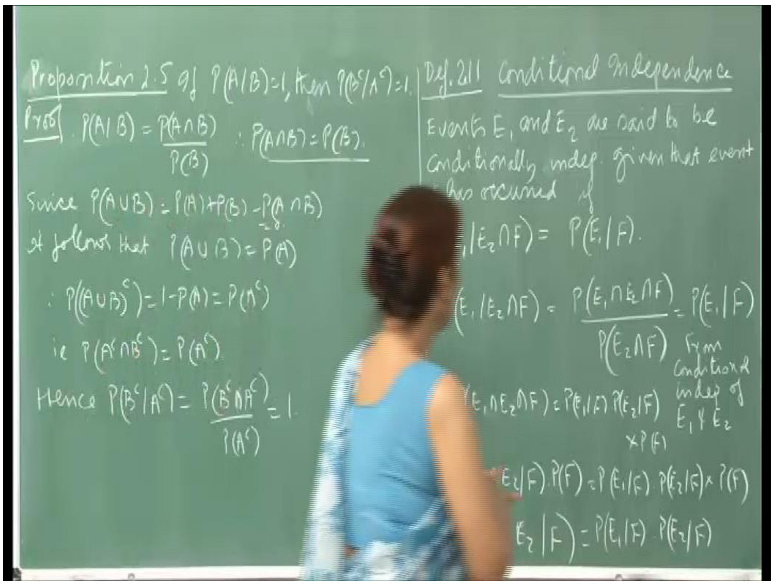

Now again continuing with some more results on conditional probability; so this is proposition of 2 point 5. What we are saying is that suppose you are given that conditional probability of a given b is 1, then you can show that conditional probability of b complement given a complement is 1. So, here I could have also given this as an exercise, but I just thought I will show you some more ways of doing it and then you can apply it to; for example, I think you can show that conditional probability of a complement given b will also be or okay, that will be independence, fine, alright.

So, right now just look at the definition. So, the conditional probability of a given b is this. Now since this is equal to 1, it follows that probability a intersection b is p b. So, the moment you get this result, you can see that a probability a union b which is written as p a plus p b minus p a intersection b this will because this thing is 0 p a intersection b minus p b. So, it reduces to p a. So, therefore, your probability a union b reduces to p a, alright, and, therefore, when you take the complement of a union b. So, probability of a union b complement will be 1 minus p a, which is nothing but probability of a complement and so then again by De Morgan’s law we had seen that a union b complement will be a complement intersection b complement.

So, this probability is, therefore, probability of a complement, alright, and hence, your probability of b complement conditioned on a complement is this. Now since these two things are the same, therefore this reduces to 1. So, one can go on, and, therefore, now the idea would be that you should get interested enough to try out many more results related to conditional probability, solve more examples, okay. Now final result on conditional probability is now this is conditional independence. Remember we defined independence of two events if the probability of the intersection of the two events is equal to the product of the individual probabilities.

Now same thing gets extended. So, we say that e 1 and e 2 are said to be conditionally independent given that event F has occurred. So, if probability e 1 conditioned on e 2 intersection F is probability e 1 conditioned on f; that means the occurrence of e 2 the same definition that we gave for independence of two events. Occurrence of e 2 has no bearing on this probability, alright. So, that means when you condition it on e 2 intersection F, it is the same as conditioning on F and e 2 has no role to play, alright. Now in fact so we will just write it out. So, this is one definition but you can come to a better result and this is we say that e 1.

So, therefore, by definition e 1 condition e 2 intersection F, this probability can be written in this way, alright. And this is given to be by the definition of e 1 e 2 being conditional independent; this is probability event e 1 conditioned on F, alright. So, therefore, from this two, you get that this probability is equal to e 1 F and this probability of e 2 intersection F I can write as e 2 conditioned on F into p f, alright. So, this is the result and this one also again now I can condition on f. So, this will give probability e 1 intersection e 2 conditioned on F into p F and this. So, p F p F cancels out, because remember, whenever you talk of conditional probability with respect to the event, then that probability has to be positive; otherwise, you cannot define it.

So, of course, it is understood that p F is greater than 0. So, therefore, I can cancel out p F here and I will be left with probability of e 1 intersection e 2 conditioned on F is the product of the individual conditional probability is that is probability e 1 conditioned on F into probability E 2 conditioned on f. So, this is you just extend the original definition of independence of two events to in the same way, and so I will take up now an example a little elaborate example to show you the use of conditional independence.

