Introduction to Probability Theory and its Applications
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Lecture - 33
Inter Arrival Times Properties of Poisson Processes

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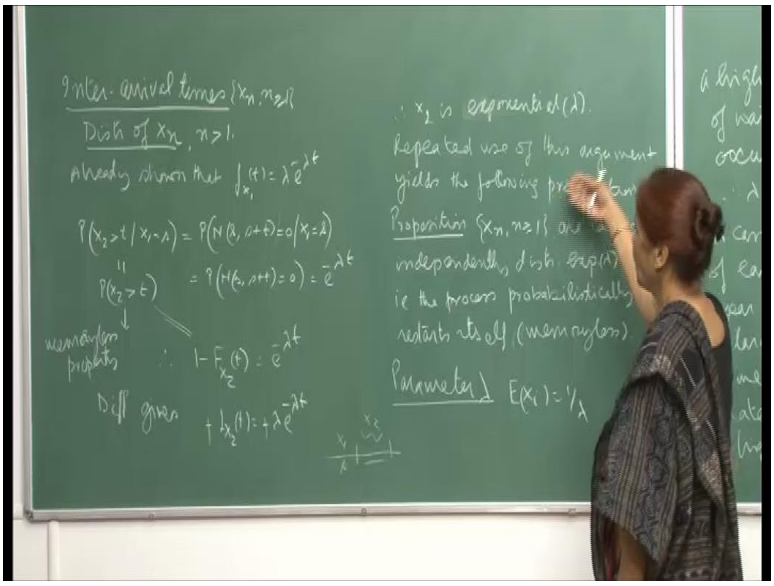

I will just quickly again, go over through the go over the inter arrival times that we discussed in the last lecture. And then I obtained the distribution for inter arrival times. So, we already said, showed that fx1 t is lambda e rise to minus lambda t; that means, the x 1 denoted the arrival time up to the first, up to the first event that occurs. So, therefore, that is exponential, in the interval is lambda the interval for the first has exponential with distribution parameter lambda. Then, if you want to now, compute for x 2 then let us look at the probability x 2 greater than t when x 1 is equal to s.

So, here you see again the idea is that the first interval the first event occurred here. And now, so this was your x 1 now, this time is denoted b y x 2. And, we are saying that this is greater than t. So, if this time is greater than t; that means, there is no arrival in this time. And, so you are looking at probability x 2 greater than t, condition on x 1 being s. But since, we have shown already, that the inter arrival times are exponential. And, so they are memory less, and therefore it does not matter. See, this probability will remain the same whether it is here or here or anywhere. So, it does not matter, when the first event took place the probability the conditional probability is the same as the, probability x 2 greater than t.

So, this is the memory less property that we have already shown. So, therefore, this is equal to this, and here the same thing probability s comma s plus t 0 given that s 1 is s. So, therefore, this is equal to probability that there no arrivals in the time interval s comma s plus t. And, that probability is e rise to minus lambda t, is it ok? Because, here again the number that you the probability that you want to compute here, is that there is no arrival in this interval.

And, so the conditional has no bearing on this probability also. So, therefore, this is e rise to minus lambda t, and here, this in term of your distribution function. You will write this probability as 1 minus F x 2 t and this is equal to e rise to minus lambda t. Therefore, if we differentiate both sides, you get f x 2 t is minus lambda e rise to minus lambda t.

So, this goes out and therefore, you have shown that for x 2 also the distribution is, exponential lambda. And now, repeated use of this argument because essentially we are using the memory less property. And, so the same argument can be repeated for x 3 x 4 and so on. So, we end up with this proposition, that when you take the sequence of inter arrival times these are identically, independently, distributed. Remember, we have assumed for the Poisson process independent increments, and stationary increments.

So, therefore, these are inter arrival times are identically, independently, distributed exponential lambda random variables. That is, the process is probabilistically starts itself, which means its memory less. Every time, event occurs it starts itself again. So, there is no memory as to, when the last event occurred, it just starts fresh from after any event. So, when you start counting the inter arrival times, it rejuvenates itself again. Now, just a word about the parameter lambda. So, you see because this is exponential lambda. So, expectation of x I will be 1 upon lambda, and I mean the theory about exponential distribution does not say anything about lambda as long as lambda is greater than zero.

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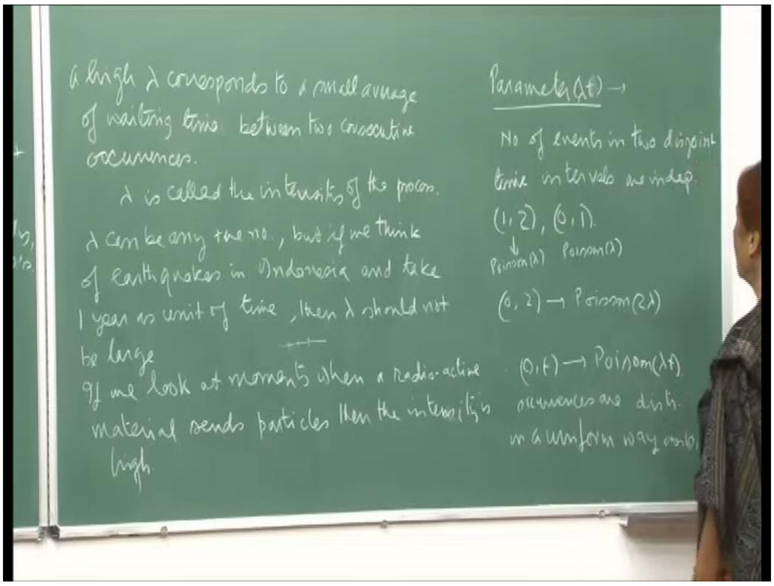

So, lambda can be any positive number. So, here before I come to that see a high lambda corresponds to a small average of waiting time. If lambda is large, then 1 upon lambda which is the expected value is small. And, so it is saying that this is the average of the inter arrival times so; that means, if the average is small, then the arrivals will be occurring in small intervals because the expected value is small.

