Introduction to Probability Theory and its Applications
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Lecture - 24
Convolutions

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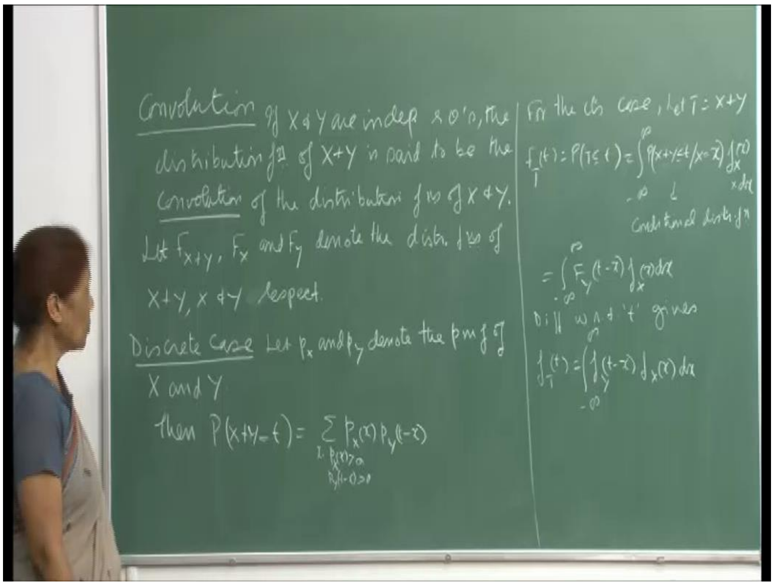

So, in this lecture I will be talking about convolution and again this is one of the tools that we will use just like m g f to sometimes compute the distribution function and their density function for different kinds of random variables of the functions of the random variables. Mostly the convolution is use for computing the distribution function of sums of random variables. So, here the definition says that if x and y are independent random variables, the distribution function of x plus y is said to be the convolution of the distribution functions of x and y. So, the distribution function that we obtain for x plus y is will be is called the convolution of the distribution functions of x and y.

Now, if highlight f of x plus y f x and f y denote the distribution functions of x plus y x and y respectively right. So, by notation it is clear that the f x plus y is the distribution function for x plus y and f x is for x f y is for capital y right. Now, for the discrete case the definition would be as follows. So, when the x and y both are discrete random variables and they are independent random variables p of x and p of y denote the p m f of x and y respectively. Then p of x plus y equal to t we will write as summation of p x x, so you fix the value of capital x then the y will take the values t minus x right. And of course, this summation will be over all x for which you are this probability is positive and also it should be such that t minus x the probability y at t minus x is also positive. Otherwise since is the product of these two probabilities whenever one of them is 0. The whole the contribution of that particular term will be 0.

So, simple definition that we will see how we can apply these definition and similarly for the case when for the continuous case that means when both x and y are independent continuous random variables. That let t denote the sum of the two random variable that is t is equal to x plus y then your distribution function for t at small t is probability capital t less than or equal to t. And this we can write as minus infinity to infinity probability x plus y less than or equal to t condition that x equal to x just as here, we chose the value of x and then the corresponding value of y got fixed at t minus x.

So, here you condition it on x equal to x and then probability x plus y less than or equal to t. So, then that into; that means, this is the distribution function into the probability that x takes the value well the p d f of x at small x which will be f of capital x x into d x. And your x reform minus infinity to infinity. So, this is the general expression and of course, it will depend on the range will depend on the values that your random variables take. So, I am use the concept of conditional distribution here and so this is the expression.

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$$F_T(t) = P(T \le t)$$

$$= \int_{-\infty}^{\infty} P(X + Y \le t / X = x) f_X \, dx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{t-x} \frac{f_Y(y) f_X(x) \, dy}{f_X(x)} f_X(x) \, dx,$$

since X & Y are indipendent.

