Introduction to Probability Theory and its Applications
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Lecture - 29
Random walk Periodic and Null States

So, in the last lecture I had told you that it is a number of states are finite of Markov process and then all states cannot be transient, right. We had argue rate out that if the system has to go on then because transient states will be visited only a finite number of times. If they are finite number of states, then the process must come to an end in a finite number of times, but since the process has to go on, therefore if the number of states is finite, then all states cannot be transient, but the argument does not hold when the number of states is in finite. So, I said I will give you an example. When the numbers of states are in finite, it is possible that all states of this process may be transient, right.

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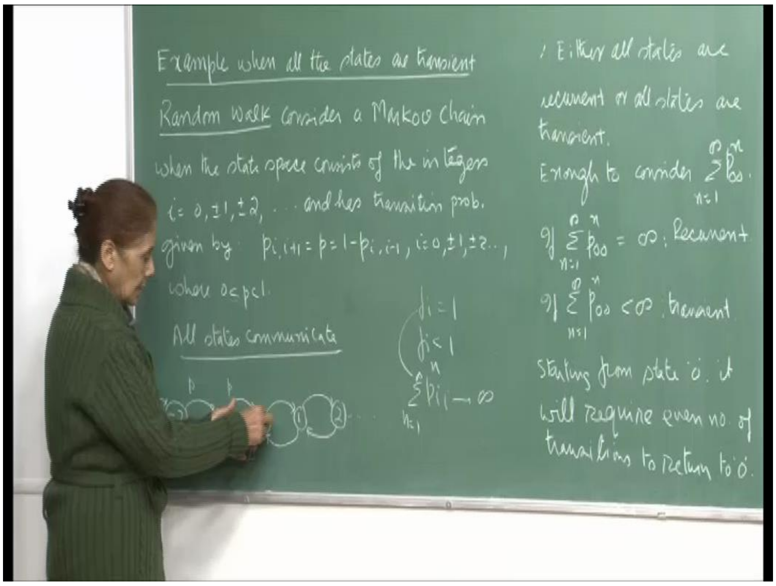

This is what we want to talk about today, so giving you an example when all the states are transient. Now, random walk is very interesting and important process and I hope that after having you know read about, now you will be able to recognize situation processes, stochastic processes which follow behavior of it random walk. So, this now a Markov chain when the states space consist of the integers i mean varying from contain the value 0 plus minus 1 plus minus 2, and so on. So, that means, I am numbering the states by minus 1 plus 1 plus 2 minus 2 0, and so on. And so this can go on for a infinite number.

The number of states is not finite. This is infinite states situation and then, the transition probabilities. So, if you go forward; that means, if you transition from state i to i plus 1, the probabilities is p and if your transition from i to i minus 1, then that means, if you go backwards, the probabilities is 1 minus p, and this is i varying from 0 to plus minus 1 plus.

So, therefore, the probabilities remain the same sensually. It is going forward. The probabilities is p and if you are going backward, then the probabilities is 1 minus p for some, any p varying between 0 and 1. So, therefore, for different values of p you will get different random walks. This is the idea and diagrammatically if you look at the transition diagram, the transition diagram simply says that. So, these forward probabilities are p and the backward probabilities are 1 minus p. So, from minus 1, you can go forward and then, it will go to state and from 0 if you go backward, you will go transition to this state minus 1 and the corresponding 1 minus p and therefore, this process go on either side. So, this is your transition diagram.

So, you see that from here that all states communicate because you can go anywhere by forward movements. If you go from minus 1 to 2, you can go here and then, you can again come here. So, any possible path is there, but you can transition from any state to any state. So, all states communicate, ok. So, therefore, either all states will be recurrent or all states will be transient because remember the recurrent states will form a class. So, if they form a class, then within a class, all states must communicate with each other. Since, it is already true that all the states communicate. Therefore, all the states will either be, so they are all in one class. So, they will either be recurrent or transient, ok.

So, here again remember I had defined for you recurrent state via the probabilities of first passage, first passage probabilities and remember we had said that if f i is equal to 1, then the state is a recurrent state because then there is positive. There is the event that it will recur back to itself is a certain event, right and if f i is less than 1, then we had said that the state is a transient state. And then, using an alternate characterization of a recurrent transient state, we had also said that if from here we had said that if p i summation, this will be summation p i i n n varying from 1 to infinity. If this goes to infinity, then the state is recurrent and this is less than infinity. Then, this state i is transient. So, this was another characterization. So, that is what we will use here.

Since, all states are behaving as exactly the same way because the probabilities are the same of going forward or backward. So, it is enough if we consider sigma n varying from 1 to infinity p 0 0 n. So, let me just look at the behavior of this expression and if I can show that this will be diversion series, then that means, it will go to infinity. Then, I can conclude that state 0 is recurrent and since, they are all the states from one single class, therefore every state is recurrent and if sigma n varying from 1 to infinity p 0 0 n is less than infinity, then 0 is a transient state which implies that all states are transient.

