Introduction to Probability Theory and its Applications
Prof. Prabha Sharma
Department of Mathematics & Statistics
Indian Institute of Technology, Kanpur

Lecture - 38
Application to Reliability Theory Failure Law

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So, now the last topic that I want to consider in this course is applications to reliability theory. That means, again the concepts that we have learnt during the course of probability theory, want to show you some applications of course, some, everything may not be new, but still we want to put them altogether, so that you have a good feeling about.

And of course, see the reliability theory is very, it has become very important now with systems, the systems, very complex systems, you know, coming in, and then besides that you know, you have lot of dependence on the systems' functioning, otherwise if something fails, then lot of things connected with it also fail and you know, there is a chaos. So, so there is a very growing area and we see that the applications are really interesting and very meaningful. So, you can model situations here also through the tools, that we have learnt during the course.

Now, so first of all let us first understand what we mean by reliability of a system. Now, just for example, consider a steel beam under load. And so you have whole structure, you know, resting on a beam and there is a heavy load, then you have a fuse inserted into a circuit, so the source of stress. So, you know, there are systems and then there are some source of stress. So, here and for example, an aeroplane wing, the planes fly at high speed, there is a lot of friction and so there is some sort of stress on the aeroplane wing also. Then, electronic device you have, this is, when it is functioning again, that is, that is sort of a stress and therefore, so one can go on writing. The number list can be very, very big, you know, anything that you use as a tool and then.

So, when, when you are using it there is some stress, some kind of stress on the tool and so things will change or things will happen. So, that means, so you have source of stress, you have a system that means, a component or a whole system, and then you talk of state of failure. So, what can happen? The beam can crack, right and similarly, a fuse can burn out or the aeroplane wing can buckle because of pressure or stress and an electronic device can just fail. You suddenly find, that your PC is not working or your, you know, electric kettle is not working or whatever it is, right. So, therefore, you know you can write down your own this things. You can take a component or a system, you can talk about what kind of stress that system is facing, and then you can also define the way the system will stop functioning.

So, essentially what we are saying is, that we have this. So, now here we can and suppose we, I define, so therefore, so we need to talk about time to failure or life lengths and it can, it can, it will be a random variable and why because you see, if you take identical components under identical stress may fail at different and unpredictable times. You really have no idea, sometimes things just stop working. So, some fail early, some may fail at later stages and so on.

So, therefore, you can see, that whatever the systems we have talked here and talked about and many other, you see, that you cannot really, of course, sure say, that ok, this component or this system will work for so long. So, there is lot of unpredictability if they are not. So, these things cannot be predicted very well. And therefore, because you never know how the stress works on a particular component or a system and also the manner of failure also varies, as I told you, because the beam will crack, the fuse will burn out and so on, ok.

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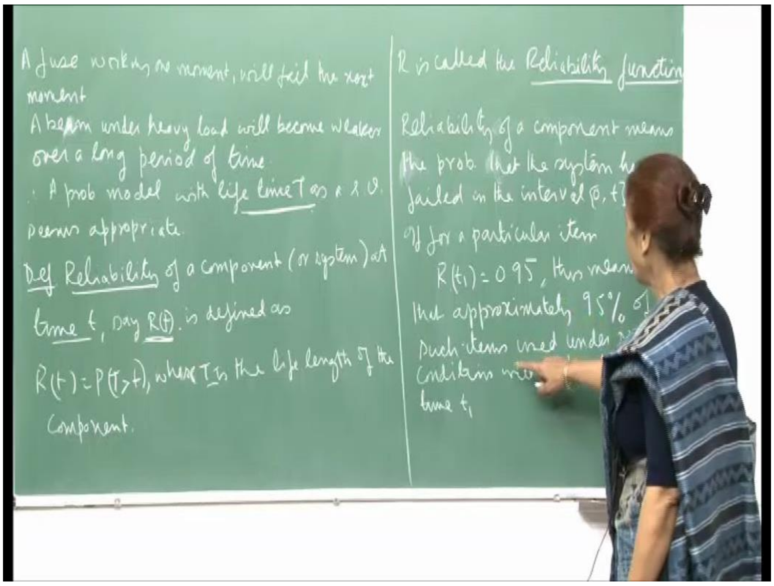

So, yeah, so the manner of failure, you can say, fuse working one moment will fail the next moment, right. Suddenly you find, that the thing is not working. You are having good output, the radio was functioning very well, broadcasting and suddenly it goes away, goes out. So, that the beam, you know, steadily over a long period, it becomes weaker and weaker and then cracks, right. So, here again and and many other situations you can talk about. So, therefore, it is, it is reasonable or it is appropriate to, to construct a probabilistic model and then treat the life time of the system or the component as a random variable. So, this seems to be very appropriate.

And now, we will, what we will do is, we will talk about different models, that we can, that we can use for predicting or for the life time of a component or a system and we will be using lot of tools that we have learnt so far. But so first of all let us define because now with the understanding, that what we mean by system failing and the manner of failure can be very different and the unpredictability of these components or systems failing under different situations. So, we can now define reliability of a component or a system at time t say by R t. So, we will define.

So, R t will be the, we will define the reliability of a system and we will say R t is equal to probability T greater than t. So, at time t we want to know the reliability of, of a system, then this is equal to probability T greater than t, where T is the life length of the component. So, suppose let us, we are talking about a component, then T is the life length and you want to know the probability, that T will be greater than t and R t is defined as the reliability function. So, this is the reliability function. And so for different values of T we can get the value of this function, which will tell us the probability that the system is functioning at that time, right.