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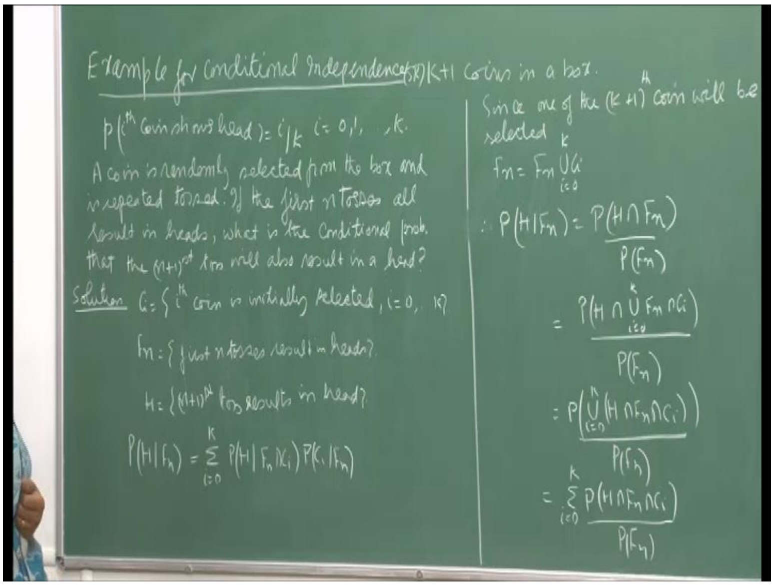

After defining conditional independence of two events respect to the occurrence of another event, I will now take up this example. It may look a little complicated, but it shows good use of concept of conditional independence. So, here again this example I have taken from Sheldon Ross. Surprisingly, this book I will give you the reference later on. It has lot of new and innovative examples, and, therefore, I am using a quiet few of them here in this course. So, now consider the situation when there are k plus 1 coins in a box, right, and the probability of choosing the I th coin there k plus 1 coins in the box and if I pick up the I th coin and toss it.

Then the probability of its showing a head I th coin shows a head is I by k and I varies from 0 1 to K. So, that means, you can see that when i is 0; that means if you pick up the zero th coin, then the probability of its showing a head is 0, which means both sides must be tails. Then if you happen to choose the second coin, the probability of its showing a head would be 2 by k, alright. And similarly, if we choose the coin the k plus 1 th coin which has the number k, then the probability of showing a head would be k by k which is 1. So, probably this particular coin the k plus 1 th coin is having head on both sides. So, whatever it is, the situation is this.

Now a coin is randomly selected from the box and is repeatedly tossed, this should be repeatedly if the first. So, let me just make the correction here. So, it is repeatedly tossed, okay. If the first n tosses all result in heads, what is the conditional probability that the n plus first toss will also result in a head. So, that means I pick up a coin at random from the coins which are there in the box. Then I repeatedly toss it and if the first n tosses have shown heads, then I want to compute the probability that the n plus first toss will also result in a head. So, let us see; we will start finding out how to compute this probability.

So, suppose c I is the event that the i th coin is initially selected and this could be any of the 0 1 to k numbered coin, alright, then F n is the event that the first n tosses resulted in heads. And then h is event that we want. So, n plus first toss results in head. So, I want to compute the conditional probability of h given F n, given that F n has occurred. So, we have had n tosses and now I want to compute the probability that the n plus first toss will also be a head; we will show head. And this is the expression. So, I am going to derive it for you.

Since, one of the k plus 1 th coin will be selected, right. So, then F n can be written as F n union c I because at least one of the coins. So, this probability, probability of c I union c I, I varies from o to k will be 1, alright. So, F n can be written as F n union I varying from 0 to k c I, and therefore, probability h conditional F n which can be written as this, then from F n I can write this expression union F n intersection c I, alright, and again by the distributive property of intersection and union, I get that this can be written as probability of union I from 0 to k h intersection F n intersection c I, alright, because this is this and then this because you see all the c I's are mutually exclusive, right, because one of the coins will gets selected.

So, therefore, these events become mutually exclusive and so probability of the union can be written as sum of the probabilities, I varying from 0 to k h intersection F n intersection c I divided by p F n, alright.

