Introduction to Probability Theory and its Applications
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Lecture - 26
Transition and State Probabilities

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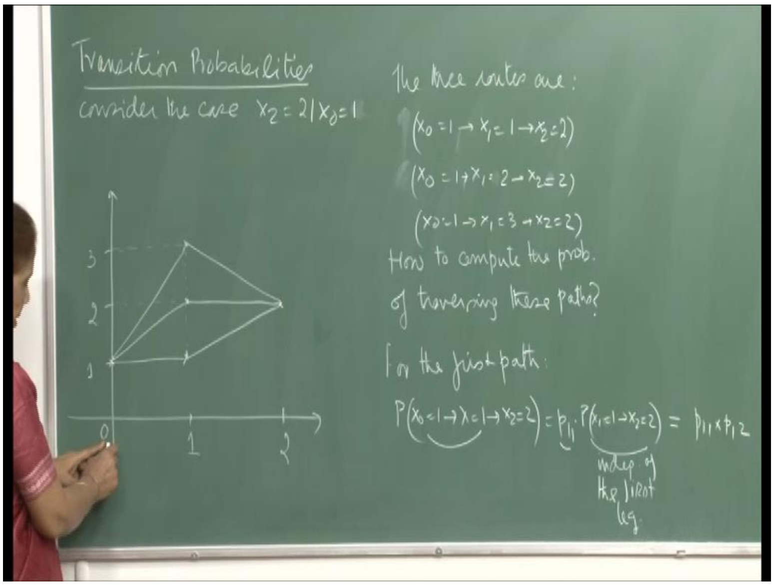

So, I will continue the discussion about computing the transition probabilities one step transition probabilities I showed you and we have just started talking about two step transition probability. So, just consider a case in last lecture I considered the case x 2 equal to 1 x naught equal to 1 now let us look at x 2 equal to 2 and x naught equal to 1. So, in that case you see that what are the possible routes of transitioning from the initial state of 1 2, the state 2 in 2 steps. So, I want to look at it. And of course, three possible paths would be see at step 0. So, this is a time period 0, time period 1, time period 2 and these give you the states. So, therefore, at time 0 the system is in state 1 it transitions to state 1 that is a possibility, because in one step you may transition to 1 2 or 3 and then you have to come back to state 2.

So, therefore, one possible route would be from 1 to 1 and then 1 to 2 right in one step transition in one period you go from 1 to 1 and then from 1 to 2. Similarly you could go from 1 to 2; that means, you transition to, so this was our production and this is h r. So, from production to h r and then h r to again h r right, you can continue here or the third route would be from 1 to 3; that means production to sales and then sales to h r. So, these are the 3 routes and that I have written down the 3 routes. In this way these are the 3 possible ways, no other route is possible of going from 1 2 2 in 2 steps right. Now, we need to compute the probabilities of traversing these paths, because you want to compute two step transition probabilities.

Now, look at the first path. So, the first path I know the probability of transitioning from 1 to 1 when I am initially in state 1 and I am now transforming or transitioning to state 1 in the next time period then it is p 1 1, I know that now look at this path of the leg. So, this is the 2 paths of the journey from 1 to 2 for example, so this is this. So, 1 so first 1 leg, I know you already know the other 1 step transition probabilities.

Now, we use the Markovian property, because you see the probability that you want to compute is going from 1 to 2 in; that means, you are actually looking for a probability x 1 to 1 x we are in 1 at time period 1 and you wanted to transition to 2 time period 2 right. So, then you see this is independent of the first leg y, because you see this is independent the Markovian property says that from here to here the transition probabilities independent of where I was at time zero the initial state. So, its independent of value of x zero right and therefore, I can write the probability of traversing this path as product of the first leg into the probability of the second leg. So, this is the idea and this is where I am using and otherwise I would not able to write these transition probabilities two step transition probabilities if I did not have the advantage of the Markovian property ok.

So, it is clear that, because as remember, I said that the past history is not important for computing is not required for computing the transition probabilities. So, the here it is, where you are currently and where you want to transition. So, this is the only thing that we need to compute the probability I do not need to know, where what was the value of x naught right. And so therefore, the two are independent and since by our stationarity.

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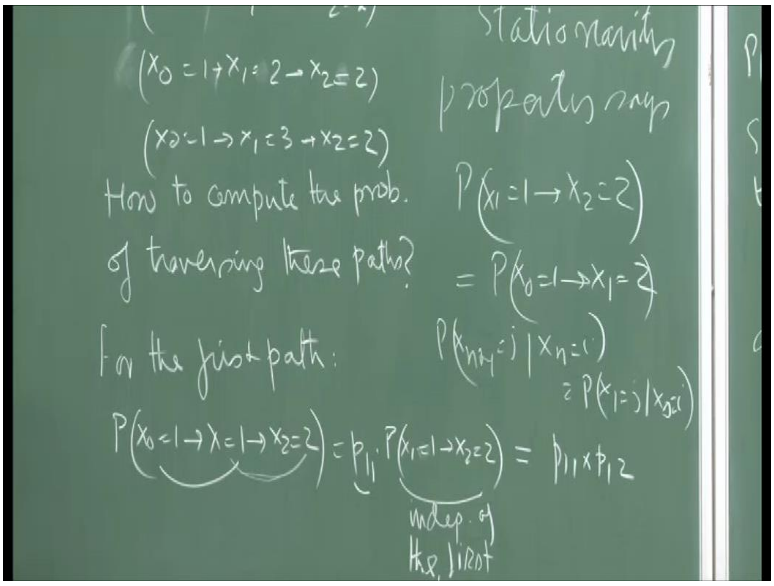

