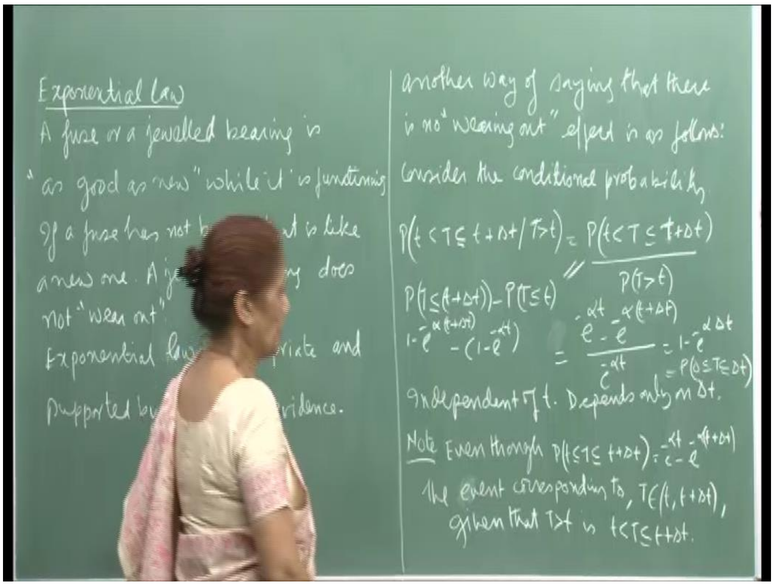
Introduction to Probability Theory and its Applications
Prof. Prabha Sharma
Department of Mathematics and Statistics
Indian Institute of Technology, Kanpur

Lecture - 39
Exponential Failure Law Weibull Law

(Refer Slide Time: 00:14)



So, I will continue discussing the exponential laws very important one and any way there are some many aspects that there may be some repetition also, but does not matter. So, we said that for example, I have been giving the example of queues, which does not show the varying out effect. And are similarly a jewelled bearing you know you have them in watches. So, there is no bearing out effect on such components and they are as good as new while they are functioning right. It does not matter how long they have been functioning they are as good as new. And you have fuse has not burnt out it is like a new one, a jewelled bearing does not wear out.

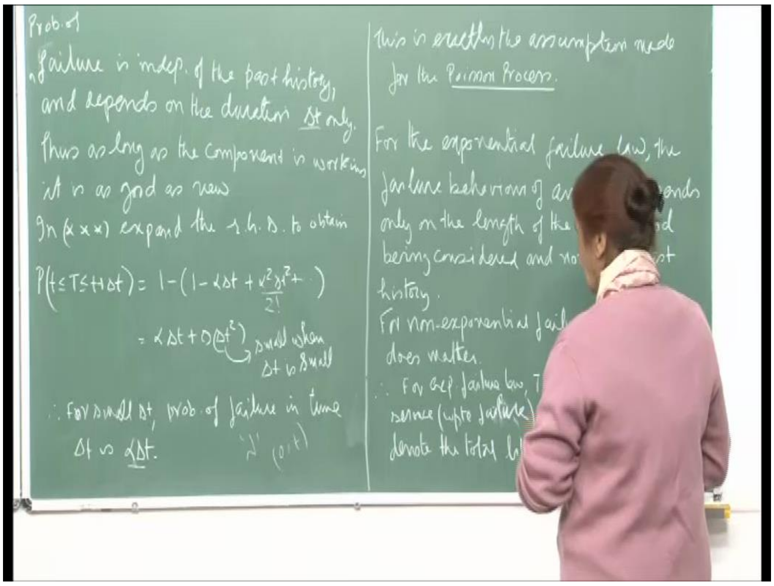
So, this is again and again I am trying to give you examples of component, which for which and therefore, for such components the exponential law is appropriate and is supported by empirical evidence; that means, when you collect the data for such components how long it takes for them to fail and so on. So, then and then fitting the curve to the data it turns out that exponential law is an appropriate one was such components. So, another way of saying that there is no wearing out effect is as follows consider the conditional probability that capital t the life time lies between t and t plus delta t, given that capital t is greater than t right. Now, the intersection of these 2 events is simply this right, because here also T is greater than t and it is less than t plus delta t. So, therefore, the intersection of these 2 events is this. So, then their conditional probability can be written as probability of the intersection of the 2 events divided by probability that t is greater than t right.

Now, this you can see immediately is the probability T less than or equal to t plus delta t minus probability T less than or equal to t so this event right. And since this is 1 minus e rise minus alpha t plus delta t minus of 1 minus e raise to minus alpha t right. So, the 1 1 cancels out and you will be left with e raise to minus alpha t minus e raise to minus alpha of t plus delta t divided by this probability, which is e raise to minus alpha t yes, of course, I am showing you for the exponential law. So, therefore, this is 1 minus e raise to minus alpha delta t, which is nothing but the probability that t lies between 0 and delta t. So, tells you that this probability is independent. So, this conditional probability is independent of t depends only on delta that is depends only on the length of the interval that you are considering; that means, you want to consider.