So, when lambda is high it corresponds to a small average of waiting time between 2 consecutive occurrences. So, we were saying that when lambda has a large value the corresponding expected value will be a small number. And, so that would mean that the inter arrival times are smaller, in the small in the sense that the average is small. So, therefore, the arrivals are occurring at smaller intervals. And, in any case lambda is called the intensity of the process.

So, there is no therefore, it simply says that lambda measures the, so if lambda is small then this will be big, so that means, the inter arrival time average is large. And, so the events are occurring at large intervals hence therefore, we are saying that lambda is called the intensity of the process also. And, lambda can be any positive number, but if we think of earth quakes in Indonesia say for example, and take 1 year as unit of time. Suppose, consider the process you know, if I am counting the number of earth quakes that have occurred, in a span of ten years say for example.

And, so if I take the unit of time as 1 year, then lambda should not be large because if lambda is large then what will it say that 1 upon lambda is small and therefore, it would mean that the earth quakes are occurring at smaller intervals of time, but we all know that the earth quakes of course, they are un predictable, but normally it does not happen that earth quakes occur very often.

So, one has to be careful that is; why this interpretation of lambda gives you an insight into, how when you go about modeling a process? When how should your choice of lambda be made? Our also like, if you look at moments this, I mean again this is the time this 1 d when a radioactive material sends particles. Then the intensity is high, the intensity is high and therefore, this is small.

So, the particles spread radioactive particles spread very fast. And, so if you are counting at any point of this thing, how often the particle is arriving? Then the inter arrival times will be small, and therefore, this will be small. So, lambda would be high. So, just to give you an idea, and then you can look at many different examples, and see how the value of lambda will reflect the inter arrival averages. Similarly, you want to look at the parameter lambda t. So, lambda t is number of events. So, this is actually on the average the number of events in time t in time period t. So, this is your mean arrival rate.

So, now number of events in 2 disjoints time intervals are independent, just now we said that they are independent increments. So, therefore, if you look at the arrival in between 0 and 1, it is Poisson lambda. And this is Poisson lambda between 1 and 2 now; if you look at the time interval 0 2 then it will become Poisson 2 lambda. Because, again from now, by now we have by, so many different methods shown you, that if 2 random variables x 1 of course, here x 1 is Poisson lambda x 2 is Poisson lambda.

Then x 1 plus x 2 would be again Poisson 2 lambda, the parameters get added. So, therefore, 0 2 would the number of arrivals, will be Poisson 2 lambda. So, therefore, in 0 t it will be Poisson lambda t, this is the whole idea. So, this is the important thing and therefore, in time 0 t we will say that the arrival rate is lambda t, Poisson lambda t, this is the whole idea, because t can be fractional and so on. So, 1 can again interpret in the same way now another thing is that since we are talking of stationery increments therefore, what we have to say is that; the arrivals over the time 0 t are distributed in a uniform way.

Because, they are random 0 to t they are anyway random events such then when we are talking of number of arrivals for example, when I am counting n t this is the number of arrivals in time 0 to t, in the span time in the span of time 0 to t. So, then we have to think the way the process is being modeled, is that in this particular time period, the arrivals can occur anywhere. And, so the best way to model that is, that the arrivals are uniform in the interval 0 to t, and through an example again I will try to make you understand this concept a little better.

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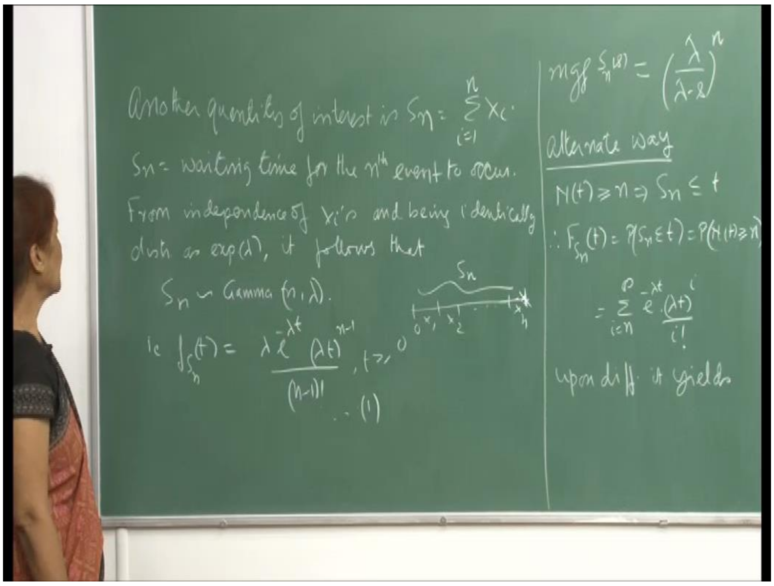

So, after computing the distribution function of the interval arrival time, let the another quantity of interest is s n, which is sigma x i, i varying from 1 to n, which means x n is the waiting time for the nth event to occur. So, because see you are adding up x 1 to x 2 to x n. So, x n is the time, when the n inter arrival time between n minus 1 and the nth event. So, therefore, s n will be the waiting time for the nth event to occur.