So, let us now start looking at this expression. For example, if you look at p 0 0 n, then that means, this is wanting to know that 0, you starting from 0, you will be back in 0 in a n steps. So, since you can see from here, from the red diagram that you can return back to 0 only in even number of steps, in even number of transition, right. If I go here, then I can come back here. So, 2 if I go from here and here and then, I need another two steps backwards, right or if I go from here, then I go back here and then, in fact you can have any possible number of forwards and backwards, but they should. So, if the number of transition if my n is odd, then I cannot comeback from 0 if I start from 0, right.

So, coming back to itself requires even number of states and you can just try to draw number of paths, and you can see that you can go from here, here, then here, then you can come back here and again here and then, here. So, all possible ways are thereof, but then you will require every time even number of steps.

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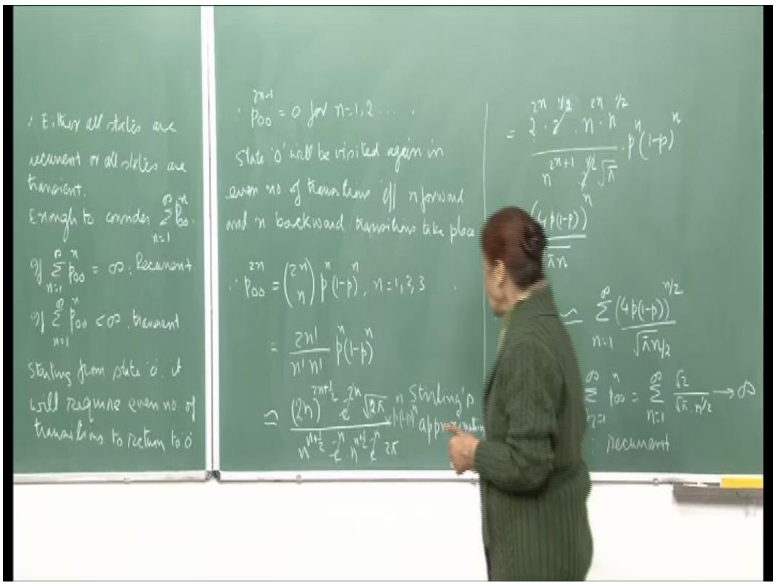

So, therefore, if you look at p 0 0 2 n minus 1, then this will always be 0, all right. You cannot transition to back to the state starting from 0. You cannot comeback to 0 if odd number of steps. So, those probabilities are 0 and for coming back, you need even number of steps. Then, you require exactly n forward transition and backward transition in any order right as I try to explain from the diagram. So, this will be equal to, therefore from 2 n you chose n forward steps, forward transition and n backward transition. So, therefore, the probabilities p 0 0 2 n will be like this, all right. Choose n from 2 n transition and then, p transition forward and 1 and 1, sorry n transition forward and n transition backward. So, probabilities of backward are 1 minus p, probabilities of forward transition is p and this is 1 minus p.

Now, let us just open up the expression here. So, this is 2 n factorial n factorial n factorial p raise to n 1 minus p raise to n. I will now use Sterling's approximation which I have already talked about in earlier lectures. So, factorial can be approximated by Sterling's formula and so, this will be 2 n raise to 2 n plus half e raise to minus 2 n and then, under root, we left out the part p raise to n and 1 minus p raise to n. So, this is and therefore, similarly n factorial can also be written by Sterling's approximation formula. So, n raise to n plus half e raise to minus n and then, there will be 2 root 2 pi. So, root 2 pi for both of them. So, therefore, it will be 2 pi and raise to plus half e raise to minus n and now, we can cross out few things. This and this gets canceled out and then, you can two raise to half. We can cancel out from here and then, I will be left with root 2 pi, yeah and oh this is root [FL].

So, root 2 pi was there because this is root 2 pi and this is 2 pi. So, root 2 pi was left and then, that root 2 gets canceled by this root 2 here and then, you can just see this simplification here and this is n raise to half. So, here this is n raise to 2 n. That cancels out with this, right and then, there is n here and there is n half. So, you are left with the root n. So, this is root n root pi and then, you have 2 raise to 2 n which I bring inside here. So, this is again 4 into p 1 minus p raise to n. So, this is the simplification, right after using Sterling's approximation for the factorials I get.

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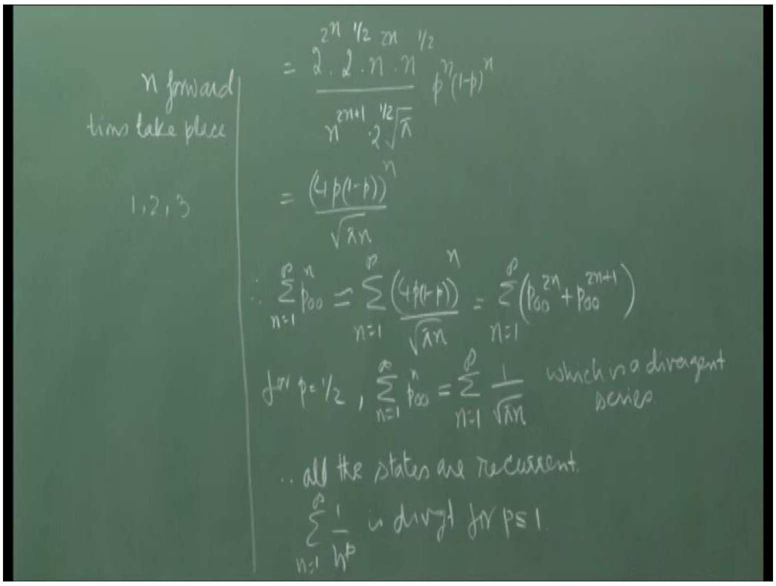

So, we saw that this series can be written approximated by the series and varying from 1 to infinity 4 p 1 minus p raise to n upon root pi n. Now, the terms I can breakup as sum of even terms and odd terms and here, even though I have written the index is n does not matter. So, dummy index. So, here it could have been m also, but the idea is that I am breaking up. I am writing p 0 0 n. Instead of this, I am writing the odd. I am adding up the odd terms and the even terms.