So, essentially when we say, that R t, when we are computing this way this probability, so if T is the life length, that means, here it is saying, that life length, this more than t. So, that means, at time t when you are computing this, the system is functioning, right. So, the probability is not 0. Well, I am not saying that. I am saying, that probability, that T is greater than t, that means, the life length is greater than… So, the system is still, that is how you will interpret this because this is the life length. So, life time of the component is greater than T. So, at time t, that means, it is functioning, the system is functioning. So, this is what, right.

Now, for a particular item, suppose R t 1 is 0.95. So, at time t 1 you compute this and it turns out to be equal to 0.95. This means, that the, that approximately 95 percent of such items used under similar conditions will be functioning at time t 1, that is what we mean. So, the 95 percent of the items will still be functioning, will be functioning at time t 1. So, this is what we mean by. So, probability being 0.95. So, probability, that the life length is more than point is more than t 1. So, this here, this you will write as equal to probability, that t is greater than or equal to t 1, sorry, T greater than t 1. So, that is equal to 0.95. So, this probability is 0.95 that means, 95 percent of such items. And the similar conditions will be functioning at time t 1 and we continue with the…

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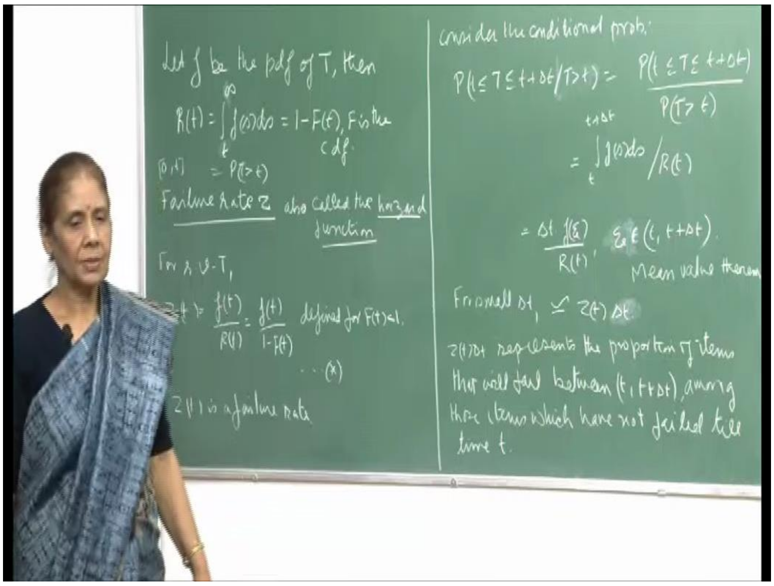

So, you see, we saw that if f is the pdf of T, the random variable t, probability density function. So, in that case we can write the reliability function R t as t to infinity because remember, this is probability T greater than t, right. So, therefore, this will be the integral of f s from t to infinity. So, because we were saying, that this definition tells us, that the reliability gives you the probability, that the system is functioning in the interval 0 t.

So, it has not, that it is not, sorry, the way to say it is, that the reliability tells you, that the system is functioning, has not failed in the interval 0 t. That is the better way to put it, right, because the life time is greater than t that means, the system has not failed in this interval, right. So, it can fail anywhere in the interval t to infinity, that is what reliability is, right, probability T greater than t. So, that will be written as t to infinity integral of f s from t to infinity, which it is a capital F as we denote the capital F as the cdf of cumulative density function of t, then this is 1 minus F t, right. So, this is what we can express the liability function as.

Now, we will associate another function with reliability and which is with the random variable t, which is the in failure rate Z and this is also called the hazard function. Now, if you recall, while talking about exponential distribution, I had introduced this hazard function and of course, we did not talk much about it, only I simply I showed you, that if the, if the pdf of random variable t is exponential, negative exponential, then the hazard function will be a constant. And so we will see many more interesting kinds of hazard functions. And, but anyway, we had come across this at that time, but now I am using this, that is why I said, that some of the things, which I talk about here may already have been discussed, but we are putting them all together, so that it becomes a complete unit.

So, now, failure rate we are defining as Z t is equal to f t upon R t. So, the pdf divided by the reliability function, which you can write as f t upon 1 minus F t, right, from here defined. And of course, this is defined for F t less than 1 because this, remember F t is your, F t is probability, T less than or equal to t. So, if F t is equal to 1, that means, and so this will say, that the life time is less than or equal to t. So, if this is 1, then this, that means, it is a certain event, f t equal to 1. So, f t equal to 1 implies, that by time t, the system has definitely failed, is a certain event, right. So, therefore, this has meaning only when the system is still functioning. And therefore, this is reasonable condition to impose, that this is defined for F t less than 1 because F t equal to 1 would mean, that the system has or the component has failed. So, therefore, so this is a failure rate.

And why, why is this, why are we calling it is a failure rate? This is the definition, but now let us understand why this does covers the failure rate of the system. So, consider the conditional probability T, capital T lying between small t and t plus delta t, given that capital T is greater than t. So, since I am asking for, see the failure rate has to be after time t. So, here you have to consider the inequality, strict inequality. I cannot allow equality here because my conditional event is, that T must be greater than t. So, that is why, you cannot write less than or equal to here.