So that means, the here from t to t plus delta t. So, the length of the interval is delta t. So, this probability this conditional probability is independent of t and depends only on delta t right. So, therefore, this is another way of saying that it is memory less or it the varying effect is not there. That means, is not it does not matter for how long the system is already will working, but now when you want to look at the probability that it will be working in the interval t to t; that means, it fails before t plus delta then that is dependent only on delta t. So, this is the whole idea right. And so we have seen that there are many situations where this is a very appropriate law. Now, see this want to make a note here that even though this will, this probability will all if would say t less than or equal to capital t less than or equal to t plus delta t this will also come out to be the same as this one right.

But since, we are considering the conditional probability that t is greater than capital t. So, therefore, we have to consider this event right and not this event. So, sometimes inadvertently may be one can I may have written it like this, but the when you looking at this conditional probability then it has to be t strictly less than capital t and here it is less than or equal to t plus delta t. So, that would be the right way to write the event and the even though, because of the continuous case the 2 probabilities may come out to be the same. This is the important. So, thus as long as the component is working it is as good as new.

(Refer Slide Time: 04:47)



Now, in this equation suppose, I expand the right hand side so expansion for e raise to minus alpha delta t right. So, all of you should be familiar that much calculus everybody has done. So, you can write down the expansion for the e raise to minus alpha for delta t, which will be 1 minus alpha delta t plus sorry, this should have been 2 factorial and so on, powers of delta t. And then when you open up the brackets 1 cancels out and then will be have alpha delta t plus higher power terms containing powers of terms like delta t square delta t q and so on fine. Now, when delta t is small, we can just ignore these terms and therefore, for small delta t probability of failure in time delta t is proportional to alpha delta t. So, this is what again just reiterating what we have been saying. So, now, in fact, we have given it better expression from here this is more you can immediately you conclude quite a few things from here. So that means, in a small interval no matter where the time interval is length delta t is the probability of a failure in that time period is proportional to time period itself delta t.

Now, this is if you again see I am saying the same thing again which I said. So, this is nothing but your Poisson process. This is the one of the basic assumptions of a Poisson process. In fact, what you can say now here is that, suppose you have a electronic device and you have lot of components, which have the same, which follow the same exponential failure law and they have the identical distribution that is the same the same parameter lambda let us say right. And then your… And of course, the components behave independently. So, that that condition also for a Poisson process is satisfied that is the, you know arrivals are independent. And so here the components will behave independently. So, their failures will also be independent of each other. So, that assumption plus the assumption that with an in a small interval the probability of a failure is proportional to alpha delta t ok.

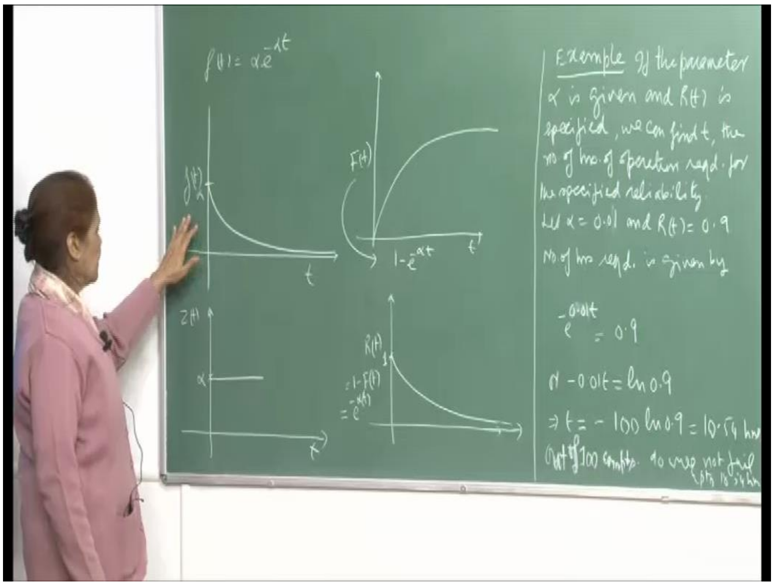
So, then in that case with those 2 assumptions you can then say that, if you are considering let us say time period 0 comma t then the number of failures within this time interval will follow a Poisson process. So, you can see that the arrival and the inter arrival times that will talk in detail later on. So, the inter arrival times and the arrival pattern. So, inter arrival times would be exponential and the arrival patterns would be Poisson and so on.