So, just added up all the see on the line, you have you are starting from 0 this is x one this is x 2 and so on. And finally, this is x n. So, at this point the nth event has occurred, right starting from here, this is the first event, second event and so on. So, at this point the nth event has occurred. So, this is the total time; that means, this the total time you are denoting by s n. So, which is you can say waiting time for the nth event to occur, for the nth you know the time the volcano has to erupt particular volcano or earth quake to occur, whatever process you are looking at, you can interpret s accordingly. Now, from independence of x i's, and being identically distributed as exponential lambda. And here, again you see in the last few lectures we have been talking about, some of independent random variables, and through convolution through m g f s and so on.

We have looked at the distributions of sums of independent random variables. So, here it immediately follows, that s n is gamma and lambda. Because, here the x i' s are identically distributed as exponential lambda, n of them you are taking some n of these random variables exponential random variables. So, the sum will be gamma, n comma lambda, and therefore, the p d f are the density function for f s n t is lambda, e rise to minus lambda t lambda t rise to n minus 1 upon, and minus 1 factorial for t non negative. So, this is the thing, but now again as I said it always helps to be able to use other tools that we have developed. And, so let us try to do it, and then of course, m g f again we has already been computed, for a gamma random variable is lambda upon lambda minus s rise to n. So, while writing m g f of s n the s got written by mistake.

So, it is actually m g f of s n, which will be lambda upon lambda minus s rise to n. So, the idea was that you were computing it at s. So, therefore, it got written there. So, this is actually lambda upon lambda minus s rise to n. Let us look at it in an alternate way, and that is also interesting. So, let us just be very clear about this n t greater than or equal to n if and only if s n is less than or equal to t, that is if the number of arrivals by time t greater than or equal to n. Then, the time s n for the nth event to occur is less than or equal to t and vice versa. That is if s n is less than or equal to t then it will imply that n t must be greater than or equal to n.

So, when you want to compute the distribution function of s n, this is probability s n less than or equal to t, which because the 2 events are the same, this is probability n t greater than or equal to n. And so since n t is Poisson distributed, a Poisson random variable with lambda t as the parameter. So, therefore, this probability can be written as sigma i varying from n to infinity e rise to minus lambda t, lambda t rise to i, upon i factorial. Now, let us differentiate this equation from both sides.

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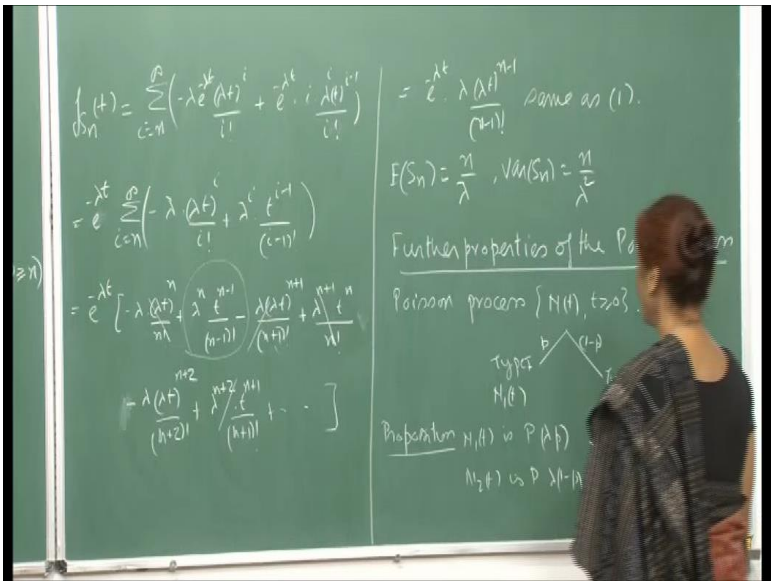

And, so on the left hand side this will be the p d f of s n, and now here, let us do it term by term. So, the derivative of this first. So, minus lambda e rise to minus lambda t, lambda t rise to i, upon i factorial, and then the derivative of this, which we are writing as. So, first function as it is e rise to minus lambda t in to, lambda rise to i remains as it is, t the power of t becomes, i minus 1 and then i factorial here, which you can you know cancel the i part here, then it will be i minus 1 factorial.

So, and e rise to minus lambda t I have taken outside then this summation from n to infinity. So, I am not writing out many terms here, we just have to show you that you see, when you take n i equal to n, the term from here you will get lambda t rise to n upon n factorial. And, this will give to lambda rise to n, t rise to n minus 1, n minus 1 factorial. So, these are the 2 terms now, put i equal to n plus one.

So, this will be minus lambda, lambda t rise to n plus 1, n plus 1 factorial, plus lambda n plus 1 t rise to n upon n factorial. So, you see this cancels with this, and then I thought, I will also write the values corresponding to i equal to n plus two. So, then that will be lambda, lambda t rise to n plus 2 upon n plus 2 factorial, plus lambda n plus 2, t rise to n plus 1, and n plus 1 factorial. So, that cancels out this. So, you can see the pattern first, and the fourth here then the third and the fifth sixth and so on.

So, all these things will cancel out except this. Because, this is the lowest degree term after that the powers of t keep on increasing. So, this is the only 1 which is left out all these will cancel out. And, so you are left with e rise to minus lambda t into lambda, lambda t rise to n minus 1 upon n minus 1 factorial, same as 1, which is a gamma density function, gamma n comma lambda.