So, this is what we have to consider this, I mean, what I am saying is, that the event should be described properly. So, then the probability, this less than or equal to, so t lying between, so this is strict inequality and that is less than or equal to t plus delta t. So, then by our rule, because this and this when you take the intersection actually means just this event, right, because capital T is greater than t is already satisfied, which is here. So, therefore, this conditional probability can be written as probability capital T between small t and t plus delta t divided by probability T greater than t.

And this integral form, if I take f to be the pdf, then this would be my failure law. Then, this will be t to t plus delta t f s ds divided by R t, right. Now, for small delta t, you see, I can take this to the, by the mean value theorem or ok, here you do not need small delta t. So, by the mean value theorem of integral calculus, this integral can be written as delta t, the length of the interval of integration into. So, there exist a value, a point, real number psi between the interval t and t plus delta t such that this integral can be written as delta t into f psi. So, this is well known for a mean value theorem of integral calculus divided by R t.

Now, for small delta t when I can say, that this approximately Z t into d because your failure rate is f psi of f t upon 1 minus f t. So, then I can, because my delta t is small, then this interval is very small. So, I can treat this as the value at t. And therefore, this will become f t because if this interval is small, then your psi is close to t. And so approximately I can say, this is equal to Z t into delta t because your Z t is f t upon R t. And so this is what your and therefore, Z t represents the proportion of items, that will fail between t and t plus delta t, right. That means, during the time span delta t, this is the proportionate, proportion of items and that is why, we call it the failure rate. So, among those items, which have not failed till time t, remember, because we are computing this, so this is P T greater than t.

So, all items, which have not failed till time t, then the proportion of those, which continue to work in the interval t t plus delta t. So, that is represented by Z t. So, this is, you know, an interpretation of what I defined here, Z t as f t upon 1 minus f t right. So, this was the conditional probability. So, life time is more than t and therefore, this is, so therefore, Z t is the rate of failure, right. So, that make sense, right.

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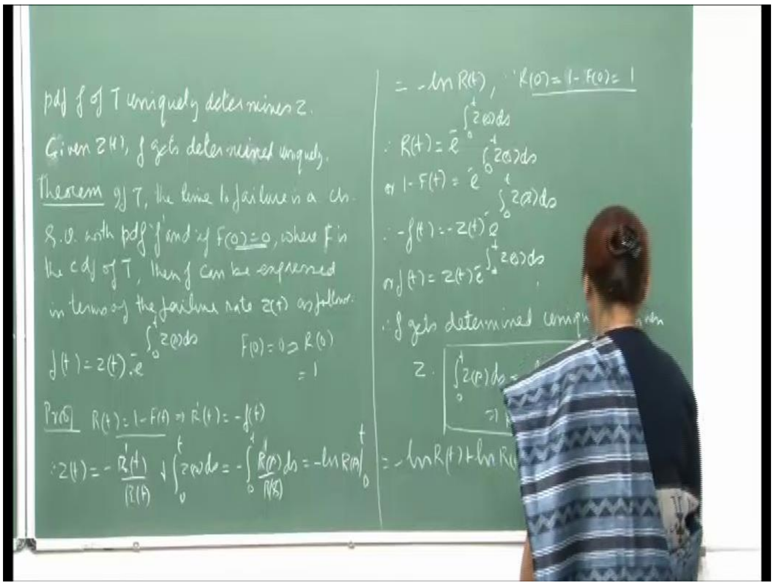

Now, we could, we could define Z t given the pdf f of T, right. So, given the pdf of T, that means, the f determined Z t, Z uniquely, the failure rate. The converse is also true, that means, if you are given the failure rate, then you can determine your T uniquely, determine your pdf f uniquely. And once you know f, then you know, the cumulative density function also, right. So, let us see that.

So, the theorem is, that if t, the time to failure is a continuous random variable with pdf f and if f 0 is 0, where f is the cdf of t, sorry, where capital F is the cdf of t, then F can be expressed in terms of the failure rate Z t as follows. So,, then F t can be written as Z t into e raise to minus integral 0 to t of Z s d s. So, that means, once I know z, then through this function I can compute my pdf. So, if, if suppose you are, you somehow know the failure rate, you know, empirically or somewhere, then you can sort of compute the pdf also of T, right. And of course, this make sense.

What, what are we saying, that f 0 0 implies, that R 0 is what, 1. So, that means, certainly the function, this is, we are not going to be, the time is 0, then your system is not going to fail. It will take some, require some time. We will start the function, the system working or the component is working, then only after little lapse of time this is a possibility, that the system may fail or something. So, the reliability will be 1 at 0 time. And so this is not< you know, this is a reasonable assumption, that F 0 is 0 because you are talking of reliability and the reliability comes only when the system starts functioning. So, some time has to lapse before you can say, that ok, the component has failed.

So, with this condition, now let us start computing f from given z from the given failure rate. So, let us see, R t is equal to 1 minus f t, differentiate both sides that will give you R prime t. So, the derivative of F t is the cumulative density function is the pdf. So, this is minus F t, right. So, R prime t is minus f t and therefore, your Z t, which is defined as F t upon R t. So, for F t you are going to write R prime t. So, this minus R prime t upon R t, right. So, f t is minus R prime t and therefore, if you integrate this from 0 to t, this is of the kind 0 to t R prime s because that is what you have here. So, this is 0 to t Z s ds and this will be.