So, the stationarity property, I should say that stationarity property gives us says that probability x 1 equal to 1 and transitioning to x 2 equal to 2; that means, the conditional probability of being in 1 and transitioning 2 this is same as probability x naught equal to 1 transitioning to x 1 equal to 2 right.

So, remember because we said that this stationarity say that probability x and plus 1 equal to j given x and equal to I is a same as probability x 1 equal to j given x naught equal to I right. So, the stationarity property says that these transition one step transition probabilities remain the same no matter in which time period you are and therefore, this transition probability of going from 1 to 2 was the same as the probability x zero equal to 1 so going from here to 2 in initial stage. So, therefore, we are able to write down the probability of traversing these paths. And so I have written them down here for the all 3 paths.

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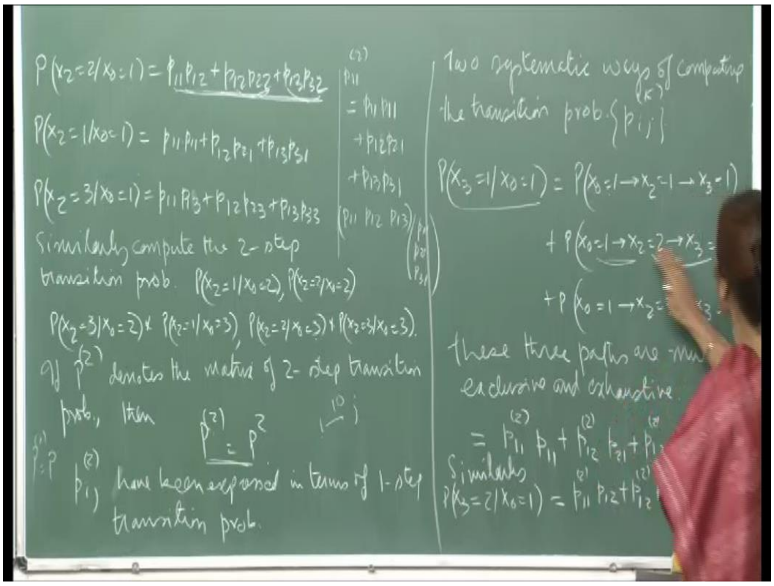

So, this is for the first path, this is for the second one, because you are going from 1 to 2 and then 2 to 2. Why have I written it this 1 p, p 1? Ok this was to 1 to 2 and here I am using the fact that you are going from what is this path x 2 equal to 2. So, what is the path I have written here x naught equal to 1 so 1 1 and 1 2. So, this is a path right and then you have 1 2 and 2 2.

So, that should be, so this is that one and this is this one and this is a third one, 1 to 3 and then 3 to 2. So, this is 1 1 and 1 2 and this is 1 2 and 2 2. So, that is the 1, which I have written here and this is the 1 corresponding to this and then this is 1 to 3 and 3 to 2. p 1 1 say, I do not know why am I writing this as x naught is 1 and then you are going to what is this path? This will be 1 3 and then this will be 3 2, ok sorry, all these 3, I am sorry, these 3 paths together give you this probability right of x naught equal to 1 and x 2 equal to 2 yes this is. So, these are the 3 possible ways of going from 1 to 2 in 2 steps right 2 steps transition. So, all these 3 add up to this, I am sorry, this is the 1 right, p 1 1, p 1 2 and then p 1 2, p 2 2. So, this is p 1 1 1 2 then this is 1 2 2 2, which is this path and then it is 1 3 and 3 2 so 1 3 and 3 2. So, these are the things.

So, now similarly when you want to when your x zero is 1 and then x 2 is 1; that means, in 2 steps you want to transition 1 to 1. So, again you will have 3 possible such paths right, maybe I will just again repeat the whole things then there is no confusion. So, for example, you can go from 1 to 1 and then 1 to. So, this is here see the path corresponding to this will be this right. So, maybe I make it this thing yeah just 2 and then what are the possible way you can go from here 1 to 2 and then 2 to 1. So, 1 to 2 yes and then 2 to 1 so this is the 1 right.

And then the third path would be when you 1 to 3 and then 3 to 1 right. 1 to 3 so you would go from 1 to 3 and then 3 to 1. So, this would be 3 paths and then for the each path you would write down the probability. So, this corresponds to the three exclusive paths through which you can go from 1 to 1 in 2 steps and similarly this will be 3.