So, I just wanted you to sort of you know, make use of this also. And therefore, you can even do it directly. So, once when you generate, so many tools, it is always possible to prove result by more than 1 way, and it also helps gives you a better insight, if you can do that. So, then expected value of s n will be n upon lambda, and variance s n will be n upon lambda square. Now, we will further prove some more properties of the Poisson process, and then you know work out examples to show you how you make use of these all these machinery, that we have developed.

Now, for example, if you take a Poisson process n t, t greater than or equal to 0 then they can and they can be 2 sub processes. If you remember while discussing, the joint m g f I talked about Poisson process, and then I said, that if all the events that are occurring are being counted. Then they the probability of an event being counted was p, and event not being counted was 1 minus p. And then I showed you through the m g f, that each of them, each of these process again would be a Poisson. So, see while talking about the Poisson process, having 2 sub processes, which we call type 1 and type 2. And, so the probability that the type 1 would have occurrence with occur with probability p, and type 2 with occur with probability 1 minus p.

So, I have already. So, the only correction I want to make is that see here, you n t is lambda p t, because we are talking about with respect to t. So, then we are talking of arrival time arrival rate in the interval 0 to t. So, now, here similarly your n 1 t will be then Poisson, and this we showed through m g f processing of we showed that it can be both will be again Poisson. So, the sub processes n 1 t would be Poisson lambda p t, and n 2 the process type 2, which will be. So, the random variable is n 2 t will be Poisson lambda 1 minus p t. So, the we have to attach. So, what I wrote in the lecture was, without the t part everywhere here. So, this is what the correction is being made, otherwise I have explained, what we mean by these sub processes and so on.

In the lecture itself, so exactly the same thing, but here again I will do this, I will try to prove the same result by using the machinery that the definitions that we have made here. I will try to do that because the m g thing we know. So, here it is saying that the 2 types of; that means, you must be considering, let us say immigrants from another country, and the immigrants may be Hindus, Muslims whatever it is. So, therefore, the total process of immigrants coming from another country, may be a Poisson process and then the kinds of people that are arriving, you may want to separate them into 2 streams. One may be let us say Hindus, the other may be Muslims. So, there will be type 1 arrival and the probability of 1 of the arrival immigrants being Hindus is probability p, and the 1 minus p is the probability of the immigrant being Muslim.

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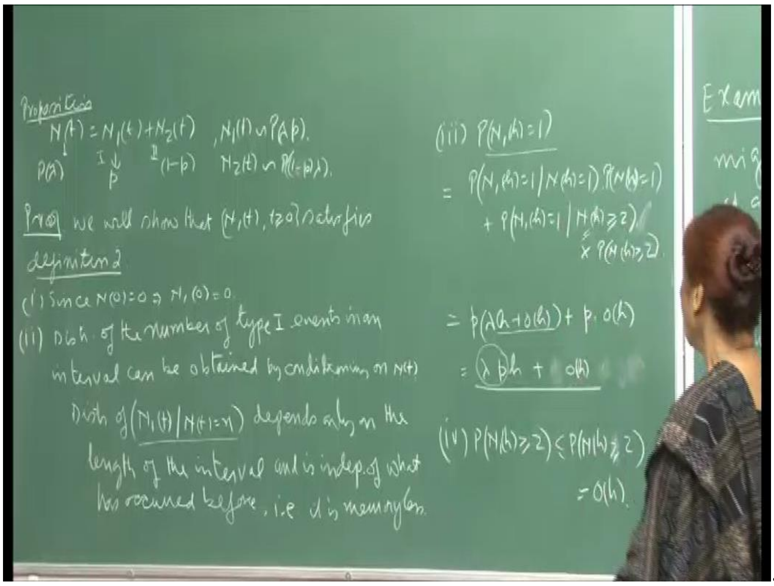

So, we will now, prove it in a different way, in an alternate way. So, as I said the proposition is that there is a Poisson process, and n t is the number of arrivals in time up to time t. Then if there is type 1 and type 2 processes, sub processes. And, so type 1 process; that means, the type 1 event occurs with probability p, and time and the type 2 event occurs with probability 1 minus p. We want to show that n 1 t is Poisson with lambda p as the parameter, and n 2, n 2 t is Poisson with parameter 1 minus p lambda. We want to show this. And as I told you, that we have already shown this result using m g f s joint m g f, but let me do it through. So, we will show that n 1 t, t greater than 0 this satisfies your definition 2, which remember we said is more easily verifiable.

And, so let us do it quickly since n 0 is 0, this implies n 1 is 0. Because your n t is n 1 t plus n 2 t. If this is 0 then both of them must be zero. So, n 1 is 0 now, the other part is that you know, independent and stationary increments. So, which can also be easily seen because if I condition this by fixing n t equal to n. Then, the arrivals here are also you know they only depend on the length of the interval, and are independent of, what is occurred before. So, that is the memory less. So, by conditioning also you do not change the independent increment, property and the stationary increment property. So, therefore, n 1 t satisfies both, now we just want to show that your probability n 1 h equal to 1. So, the property 3 should be satisfied.

So, let us just look at this event. So, if type 1 arrival in time h is 1 then ,we can write this break up this event as saying that n 1 h is 1 given that n h is one. So, total arrival is 1 and then n 1 is 1, and. so this will be condition this in to probability that n h, n h is 1. All probability n 1 is h and n h is greater than or equal to two. So, these are the 2 possibilities, because either n h 1 has is 1 or n h 2 is greater than or equal to two. So, this would be this into probability n h greater than or equal to two.