Now, the odd terms do not contribute anything to this sum. So, it is only the even part and so now, let us consider the case when p is equal to 1 by 2. So, in that case, this will become equal to 1, right. So, this series will reduce to summation and varying from 1 to infinity 1 root by n which we know is a diversion series because the power root by of course is constant. So, this will be n raise to half 1 upon n raise to half and we know that this series 1 upon n raise to p n varying from 1 to infinity is diversion for all values of p less than or equal to 1. This we already know.

So, therefore, this is a diversion series and therefore, since all the state, we will immediately conclude that all the states are recurrent because the time to return to this is infinity. So, all states are that mean sigma n varying from 1 to infinity p 0 0 n. So, this is recurrent and therefore, all other states are also recurrent, ok. Now, we have to consider the case when p is not equal to half.

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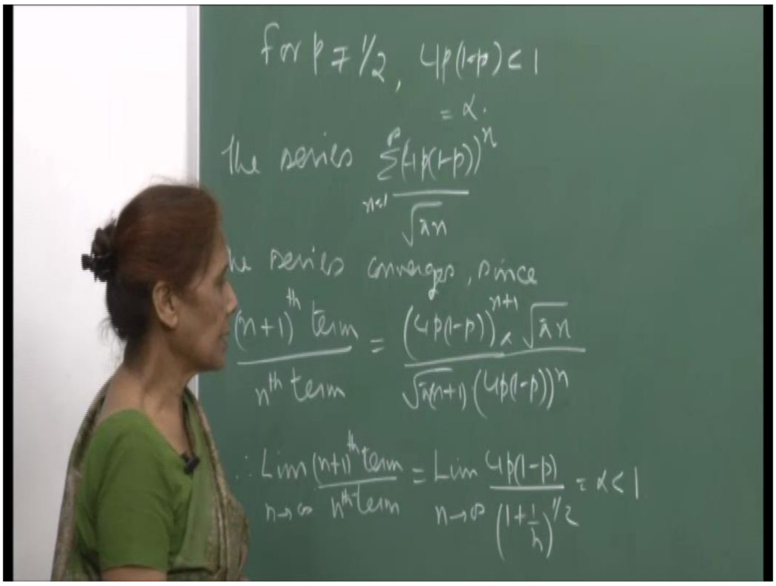

So, for p not equal to half, your 4 p into 1 minus p will be less than 1, remember because for p equal to half this, the maximum this as the maximum value for this term for you p into 1 minus p is for p equal to half. So, for p less than half, it will be less than 1, right. So, let me call this term as alpha. Let me denote it by alpha. Then, we will show that this series converges and simple. You just apply the ratio test. So, take the n plus 1 term divided by the nth term which will be you know the n plus yth term will be 4 p into 1 minus p raise to n plus 1 under root pi into n plus 1 and here you dividing by the n th term which is this. So, you have this.

So, here you see you are left with 4 p into 1 minus p and then, this root n i bring in the denominator. So, this will be 1 plus 1 by n raise to half and therefore, the limit of this ratio of the n plus 1th term and the nth term as n goes to infinity will be you see, this will go to 1. And therefore, it will just converse to 4 p into 1 minus p which is a number equal to alpha less than 1, whatever the value of p. Since, p is not equal to half, this number is will be equal to something less than 1, all right and yeah, so in that case we will conclude that all states are transient states.

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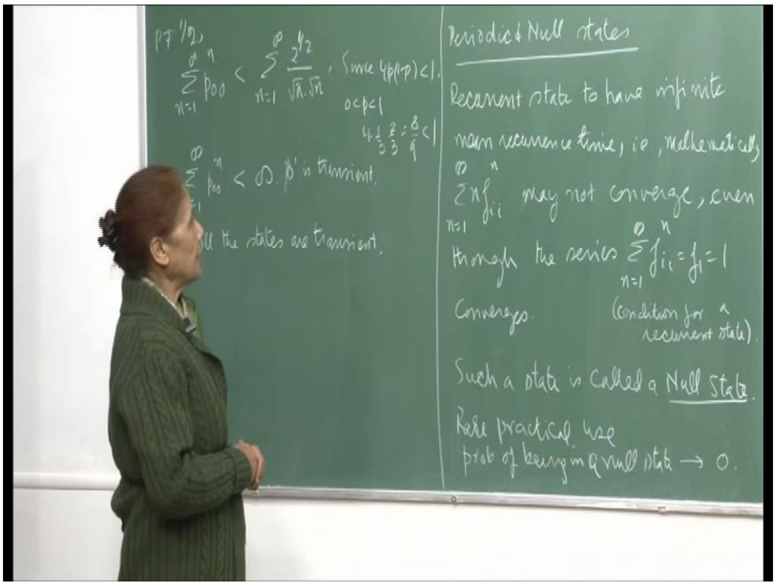

Now, of course, I had told you that you can look up for whenever now that you know this random process which backward forward and of course, for the transient case, the probabilities of going forward is different from going backwards and for recurrent states, it was both the probabilities are the same.