So, we will continue with that discussion. So, now, because again we want to say this thing now that since the for the exponential failure law, the failure behavior of an item depends only on the length of the time period being considered and not on its past history right. So, therefore, and for the non exponential just like, we have so far considered the normal failure law. So, for non exponential failure laws, we pass this matter it does matter right, when you are under we know stress then the varying out effect is there and so it depends on how long the stress has been and so on.

So, for non exponential failure laws we pass thus matter. So, therefore, it is important to understand that when you talk of the time t, you know life length. So, for the exponential failure law t will denote the time in service up to failure right, because it does not matter when you start counting it is if the component is functioning then you start counting the time from then. So, up to failure t will denote the time it does not matter when you start counting it right, but for other for non exponential components t will denote the total life length up to failure. So, you started functioning from whatever time you count your time form that and. So, capital t will in that case the normal the random variable will be will denote the total life length up to failure.

So, total means at you whenever use start surveys or whenever you start using the item then you start counting your time from here whereas, here it does not matter you start counting from any point and then up to failure. So, the life length will denote that. So, t will denote that time period. So, it is important understand these 2 differences between you know, how you count the time for exponential failure law and for a non exponential failure law.

(Refer Slide Time: 10:09)



Let us look at the various graphs connected with the exponential failure law. So, this is the familiar one right, at t equal to 0 this will be 1 it will be equal to alpha right. And then it will go down this way right, as t goes to infinite this will go to 0, and then correspondingly you see z your failure law of a rate of failure z t is alpha right.

Which is the constant right and. So, it continuous to be alpha value alpha for all values of t this is your t axis this is your z t axis then f t would be of course, so 1 f t would be what 1 minus e raise to minus alpha t right, oh sorry, this function right. So, 1 minus e raise to minus alpha. So, that t equal to 0 this will be 0 and then s t goes to infinity this goes zero. So, the function finally, goes up to 1. And then your r t the failure function ho sorry, the reliability function yes, the reliability function r t, which is 1 minus f t and therefore, this is equal to e raise to minus alpha t. And so here again f t equal to 0 the value will be 1 and then it will go down s t goes to infinity. So, the reliability goes down as t goes to infinity ok.

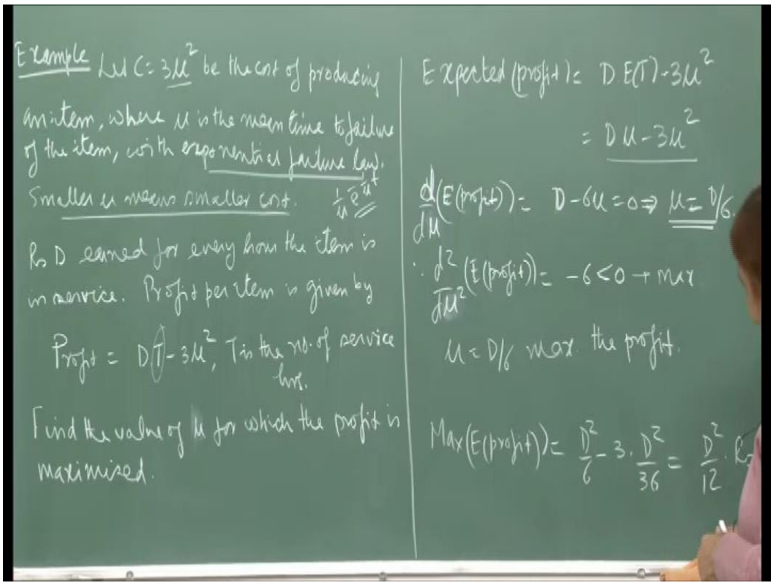
So, this is the graph for the p d f of an exponential failure law. So, t equal to 0 t equal to alpha and then it goes down to infinity as t goes to infinity then we know that the failure rate z t the hazard function this is a constant right. And the constant value is alpha then cumulative density function which is f t equal to 1 minus e raise minus alpha t would again at t equal to 0 it will be 0, because this will be 1. So, 1 minus 1 is 0 and then it goes up to 1, the reliability function would e rise to minus alpha t so at t equal to 0 this is will be 1 and then it goes down to infinity. So, these are the 4 graphs the 4 functions relate with the exponential law and you have picture of the all 4 of them. Now, let us look at this example if the parameter alpha is given and reliability is also given; that means, r t is specified we can find t.

So; that means, given alpha; that means, your specified the failure law and then you are asking for certain level of reliability. So, you want to know what would be the time required for the equipment or the component to operate to achieve that reliability right. So, the number of hours of operation required for the specified reliability level I should say. Now, let alpha be 0.01 and reliability is 0.9. So, you want this to be obtained and your parameter is 0.01. So, if alpha is 0.01 then your mean will be what? So, for exponential distribution the mean is. So, expected t would be 1 by alpha, which is 1 upon 0.01, which is hundred hours, if you are talking if a unit of time is hours then this is hundred hours ok.