So, now, you have this integrant of the kind where you have numerator as the derivative of the denominator. And so immediately you know, that this is the integral of this is minus ln, minus sign is here. So, ln of R s log of R s from 0 to t, right. And now, here you see this that means, this will be, yeah. So, if you write it out, this will be minus ln R t plus ln R 0, right. This, what this integral will be, but then since f 0 is 0, R 0 is 1 and log of ln of 1 is 0. So, therefore, this is, there is no contribution from here and you have minus ln R t. So, this is what, since R prime f 0 is 0, therefore r 0 is 1, right. And so from here, yeah Z t is this.

So, I should write, therefore R t. R t, I wrote down this way and this, this I just integrated 0 to t. So, R t is, so this is ln R and this is ln R t. So, you have the equation, that Z t, Z t, I am sorry, yeah. So, what about have you obtained, yeah? Now, let that that be there, yeah. So, the long, equation has become long that is why. So, what we obtained is 0 to t Z s ds is minus ln R t, right. So, from here we are, I am saying, that this implies that R t is. So, ln is. So, e raise to, remember when we write ln, it means, to the base e. So, this could be e raise to minus 0 to t Z s ds.

And now, you have it from there. See, from this equation, our original definition because you want to compute, yeah. So, I should have finished it here. So, once you have this, now your Z t is f t upon R t. So, therefore, f t is R t into z t and R t we have just computed as this. So, therefore f t is Z t into e raise to minus integral 0 to t Z s ds. This is what our result was, we wanted to show this. And so given a failure rate z, I can compute the pdf of the life length random variable t uniquely.

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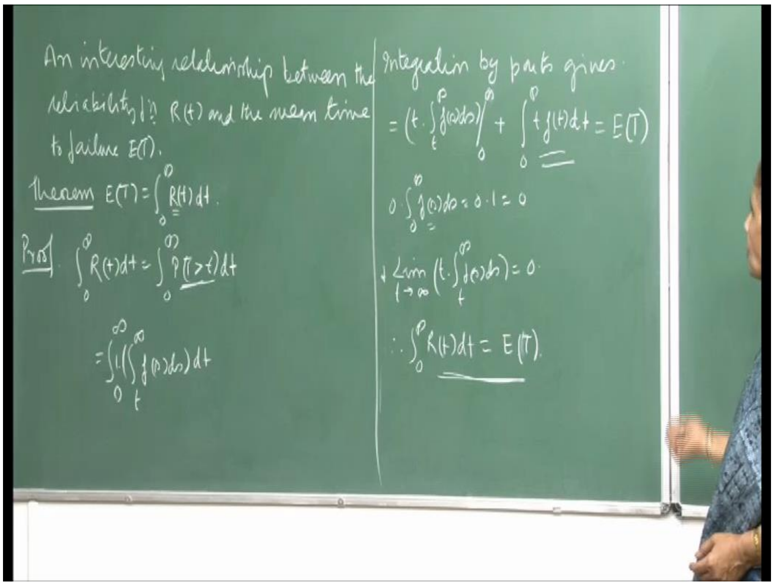

So, there is an interesting relationship between the reliability function R t and the mean time of failure. So, expected T would be the mean time to failure, right. And so why I want to show you that. So, this is the theorem, it says that. So, the t is the random variable and R is the corresponding reliability function. So, want to show you, that E T, the expected mean time, expected value of T is nothing but the integral 0 to infinity R t dt. So, in other words, if you have this, then you can integrate this function and also compute E T. There is nothing much so great about it, we, it can use the same concept that we have.

So, remember your definition of R t is, so probability T greater than t. So, integral 0 to infinity R t dt will be integral 0 to infinity probability T greater than t dt. Now, this was, so I think I gave you this as an exercise in one of the earlier exercises when we were defining expected value of a random variable. But let me now just spend time and show you why this will be true. So, T greater than t, f integrating, now this I can write as integral t to infinity of f s ds, right, probability T greater than t was per, thus what we wrote down just now. So, it is a t to infinity f s ds and then 0 to infinity.

Now, let us integrate by parts. So, I will treat, you know, this one as a first function and this as a second function. So, integral of the first function would be simply t, right. And so we want to have this t into integral t infinity f s ds from 0 to infinity, right, plus, why will it be plus? Because when you differentiate this, so integral of the first into the derivative of the first, second and the integral of the whole.

So, when you want to differentiate this, you see the lower limit is the function of t. So, it will be as it as when you do the, you know, first integral, the integral of the first function into the second function minus, it is a minus sign. But since this is the limit is the function of t, the lower limit is a function of t. So, there will be another minus sign. So, and the derivative of this will be 1. So, then f t, so plus. So, therefore, this sign will become plus and this will be 0 to infinity t f t d t.

So, if you apply integration by parts to this, treating this as the first function and this whole thing as a second function, then you can write this. Now, you can immediately see, that there is a limit at the 0 point. So, this whole thing has to be integrated, has to be computed between 0 and infinity. So, at 0, this is 0 and this will be 0 to infinity f s d s, which is equal to 1 because remember, f is the pdf and the, and that variable t varies from 0 to infinity. So, this integral is equal to 1. So, the f is 0.