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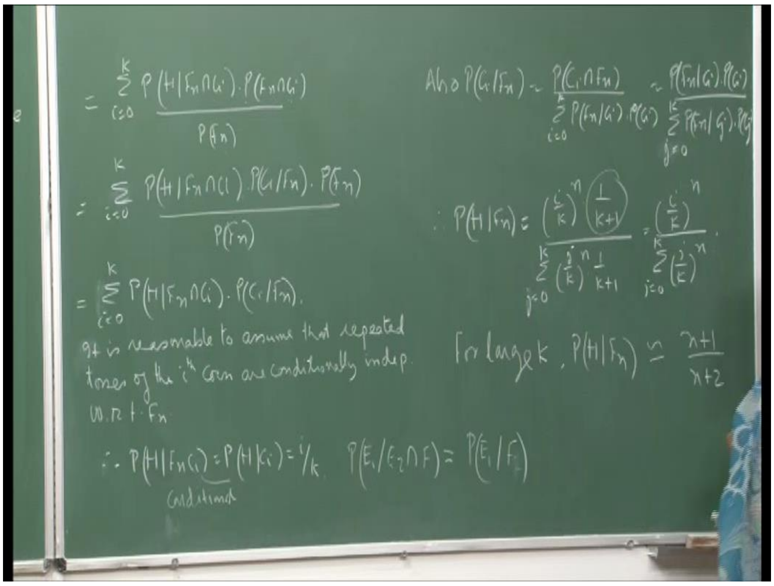

Now this cone I can write in terms of conditional probability as h condition F n intersection c I into probability f n intersection c I divided by p F n, alright. Then this remains this; this again I can write in terms of conditional probability c I condition f n into p f n. So, p f n p f n cancels because f n is given event. So, therefore, probability f n cannot be zero. And so I get this expression which is written here, alright, okay. Now it is reasonable to assume that repeated tosses of the I th coin are conditionally independent. This is I am using the concept; that means to assume that the repeated tosses of the I th coin are conditionally independent with respect to of f n.

That means see I am considering the case when coin was picked up randomly, then it resulted in tosses showing heads. And now when I tossed further, so then they will be conditional independent of f n, okay, which means that probability h condition f n intersection c I is actually probability h condition c I only. So, f n has no main role to play here, right; this is what our definition of remember we said that e 1 given e 2 intersection F. If I wanted to say that e 1 and e 2 are conditionally independent with respect to F given that F has occurred, then this is probability e 1 F; that means e 1 and e 2 are conditionally independent when F has occurred.

So, then e 2 has no role to play on the occurrence of conditional happening of e 1 given F. So, the probability would be independent of the event e 2. So, the same thing here we are saying that here F n because it is conditional on F n. So, this probability is F n has no role to play here and so probability h given c i. So, this is what we are assuming here, and therefore, each of this probability is I by k, alright. So, now that means I can now apply this in the formula here and this is I by k. So, that means you are getting because you have got n heads have shown up and probability of each head is i by k, and I am assuming that the tosses are conditionally independent.

So, therefore, this is I by k raise to n and the n plus first toss gives you a head will be 1 upon k plus 1 because there are k plus 1 coin there. Sorry, so this probability is what am I writing here, yeah, c I f n. So, h given f n so let me just check out here. This is c I f n, right. So, a probability of picking up the I th coin. So, again if we just pick up the I th coin that probability should be 1 upon k plus 1, because any of these coins are equally independent, remember. A coin is randomly selected; so that means any of the coins is equally likely when you pick up from the box. So, therefore, the probability of picking up a coin is I by k plus 1. So, therefore, this becomes this.

And, similarly, here, yeah, so actually what is happening is that c I f n; yes, I missed out on the spot. So, probability is c I given F n I have rewritten as this. So, this is probability F n condition c I into p c I and then this is sigma j varying from 0 to k F n. So, the probability f n I am writing in this way f n condition c j into p c j. So, this is where so now probability f n given c I is since the things are conditionally independent, the probability of picking up a head remains the same. So, when you want to pick up n heads, this will be I by k raise to n and probability c I will be 1 upon k plus 1, right. And then here similarly this is summation.