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{t-x} f_Y dy . f_X(x) \, dx$$

$$= \int_{-\infty}^{\infty} P(Y \le t - x) f_X(x) \, dx.$$

And therefore, in terms of f f f y capital f y in small f x you can write this as minus infinity to infinity probability x plus y less than t condition on x equal to x into f x x d x. So, this we can write as f y integration of minus infinity to t minus x of f y y into f x x d x upon f y x. Then this we can do because x and y are independent; that means, the probability x plus y less than t conditioned on x equal to x, I am able to write break it up into f y y into f x x upon f x x d y. And so because x and y are independent and there into f x x d x. So, we are f x x cancels out and we are left with the integration minus infinity to infinity again integration minus infinity to t minus x f y d y f x d x. So, therefore, the first part I can write, I mean the integral minus infinity root t minus x f y d y I can write as probability y less than or equal to t minus x.

So, whole integral is then minus infinity to infinity probability y less than or equal to t minus x into f x x d x. So, I hope there is no doubt about going from here to here right, because when x is equal to x your y will be required to be less than or equal to t minus x and so that is why I have written this probability as a capital f y t minus x into this right. Now, differentiate respect to t since the limits are independent of your t therefore, the differentiation will just go inside. So, therefore, this will become small f t t and this will be equal to. So, only this thing gets differentiated this is the function of t and therefore, it will be p d f of y at t minus x and then integral from minus infinity to infinity of this function, product of these two right. So, this will be your convolution you can either write it in this form or you can write it in terms of the p d fs. And of course, the understanding is that wherever you know for all values of t for which this is defined as and you also take the values of x for wish this is define right.

Now, let us so the basic definition is this and then we will just see how we applied to different cases and of course, there will be reputation in the sense that for sums of independent random variables quite a few cases by other methods by using the transformation method or by the movement generating function.

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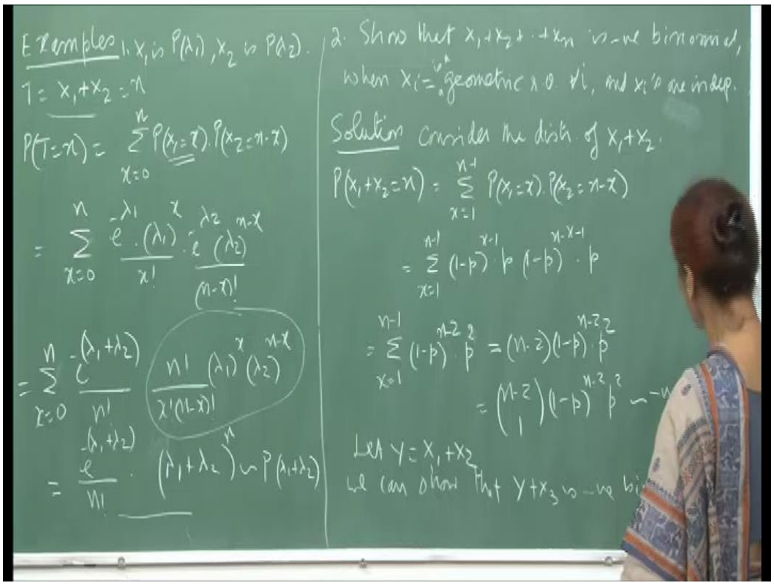

We have already obtain the density functions for the sums of independent random variables, but here I just want to show you the working of this particular method and therefore, we will just through a few examples. Now, suppose x 1 is the Poisson lambda 1 and x 2 is Poisson lambda 2. So, then t is the sum of the two poison random variables and now you want to find out probability t equal to n.

So, then if I chose x 1 is equal to x then x varies from 0 to n and this will be probability x 1 equal to x into probability x 2 equal to n minus x right. See from here if this is end I fix x 1 at x then x 2 will be n minus x. So, you do some and now since there independent and therefore, I written it this way product of the probability. So, now, this particular probability can be written as e rise to minus lambda 1 and we have missed out on the summation should have been there sorry, summation x varying from 0 to n right, e rise to minus lambda 1. So, this probability is e rise to minus lambda 1 lambda 1 rise to x upon x factorial.

And this probability would be e rise to minus lambda 2 lambda 2 rise to n minus x upon n minus x factorial and so here also summation x varying from 0 to n right. Now, then just rearrange the terms e rise to minus of lambda 1 plus lambda 2 then I have divided and multiplied by n factorial. So, this is n factorial and then here it will be n factorial divided by x factorial and minus x factorial and then you have lambda 1 rise to x and lambda 2 rise n minus x. Now, this is the you can see that this is the expansion of the binomial expansion of lambda 1 plus lambda 2 rise to n, because your summing see thing this the independent of x, e rise minus lambda 1 plus lambda 2 divided by n factorial. So, this I take outside and then this summation x, x from 0 to n of this will be your lambda 1 plus lambda 2 rise to n.