So, now just on the lighter side, we know you can say that if there is a drunken man and he is trying to walk on along a straight line, then he will takes step forward. Then, he will take two steps backwards or he may take two steps forward and one step backward or something like this. So, you know the wonderings of a drunken man. You can sort of say that the process of the walking of a drunken man can be modeled as a random walk and of course, it will depend on the p. Then, value of p will depend on how drunk he is or something like that, ok. So, that is one of the examples. Then, we may be we will come across some more in the process of this or otherwise, you can you know now be aware of such a process, ok.

Then, we will continue with the classification on the states and after the transient states, they are also states which are periodic and null states which are both of which are not of much practical use and of course, there occurrence is also not that often. So, now, you see we have talked as we said that the recurrent state, it is possible that recurrence state may have infinite mean recurrence time that is mathematically, this sum may not converge and remember for recurrence states even you wanted to find out the first passage time and so on. We did it by finding the mean recurrence time, the m i j s and m i i and so, when we did this, then we assume that this is finite; this series will converge, but it is possible mathematically that this series may not converge even though the series, this of course, this is your condition for state to be recurrent that is sigma f i i and n varying from 1 to infinity which is equal to sigma which is equal to f i. That is the recurrence time probabilities of the state coming back to itself. Then, this is equal to 1.

So, for recurrent state, these probabilities have to be 1 because coming back to a recurrence state is a certain event. So, this series converges, but this series may not converge which is your mean recurrence time and this is such a state, we will define as a null state, ok. So, not much is talked about it. So, we will also not spend much, but we must complete the presentation of the states of the classification of the various states. So, for the recurrence states where we assume that this is finite, because then we were solving for m I j s or m I i s, but there is possibility that this series may not converge, ok and this as I have already said these are a frail practical use and the probabilities of being in null state will also go to 0. So, therefore, we will not talk about such states, we will not spend much time on it, but just to complete the discussion, we have also considered the case when this may not converge. So, this may not be finite.

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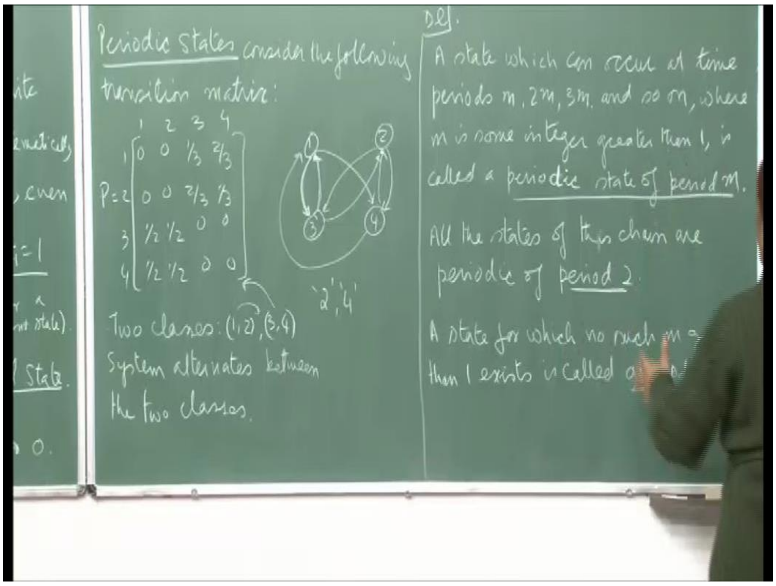

Now, let us talk about periodic states. So, consider the following transition may tricks and this is the corresponding transition diagram, and you can see immediately from here that from 1 you will either go to 3 or 2, 4 right and also from 2, you will go to 3 and to 4 and then, again from 3, you may go to 1 or you may go to 2 and from 4, you may go to 1 and from 4, you may go to 2. So, that means, there are two classes. You can immediately see because there is no communication between the states here, between among the states and states here, all right. So, it is you can just I should not say classes exactly because these two are not communicating, but they are communicating to the. So, you have communication between the states of this set and this set and vice versa, all right.

So, your system will alternate. That means at any time, either the system will be occupying states 1 and 2 or it will occupying 3 or 4, right because once if you start in 1, then you will either be in 3 or 4 and then, if you are in 3, then you will either be in 2 or back to 1, all right. So, you either communicate this way or you communicate this way. So, your system alternates between these two classes and that you can see by these probabilities also, ok.