So, I like the probe because just by basic definition of the process, we are able to show this result. So, here see n 1 h is p, and then probability n h equal to 1 is because n is anyway Poisson process. So, we already satisfy the definition. So, therefore, probability of n h equal to 1 is n h plus order h, this when given that there is 1 arrival. So, then n 1 h the probability is p plus now again here arrival n 1 h is one.

So, that probability is p and then n h greater than or equal to 2 n satisfies the condition four also. So, therefore, order h right and. So, this will be lambda p h plus order h because p see remember, when you say a function is of order like this, then constants are all allowed. Because, it is only the power of h which is important, it is higher power and. So, as h becomes smaller this goes to 0. So, therefore, that p gets absorbed here and therefore, this is it. So, this is what?

So, therefore, this satisfies definition because lambda p is the probability now of arrival. So, therefore, this will be lambda p in to h, and this n 1 h is greater than or equal to 2 is satisfies. Because, this probability is less than or equal to probability of n h, greater than or equal to 2 right by the definition because n h is n 1 h plus n 2 h, and since, this is order h. So, this has to be also order h.

So, nice simple proof and I like it, of course, you we have already done it through the m g f method, but that was for a general situation. Now, here, we are doing it for a Poisson process. So, therefore, the type 1 and type 2 you can see if you have sub processes, and certainly if this can be extended to more than 2 sub processes. So, if you have more than 2 sub processes, each of them and of course, the some of the probabilities must add up to 1, which it will and therefore, you can say that all these sub processes will be independent. Now, how do I show that n 1 and n 2 are independent, that part is also there that n 1 and n 2 are independent. Once we have shown and similarly, by this similar by this similar argument you will show that n 2 h is also Poisson with parameter lambda into 1 minus p.

Now, you can now use the joint m g f method to show that they will be independent. So, this method I use to show that n 1 t will be Poisson with lambda p s the parameter. And, n 2 will be Poisson with parameter lambda in to 1 minus p, to show independence you can use the m g f method.

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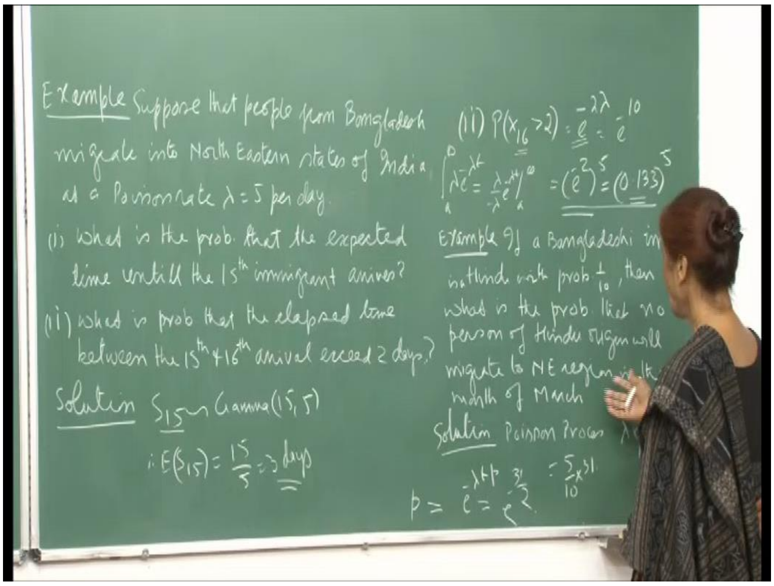

So, let us look at this example, suppose that people from Bangladesh migrate in to north eastern states of India at a Poisson rate of, lambda equal to five per day. So, the question asked is, what is the probability that the expected time until the 15 immigrant arrives? So, what is the probability that the expected time until of the expected time that the 15th immigrant arrives?

So; that means, you are asking for s 15. So, s 15 is gamma 15 comma 5 by the result that we arrived some time ago. Because, it will be x 1 plus, x 2 plus, x 15 and. So, that will be gamma 15 comma 5, and the expected value is 15 upon 5. Remember because this is gamma this is n upon lambda. So, this will be 15 upon 5, which is 3 days. So, the expected arrival time that the expected time until the 15 immigrant arrives.

Now, what is the probability? That the elapsed time between the 15th and 16 arrival exceed 2 days. So, here you are asking for x 16, because x 16 is the inter arrival time between the 15th and the 16 arrival. So, you are asking for the probability that x 16 is greater than 2. And, that will be e rise to minus 2 lambda, again this is from your, this is from your exponential distribution. Because, when you have lambda e rise to minus lambda t, and if you are asking for this thing from let us say a to infinity. Then, this is, what lambda upon minus lambda e rise to minus lambda t a to infinity. And, so this is e rise to minus a lambda.

So, this is it. So, probability x 16 greater than 2 will be e rise to minus 2 lambda, which because lambda is 5. So, this is e rise to minus 10. And, I have just computed the value here, whether you can write this as e rise to minus 2 rise to 5, and e rise to minus 2. I knew the value is 0.133. So, we just rise it to 5 anyway now, if a Bangladeshi immigrant is a Hindu with probability 1 by 10 then what is the probability that no person of Hindu origin will migrate to north eastern region in the month of march. Just to show you the use of you know the, what we just discussed. So, here; that means, it is lambda p t. So, lambda is 5 p is 1 by 10 and the time is 31 days, March has 31 days.