So, now you have computed all the two step transition probabilities for when x naught is 1 right and then in the in 2 steps you can be in 1 2 or 3. And so similarly you will have a six more such transition probabilities, when x naught is 2 and then you want to transition to 1 or 2 or 3 in 2 steps starting from a step 2 at time 0. And then finally, it will be x naught equal to 3. So, x naught is equal to 3. So, because you do not know you could be in any of the 3 states or the beginning of the process. And so x 2 is equal to 1 you are transitioning to 1 from 3 in 2 steps from 3 to 2 in 2 steps and from 3 to 3 in 2 steps right.

So, let p 2 denote the matrix of 2 step transition probabilities just as p we are not writing p 1. So, understood p 1 is said this is p. So, this is a transition matrix of 1 step transition probabilities now let p 2 denote the matrix of 2 step transition probabilities. Then we will just quickly notice that p 2 is actually p square, because ya here you have written the components of this, let me just write down say for example, if you want to write. So, p 2 this here will be the first one will be a p 1 1 2 you want to look at right. And so that will be going from so p 1 2 p 2 1 plus p 1 3 ok, first let it be 1 1, 1 1 then it will be 1 2 p 2 1 plus p 1 3 p 3 1. So, this will be a first element right, these are the 3 paths, which I had drawn in the last lecture ok.

So, then you see this is multiplying a p 1 1 p 1 2 p 1 3. So, first row of p and with p 1 1 p 2 1 p 3 1. So, if you multiply the first row of p is the first column of p you get the entry p 1 1 2 right. And now, you can also verify this for example, this is the first row p 1 1 p 1 2 p 1 3 you are multiplying with the second column p 1 2 p 2 2 p 3 2 right. So, to get the entry 1 2 in; that means, it here if you want to get the entry 1 2 then you are multiplying the first row of p with the second column of p. And therefore, you can just verify yourself quickly that p 2 the second step condition maybe nothing, but the product of p and p. So, that makes like very easy and we will show that this is valid for all values of for a higher powers of p also; that means, if you want to look at yeah.

So, let me just show you a systematically that we would be you are really on the path of getting a very interesting and very useful result. Because to be able to compute these transition probabilities any step transition probabilities by raising the power of p is a very convenient way of getting the transition probability right. So, here you are want to know that what is the probability that you will be in tenth steps you will be from i to j starting in state i you will be state j then the tenth the i j th entry of p ten will give you the at probability and so on. So, you will see that yeah. So, basically what we have we have shown is that your 2 step transition probabilities can be expressed in terms of 1 step transition probability right, because we are just multiplying the 1 step transition matrix with itself and we are getting computing the 2 step transition probabilities right ok.

Now, same way we can do it for any power and here again I will just want to spend time it may look like little reputation, but it does not matter, because you must get the ideas very clear. So, for example, now yeah so the whole a target is now to compute p i j k; that means, k step transition probabilities we want to compute. And so here again I will just start from x 3 equal to 1 and x naught equal to 1. So, suppose now the 3 steps, you want to transition from 1 to 1. So, this again I can break up like this x naught starting with 1 you are in 2 steps in 1, state 1 and then you will be going again transition to 1 right. So, this is your 2 step transition probabilities and then this is your 1 step.

So, again I can write down this and now these 3 paths are mutually exclusive and exhaustive right. This is not because there you can go to x 2 you can be after 2 steps after 2 transitions you can be in state 1 2 or 3 right. Starting from x naught equal to 1 and then you have to transition finally, to a state 1. So, therefore, these are the 3 paths right. And here again we are using the Markov property because this one is this then this part of the path will be independent of the probability for this, because here you have x 2 equal to 2 and x 3 equal to 1.

So, there only the current state is needed to compute the transition probability and this is not dependent on where you were earlier either at x 1 or at x naught, that is not important. So, therefore, we will write this and again using stationarity I will simply be able to write this also again as a 1 step transition probability of going from 3 to 1 right.

So, therefore, you see that these 3 probabilities can be written as p 1 1 2 yes in 2 steps you are going from 1 to 1 and then again 1 step you are going from 1 to 1. So, this is p 1 1 2 and p 1 1 right. Similarly 2 step transition from 1 to 2 1 to 2 p 2 and then 2 2 1 p 2 1 plus p 1 3 2 and p 3 1 right, this is yeah right.

So, and then I can write down x 3 equal to 2 when x naught is 1 and then of course, probability x 3 equal to 3 x naught equal to 1 right. This will also be equal to so p 1 1 p 1 3 plus p 1 2 p 2 3 plus p 1 3 p 3 3. So, this is the first row of and you can see that since this is your element of p square. So, therefore, if you write these are the entries of your p square and then you are again multiplying by p 1 1 p 2 1 p 3 1 so; that means, so essentially yeah ok.