And now, here this is little complicated, but we can see, that limit here as t goes to infinity. So, you see here, as t goes to infinity this part becomes 0 and this is going to infinity. So, it can be shown, that the product here, the limit of this product will go to 0. So, once that happens, then that integral reduces to simply 0 to infinity R t d t, which is, I mean, it reduces to simply this, which is t f t d t from 0 to infinity and this is your P T, right, and so you have this relationship.

So, either way you can use, I mean, if you, if you know this, when you can say, that this integral is equal to this or if you know R, then you can, by integrating this you can compute the expected value over random variable t.

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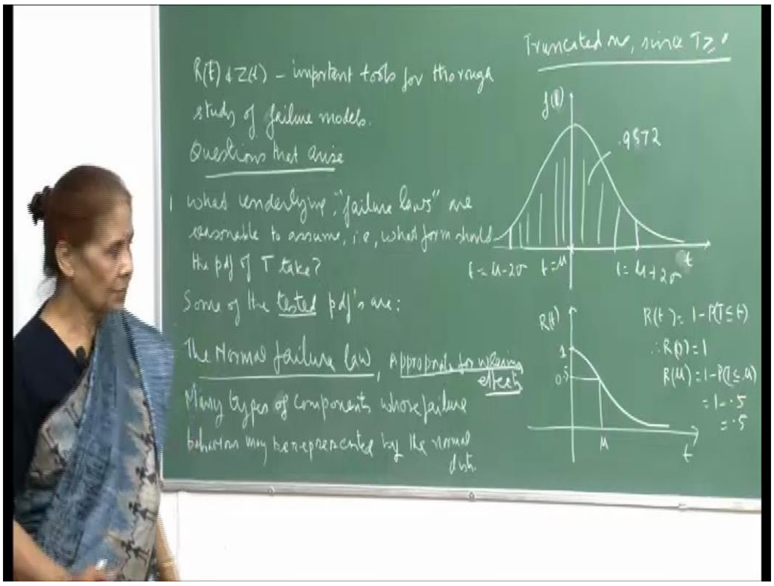

So, now it is we want to study various failure laws. That means, we want to find out the different pdf’s, which will be suitable for different situations where we want to study. And of course, we will keep it very simple here, not going to complicated results. But just look at various distribution, some of the distributions and we will show why they are appropriate for modeling certain situations for reliability, ok.

So, after defining your reliability function and your failure rate we are going to show how important tools they are to, you know, study these failure models, that we will be discussing. So, these are the two basic tools that we need and continuously will be making use of them.

Now, the questions, that arise, the first question, of course, then we will ask what underlying failure laws are reasonable to assume there is, what should be the appropriate form of the pdf of t we want to know. And of course, people have, you know, computed data, I mean, collected data and then try to fit this various probability laws, which are, which are normal exponential and so on. So, that is what, say, at some of the tested pdf's we are going to look at. And tested, by tested we mean, that you know, you try to, you have the data, you know system is going on and then you compile the failure times and so on, of the components and then you try to fit curve. And these, the ones test we discussed, now have been very well tested and you know, found suitable for the some particular data that is we are going to talk about, right.

So, when there is wearing effect on the components, just as we said, that you know, been under heavy load, so slowly, gradually there is a wearing effect, and then breaks down. So, for such situations where the wearing effect is the one, which is the cause of the failure, then normal failure law is considered to be appropriate; considered in the sense, that again it has been tested. And by fitting the data, by fitting, you know, having the particular kind of data, which is, you know, which is for components failing under the wearing effect, and then you know, finding out, that yes, normal failure law, the pdf, normal pdf seems to give the quite accurate results.

So, now, therefore, we want to and there are many situations where the wearing effect is the prominent reason for the failure of the, or the breakdown of the component or the system now when you look at the normal law. So, because the normal law is such, that you know, if mean is the mean expected value, so mu is, yeah. So, I am looking at the normal law, which is mu sigma square. So, mean is mu, the expected value is mu and variance is sigma square, then this has the bell shape, the normal. So, we have already studied in this course, I do not have to spend time on describing the normal curve to you, right.

And we know, that if you, if you take the area between mu minus 2 sigma and mu plus 2 sigma, then area under this, this two is 0.9572 and if you go up to 3 sigma mu minus 3 sigma mu plus 3 sigma, then of course, very little area is left out. So, almost, I think, 0.99

something the area within these two limits. And you also see, that for t is equal to mu, this point is the mean as well as the mode. That means, the maximum failure will occur at around the time t is equal to mu, right. But since our variable t takes only non-negative values, therefore we will consider the normal distribution, that is, only the portion from 0 to infinity and not from minus infinity to infinity. So, this is the.