So, therefore, the no Hindu will arrive in that period, again will be e rise to minus lambda t p, which is e rise to minus 31 by 2. And, so you can compute this number. So, this is the whole idea and then of course, we will look at some more properties of the poison process, and work out if you more examples.

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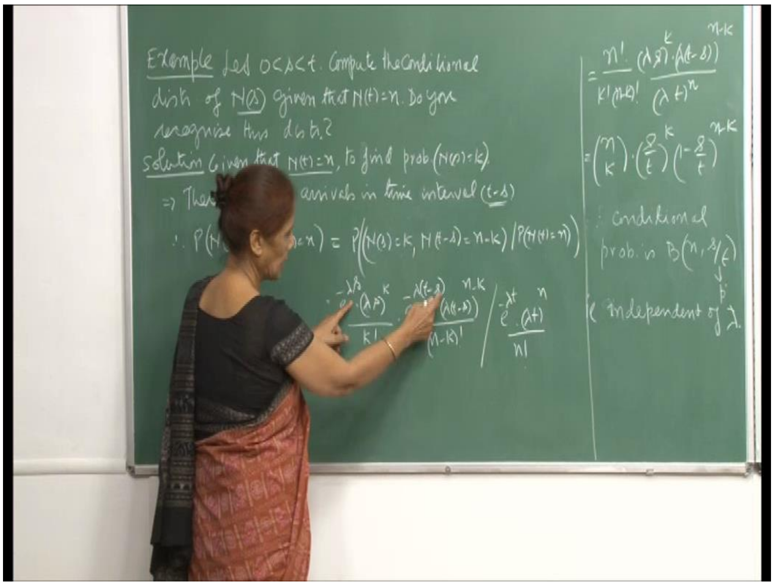

Now, let us look at this example, where we are trying to compute the conditional distribution of n s, given that n t is n. So, given that n arrivals are there in time 0 t, and s is less than t. So, now, you want to look at the conditional distribution of n s, given that see through all these examples, I am just trying to familiarize you more with the working of the process, and the machinery that we are developing. This is the whole idea, and is the process that makes subject quite interesting. So, the solution is that and then the of course, it is been asked do you recognize this distribution, once you get once you obtain the conditional distribution, the question asked is, do you recognize this distribution?

So, you have given that n t is n, you have to find the probability, that n s is equal to k given that n t is n. So, now, that means, if up to time 0 s the arrivals are k, and up to time t the arrivals are n so; obviously, the number of arrivals in time t minus s is n minus k. That is, how it will make up the number of arrivals up to time t as n. So, time s the number of arrivals is k, and then in the interval s 2 t, which is the interval length. And, by now, we know that we just have to worry about the length of the interval, and not exactly, where that interval is occurring.

So, the number of arrivals in time t minus s is n minus k. So, when you write down this probability, probability n s equal to k given that n t is n. This you can write as probability n s is k and comma n t the joint probability of n of t minus s is n minus k. Condition on the n t equal to n, and since again from the dependent increment property for disjoint intervals, s in to minus s the probability can be written as the product of these 2 probabilities. So, therefore, this will be the product of these 2 probabilities, the numerator and the denominator will be probability n t equal to n.

So, I suppose I hope this is clear. So, therefore, e rise to minus this probability is e rise to minus lambda s, lambda s rise to k upon k factorial, and this probability will be e rise to minus lambda t minus s. Lambda t minus s rise to n minus k upon n minus k factorial, divided by e rise to minus lambda t, lambda t rise to n upon n factorial. So, as long as this part is clear that when you see this probability of course, I can write in this way and in this I can write as the product of these 2 probabilities.

And, so now, you see that this is e rise to minus lambda s, and here you get, e rise to plus lambda s, which cancels out. Then e rise to minus lambda t, and in the denominator you have e rise to minus lambda t. So, the e terms all cancel out, and you are left with. So, this n factorial will come in the numerator.

So, the first term that I have written is n factorial upon k factorial n minus k factorial, this all put together. And then you have lambda s rise to k, and lambda t minus s rise to n minus k, divided lambda t rise to n. So, this is what I have written here. So, this whole expression simplifies to this.

And, now here what you can do is see again the lambda rise to k and lambda rise to n minus k is lambda rise to n, which cancels with lambda rise to n here. So, you are left with s by t rise to k now, the t rise to n, I can write this as k plus n minus k. So, the k t rise to k couples with this, which is s upon t rise to k and this here it will be 1 minus s upon t rise to n minus k.

So, see that now you can recognize this, if you treat p equal to s by t then this is a binomial probability, when we are choosing k items out of n; that means, you are asking for k successes out of n trials. And, n independent trials, and your probability of success is s by t. The more important thing is that this whole expression that the conditional distribution is independent of lambda; that means, no matter what the parameter of the poison process is these conditional this conditional probability is independent of lambda. It is only dependent on the length of the time intervals; that means, here it was n s and there it was n t. So, that s and t... So, I am sure there are many more interesting implications of this result, but again know you can get it nicely, by just using the definitions and so on.