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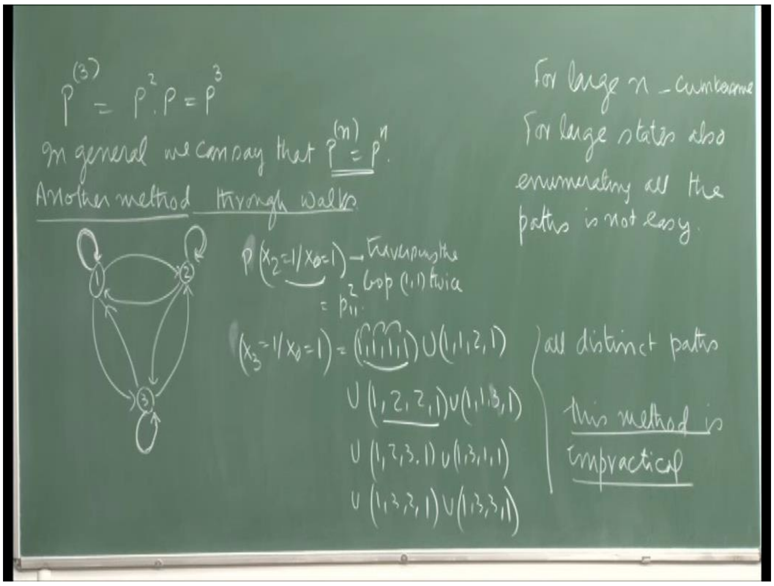

So, we saw that p 3; that means, if you want the 3 step transition probabilities, this will be given by p 2 p square into p. So, again a third power p. And so in general we will should be, we can now say that if you want to n step transition probabilities then this will simply be raising the matrix p to power n. That means, multiplying p n times and the entries in p n will give us all the n step all the n steps transition probabilities that we need right.

Now, there is a another method through walks on the transition graph, but you will soon realize that it is not a very efficient method it is fine to see what is happening when your graph is small in number of states are also not too many. So, for example, when you want to look at this probability of 2 step transition probability from going from 1 to 1 right.

So, which would mean that you traverse this loop once and then again one more times, 2 times you traverse this loop and you have this then you can compute the probability, which would mean that this is a traverse of probability of traversing the loop once is p 1 1. So, when you do it twice it will become. So, the probability would be. So, the probability this would be p 1 1 square right, fine. Then if you want to for example, compute the 3 step transition probability of going from 1 to 1 then let us see what are the we will try to see all possible paths on this transition graph going from 1 to 1 right.

So, of course, I have not written the detail for example, x naught equal to 1 x 1 equal to 1 and x 2 equal to 1 and x 3 equal I am just you know made the notation simpler. So, here this tarsus the, you know 1 path which is you are traversing this loop four times right, I mean 1 to 1 sorry, 3 time. 1 to 1 then 2 and then 3, three times, because 3 to 3 transition probability so this path you would traverse 3 times; that means, the loop you will traverse 3 times. Then the probability is that you go from you traverse the loop once then you go to 2 and then from 2 go to come back to 1 right. So, you traverse this loop twice then you go to 2 and then 2 to 1. So, this is 1 path and therefore, here again you can write down the probability as p 1 1 square into p 1 2 plus p 2 1 right.

So, once you write down. So, the eight possible paths, you can see that right 3 step transition and you this 3 possible states so therefore, eight possible paths. And similarly like 1 to 2. So, you go from 1 to 2 then you traverse this loop 2 to 2 then 2 to 1. So, this will be p 1 2 into p 2 2 then p 2 1 right. So, this way, I can write down all the probabilities and then compute the. So, therefore, they are all distinct paths and you can compute the probability of each path and as I told you each leg of the path is independent. So, we are multiplying the corresponding probabilities of traversing each leg of the path. And so you can write down, but you see that this will become very conversion the moment your number of states become, you know if you had four to five states or even, you know easily any Markov process that you consider we have seven to eight or ten states.

Then you see the possible number of the number of paths were really go up right, exponentially. And then you may it is a since its very likely that you will miss out on some of paths, because you have to inomulate all the possible paths. And here in this case only you know for x 3 equal to the 3 step transition probabilities for 1 you are going from 1 to 1 you had to inomulate 8 paths.

Now, if it becomes x four it will be 16 paths. So, the number this will blow up and. So, therefore, this is for large n and for large number of states; that means, if you want to ten step transition probabilities and the number of states is also ten. Then it just not possible for you to inomulate all possible paths and then compute the probabilities, but for small cases and. In fact, to see actually what is happening this is a good way right. So, I thought I have just talked to you about it and when you are working out with small problems you can actually see this, but otherwise this is really a very efficient way of computing the higher order transition probabilities. Now, this is fine. So, therefore, we have now a method of computing any step transition probabilities provided we are giving the 1 step transition probabilities right.

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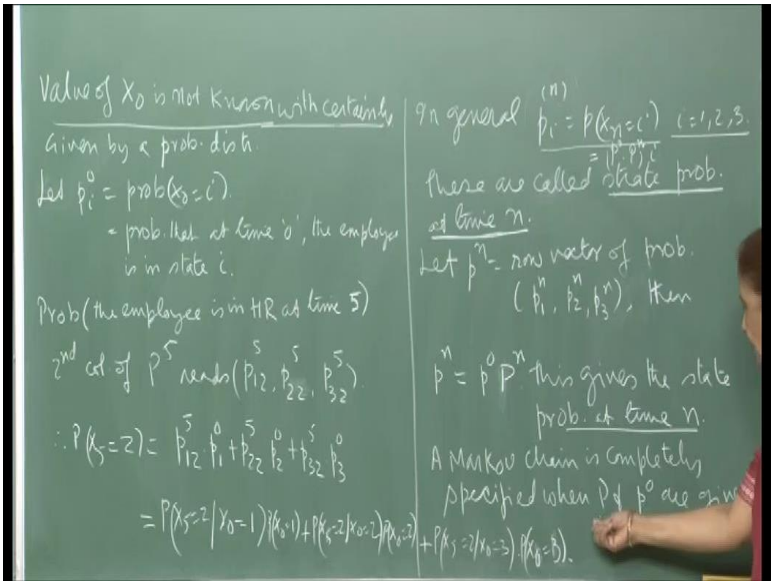