So, now, so therefore, the number of offers that are required is given by to achieve this reliability level then you are saying that e rise to minus 0.01 t, remember this is the function reliability function. So, e rise to minus 0.01 t should be equal to 0.9. So, you want that value of t, which will satisfy this equation. So, you take log of both sides and this will then give you minus 0.01 t is equal to l n of 0.9 and remember l n of number less than 1 is negative. So, therefore, this is so this is minus sign here right. And so if you, now, divide by 0.01 then you get that the t is equal to hundred into l n of 0.9.

Which comes out to be if you look up the values of l n 0.9 then multiplied by hundred that gives you 10.54 hours. So, therefore, the way you can depict this is that out of hundred components all above working simultaneously and ninety what we are saying is and if they all operate for 10.54 hours then our expectation is that ninety of them will not fail. So, after 10.54 hours ninety of them will be working. So, this is what we mean by the reliability level and so on. So, therefore, you know essentially it is question of given what and then what will you can compute. So, sometimes you may be given the time then you can compute the reliability level right. So, if you given t then you will be able to determine this number and if you are given this reliability level then you can give determine the time. Or if you given time and reliability then you can determine the parameter; that means, you can uniquely determine the exponential failure law, because you only need the parameter alpha to determine the failure law, exponentially failure law ok.

(Refer Slide Time: 15:55)



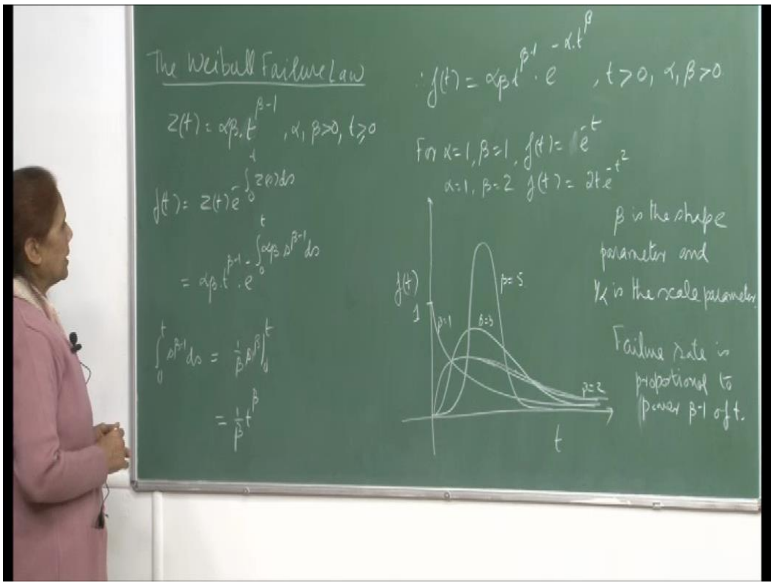
Let us look at another example. So, here you given the cos c interms of mu so c is 3 mu square be the cost of producing an item, where mu is the mean time to failure. So, again we talking of exponential distribution, exponential failure law and mu is the mean. So, therefore, if mu is the mean then remember the distribution the failure law would be minus 1 by mu t right, this is what you have. Now, this cost is 3 times mu square. So, which means that, if mu is small then the cost is small, if mu is large then your cost would be accordingly large right ok, which makes senses.

So may be, because if your mean time to failure is small then you expect that the cost is also small. And if the mean time to failure is big is large number and; that means, you expect the component not to fail very early I has a long life time and then that case the cost of producing that item would be also high. So, it is reasonable to assume the cost in this way right. Now, suppose rupees d are earn for every hour, the item is in service. So, you earn a profit of d rupees per hour, when the item is functional. Now, you want so therefore, profit for item is given by the profit would be d in to t, if the life time is t hours then d into t minus the cost 3 mu square right. And t is the number of service hours right. So, find the value of mu for which the profit is maximized.

So, expected profit so certainly, because this is a random variable so we will maximize the expected profit. So, the expected profit is these right, because d of e t this is a, I mean this is not a random variable this would be some fix number. So, then d of e t minus 3 mu square and e t is mu. So therefore, d mu minus 3 mu square so exactly what ever, I saying that yeah. So, this is, so therefore, to maximize this expected profit, I would differentiate this is respect to mu is a function of mu and put it to zero. So, that gives me d minus x mu is equal to 0, which implies that mu is d by 6. And of course, only critical value and, but still you need to verify the d square by d mu square is of the function is minus 6, which is less than zero. So, therefore, the critical point is a point of maxima. So, the value that we get of mu here is value which maximizes the expected profit ok.