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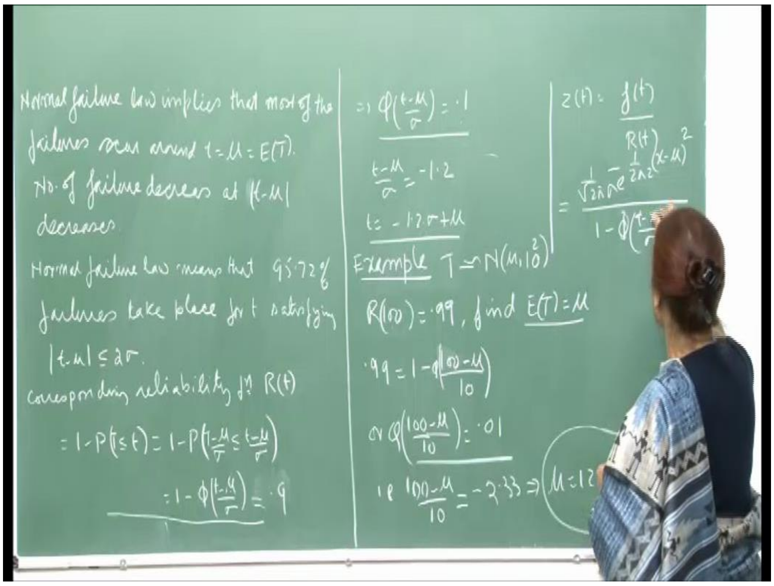

So, normal failure law implies, that most of the failures occur around t is equal to mu, which is the expected value and number of failures decrease as t minus mu decreases in absolute value on either side, right. So, the number of failures will decrease and the probability goes down.

And so for example, if you take normal failures law, means, that 95.72 percent failures take place for t satisfying this mod t minus mu less than. So, that means, if your t is in this area,, then so it will be from 0 to infinity only, only this portion. So, we do not have to worry about this, yeah. So, whatever the mean, the expected value, the maximum failures will occur this time.

Now, if you look at R t, the reliability function, then the reliability function, yeah, so R t is equal to 1 minus probability capital T less than or equal to small t, right. Because this is R t is equal to probability T greater than t, which can be written as this. And now, here you can write this probability for in terms of the standardized normal variant, which we have been doing all along in this course. So, this would be equal to 1 minus probability T minus mu divided by the standard deviation. So, this is less than or equal to t minus mu by sigma, right. And so this becomes 1 minus phi t minus mu by sigma, which is equal to 0.9, that I was just considering the value. So, this is the, this is the function, functional form for your reliability function.

((Refer Time: 26:49)) And if you plotted here on the t axis, then it will be something like this. So, at, at 0, t equal to 0, your probability T less than or equal to 0 is 0, right, because t has, takes only non-negative values. So, this probability is 0, at small t is equal to 0. So, therefore, this is equal to 1. So, at 0 your value of R t is 1 and when you take R mu, then this will be t less than or equal to mu.

Now, since t is normally distributed, we know, that area, this, this area, that means, this whole area is equal to 0.5. Right half, the area is on this side and half the area, it is a symmetric curve. So, therefore, this would be 1 minus 0.5, which is 0.5. So, therefore, for mu equal to, so this is at mu t equal to mu, the value of R t is equal to 0.5. So, this is the kind of curve when, then it goes to, as t goes to infinity, this goes to 0. So, the reliability function decreases with time.

Now, yeah, so I was saying, that see what you can see from here is, this is your reliability function. If you want high value for the reliability function, then obviously, so that means, you want this whole thing to be high, which that is, what I was saying here. So, suppose this is equal to 0.9 and this implies, that this must be small and if, for this to be small you can again tell by the graph, that t must be away from mu. This is the whole idea, right. Because as we said, that maximum failures will occur around t equal to mu, and as you get away from mu, the values become smaller. So, if you want this whole thing, if you want high reliability, then your value of t must be removed from mu, right.

So, for example, if this is 0.9, then this implies, that your phi of t minus mu by sigma should be 0.1, right. This you bring here, then 1 minus 0.9 would be 0.1 and so that will make it, make t minus mu by sigma equal to minus. So, you look up the tables, right. Again I do not have to spend time on this, the standard norm because this is now standardized normal variant.

So, you look up the numbers table, among the tables corresponding to 0.1 area you look up. So, it will be somewhere here for the standard normal thing. So, it will be somewhere here point because you want only area 0.1 up from up to this point. So, it will be very small, right. So, this is minus 1.2 and so t comes out to be minus 1.2 times sigma plus mu. So, that much remove from mu, your value of t is, if you want reliability of the order of 0.9.

So, take another example. Now, here suppose the failure rate, the time, life line, life time of a component is normally distributed with mean mu and variance 100, that is, right. So, the standard deviation is 10. Suppose, this is this and you are told, that the reliability for the 100 hours is 0.99. So, R of 100 is 0.99, you have to find the value of mu, that is, the expected value of T you have to find. So, since we know the functional form for R t, that means, again 0.99 is 1 minus phi of. So, we standardize the normal this variant t, which is 100 minus mu upon 10 standard deviation, right. So, this is phi 100 minus mu by 10. So, that gives you, that phi of 100 minus mu by 10 is 0.01.

So, again we look up the standard tables corresponding to the area 0.01, this value, the value of Z is equal to minus 2.33. So, the tables. So, see this smaller, this becomes the, further away you go from the mean value, that is what we are saying. And so this implies, that mu is, see here you will multiply this with this, and then so it will become 23.3 and mu comes to this side, this gets added to 100. So, this will be then 123.3 hours. So, the mu, that means, the mean is here and so you see, this is removed from the mean, this is 123.3 hours. So, at 100 hours if you are asking for a reliability 0.99, then your mu is this.

So, this is the whole idea, that you, if you have, if you, if your data or your experience with the system that you are working with is, that the normal distribution is appropriate for studying the failure rates then. So, here of course, you should also look at what your Z t will be, right.