But again we there is some more information that we need and that is see the value of x naught is not known with certainty we do not know when which a state the system has was initially or for where it started right. So, this will be normal given by a probability distribution; that means, you will be given these values. So, p I naught is a probability that x naught is an state I right. So, this at time 0; that means, that initial time the employee is in state i. So, this is there is a probability attach to it.

Now, if you want to compute the probability that the employee is in H R in human resource section division at time five. So, you know; that means, you want to compute yeah, so let us see you want to compute the probability that x 5 is equal to 2. So, the particular employee at time period five is in h r right. Now, let us see the column of p 5 remember the 5 step transition probabilities are given by p 5. So, the column would be p 1 2 5 p 2 2 5 and p 3 2 5. So, actually I should have written this let me write this instead of you know immediately jumping to this. So, we want to say that the probability x 5 is equal to 2 given that x naught is 1 right. Then you will multiply this by probability x naught equal to 1 right. So, I am writing this as a conditional probability then into the. So, then I will have to do it for all possible values of a x naught.

So, to get the probability that x 5 is equal to 2. So, this will be probability x 5 is equal to 2 given that x naught is 2. So, probability x naught is equal to 2 right, plus probability x 5 is equal to given x naught is equal to 3 into probability x 3 is equal to oh sorry, x naught equal to 3. So, I break up this. So, again you see mostly what you have seen that, you know the basic probability theory that we need for analyzing these chastic processes. So, this is a writing probability as a, you know breaking it up into conditional events and then writing them down as the some of these conditional probabilities right. And so this one gives you this is your five step transition probability from 1 to 2. So, 1 to 2 5 and then into probability x naught is equal to 1, which is given to you from here.

So, p 1 naught so this will be p 1 naught right. Similarly this will be a five step transition probability of going from 2 to 2. So, this is p 2 to 5 into probability of being in state 2 at time 0. So, which is p 2 0 and this is third. So, this is how you can write down the probability if you want to know that the in time period 5 the employee will be in H R right. And so you can now compute in general you can say that this is now p i n will be the probability at the system is in state i at time n and see here just to make the presentation simple. I am just taking all the time referring to our job assignment example, but you know that this can be made to whatever the number of states general you can have a simple k here. So, k states and everything can be argued with respect to general number of states, but here I am doing it for this particular example just show that you can fixed your ideas better ok.

So, then p i n is probability x n equal to i; that means, the system is in state i at time period n. So, these are the probabilities and I have shown you how you will compute them. So, you can write them down right here. So, this will be your, I am writing this small p naught into p n. So, when you want the probabilities of the system being in a particular state and time period n then this is a formula and these are called the state probabilities at time n. So, this is the important right.

So, now we have we do also have the see we have the transition probabilities for any time period and we also have the state probabilities at for any time period n right. And yes so I have written it here where these I am saying that p n is your row vector of probabilities. So, this is the i th component. So, then p n is ya, I should have written here so this is the i th component ya. So, in general so if this is a row vector is p 1 n p 2 n p 3 n. So that means, state probability of the system being in n being in 1 at time period n this is probability of the system being in state 2 at time n this is the probability of the system being in state 3 at time n. So, I denote this by a row vector p n and then this can be written as p naught into p n. So, this gives these state probabilities at time n right.

Now, the Markov change is completely specified when you are given the first step transition matrix p and the initial probability, initially probability was being system being in particular state. So, p naught is the vector which gives you p naught 1 p naught 2 and p naught 3. So, this will be the probability in initially when the system is in state 1 2 or 3. Then you can compute these and of course, you can compute the transition probability in step transition probability also.

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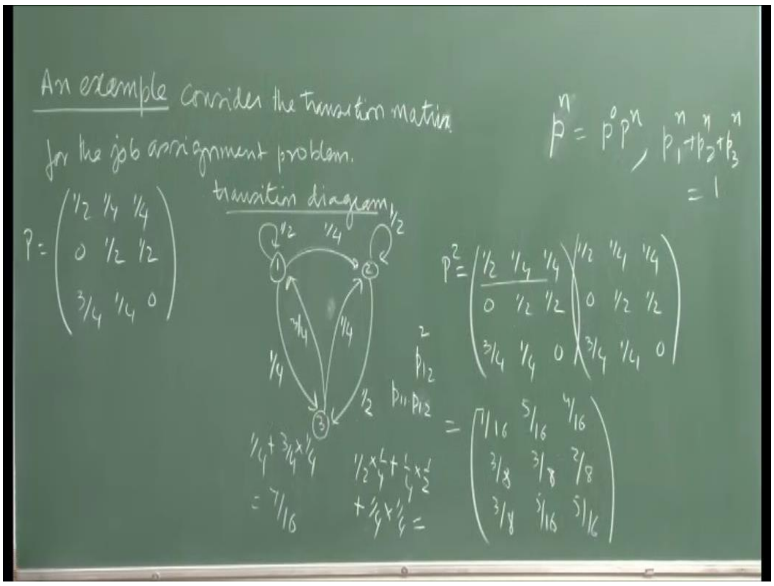