And so this will be now Poisson with the parameter lambda 1 plus lambda 2 right, this is probability equal to n. So, therefore, e rise to minus lambda 1 plus lambda 2 divided by n factorial into lambda 1 plus lambda 2 rise to n right. So, you can immediately conclude that this is now Poisson with parameter lambda 1 plus lambda 2 yeah. Another example I want you to show that x 1 plus x 2 plus x n is negative binomial and so here now we are extending this concept to more than two, some of more than two independent random variables. Where x size are actually I could have straight away said that x I are independently identically distributed geometric random variables and geometric random variables with probability of success is p.

So; that means, probability x i; that means, probability this is just a geometric random variable and so and the probability of success is 1 that is it. When you describe a geometric random variable you just need to known the probability of success and then you want to find the probability that you will get a success in n trails. So, now, here we first consider this some of two random variables x 1 plus x 2 and so probability x 1 plus x 2 equal to n by convolutions we will write as x equal to 1 to n minus 1, because they should be I mean I cannot make this 0. I cannot have x going to from 1 to n, because in that case when x is equal to n this will be 0 and so anyway this will be not defined on the probability will write it as 0. So, there will no contribution to this sum.

So, I have to take this summation from 1 to n minus 1 right. And then if I fix x 1 at x that means, for the first geometric random variable the success, the first success occurs for x trails then for the second one the first success will occur at the n minus x th trial right. So, the probability of x 1 equal to x is 1 minus p rise to x minus 1 into p right at the x th trail the success must occur, otherwise before that all failures. And similarly here it will be 1 minus p rise to n minus x minus 1 into p. So, when you rearrange the terms here see x minus 1 and this is minus x. So, x minus x cancels out you left with the n minus 2 and this is p square. So, this is equal to sigma x varying from 1 to n minus 1, 1 minus p rise to n minus 2 p square, but you see x is not present here therefore, you just add up this terms n minus 1 times. So, this is n minus 1 into 1 minus p rise to n minus 2 p square.

Now, this you can see is the probability that out of n successes out of sorry, out of n trails to or successes and n minus 2 are failures. So, that means, this is the probability of 2 successes in n trails right, where the law success occurs, where the second success occurs at the end so when the one success can occur anywhere and the n minus 1 trails. So, therefore, this is n minus 1 1, you can write this n minus 1 1 choose 1 and then 1 minus p rise to n minus 2 and p square. So, 1 success occurs anywhere in the n minus 1 trails and then one success occurs at the end and therefore, this is probability of two success is in n trails. So, therefore, x 1 plus x 2 if you let y be equal to x 1 plus x 2.

We show that y is negative binomial with the parameters 2 comma p right. The probability of success is p. Here in the example I probably denoted mention, but it is understood that probability that x i is equal to the probability of success sorry, I should say that, probability of success is equal to p I should have mention that here. So, it is understood anywhere right. Now, we can iteratively show that if you now consider the random variable y plus x 3 then by the same argument will be able to show that this is negative binomial 3 comma p and so on. So, therefore, you can show that this will be negative binomial when x I is a geometric random variable when each x i is a geometric random variable for all i and x i s are independent.

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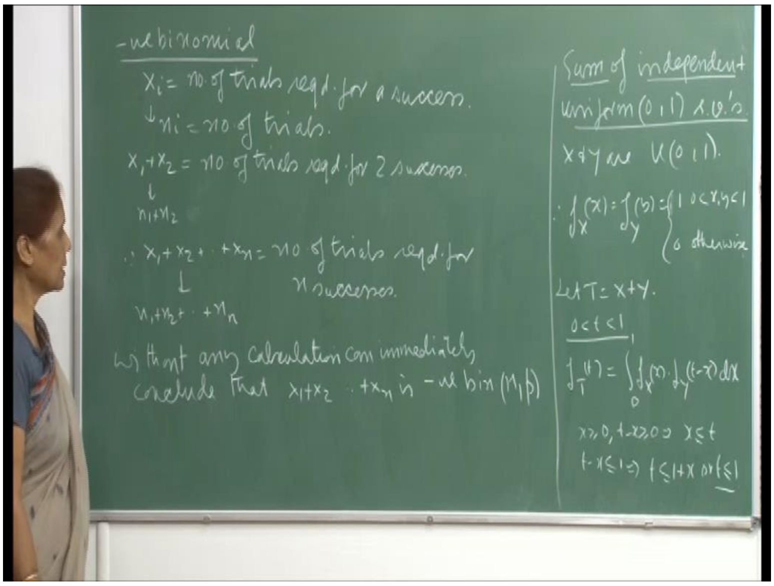