Now, in fact, you can see from here that suppose from 1 I go to 3, then I can come back to this. So, that will be in two transitions. I can come back from 1 to itself or I can go from 1 to 3, then I can go from 3 to 2 and then, 2 to 4 and then, 4 to 1. So, that means, it will be then in case it will be four transitions that will be required to go from to start from 1 and come back to 1, right or look at the other thing. If you may go to from 1, you may go to 4 and then, again you can come back. So, again it will be two transitions, but if from 1 you go to 4 and then, you go to 2, then you will go to 2 to 3 and then, 3 to 1.

So, that means, you can recur back to state 1, either in two transitions for in four transitions and the same story true for state two. That means, from 2 you can come back to itself, either in two transitions or in four and the same again. What? It is three and four, right. From 3 you may go to 2 and then, come back, right or you may go to 2 and then, you may go to 4, then 4 to 1 and 1 to 3. So, all the states can be visited, re-visited either in two transitions or four transitions. Revisited I mean starting from that state, you will revisit either in two transitions or in or four transitions, right. So, you can see there is a periodicity. So, let us make a definition. So, we will say that a state which occur at time periods m2, m3 m and so on and m is sum integer greater than 1. This is called a periodic state of period m, right.

So, therefore, using this definition you can say that all the states of this particular chain are periodic of period 2, all right. A state for which no such m greater than 1 exist is called a periodic. So, here of course, the understanding is that it will actually the word should have been a state which can occur a times period m 2 m. So, that means, if you are starting from that particular state and then, it occurs again at period m2 m3 m, and then the period is m. So, that part is understood here, right that you are starting from a particular state and then, if you can visit it at regular intervals of periods m, m, some integer which is greater than 1, then that state will be periodic.

So, in this case of course what is happening is that all the states are periodic. So, this is one particular example, but of course you can have situation where you may have some recurrence states. So, these are also recurrent, but not in that sense. See here yeah. So, we will just see that there will be no this thing. There will be you can say that this series, in fact the periodic state you already know that you are going to visit it at a particular time. The probabilities of visiting it at regular intervals are there. It is positive probabilities.

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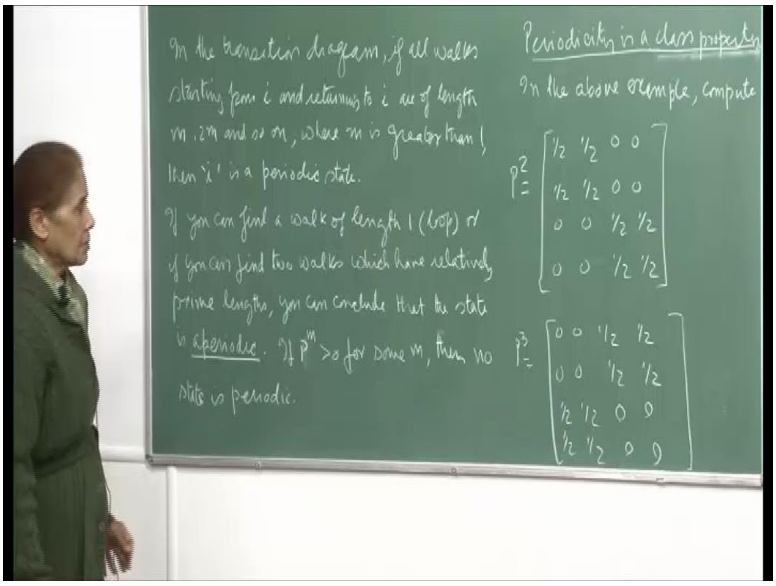

So, for periodic states we said that the coming back number of transition has to be a factor of some integer greater than 1. That means m 2 m 3 m and so on. Then, we will say that it is periodic state of period m. Now, of course, on the transition diagram if you can see that all walks starting from walks or paths, whatever we have been saying starting from i and returning to i are of length m 2 m and so on, where m is greater than

1. Then, i is a periodic state and the period is m, right. So, for example that we just considered, I showed you that any state you start from either 1 or 2 or 3 or 4, you can come back to them in either two transitions or four transitions or six transitions and so, we concluded that the period for each of the state was m, right.

Now, if you can find a walk of length one, that means, if there is a loop for a state that means, you can come back to it one step or if you can find two walks which have relatively prime lengths. That means, one step one walk may be of length m 1 and the other may be of m 2, and they are relatively prime. They have nothing in common. Then, you can conclude immediately that the state is not periodic. That means, it is periodic, right because if 2 there can be in two parts of length that is starting from that state and coming back to it, either in m 1 transition or m 2 transition and these are relative. Then, certainly there cannot be anything common. So, there cannot be a period to that state, right or if there is a loop, then certainly it is not periodic state, right. That is another way.

So, we are just trying to look at various ways in which you can characterize a periodic state and then, again if it turns out that your mth power, that means, the p m transition matrix raise to power m. If this is greater than zero, that means, all components are all entries of this matrix are positive from some m, then no.