So, therefore, u mu is equal to d by 6 is maximizing the profit and the maximum profit is d square by 12 rupees. So, just trying to give you a feeling for the failure law, exponential failure law and the kind of problems that can be discussed and that arise corresponding to these. So, again the level is at very basic you know, the level is very basic, because this is just trying to give you a gleams of how these probability tools at we have learnt can we used for answering so many questions about day to day operations of you know systems, service systems and so on. So, this is the whole idea right. Otherwise reliability theory has become very complex and in fact, the next failure law that we will discuss is a complex one. And we just try to understand the basics of that of the Weibull failure law.

(Refer Slide Time: 19:55)



So, let us the talk about the Weibull failure law and you can see that. Now, the degree of complexity is going up and we have not so for discuss this distribution probability law also. So, let us look at it and the idea here is to, because the constant failure rate was only applicable to a special kind of components, which were not let us say, for which there was no wearing out effect right. So, that was a special situation and we have seen that there are so many almost electronic devices or you know behave that way and so the exponential noise appropriate for them. So, now, if you want to modify this constant rate then the Weibull failure law was thought of and this is alpha beta t raise to beta minus 1. So, now, you have introduced power of t of the time and of course, two parameters and they can more than 2 also. So, we are talking of a 2 parameter Weibull failure law. And so here alpha and beta are positive and t is greater than 0 as usual, because its 3 time lifetime wherever. So, therefore, has to be non negative.

Now, you can look at I have drawn various pictures of z t when for values of beta and different values of alpha and beta. And so may be will just look at it, let me just first compute your f t. So, remember we said that we can compute f t uniquely given the failure rate functions z t. So, then this z t e raise to minus integral of 0 to t of z s d s integral 0 to t of z s d s right, this is our formula for computing f t given z t. So, then here if you substitute for z t this is alpha beta into t raise to beta minus 1 e raise to minus integral 0 to t alpha beta s raise to beta minus 1 d s right. So, if you just look at this integral let us just compute. So, this will be s raise to beta minus 1 d s from 0 to t and the integral here is 1 by beta is raise to beta 0 to t. So, therefore, this is 1 by beta t raise to beta.

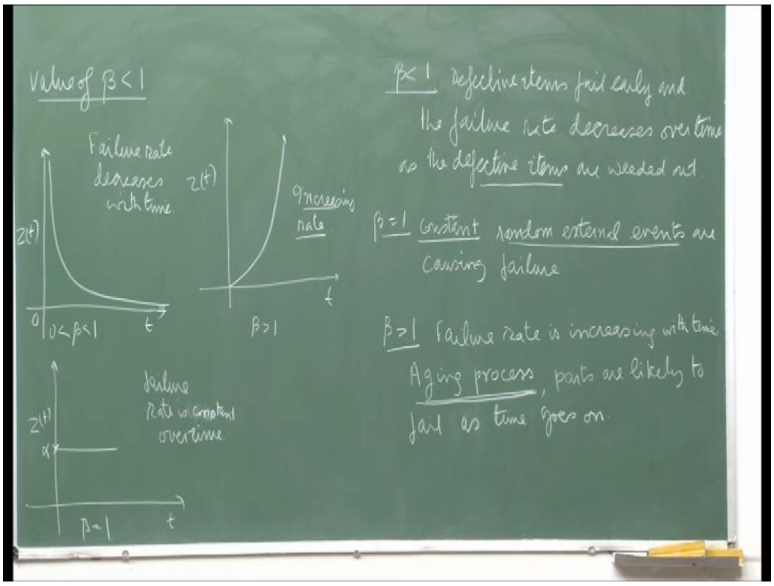
So, if I make that substitution here, I get my f t s this. So, this is the p d f connected with a Weibull failure law and the failure law is specified there. So, yeah the weight looks it is it will complicated, but just see when you put alpha equal to 1 and beta equal to 1, alpha equal to 1 beta equal to 1 then this is 1 and this will be e raise to minus t. So, e raise to minus t would be your yeah, so this would be your p d f right. The exponential with parameter 1 and that is the 1 I have drawn here right. Then if you look at the value alpha equal to 1 and beta equal to 2. So, therefore, in this case the, beta equal to 1 this will be exponential and the failure rate will be constant which will be 1. So, of course, here I have drawn it only for beta equal to 1. So, alpha is as it is, this 1 is of course, you put alpha equal to 1 also. So, anyway this is your function for the failure rate then when you put alpha equal to 1 and beta equal to 2.