So, j varying from 0 to k j by k raise to n and 1 upon k plus 1, because now here your j is varying; this corresponded to the I th coin that you have chosen. So, this is the thing. So, this is just a computation I wanted to illustrate maybe you can say that it is an engineered problem or whatever it is, but somehow you could make use of the concept of conditional independence and arrive at this result. And again through methods of calculus, you can actually show that if k is large, then this probability is approximately equal to n plus 1 upon n plus 2, okay.

So, therefore, it gets simplified when you have a large number of coins in the box, but, otherwise, continuously you broke up the. So, here in this expression, yes, c i conditional f n; this also I had to rewrite in this decomposed form and then apply the probabilities to get this expression. So, that was missing here, yeah, okay, fine. So, this is an example and often there will be situations when you would be coming across the concept of conditional independence. Let me discuss exercise two with you; again I will just try to give you brief hints.

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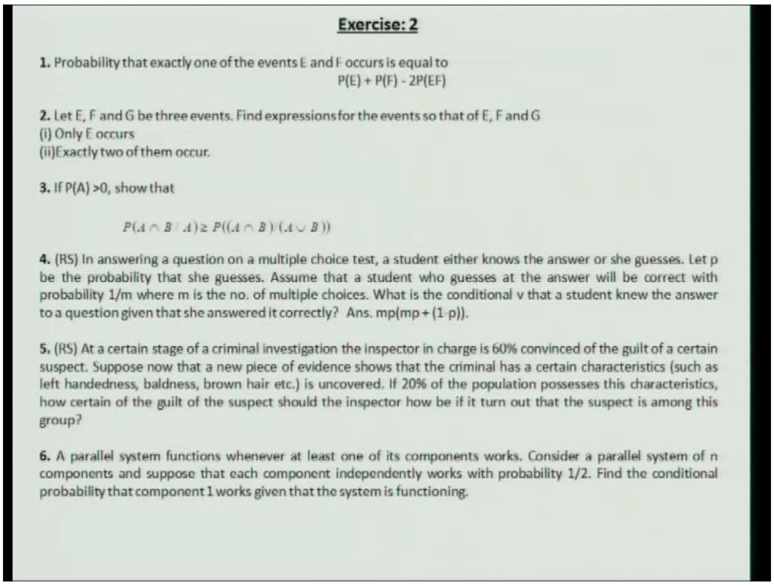

Question one says that you have to compute the probability that only exactly one of the events e and F occurs and that is equal to probability e plus probability F minus 2 and of course when we say minus 2 probability e f; that means e intersection f. So, that notation is also acceptable; you do not write the intersection sign. You simply say e F; so that is what it means, right. Now if e F and g are three events, then you have to find expression. So, again I am just wanting you to be familiar with how you write express events in terms of your complements, union and intersection. So, here I want you to write, find expression for the events, so that of the three events e f and g, only e occurs.

So, if you want to write this, then in the two you have to write exactly two of them occurs, right; exactly two of them occur. So, the third one should not occur. So, you can imagine that you will have to use unions and complements, right. Now in question three, this is actually very simple; the slash sign the condition sign is sort of dim but anyway. So, this says that if probability a is greater than 0, then show that probability of a intersection b condition on a is greater than or equal to probability a intersection b condition on a union b. So, that is very straightforward actually.

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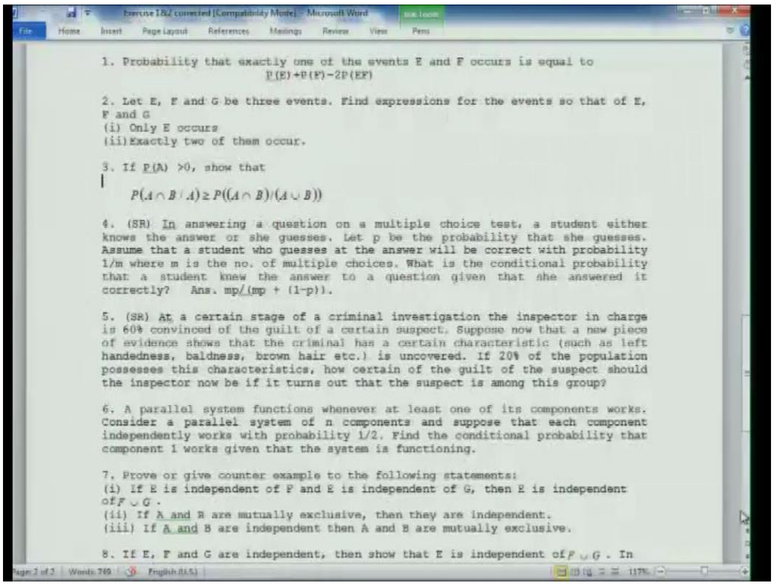

1. Probability that exactly one of the events E and F occurs is equal to

$$P(E)+P(F)-2P(EF)$$

2. Let E, F and G be three events. Find expressions for the events so that of E, F and G

(i) Only E occurs

(ii) Exactly two of them occur.