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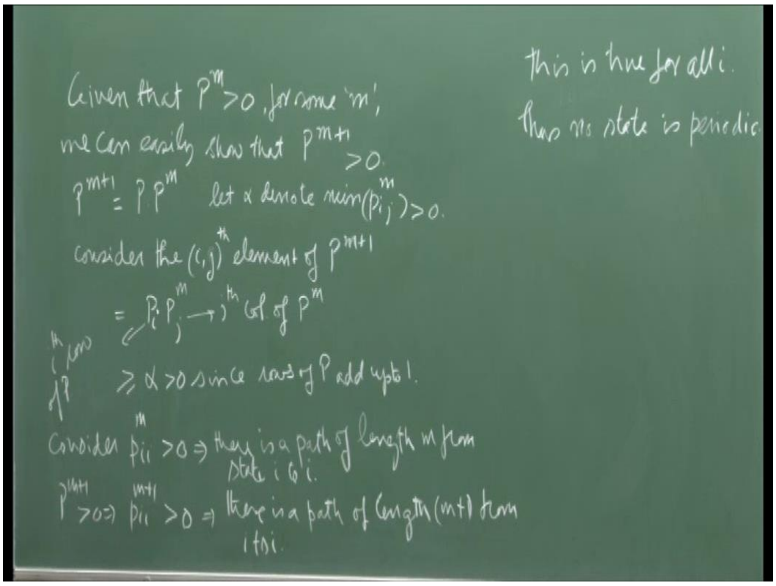

So, actually it is not difficult to show that if you are p m s or components of the matrix p m are positive for some m, then it can be easily shown that your matrix p m plus 1 will also be positive. That means all entries of the matrix p m plus 1 will be positive and this you can see immediately from here. See p m plus 1 can be written as p into p m. Let alpha denote the minimum of p I j m greater than 0 and then, we are saying because for every element of p m is positive. So, take the smallest and that smallest one, I am denoting by alpha and that will be positive, all right.

Now, consider the Ijth element of pm plus 1. So, that will be ith rho of p multiplied by the jth column of pm, right and so this is p i into p j m, right. Now, you see that when you multiply the ith rho with the jth column and replace each entry of the jth column by alpha because all other elements are bigger, right. So, I am writing the small as possible number for each of the entrees of the jth column. So, it will be p i 1 plus p i 2 plus p i n times alpha because since the rho's add up to 1, so this will be equal to alpha. So, therefore, the i j th entry of p i j of the matrix p m plus 1 is greater than equal to alpha which is also positive and this holds for any element i j of p m plus 1.

So, therefore, matrix p m plus 1 is also positive. Now, since all the entries of pm are positive. So, it seems that pick up any one, then p i i m is positive. That means, there is a path of length m from state i to i and since, p m plus 1 is also positive, matrix p i i m plus 1 is also positive which implies that there is a path of length m plus 1 from i to i. Now, by a definition you see m and m plus 1 for any m integer, positive integer, there are c prime numbers. So, we have shown that there are two paths from i to i of co-prime lengths and hence, by our definition, the state I cannot be periodic. It will by a definition, it will be periodic. So, n says this is true for all i. Therefore, no state is periodic. So, the proof was simple. I asked you to do it on your own, but I realized that we can show it right here and figure it out. Think about it.

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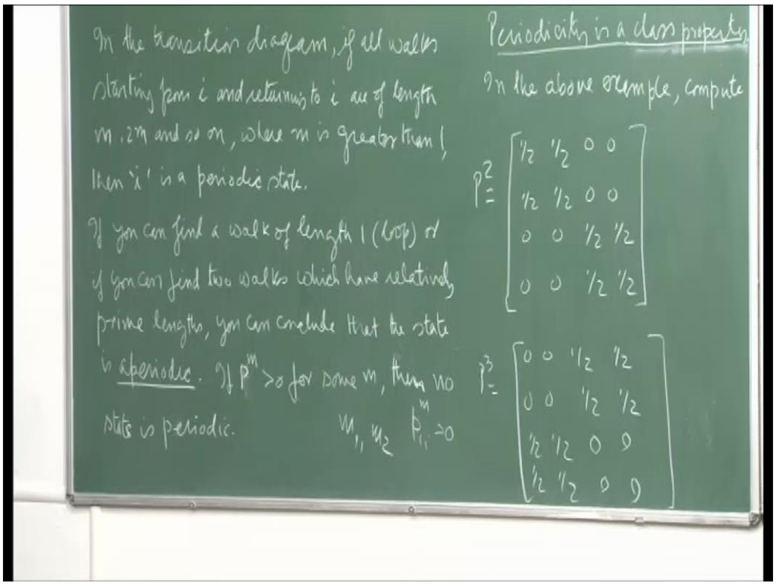

Now, of course, periodicity is a class property as we have shown. That means, you will have just as we defined that all states in that class would be periodic and having the same period, right. So, we can put together all states which are periodic of the same period, right.

Now, consider this example that we looked at the transition diagram for four states, and we saw that everything I did not write down. I think the probabilities or let us not so let us see given that matrix p. Then, you have p square. If you now multiply p with itself, then you get this matrix, right. So, your p was if we just wrote down in the last we said that 1 is going to 3 or 4. So, there were numbers here 1 by 3, 2 by 3 and so on and you had positive numbers and you had positive numbers here. These were zeros and then, when you take p square, the numbers, non-zero numbers shift here and the non-zero number from here shift here, right and each entry becomes half, right. Then, p 3 when you now take p 3 that is you multiply p 2 with that transition matrix p and then, you will get these entries will shift here and these entries will shift here. So, this is how.