In the in the example, where we consider the sum of the geometric independent geometric random variables all with probability of success is p. Then you see x i was the number of trails required for a success for one success. Therefore, see ah if the corresponding number is n i; that means, the number of trails required for one success for the i th geometric random variable. So, then you see when say x 1 plus x 2 this will be the number of trails required for two successive right. And that number will be then n 1 plus n two. So, essentially when I wore that the sum x 1 plus x 2 is 2 comma p negative binomial.

So, that the understanding is that you want to two success and therefore, the number of trails of course, will be equal to n 1 plus n 2. Then so therefore, finally, when you go and doing iteratively this procedure of in adding on or convoluting these random variables then x 1 plus x 2 plus x n will finally, give you the number of trails required for n successive right. And therefore, this was negative binomial n p right. And the total number of success is trails required would be n 1 plus n 2 plus n n right. Now, the next part was that without any calculation you can also conclude see this was just to show you how you would apply convolution and now here we can also because by just description, because when you say x 1 plus x 2 plus x n that is means you are asking for n success and your probability of success is p. And therefore, x 1 plus x 2 plus x n gives you the number of trails required for n successes when the probability of successes is p.

So, just by saying it aloud you know since each x 1 x 2 x n is the, is geometric random variable independent therefore, we you add up you can say that this will be a negative binomial and comma p so that is all. We just do because here (( )) required to use convolution right. Now, lets us consider again applied convolution to some of independent uniform random variables both are 0 1 right. So, therefore, the corresponding p d fs all both be 1 as long as x and y are between 0 and 1 and it will be 0 otherwise right.

So, let us define the random variables t equal to x plus y now the thing is that you see will have to and this where the this part comes that sometimes of course, you can easily determine the ranges, but sometimes you cannot be that easy. So, here for example, you see when I write the formula f x x x and f y t minus x. Now, you just see this is define only for between 0 and 1. So, by t minus x also should vary between 0 and 1 and therefore, I have to get this have to write I mean after do this computation for first for t between 0 and 1, because t minus x greater than 0 implies that x is less than or equal to t. See from here this is not define for t minus x being negative.

So, therefore, t minus x has to be non negative, which requires the t should be greater than or equal to t right, greater than or equal to x. And then also t minus x should be less than or equal to 1. So, you cannot take a value of and therefore, t must be less than or equal to from here t must be less than or equal to 1 plus x. Since x can take 0 value therefore, you see immediately from here that your t cannot be more than 1. So, here we will have to the way we are defining this, it will have to be the limits for t would have to be from 0 to 1 right. And then you are x can vary from 0 to 1 when you write now; I mean I am writing the integral this way, but since for x non negative and x between 0 and 1 this is 1.

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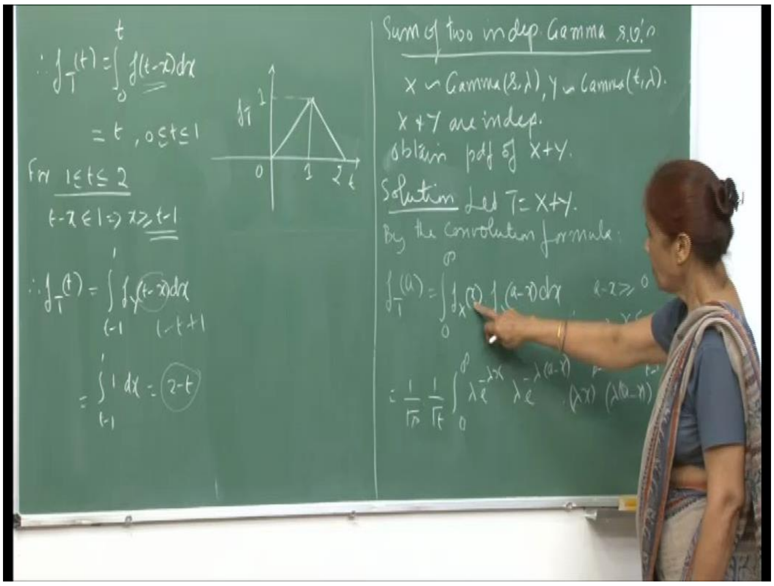