So, let us consider an example the same example now with numbers. And so we will just go through all the concept that we have talked about the transition probabilities and state probabilities I mean higher order transition probabilities and the state probabilities. So, suppose the transition matrix is given as this. So, then the corresponding diagram you see for example, there is no arc from 2 to 1. So, therefore, this is missing here and similarly you do not have a loop from 3 to 3. So, it is missing the probability is 0 right, otherwise and you see that they add up to 1 all these probabilities or the rows must add up to 1. So, this is a valid transition matrix and these are other positive or zeros and they row entries and all add up to 1. So, this is valid and a comparing diagram is this, the transition diagram is given by this right.

Now, let us compute second order transition probabilities. So, I multiply p with p and I get these numbers. So, for example, here you can transition from 1 to 1 in 2 steps this will be 1 to 1. So, 2 steps therefore, again the same thing as I showed you that if you just want to sit on path it will be 1 by four this plus then you can go to you cannot go to 2 in 1 2 steps transition, because then you cannot come back to 1 right. So, then it will be you can go to 3 and then 3 to 1. So, this will be 3 by 4 into 1 by 4 right.

See you can either stay with 1 to 1 and then you can go from 1 to 3 and then 3 to 1 this is what you can do in 2 steps if you want to transition from 1 to 1. So, 2 possible paths and so therefore, this is 3 by 16 and this is 4 by 16 so 7 by 16 right. Similarly you can maybe look at this 1 here 5 by 16. So, this is from 1 to 2, 1 to 2 you want to transition in 2 steps. So, then the possible path is 1 to 2 then 2 to 1 right. So, that will be what 1 to 2 is 1 by 4 into you transition from 2 to 2 will be 1 by 2 plus or you can stay from 1 to 1 and then go from 1 to 2. So, that will be a 1 by 2 into 1 by 4 right. So, what how much will this be 1 by 8 plus what or what I miss something out. So, this is you are going from 1 to 2, 1 to 2 you are going in 2 steps yeah.

So, 1 by 2 yeah this is a 5 by 16 and you are computing p 1 2 right. So, you can go from p 1 1 into p 1 1. So, that was 1 by 4 and why did I multiplied by I want to go from 1 to 1 sorry, this is not correct, why should I have say 1 to this is simply 1 to 1 so this into p 1 2. So, therefore, this is half ha that is I wrote this plus 1 by 2 into 1 by 4 or I can go from 1 to 2, which is 1 by 4 and then transition 2 to 2. So, that comes out to be 2 by 8 and I am getting it as 5 by 16. So, let us multiply this; that means, you want 1 and 2. So, that will be, (()) I am missing out on the path 1 by 8 plus 1 by 8 plus 1 by 16, so 1 by if you want to go from 1 to 2. So, right I missed out on the path. So, this is 1 to 1 and then 1 to 2 then 1 to 2 and 2 to 2 and then there is a another path 1 to 3 and 3 to 2. So, plus 1 to 3 is 1 by 4 and then from into 3 to 2 is 1 by 4.

See that is what I am trying to say that you know even in such a small diagram I was missing out on a path. So, just imagine if you had these five or six states and then you had you know, that many nodes so and then you have these so many arcs. You will certainly miss out it will be very difficult to it will be very time consuming to enumerate all possible paths. So, as it is you know such a small example, I could miss out on the path 1 to 3 and 3 to 2. So, that will give you now 1 by 8 plus 1 by 8 plus 1 by 16. So, that will be 5 by 16 right, 1 by 8 plus 1 by 8 which will be 1 by 4. So, that is 4 by 16 plus 1 by 16 5 by 16.

So, anyway you can now interpret all these probabilities you know by looking at the path or by find. Now, I make further computations took powers of p. So, this is p 3 comes out to be this and if you make compute fourth power then these are the numbers. And another aspect that I want to point out here is that for example, in this particular case you are able to transition from any state to any state. Even though some of the arcs are missing, but even after that, you can go from 1 to 2, 2 to 3, 3 to 1 again and so on. So, you can transition from any state to any state maybe not in 1 step always, but in fact, here it is happening that you are able to go yeah for example, from 2 to 1 you cannot go in 1 step, but you will able to go from 2 to 3 and then 3 to 1. So, in 2 steps you will be able to transition.