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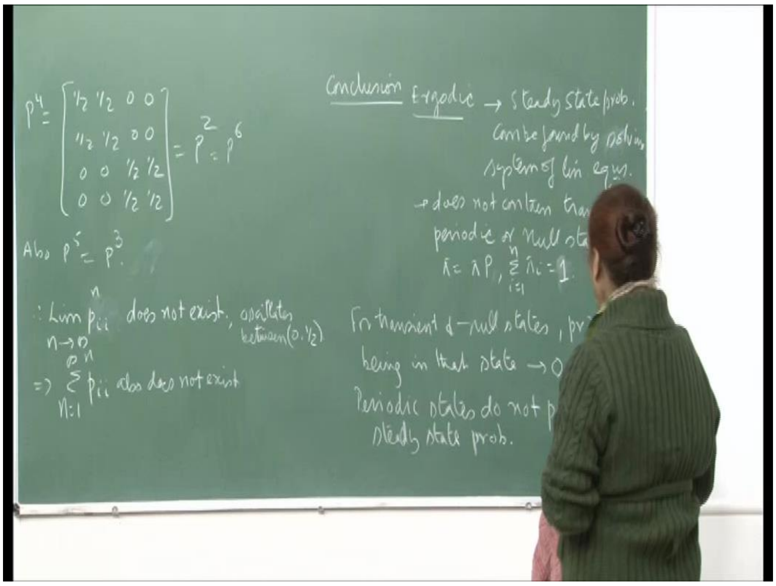

So, I just want to go through powers of a transition matrix. I am just trying to show you what is happening and then finally, when you take p4, the half will again shift here, ok equal to p square which you would expect because every state of system had period two. So, therefore, after two iterations of two transitions, this will be the transition matrix will be the same. So, p 2 will be p 4 will be equal to p 6 and so on, all right.

Similarly, p 5 will be equal to p 3 and this is equal to p 3. This will not be equal to p. That is true. So, p 5 will be p 3 and then, all other powers will, all odd powers will continue to be the same because then you see p 3 tells you probabilities of going from 1 to 3 in three steps. That is what will happen, right. It will come back to itself and then, go back. So, from here it can go to 3 and 4 and again in three steps. So, either it happens you see you had 1 here and then, you have 3, right and you had 4. So, if come to 3 in one step and then, 3 you cannot go back to it. You cannot come back to from 1 to 3. You want to comeback in three steps. So, you will either go here or you will go here, right to 2, right and then, you may come back to 2, 3.

So, it will take three steps to come from 1 to 3. So, starting from 1 if I want to come back to 3, then you will have to require three steps because in two steps, you can either starting from 1, you can either come back to 1 or 2. So, therefore, you will need odd number of steps to go from 1 to 3 or to 4. So, either in one step or in three steps, five steps and so on because even transition are reserved for coming back either to these things. So, that means, within a class and that is why we said that this and this of course, they all had the same period, but here what we are saying is that you can go from 1 to 3 and then, 3 to back or 1 to 4 and 4 to 1 and so on, ok.

So, this is what is happening and so, you see that p i i n. So, if you see here the first matrix, your p i i, so 1 1 1 was 0. Now, it is half and then, again in p 3, this number is 0 and similarly, all these numbers are 0 and all these numbers here in p 2 are half, right and then, in p 4 again, they are half and in p 5 when you again look at it, all the numbers, all these p i i will be 0. So, this limit does not exist, right. So, limit p i i n n going infinity does not exist. It actually oscillates between 0 and half. So, therefore, this is for the first condition for this series will also not converge. If you want to take this summation, but anyway these are not the first step transition probabilities. So, all we are saying here is that limit p i n as n goes to infinity does not exist this limit.

So, this summation will also not exist. This is not convergence series p i i n n varying from 1 to infinity, but will not be a conversion because the necessary condition for this series to converge is that the lth term must go to 0 as n goes to infinity. So, here either limit does not exist. So, therefore, I cannot say anything about this series also, ok. So, now after having looked at all possible states of Markov process, the conclusion is that you know if the process is erotic, then of course, finite number of states, then the study state probabilities can be found by solving system of linear equations as we saw, right. By solving this matrix equations pi equal pi p and sigma pi i i varying 1 to n is 1 with this condition because otherwise the solution here is not unique. So, we saw that when you put this condition, you will be getting unique solution if your system is erotic, that is finite of number of states and all states are recurring, ok.

So, therefore, the system does not contain any transient periodic or null state. So, this was one convenient way, but then we saw that there were other states also. So, for transient and null states probabilities of being in that state is 0 and of course, periodic state also do not possess steady state probabilities, all right. So, when the state possesses as study state probabilities, then we saw we can solve it by this linear equation adding this equation to it, but otherwise periodic as we have seen do not possess steady state probabilities for transient and null states, the probabilities of being in that state goes to 0 as the process continues for a longtime. So, we have finally sort of completed this argument and said that when you know that the system is erotic and so on, then you can solve prostate, the state probabilities using system of linear equations. We have methods how to classify and how to decide that the state is periodic and it is null or transient, ok.