So, therefore, this will reduce to simply integral 0 to t f t minus x d x now, when I say that my x varies for 0 to t. So, this is also define and therefore, this is also equal to 1 right. As long as x is varying from 0 to t, this is well defined and it is within the range of the values for y and therefore, this also is 1. So, this integral comes out to be t and t between 0 and 1 with this is what I have to drawn you, the curve here this value is also 1 right. So, therefore, the p d f for the some of the two random variables and both are uniform will be given by this.

Now, because x plus y and both can take values between 0 and 1. So, of course, the range this is from 0 to 2 right. And so we have to consider the values of align between 1 and 2 and this case you see again the convolution formula is this. So, t minus x less than or equal to 1 will imply that your x is greater than or equal to t minus 1. So, here immediately you see that t must be greater than or equal to 1 right or x is greater than or equal to t minus 1. So, that is why the range t greater than or equal to 1 right. And then of course, it cannot go beyond two. So, yes therefore, again the same reasoning that this value is 1 and the valid portion region we you can define this and why I am writing. So, this is t minus 1 to 1; that means, my x can vary from t minus 1 to 1 so t minus 1 yes. So, this thing you see at t minus 1 this will be 1.

Because t minus t plus 1 and when this x is 1 this will be t minus 1, because your x has to be greater than or equal to t minus 1. So, therefore, the range is this and again in this range this function is equal to 1. So, the integral is t minus 1 to 1, 1 into d x which comes out to be 2 minus t and that will be your, this graph will be, the function will be represented by this great line. So, therefore, the sum of the two uniform random variables both independent and defined on 0 1, the p d f is triangular distribution yes. So, therefore, you see here you could not have just straight away done this integration from 0 to 2; that means, you could not allow t to vary from 0 to 2, because that could not have given you the valid answer and here you have to break up the region of integration from 0 to 1 and then 1 to 2. And I suppose yeah, because we are writing t minus x. So, we have to do it this way that it x has to be greater than or equal to t minus 1 and we are integrating with respect to x, x cannot go beyond 1, therefore, this will be t minus 1 to 1 right yeah.

So, another example is now sum of two independent gamma random variables. So, suppose x is gamma s comma lambda and y is gamma t comma lambda then and x and y are independent. So, we have to obtain the p d f of x plus y. So, to define t as x plus y and then by convolution formula f t a we will write as this now, here again we have to apply the same thing, because gamma p d f is defined only for non negative variables. So, therefore, this has to be a minus x has to be non negative. So, therefore, this a minus x no negative, this implies x less than or equal to a yes. And so I can; that means, here while integration has to be from 0 to a now, why I am writing this as yeah.

So right now, I am I will now write the correct range what we just right now substituting for f x x, which will be lambda e raise to minus lambda x and lambda x raise to s minus 1, because x is s comma lambda, gamma s comma lambda. So, the p d f is lambda e raise to minus lambda x into lambda x raise to s minus 1. And this is p d f for y is lambda e raise to lambda a minus x into lambda of a minus x raise to t minus 1. You see as we said x has to be less than or equal to a and now we therefore, this infinity will get replaced by a and you see here e raise to minus lambda x then e raise to plus lambda x that will cancel out you will be left with e raise to lambda a and then here it is lambda x raise to x minus 1.

So, if you just take out the lambda here it will be lambda raise to s minus 1 and this will be lambda raise to t minus 1 and you have lambda square here. So, the whole thing will be lambda raise to s plus t minus, because this is 2 and this is minus 2. So, you have lambda raise 2 s plus t right and that is what we have written here.