3. If P(A) >0, show that

$$P(A \cap B / A) \geq P((A \cap B)/(A \cup B))$$

4. (SR) In answering a question on a multiple choice test, a student either knows the answer or she guesses. Let p be the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability 1/m where m is the no. of multiple choices. What is the conditional probability that a student knew the answer to a question given that she answered it correctly? Ans. mp/(mp + (1-p)).

5. (SR) At a certain stage of a criminal investigation the inspector in charge is 60% convinced of the guilt of a certain suspect. Suppose now that a new piece of evidence shows that the criminal has a certain characteristic (such as left handedness, baldness, brown hair etc.) is uncovered. If 20% of the population possesses this characteristics, how certain of the guilt of the suspect should the inspector now be if it turns out that the suspect is among this group?

6. A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component independently works with probability 1/2. Find the conditional probability that component 1 works given that the system is functioning.

7. Prove or give counter example to the following statements:

(i) If E is independent of F and E is independent of G, then E is independent of $F \cup G$.

(ii) If A and B are mutually exclusive, then they are independent.

(iii) If A and B are independent then A and B are mutually exclusive.

8. If E, F and G are independent, then show that E is independent of $F \cup G$. In

And now you have to show that the conditional probability of a intersection b given a is greater than or equal to conditional probability of a intersection b given that a union b has occurred. So, it is very simple, because you see a is a subset of a union b, and as we have already discussed that probability of a union b will be greater than or equal to probability of a, right. And in the left hand side when you compute the conditional probability, you will have a in the denominator. Numerator is probability a intersection b because a intersection b intersection a is again a intersection b. So, this is what actually you have to figure out.

And, similarly, on the right hand side, the numerator is the same, but denominator would be probability of a union b. And since probability a union b is bigger or equal to probability a, you have the required inequality. So, I just gave it to you to be able to just you know figure out these things, and therefore, then you can answer the question. So, if you write the expressions, you can immediately give the answer to this question. Now the problem four is from Sheldon Ross; actually it should say Sheldon Ross or Ross Sheldon. In answering a question on a multiple choice test, you know where you have more than one choice and you have to take the right one.

A student either knows the answer or she guesses. Let p be the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability 1 by m, where m is the number of multiple choices, right, because if she is guessing, she does not know. So, out of the m choices, any one of them is equally likely; so the probability is 1 by m. What is the conditional probability that a student knew the answer to a question given that she answered it correctly and I have given the answer here. So, now what are we saying what is the conditional. So, please enter probability.

What is the conditional probability that a student knew the answer to a question given that she answered it correctly? So, use the concept of conditional probability and then you should be able to do it, right. And now again this problem is from Sheldon Ross; at a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect, okay. So, his conviction is that the 60 percent; that means 0.6 is the probability of the person being guilty. Now suppose that a new piece of evidence shows that the criminal has a certain characteristic such as left handedness, baldness, brown hair, etc.

So, it tells out that through some eye witness who may have seen the criminals of doing the act, the eye witness can only say that the person was either left handed; one of the characteristics is owned by the criminal is uncovered. So, through some facts, it is found out that whoever committed the crime has one of the characteristics, right. If 20 percent of the population possesses these characteristics that means in a town, crime has taken place and so what they are saying is that 20 percent of the population possesses this characteristics. How certain of the guilt of the suspect should the inspector now be? Instead of how it should be now be, if it turns out again, yes, if it turns out that the suspect is among this group.

So, now this is the situation for computing the base probability because you see first initially the inspector is 60 percent convinced of the guilt of a certain person. Now it has been know that the criminal possessed some characteristic which it turns out that this suspect has that characteristic. So, therefore, the probability of the suspect being a criminal would go up. So, the posterior probability after knowing that the criminal possesses the characteristic will go up. So, therefore, I want you to compute the posterior probability here.