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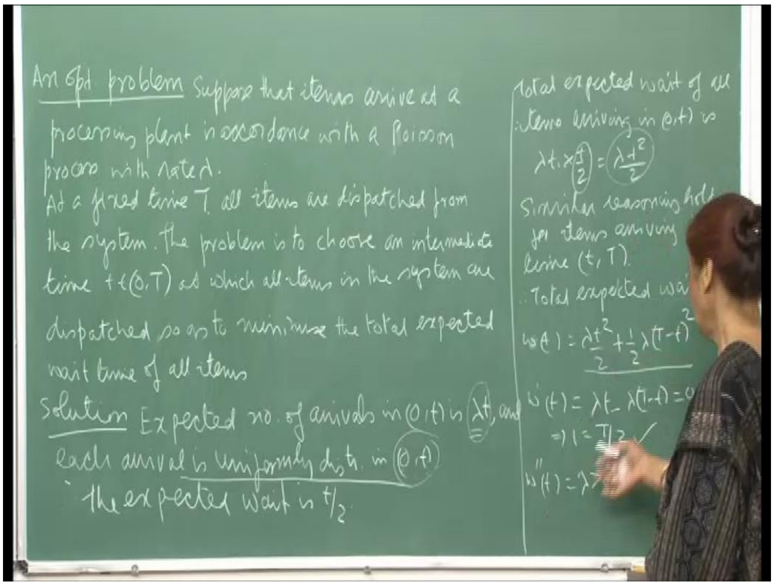

Now, let me again take up this interesting optimization problem, and here again the machinery is not very complicated. Suppose that items arrive at a processing plant in accordance with a poison process with rate lambda, so that means the items are arriving at the you know, at the gate of the processing plant, and arrival process is poison with rate lambda.

Now, at a fixed time all items are dispatched from the system. So, the items get processed and after if time t, when they all collect they are dispatched to they came. Now, the problem is to choose an intermediate time t, belonging to 0 comma t, at which all items in the system; that means, all the items which have been processed by time small t they get dispatched and then the remaining, which get processed from time t to T they will be after being processed they will be dispatched.

So; that means, they have now they are not going to wait till the end of up to time t. So, in between also they would like to dispatch the items, and the idea here is that this will this way, they want to minimize the total expected wait time of all items. So, total expected waiting time. So, this is what we have to write down the expression and then see how we minimize. So, the choice of small t; that means, the intermediate time at which you want to dispatch, whatever items have been processed, this is that has to be fixed that has to be sort of obtained by this process.

So, expected number of arrivals in 0 t is lambda t, remember because this is Poisson with parameter lambda. So, therefore, in time 0 t the expected value is lambda t. And, each arrival is uniformly distributed remember, I some time ago discussed when you are looking at the Poisson process, and it is properties and we said that because of the stationary increments, when the number of items that arrive in this time, they would be uniformly distributed, over the randomly distributed over the time interval 0.

So, therefore, in time 0 t the expected any uniform variable distributed over 0 t has mean t by 2. So, the expected wait time is t by 2, all items which start getting processed from here till up to this. So, their expected wait time is t by 2, because we have said that the, you know, the processing is uniformly done. I mean in the sense that the processing is not uniformly done it is that is they arrival. So, is distributed the arrivals of these items is distributed uniformly in the interval 0 t.

So, their expected wait time is t by 2 because they will be dispatched by the end of this time period. Now, so total expected wait of all items arriving in 0 t is therefore, lambda t into t by 2. So, expected wait time of any item is t by 2. Because, they are uniformly distributed in this interval the items and therefore, for each of them the wait time is t by 2, and since the expected number of items that arrive in this time is lambda t.

So, the whole thing is lambda t into t by 2, which is this. Now, the similar reasoning holds for items arriving in time t to t comma t, and therefore, for these items because they will get dispatched at time capital t. So, the total expected wait time will be therefore, lambda t square by 2 plus half lambda T minus t whole square.

So, I hope this part is clear, and this reasoning is because expected wait time into the expected number of items that arrive. So, that gives me the total expected wait time of all items arriving in time, in the interval 0 t. So, to minimize that to find out the minimizing value of t, I differentiate this expression with respect to t, and I get lambda t minus because there is a minus here.

So, 2 is gone. So, lambda t minus lambda T minus t is 0. And, this gives me t equal to t by 2, as you would expect. Because, the arrivals are uniformly distributed over the time interval, and just to make sure that this is the minimizing value, you find out w prime t and w prime t will come out to be 2 lambda, which is positive. So, therefore, this gives you the minimizing value. So, therefore, it says that you dispatch whatever items gets processed, in the middle of the time and then wait for the others to be processed and dispatch them at t. Simple which appeals to your reasoning also, but then through this machinery also we have arrived at this result.

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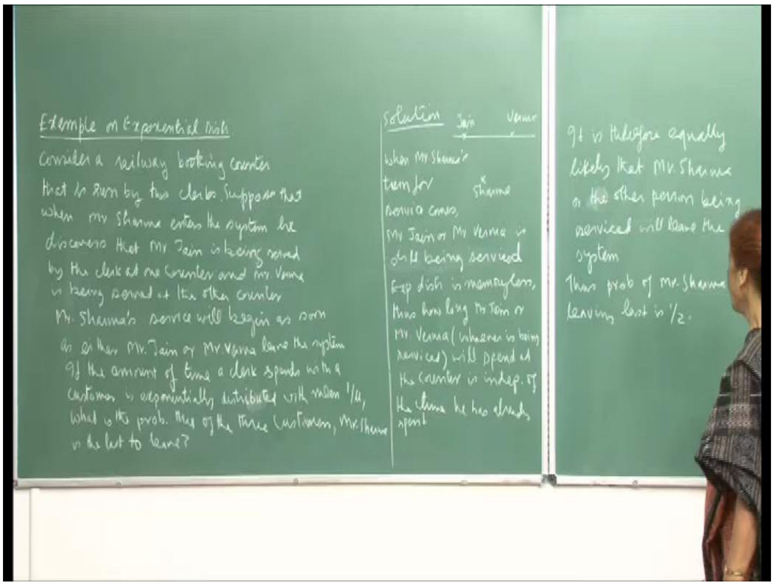

So, before I begin, you know, talking about queuing models, I thought, I will finish off the lecture on Poisson processes with this example on exponential distribution. Because, it is somehow related and part of it. And, so and this would be the right place to talk about it, because we have talked off exponential of the Poisson process. And, we have talked of people expected number of people in the system and so on.