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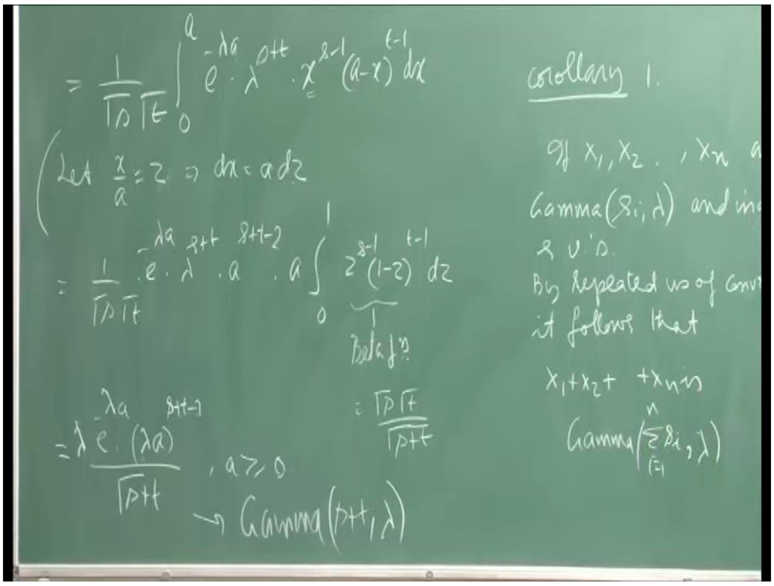

So, this is lambda raise to s plus t e raise to minus lambda a and then you left with x raise to s minus 1 and a minus x raise to t minus 1 d x. Now, make the substitution that x upon a is equal to z. So, this will imply that your d x get replaced by a d z right and your range goes from 0 to 1 instead of 0 to a, because x by a, we have put a z. So, range for z will be from 0 to 1. So, therefore, the constant terms have put outside and so this a comes from here and then you have this and this. Now, you see this looks familiar and this is the beta function. And therefore, since we want to so therefore, we know that this integral from 0 to 1 d z will be equal to gamma s into gamma t upon gamma s plus t right. So, from the definition of the beta distribution we know that this integral must be equal to gamma s into gamma t upon gamma s plus t. And therefore, I replace this whole thing by this thing. So, then gamma s gamma t cancel out you left with gamma s plus t and this is lambda raise to so the lambda be right outside here. And then it will be lambda a raise to yeah, so this makes it s plus t minus 1. So, therefore, lambda a s plus t minus 1, 1 lambda I have written out here, just to conform to the form of the gamma distribution right. And so this is therefore, this is I should have written here, gamma s this is gamma s plus t lambda.

So, we have concluded that, if you take two independent gamma random variables, x is gamma s lambda and y is gamma t lambda then this sum and they are independent then the sum will be again gamma distribution having a gamma distribution with parameters s plus t and lambda. So, the same lambda if the second parameter is same then I can go adding the gamma random variables and the first parameter gets added of course, you are adding independent gamma random variables. So, corollary first is that if x 1 x 2 x n are gamma s I lambda and they are independent random variables then by repeated use as we did earlier by repeated use of convolution it follows that x 1 plus x 2 plus x n will be gamma sigma s I, i varying from 1 to n comma lambda.

So, here of course, this thing is immediate that is your adding with distribution is not changing just the parameter is changing and that also the second parameter the first 1 that is the common one that remains same right. Another corollary, which is that if you take x 1 x 2 x n are identically independently distributed exponential lambda random variables and we know that an exponential lambda here only you can see from the gamma p d f that is a gamma 1 comma lambda right. Then exponential lambda is gamma 1 comma lambda and so when you take x 1 plus x n exponential random variables independently distributed then this is gamma and gamma lambda because the first parameter gets added in this.

So, and we will see uses of all these when we talk about random process is stochastic process is which are Poisson and Marko and so on right. So, I have tried to depict by various, because you had to you could not just blindly apply the convolution you have to make sure that you know when you are applying it you are values of the variables should be sighted this p d fs are defined and so on, so we defined this. Now of course, the question is see we have defined this for independent random variables; that means, the convolution right now we the definition says that they are independent and then you take the sum and then you can talk about the convolution, but can you answer the converse can you that is if you can show that the for two random variables x and y. The distribution function for x plus y can be written as the convolution of the distribution functions of x and y would it imply that x and y are independent. So, this is the question and I will try to answer through a counter example to say that, no it is not necessary, you may get the distribution of this sum by the convolution, but the variables may not be independent.