Then I have drawn this one, this is the 1 for beta equal to 2, of course, not a very accurate, not very accurate figures. So, you can always Google search and then you can find nice pictures very accurately drawn graphs. So, beta equal to 2 now, you look at this thing here this is alpha 1 beta equal to 2. So, this is twice t; that means, it is a linear function of t. So, therefore, you see of different values of beta things are changing. So, if alpha equal to 1 beta equal to 1 you got constant, alpha equal to 1 beta equal to 2 you get and in fact, any value of alpha, alpha does not have to be 1. Then in that case it will be a linear function of t right, if beta is beta is 2 right. And so I have here of course, I have drawn it for alpha equal to 1 and beta equal to 2, the diagram then and the corresponding p d f will be 2 t e raise to minus t square.

Then for beta equal to 3 for example, I have drawn the picture here also for beta equal to 3. So, then this starts taking a bet more well shape and beta equal to 3 alpha equal to 1 will be 3 times t square. So, that will be; that means, z t is quadratic function of t and correspondingly you are this thing will be. So, here when beta is 3 then of course, this is t square and then e raise to something right so quadratic into exponential function. Now, and for beta equal to 5 for example, it will become more steep like this and then as beta goes to infinity, you see, you can show that will simply be a spike no just a spike, because beta is going to infinity sorry, I mean yeah, beta is going to infinity then this will simply just spike into this thing, which will, which you call as a delta function. So, at the point yeah, so some way here it will become a spike as p term goes to infinity yeah.

So, the thing becomes narrow and narrow or as your beta goes up. So, here beta is the shape parameter therefore, you see beta is shape parameter and 1 by alpha is the scale parameter. So, why you scale the whole thing right; that means, when you drawing the thing when you. So, failure rate is proportional to power of power beta minus 1 of t. So, therefore, this is the generalization that we have made to the constant failure rate and so this is the failure rate is now proportional to t raise to beta minus 1.

(Refer Slide Time: 26:33)



So, you can see that yeah and then this also gives you that good feeling. Now, if an example, if your beta is less than 1, if beta is less than 1 then this will become negative. So, t will be in the denominator and so alpha beta upon t raise to 1 minus beta. And that will be for as t goes to infinity.

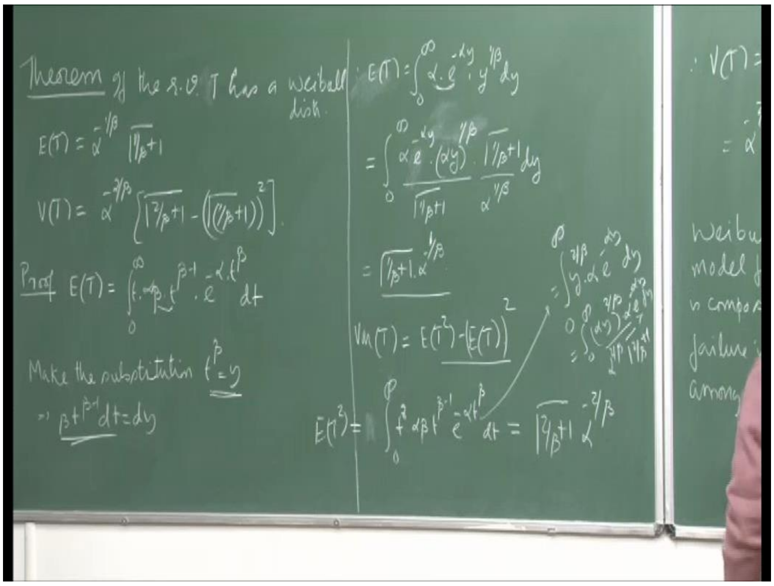
So, the denominator will go to 0 and therefore, this will let me show, alpha beta upon t raise to 1 minus beta right. So, as t goes to 0, this goes to infinity, the denominator t goes to 0, this goes to infinity and so no it should be the other way. I want to show that for beta less than 1 for beta less than 1 this is negative. So, when I take it here it will be positive this power is positive and so as yes, as t goes to 0, this goes to infinity, because this goes to 0 so this goes to infinity as t goes to 0 right yeah, this should be. And therefore, it is this way and then s t goes to infinity this goes to 0 and so failure rate decreases with time. Now, failure rate decreases with time let me see essentially defective items fail early and the failure rate decreases over time as the defective items have been we did out. So, all the defective items have been we did out from the population. So, that case the failure rate will decrease with time right. So, all defective items fail early and therefore, as time progress is your failure rate will decrease.

So, this is the situation that is gets small moderal when you by putting beta equal to less than 1. So, all values of beta between 0 and this thing here right. So, we should take this, then this is the situation it will suppose to moral and beta equal to 1 we have already discuss thoroughly now, here of course, since the failure rate is constant. So, as time goes on the failure rate does not change. So, this is because random external events are causing the failure that could be one of the reasons. So, random external for example, if a fuse, fuse will blow out if the high current comes suddenly in the line right.