So, if you want to compute, which may not be very. So, for example, your Z t, we said, is f t upon R t, alright. So, which will be 1 upon root 2 pi sigma, ok. Let me just, so this will be equal to your f t is 1 upon root 2 pi sigma e raise to minus 1 by 2 sigma square x minus mu whole square divided by 1 minus phi of t minus mu by sigma. So, something like this. This is your failure rate.

So, if you want to compute Z, you will get this complicated expression, but it seems, that here your reliability function is the one, which is more useful and as a simple form because all you have to do is to, for a fixed value, for a given value of t mu and sigma given, then you just have to look up the normal tables and compute the reliability of the system at any given time t.

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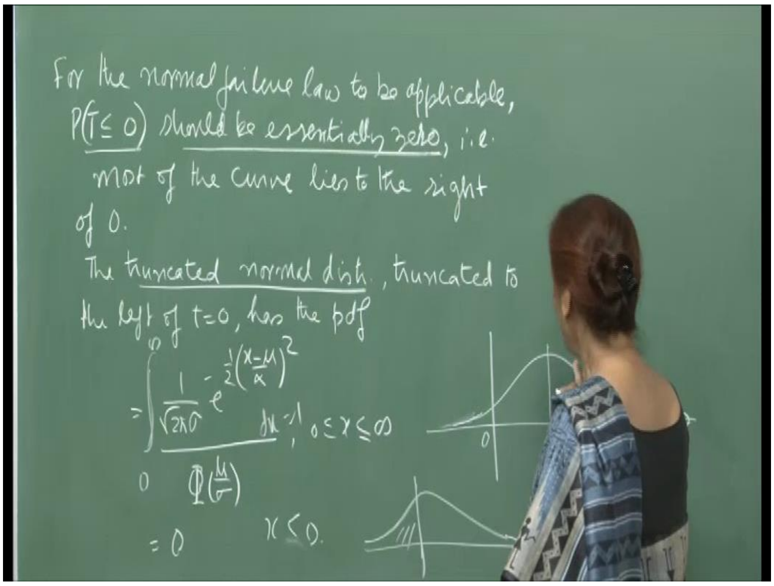

Failure law, I said, to be applicable we want this, because obviously, the time to failure cannot be less than 0 with only when this apparatus or the instruments start functioning, then you talk about its failure time and so on. So, time to failure. So, therefore, since this is less than or equal to 0 and normally, R, R, the normal distribution extend from minus infinity to infinity.

But what we saying here is, that the most of the curve should lie to the right of 0. So, that means, here is, so the curve who should be like this. So, very small area is to the left of 0. So, most of it lies to the right of 0, then the computations would be fine. So, this is very important. So, that means, this probability T less than or equal to 0 should be essentially 0, negligible, very small. That is what you want to say.

Now, another way to handle this situation could be, that I consider the truncated normal distribution that means, you truncate. When you, you may have your normal distribution, this and then you truncated this portion and so that means, then you would consider. But, then since it has to be a pdf, so then you will have to. So, that is what I am trying to say, that you know, truncated normal distribution.

So, if you consider the truncated normal distribution, truncated to the left of t 0, then the pdf would be this. But that is not defecting the actually situation because by this what we are doing is, since I want to call this a pdf, so this integral, and of course, is 0 for x less than 0. So, this is from 0 to infinity. So, what we are saying is, that this will be from 0 to infinity. This will integrate to 1, but this is not what I want because I want the normal failure law, but it should be side that the most of the curve lies to the right of 0 of t equal to 0. So, that is the meaning.

And therefore, this of course, this will complicate your computations also, but besides that it will not give you the desired results. So, therefore, the truncated normal distribution is not to be considered. It is simply, that keeping this in mind we have to make, show, that you know, most of the curve lies to the right of 0. So, therefore, well, we compute the probabilities. They would be approximately alright for as I said, that the normal failure law is for the ageing where the ageing is permanent.

So, now, we will study, we look at the other failure laws, which have again been tested for different situations and have proved very, very, have to prove to show good results. So, this would be exponential distribution and viable distribution and some others.

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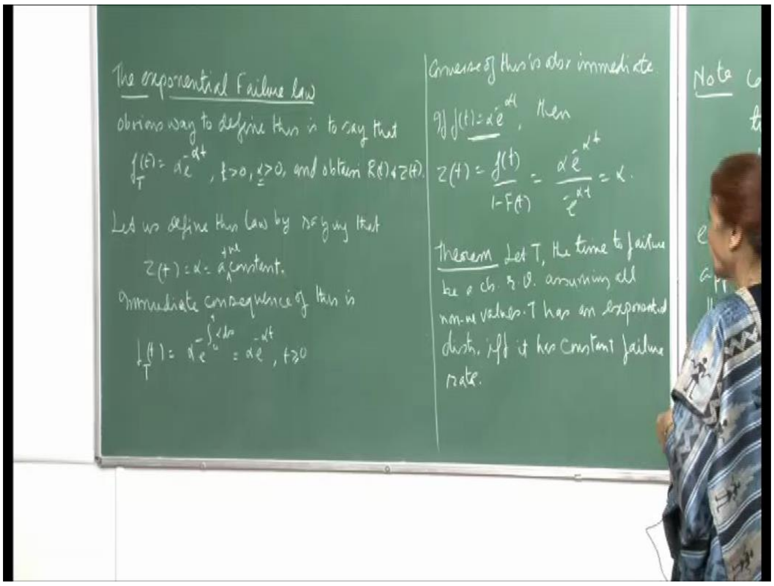

So, let us look at the exponential failure law. Now, of course, obvious way to define the exponential failure law would be by defining the pdf. So, we say, that the pdf T is alpha is e raise to minus alpha t where t is positive, takes positive values, and then alpha is also some, positive constant, right.