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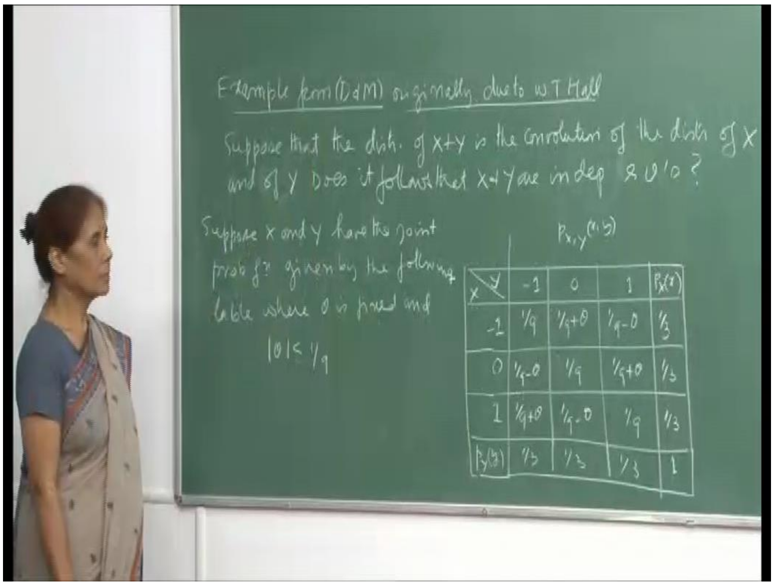

So, this example is I have taken it from dude witch and Mishra and I told you I have often taken examples from this book dude witch and Mishra and Sheldon and rose and I have give you the references also at the end of the course. So, here and this example is originally due to w t hall and you see how cleverly this has been constructed.

So, see as I said the, we try to answer the question that if the distribution of x plus y is the convolution of the distribution of x and y does it follow oh, does it follow that x and y are independent random variables. So, you want to answer this question, because we have defined the convolution for independent random variables. So, the table here gives you the joint, I should make the at least to look nice. So, this is it. So, therefore, this table gives you the joint probability function, probability maths function of x and y and theta is fixed number, but it is absolute value less than 1 by 9, because otherwise, the entries will become negative.

So, we want this to be valid table for joint probability maths function of x and y right. And you can see that when you add up this for the marginal p d fs are independent of the marginal probability maths functions are independent of theta and this where I am saying that thing has been very well constructed. So, see these will 3 will add up to theta, theta will cancel t plus theta n minus theta it will be 1 by 3. Similarly here this minus theta and plus theta will cancel so this will also be 1 by 3. And finally, these will also 1 by 3 and your column sums also give you the marginals, which are all independent of theta right. And now you want to write. So, we want to verify that the distribution function of x plus y is a convolution of the distribution functions of x and y.

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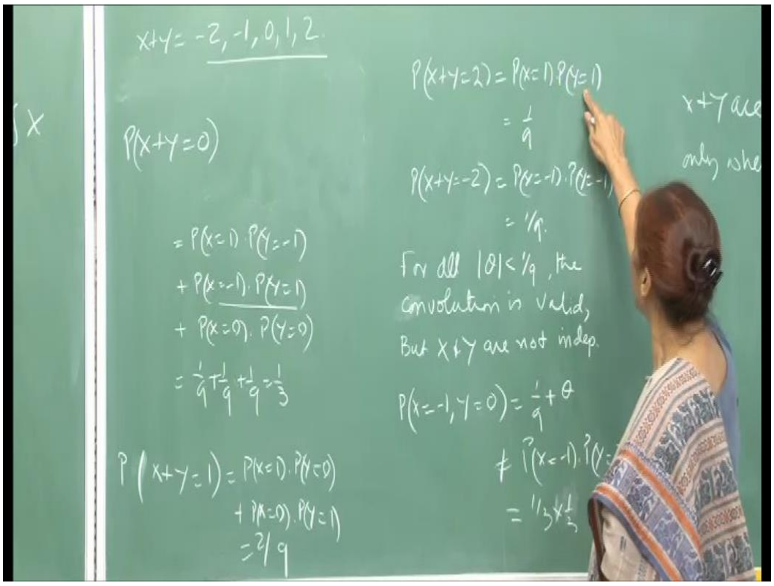