And then because the inter arrival times had we have shown that each of them were identically independently distributed as exponential random variables. So, I thought this would be also this can be part of it. So, here the whole idea is that and of course, this is a simple example on the memory less property of the exponential distribution. So, consider a railway booking counter, that is run by 2 clerks suppose that mister Sharma enters the system he discovers, that mister Jain is being served by the clerk at 1 counter, and mister Varna is being served at the other counter.

So, both the counters are busy, when mister Sharma enters the system. Now, mister Sharma service will begin as soon as either mister Jain or mister Varna leave the system. That whenever as soon as 1 of them is completes service, they will, he will leave the system and then mister Sharma's turn will come to be serviced by the clerk.

So, if the amount of time a clerk spends with a customer is exponentially distributed, with mean 1 by mu; that means, the parameter of the exponential distribution is mu. And, therefore, the mean time that a clerk spends with a customer is 1 upon mu, what is the probability that of the 3 customers' mister Sharma is the last to leave? So, of course, here mister Sharma will only get serviced once 1 of the customer is left.

So, the actual question is that when mister Sharma's turn comes for being serviced there is 1 person 1 of mister Jain or mister Varna 1 of them is being serviced. And, so mister Sharma goes to the clerk for getting his job done, and then the idea is that who will leave the system first? So, suppose mister Jain is being serviced while mister Sharma goes to the clerk because mister Varna has left. So, the question being asked is, what is the probability that mister Sharma would still be in the system? When mister Jain leaves? So, essentially you are asking, who will be the first 1 to leave? Either mister Jain or mister Sharma mister Varna has already left.

So, but exponential distribution is memory less. So, therefore, how long mister Jain how long more, will mister Jain take is independent of? How long he has already been at the counter? Because we have said that it is memory less and therefore, the service gets completed is not dependent, on how long it will take for the service to be completed? It has not depend on, how long he has already been serviced and. So, therefore, its equally likely that either mister Sharma will complete his service, before mister Jain or mister Jain will complete. I am just assuming that mister Jain is still in the system mister Varna has left, but you can do it either way.

So, therefore, very simple you know use of the memory less property of exponential distribution. And, so therefore, the probability of mister Sharma leaving last is half, because it is equally likely, whether mister Jain completes his service first or mister Sharma completes his service, because of the memory less property. So, I thought this will just add to the Poisson process, and the other systems that we have been talking of birth and death process; that means, when you have people arriving at a service station, and then they are being serviced. And, so then we want to talk about, the you know number of people average number of people in the system then what is the average waiting time? And so on.

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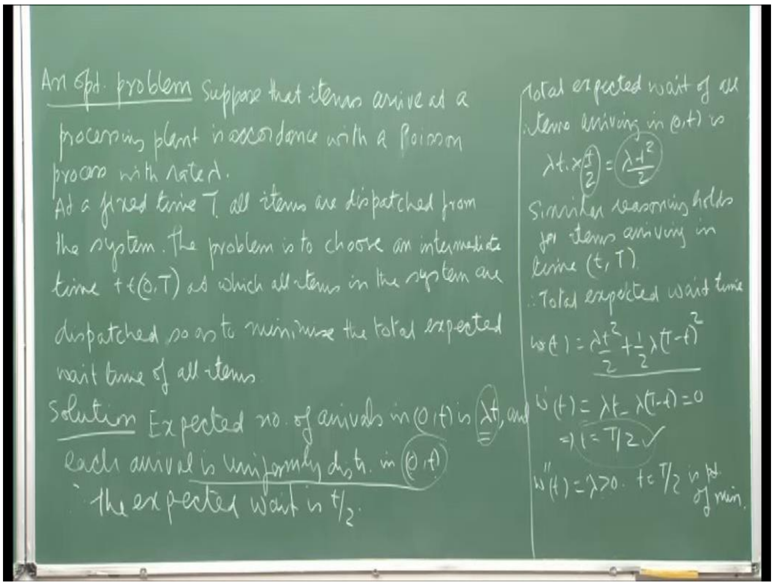

I would like to take this further because once we have been able to compute, you know arrivals, if we have been, we have discussed the Poisson process. One of the arrival processes is under these conditions that we have laid down. And, then now, we want to look at you know for example, you have a service center, and there you have people arriving for service.

Then you have people providing the service, and then that process is also random. So, now, you want to combine these 2 and therefore, the whole the theory that you, when you study such processes is known as, queuing process. And, then you want to for example, know when you have a post office you want to know, because if the average number of people arriving in the post office is large, then you would want to 1 clerk may not be enough to serve everybody. And, you would and then facility how they should be and so on.

So, the very interesting question, but of course, we will study them at very basic level. So, it will be the queuing process is where you want to compute the average waiting time of a customer. You want to compute the average service time of a customer, then you want to look at the average number of people they are at any time there are in the system and so on. So, such interesting questions we would want to answer. And therefore, we will model the situation, where you have people arriving for service. Services are being rendered, and then people leave the system. So, the whole thing we would want to study, and this we will try to do in the next couple of lectures.