So, here in fact, the moment all your entries see at p 2 yeah. So, at p 2 all your entries are positive, which shows see therefore,; that means, your p i j 2 is positive for all i j. So, this immediately makes you conclude that you have a 2 steps path from each state to from every state to every other state. So; that means, this was the word for this is communicate; that means, all states communicate with each other and maybe not in 1 step, but at least in 2. So, if all the entries of p i j 2 are positive then; that means, all states communicate with each other, I will formulate define that also, but essentially what I want to saying here is that you can go from any state to another state in 2 steps in this particular example. And in some other example, if it is you know there is some k for which this is positive then; that means, again there is a k step path from every state to another state.

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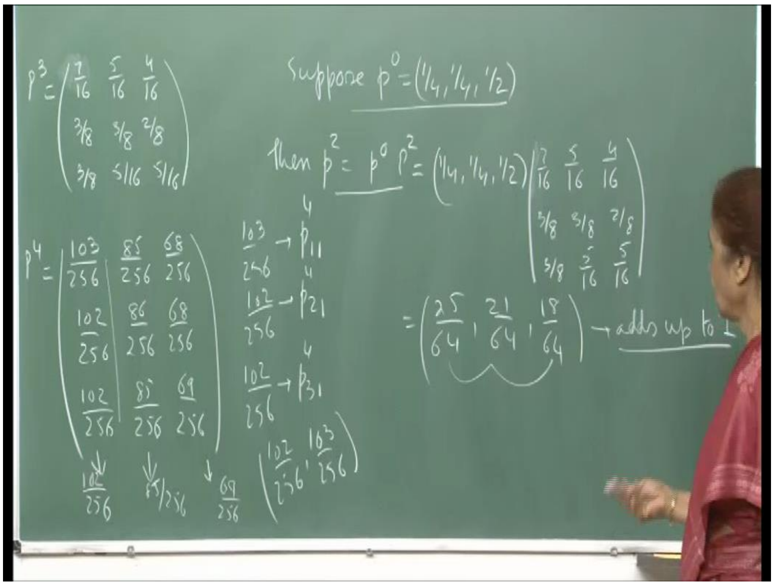

And ya, so now when you look at the fourth power of p these are the numbers and you can now you should carefully look at the column numbers this is 103 upon 256, 102 upon 256 and 102 upon 256 so the numbers are getting closer right. And you can say that almost conversing to 102 on 256. In fact, you might say that why not 103 or 102, but any one of these numbers. So, in other words that you know, if you want to interpret these numbers; that means, now the probability say for example, what is this number. So, 103 upon 256 is the probability of 114; that means, four steps you are from 1 to 1 right. And then 102 upon 256 this is the probability 2 1 4 right; that means, if you, so here it says that, if you are initially in state one then in four steps you will be back in state one. So, this is the probability.

Now, this says that if you are in state 2 in the beginning and you are transitioning to 1 in four steps then this is a probability. And third one is 102 upon 256, so this is p 3 1 4 so; that means, you are in state 3 and you are transitioning to 1 in four steps. So, you see that it is becoming almost immaterial to know, where you started, because these probabilities are getting closer and we will show you then later on you will also formula is all this discussion. So, essentially the state in which you were the initial time is becoming unimportant these. And similarly the same thing you can interpret for the second column because it is a 85 by 256 86 by 256 and 85. So, these numbers are also getting closer to 85 and here I have written, because here its 68 by 256 68 by 256 and 69 by 256.

So, you might say that why not 68, but then see remember the probabilities, which when you finally, say that all these are conversing to they must also add up to 1 right, because the system has to be in 1 of the states. So, the same argument we will continue using and so here I have if I say that this converges to 102 this 2 upon 256 this is 85 by 256 then this should by 69 by 256. Of course, it is possible that the, so essentially right now the probability of being in state 1 after four transitions is hovering between 102 256. So, which is a more exact statement, this is a more exact statement right. Similarly here also I can say the probability of being in state 2 after four transitions is between, is in the interval 85 upon 256 and 86 upon 256 right, which is a very small interval right.

So, difference of 1 upon 256 and similarly here it will be 68 upon 256 and 69 upon 256. So, these probabilities and if you take the fifth power then certainly you will see that the numbers are conversing and you will take about this formulae anyway yeah. And also while we are talking of this see I wanted to point out that here again these probabilities, because this is now row vector, this is a row vector remember 1 by 3 we are writing. So, this is 1 by 3 and this is 3 by 3. So, then again this is 1 by 3. So, the 3 atleast must again add up to 1, because at you know n after n transitions your system will be either in 1 2 or 3. So, therefore, these probabilities also must add up to 1 right, which you can see also that.

So, anyway these also add up to 1 and then now, suppose for this job assignment problem, if your initial probabilities are given by 1 by 4, 1 by 4, 1 by 2. That means, probability of being in production is 1 by 4, probability of being in H R is 1 by 4 and probability of being in sales is half then you want to ask the question that what are the probabilities of being in state in 1 2 or 3 after 2 transitions. So, that will be given by this right. So, 1 by 4 1 by 4 1 by 2 multiplied by p square and so these are the probabilities of being in you know. So, this gives you the probability of being in a production after 2 transitions right and this is the probability of so these are the state probabilities after 2 steps, after 2 time period. So, this is 21 by 64 and 18 by 64 and these probabilities also must add up to 1 right. So, now, we are trying to give you some more characteristics of the Markov process and we will do lot of analysis in terms of these transition probabilities and the state probabilities.