And, then we can obtain R t and Z t, but, but that is not really have that dramatic effect as when you say, that the law, we say, that the failure rate. That means, z t is a constant, is equal to alpha, right. So, here I have just said, that alpha is some constant, but actually when your law is exponential law failure law is exponential, then negative exponential, then your failure rate, sorry, yeah your failure rate would be a constant, right is a positive constant, is equal to alpha actually. So, alpha is your failure rate. So, this is constant, right.

And immediate consequence of this is because remember we said, that we can, given Z we can uniquely determine the pdf. And also, given the pdf, we can uniquely determine the function Z the failure rate. So, here you see, the definition was, that f t would be Z t e raise to minus integral 0 to t of Z s f s. So, here z is a constant. So, therefore, this is alpha into d s. So, integral, this leads to t. So, t alpha and here z t is alpha again. So, this is alpha e raise to minus alpha t for all t non-negatives. So, therefore, this is your, so immediately you compute the pdf of t. And the converse is also immediate because if you are given, that the f t is alpha e raise to minus alpha t, then this we have already done this computation, but anyway let us just go through it again.

So, Z t is f t upon 1 minus f t, which is R t. And so f t is alpha e raise to minus alpha t and your, you know, that 1 minus f t is e raise to minus alpha t. This is probability t greater than capital T greater than small t and so it will be e raise to minus alpha t and so comes into alpha. So, this is the constant. So, your Z t, the failure rate is constant. So, immediately we can write down this theorem that. And so if I remember correctly, we had not done this part, that means, given the failure rate when I had talked about the exponential, even we would, we had just introduced the exponential distribution, I had defined the hazard function or the failure rate.

But I did not, at that time discuss this part, that given Z t you can obtain your pdf uniquely, right. So, now, we can immediately write down this theorem because I have shown you both ways, that is, if and so for constant failure rate the only pdf that can be there is your negative exponential. And if the pdf is negative exponential, pdf of the life time random variable t, then it has to be, the failure rate has to be a constant.

So, immediately we have the theorem, that let t the time to failure be a continuous random variable assuming all non-negative values, then t has a negative exponential distribution if and only if it has constant failure rate. So, this is now neat, neat way to present the whole thing, that is, there can be no other pdf, which satisfies the condition, that the corresponding failure rate is a constant. So, constant failure rate would always mean exponential pdf's.

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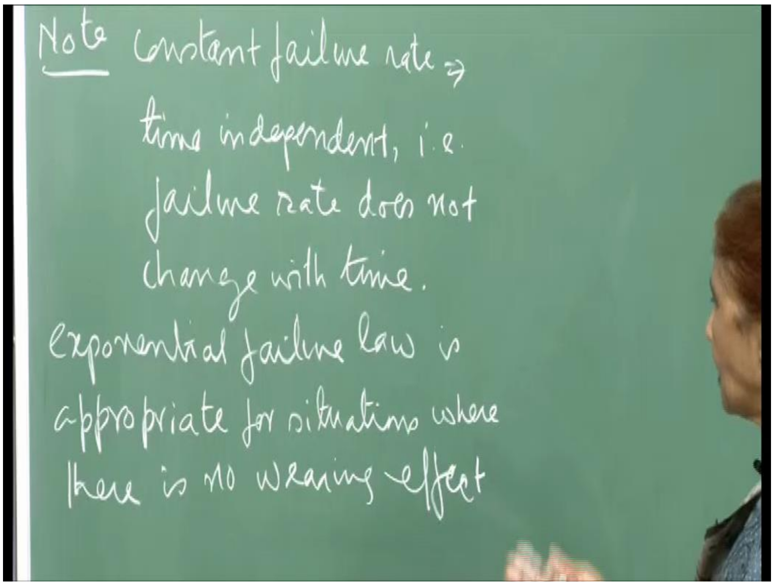

And constant failure rate implies, that it is time independent, right. So, no matter, that is, the failure does not change with time, the failure rate does not change with sign. So, no matter. So, therefore, this will be appropriate for situations where there is no wearing effect, right. So, that means, no matter how long the component has been working, it does not matter, it has no bearing on the failure of the component, it will only be some external, may be, I will come out with in the next lecture. I will try to show you some more, you know, ways of describing the situations where these different failure laws can be appropriate.

So, essentially now try to think of this situations where the failure is not because of because of the wearing effect. That means, not because of some kind of stress or load, but it is. So, therefore, as we try to say, that if you take a fuse, then fuse can be working fine for a long time and then suddenly it fails. So, it is not because may be, because of high current or something high current is come, then the fuse burns out otherwise it may continue for a long time.

So, therefore, such situations would be very appropriately modelled by the exponential failure law and one can, you know, the now, that you read about this yourself, you know, try to think of components or systems where the failures are not because of the wearing effect or this kind of some kind of load or stress, but it is a different kind of failure and therefore, this can be model by exponential law. And you can see, that how effectively it will give you the parameters that you will require for, you know, predicting things about the model.