So, therefore, that is an external event and many others can be explained high wind and so on, for other you know ten high tension wires you can snap and so on. So, therefore, beta equal to 1, because the failure rate is constant it is understood that external events would cause the failure could be the reason for the failure. And for beta greater than 1 as we have seen failure rate is increasing with time. And therefore, this modules the situations were aging process has a role to play in the failure of the system. And that is parts are likely to fail as time goes on and this is when the stress part.

So, you see this certainly captures it is a more complex and it will more comprehensive failure law, which captures more than one situation. And you can play around by manipulating the value of beta and alpha and try to get accurate results. So, this is the whole idea and we will continue with the discussion on Weibull distributions.

(Refer Slide Time: 30:13)



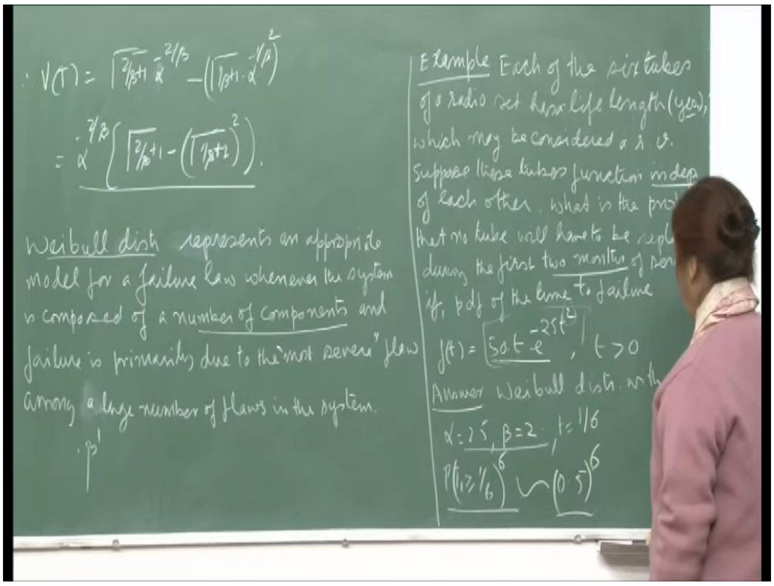
So, let us make this have computations about the expected value and the variance of a Weibull distribution. So, e t the theorem says that e t is alpha raise to minus 1 by beta gamma of 1 by beta plus 1. So, we are know the gamma function and then v t is the variance would be alpha raise to minus 2 by beta, gamma of 2 beta plus 1 minus gamma of 1 beta plus 1 whole square right, which is we are using the formula that variance is e t square minus expected value of t square minus expected t whole square right. Now, so just apply the, because we have already computed the f t the p d f for t when t e has a Weibull failure law. Then this is t into alpha beta t raise to beta minus 1 e raise to minus alpha into t beta d t right ok.

So, yes so now, you can see that, this t beta minus 1 and t beta. So, this prompts you to make the substitution that t beta is y no so we will do this. So, again you are familiar with this part of the calculus you can do this integration. So, t raise to beta is y that will make beta t raise to beta minus 1 d t is equal to d y the limits will not change will remain from 0 to infinity, since beta is positive. So, for t equal to infinity y will also be infinity and t 0 y 0. So, now, therefore, this thing whole thing gets replaced by d y. So, you have a beta t beta minus 1 and d t. So, this we will replace by d y, when you are left with t and alpha and then e raise to minus alpha y, because t raise to beta is y. So, therefore, this, what you have, alpha e raise to minus alpha y and y raise to 1 by beta and there is a yeah, so y raise to 1 by beta, because you have a t here. So, t will be y raise to 1 by beta. So, therefore, this is what you have. Now, you see this looks familiar, because you can now relate this with the gamma function gamma p d f. So, here alpha e raise to minus alpha y then you have to have alpha y here as the variables alpha y raise to 1 by beta.

Now, y 1 by beta is there so alpha 1 by beta m adding here. So, therefore, I will divide by alpha 1 by beta and then I need a gamma of 1 by beta plus 1. So, this integral d y to be 1, because this is the p d f of a gamma distribution with parameters alpha and 1 by beta. So, then be I am left with this and this final. So, therefore, the expected value of the random variable t where t is has a Weibull failure law is given by gamma of 1 by beta plus 1 into alpha raise to minus 1 by beta right. And then the second part should be straight forward. So, you had this. So, therefore, I need to compute expected value of t square and so that will be t square alpha beta t raise to beta minus 1. So, now, you see you are again beta t raise to beta minus 1 and e raise to t raise to beta would become this thing yeah I am sorry, I mean beta t raise to beta minus 1 d t that will be d y from here right.