Now, the thing is that periodic and null states are not much of practical use and they do not or there occurrence is rare. So, we will not talk about them, but transient state you know transient processes are there. That means there are lots of practical situations where the process does not continue for a long time. So, therefore, we call them to reduce Markov processes and so, there transient in the sense that after a while the process comes to an end. So, we would look at in more detail because there are situations where your processes are not supposed to continue for forever. So, we will define them as reduces Markov processes and we will talk.

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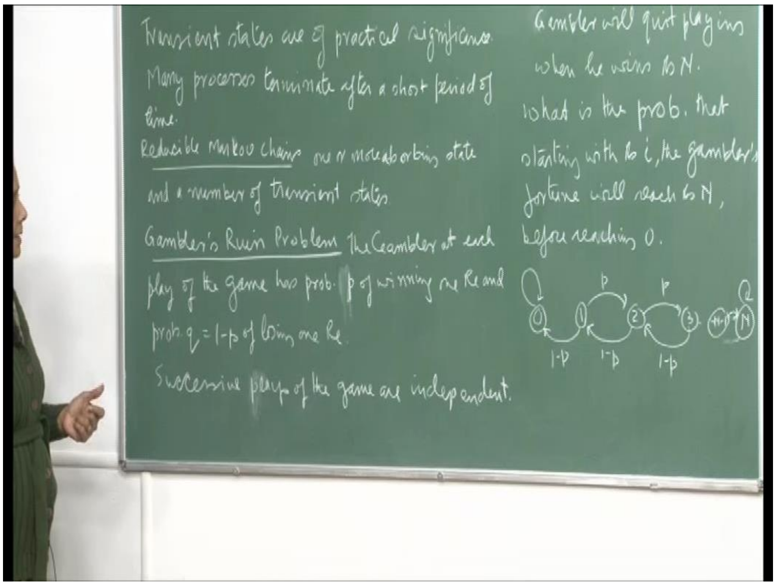

So, reducible Markov chains, one or more absorbing state and number of transient states, right because we are saying that the process will not go on. It will terminate after a short period of time. So, therefore, there will be either one or more absorbing states and remaining will be transient states. So, again by our discussion, we have seen that once you reach an absorbing state, you will not go out of that states. So, the process will terminate or if you are in a transient state and the numbers of states are, then the number states are finite. Then, again after a finite period of time, the process will be over.

So, this is what we are talking about. So, the interesting example and that is the gamblers ruin problem. Now, the idea is that the gambler at each play of the game has probabilities p of winning 1 rupee and probabilities q of losing 1 rupee. So, there is a game. Why I will tell you? Why it is called gamblers ruin problem? So, now, successive plays of the game, yeah it should be plays of the game are independent. So, successive plays are independent. That means, whatever the outcome of one play, the game goes on independent of what has happened and the gambler will quit playing when he wins rupees n.

So, he wants to make a fortune of rupees n. He is hoping for that. So, he will quit the moment he has earned n rupees. So, we want to find out the probabilities that starting with rupees, I suppose he has this much money with him. The gamblers fortune will reach rupees n before reaching 0, and that is what being by the ruin because if he allows the process to go on, then he will ultimately loose all the money and that will be the end of his playing gambling because then he cannot bet anymore.

So, let us look at the transition diagram. So, here what we are saying is that see with 0, he cannot play because he has no money to bet. So, he stays here, otherwise if he has rupee 1, then he can either loose that rupee and come back to state 0 or he will win and he will go to with probabilities p and then, he will go to 2. That means he will have 2 rupees. So, the state is described by the amount of money he has, and that is how we are using these integers to describe the situation, right and then of course, again if he has 2 rupees, he bets and he loses that rupee. Then, he will again revert back to having rupee 1 and so that will be the state.

So, this is the diagram and finally, at n minus 1 when he has n minus, 1 rupee, he will bet again and if he wins, he will get n rupees. He will make his fortune. So, we want to compute that and of course, what we are saying is that he will stop playing. So, there was no one going back from here because he will just quit the game, right.

So, this is the whole idea. So, this is now if you look at it, this is you know the duration of the processes is finite. And you can see that this is an absorbing state and you may call this as an absorbing state, and all other state are transient, because the moment he has some money with him, he will bet and then he will either convey, he will either go back to 0. For example, here he will whether win and he will go to transient to this transition to this state or he loses and then, he transition here. So, again this is an observing state and he loses. He just looses all the money. Therefore, the game is over. So, this is what we want to talk about and I will show the end of course. The whole idea is to compute the probabilities that starting with rupees i, the gamblers fortune will reach rupees n and you can say that here of course.

So, the process here you can immediately see that this is the Markov process because your transition and it just depends on where you are. So, your transitioning to the next state just depends on where you are and it does not matter how you reached one, or how you reached two. It just shows. We can immediately conclude that this is the Markov process and this is a short period duration. That means, it will terminate if the gambler ends up with rupees n, otherwise of course or if he just loses everything and he is back here. So, starting with rupees i, you want to find out the probabilities that the gambler will reach the fortune that we want to make.

So, the gamblers ruin problem. I have to put as an exercise. In exercise 10, which I will be discussing after some time. So, I have after explaining the problem, I have now left it to work out the details and let us hope that you enjoy doing it, and you are able to compute the probabilities, but while discussing the exercise, I will also give some hints and try to show how to go about it.