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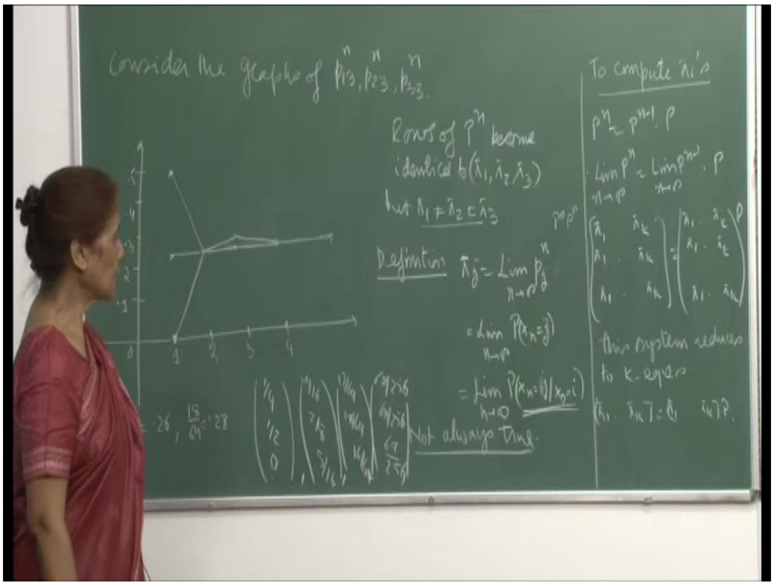

So, let us I just want to show you the limiting behavior of these steady state probabilities. So, let us just graphed the values of p 1 3 p 2 3 and p 3 three for different values of n. So, you see the starting vector is 1 by 4 1 by 2 and 0. So, p 1 3 for example, 1 by 4 is 0.25. So, I am just start it. So, these are time periods and these are your probabilities. So, numbers from 0.1 to 0.5. So, 0.25 see at time 1 it is in period 1 it is 0.25 then here also 4 by 16 is 0.25. So, in period 2 also this is a same then it will very slightly goes up in period 3 to 17 by 64. So, I am just showing it like this and then it comes down to 68 upon 256.

Now, 68 upon 256 is 0.26. So, therefore, it comes down to a point this is this is my value for a 0.26. So, this is the graph for p 13 as it goes to different periods. So, the values transition probabilities then, if you look at p 23. Now, p 23 is starts from half right then it immediately comes down to 0.25, because this is 1 by 4 so 0.25. So, this is where it is and then in the next 1 it goes to 18 by 64. So, actually sorry, my graph for p 1 3 is this 1, because very slightly it goes up and then again it settles to here this is the graph for p 2 3.

So p 2 3 the p 2 3 graph is this. So, this is from 0.5 to 0.25 then it goes up to 18 by 64. So, this is above more than 17 by 64 this and then it comes to 68 by 256. So, the same value as for p 1 3, p 1 3 4 and p 2 3 4 are the same. So, this is how it is and for the for p 3 3 see it is 0 in the beginning and in stage time period 1 then it jumps to 5 by 16, which is little more than 1 by 4. So, little more than 0.25 and then it comes to 16 by a 64, which is exactly 0.25, right.

So, I have try to just parallel it with this 1 here and then it will be 69.256. So, the line is the slightly above this. So, therefore, you can see that and then as you take higher powers all 3 will settle down to this number, which we have to compute and we will do it when we find out the steady state probabilities. So, you see this is the limiting behavior and so it does not matter even though the 3 had different very different starts all the three, but finally, they must to the same. So, therefore, the relevance of the starting state of the process is not at all a relevant here this is what you want to show. And of course, this will show this will not always be true and we will now then find out during the course of few next few lectures as to when this is valid on when this kind of limiting behavior is valid ok.

So, what we are going to say is that rows of p n become identical to pi 1 pi 2 pi 3 all the rows, because it does not matter the starting state of the starting state of the system. And so all the rows will be pi 1 pi 2 pi 3, but it is not necessary that the 3 probabilities are the same right so; that means, it is being. So, essentially what we are saying is this is now a long term behavior we are saying that the system will be in state one. So, that is a probability right then this is the probability of the system being in state 2 and this is the probability of the system being in state 3. So, the starting probabilities are not relevant, but the values long run values will not necessarily be the same. And so the definition for the steady state probability is that pi j is the limit of p j n, as n goes to infinity right, remember we defined this as the, you know multiply by p 0 and p n.

So, this is a limiting a value of p j n, as n goes to infinity, which is actually probability x n in j right. In the long run your system is occupying state j as n goes to infinity and what we are saying now is that this actually is equal to this conditional probability, but this part is becoming irrelevant right. So, the i c the probability finally, is going to pi j so the i part is irrelevant here this is what we want to show you.

And so we will in the next lecture discuss the under what condition of course, that will come in the due course of time, but first of all we would want to know how we go about computing these steady state probabilities when we know that they are they exist. So, right now we will assume that they exist, and then we will find out the method of computing them. And then we will continue with the discussion as to under what conditions they always exist.