Problem six says a parallel system functions whenever at least one of the components work. Consider a parallel system of n components and suppose that each component independently works with population 1 by 2, find the conditional probability that component one works given that the system is functioning. So, here you have to use concept of independence and the conditional probability.

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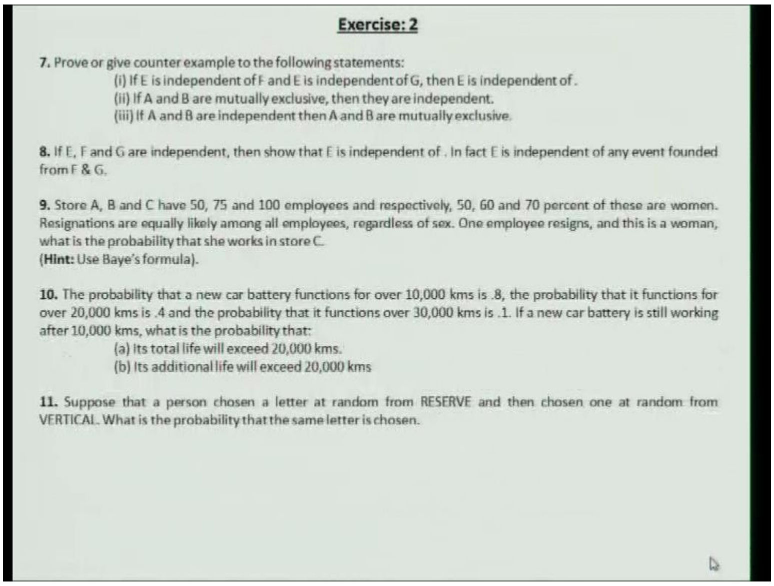

Problem seven says that you have to either prove or give counterexamples to the following statements which are self explanatory. You should be able to either show that the statement is valid; otherwise, you construct examples to show that it is not. Now problem eight I am asking you to show that if e F and g are independent, then you have to show that e is independent of F union g. See remember now here I am using the definition of three events being independent. So, you have these four conditions that will be satisfied and then you can easily show that e is independent of F union g. In fact, whatever subsets you find by operation of you know taking intersection, union or complement and then taking operations on those.

You can show that e will be independent of any of such event which is obtained by doing the operations of intersection, complement and so on from F and g; this is what we are saying here. Now store a b and c have 50, 75 and 100 employees and respectively 50, 60 and 70 percent of these are women, alright, okay. So, that means store a has 50 employees and of which 50 percent are women. So, you can immediately say that 25 are women. Similarly, 75, so 60 percent of the employees in store b are women and then 70 percent of the employees in store c are women, alright.

Resignations are equally likely among all employees regardless of sex. One employee resigns and this is a woman; what is the probability that she works in store c? So, now here again I am asking you to use Bayes formula to compute the probability, okay. So, one employee resigns that is given, and this is a women. So, this is also given; that means a women employee resigns, you have to find out the probability that she works in store c. Tenth, the probability that a new car battery functions for over 10000 kilometers is 0.8. The probability that it functions for over 20000 kilometers is 0.4, and the probability that it functions over 30000 kilometers is 0.1.

So, these are all conditional probabilities. If a new car battery is still working after 10000 kilometers, what is the probability that its total life will exceed 20000 kilometers and then its additional life will exceed 20000 kilometers? So, problem ten, we will have to answer; yeah, okay, maybe we will leave out problem ten from here and we will revisit it later on. But problem eleven you can answer easily. Suppose that a person chooses a letter at random from reserve. So, instead of chosen suppose a person chooses a letter at random from reserve; that means it can be any of the letters r e s v and then chooses one at random from vertical.

What is the probability that the same letter is chosen? So, this of course is your earlier from counting the number of combinations that are favorable to this thing; that means see the two letters that are common between these two words are r and e; that is it, right. So, you have to now find out the number of ways in which r will get selected from both or e will get selected. And you can see that for example in the first word reserve, r appears twice out of r e s e r v e and r in vertical appears only once. So, you can accordingly find out the probabilities and then find out that and since the operation of choosing letter from reserve and from vertical are independent events. The required probability would be the product of these two.