And then you have t square so that will be y raise to, I am not written the integral here. So, anyway this will that is reduce this will be equal to 0 to in, I have written t this is 0 to infinity yeah, this will be 0 to infinity. So, y raise to 2 by beta and then you have an alpha and then you have t raise to minus alpha y d y right. As so here again I will do the same trick that I did here. So, alpha e raise to minus alpha y is there, then this you need to write this 0 to infinity alpha y raise to 2 by beta alpha e raise to minus alpha y this is the same thing. And then you will divide by alpha raise to 2 by beta then you need a 2 by beta plus 1 right and you will multiply by 2 raise to beta y gamma of 2 beta plus 1 2 by beta plus 1 and so this whole thing will be and there is a d y right. So, then you will be left with gamma of 2 by beta plus 1 into alpha raise to minus 2 by beta. So, this is the expected value of t square right. And therefore, the variance will be this minus the expected t whole square.

(Refer Slide Time: 35:01)



So, you take out alpha raise to minus 2 by beta common and then you have so this is just for you now come. And they here again we could make use of the gamma distribution computations to compute the expected value on the variance. Now, again so we have seen that Weibull distribution represents and appropriate model all if failure law whenever the system is compose of number of components right. And the failure is primarily due to the most severe flow how many large numbers of flows in the system. So, this is what is happenings, you have lot of components and lot of parts in the device that you have using and each one of them has a flow, but then it will be govern by the most severe flow of among all the components.

This is what the, you know you can say that this is the representation of there is a situation, but which Weibull distribution represent and your alpha beta. And of course, you have seen that you know by changing the values of beta you can either have a increasing failure rate or a constant failure rate or a decreasing failure rate. So, this we have already seen right and yeah, so here you know I have just being able to give you, you know, short glames into the how what these distribution can do and one needs to really work at lot of examples to understand the implications or the importance or the importance of this distribution.

Now, let us just look at this example. So, each of the 6 cubes of a radio set has a life length e years. So, our time unit is year which may be considered a random variable. So, the lifetime of a radio tube in a radio set in number of years should we consider as a random variable suppose these tubes function independently of each other. So, that is important where working independently of each other what is the probability that note you will have to be replaced during the first 2 months of surveys. So, will after you know translate this to years, because the, our unit of time is and years. Now, the p d f of the failure time to failure is given by this 50 to t e raise to minus 25 t square so immediately you can recognize that, this is Weibull distribution and since t has power 1. So, your beta is to 2 right, this is beta is equal to 2, because t raise to beta minus 1. So, therefore, if beta is 2 then alpha is an of course, from here alpha is 25.

So, this is alpha beta, alpha beta into t raise to beta minus 1 e raise to minus 25 t square, because this is beta. So, this is the Weibull distribution right. Now, of course, we have not made this computation for the Weibull distribution, but certainly you can do it and may be you can use in my recover method. So, essentially yeah, it is a Weibull distribution with these are the parameters then you since the tubes of functioning independent of each other and you do not want and of course, let me say that your t is 1 by this is 2 months so 1 by 6, which I have written it here right. So, t is 1 by 6 you want to, you have capital t right.

So, yeah, so we want their 6 tubes there working independently of each other we do not want anyone of them to fail so therefore, in the first 2 months. So, the probability of let us say the first tube not failing in the first 2 months is t 1 greater than or equal to 1 by 6. So, time unit is 1 by 6. And then since all of them are independent of each other we do not want any of them to fail. So, then this would be probability t 1 greater than or equal to 1 by 6 raise to 6 so that is we will, you know this is not a difficult integral again you know, because see you have seen my computations here for e t and variance t. So, you can just use those you can use the gamma distribution computations to do the computations here. And then you can find out this probability raise it to 6. So, the answer is approximately 0.5 raise to 6. So, this integral you can handle now, since I have given you the method out computing e t was so exactly it just twice on to that. So, different per values of t and so on right ok.

So, now, this does not exhaust the failure loss, I have only as I told you and I have been repeating it that we have only considering very basic failure laws here and my of course, aim was to since we have discussed the probability theory and so the various tools you have learnt about. So, I just thought that I would like to show you the various applications also of these tools that we have learnt in the course. So, that has been the whole you know, whole idea the theme across the course that you learnt the theory and then you learnt to muse it also. And so Markov process is discrete Markov processes then we will we have talked about continuous Markov processes and the process special case is which are Poisson and exponential distributions and then birth and death process.

Since and finally, applications to reliability theory and here also the basic concepts have been given to you, but as I said it is a very growing large growing area very important and a lot of applications of. Of course, probability theory and the many more people have come up with failure laws, which probably supplement the theory that has been discussed.