So, the values that x plus y will take a minus 2 minus 1 0 1 and 2 right these are the possible values that x plus y you can take, because your x takes the values minus 1 0 1 and y takes the values minus 1 0 1 right. So, we start with probability x plus y equal to 0 and we will have to verify for all possible values to make sure that this is that x plus y the distribution of x plus y can be obtained as a convolution of distributions of x and y. So, here by convolution I means, I am taking x to be equal to 1 then y will be equal to minus 1 right. And if you take x equal to minus 1 then y will be 1 and if you take x equal to 0 then y will be also 0.

So, these are the 3 you can convolute here this right and probability for x equal to 1 by 3 right, that is what you mean by marginal 1 by 3 into probability y equal to minus 1 that is also 1 by 3. So, therefore, this is 1 by 9, similarly you can immediately see that probability x equal to minus 1 is 1 by 3 and probability y equal to 1 is also 1 by 3 and 0 zero also x equal to 0 gives you 1 by 3 y equal to 0 also 1 by 3 so this is 1 by 3 right. And then similarly, probability x plus y equal to 1 so here also when you want to fix x and then the corresponding value of y. So, here I put x equal to 1 then y will have to be 0 and I put x equal to 0 y is equal to 1 and you should have put if you put x equal to minus 1 then that is not valid right x equal to minus 1 then y can only take a value 0 1 or minus 1. So, this is these are only 2 ways you can convolute the some x plus y equal to 1 here. And therefore, this probability is 2 by 9 right, I have tried to compute just to make sure that see, because even if for one single value the convolution does not hold then I cannot say that.

And so for x plus y equal to 2 only one possibility, x takes value 1 and y equal to 1 so this is also, I have taken this example, because it shows you that when we write f x of x into f y of t minus x. Then you see you have to take only possible values of t you cannot just take any so x plus y equal to 2 will be given by x equal to 1 and y equal to 1. So, this probability is 1 by 9. And similarly x plus y equal to minus 2 will be given by x equal to minus 1 and y equal to minus 1. So, that is 1 by 9 right.

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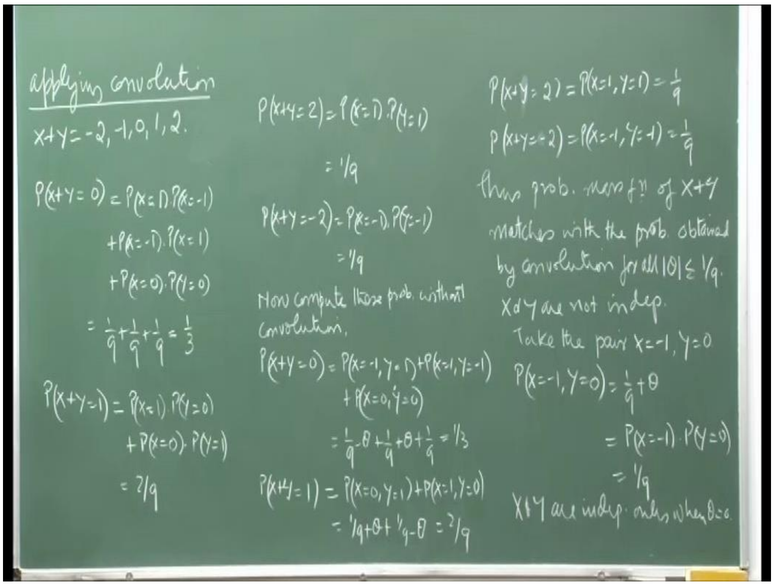

So, now we compute these probabilities without convolution. So, for example, probability x plus y equal to 0 will be probability, x equal to minus 1 y equal to 1 so all pairs that make up x plus y equal to 0 plus probability x equal to 1 y equal to minus 1 and plus probability x equal to 0 y equal to 0. And so from the table we can see that minus 1 1 x minus 1 and this 1 so 1 by 9 minus theta right. And then 1 minus 1 so 1 minus 1 is 1 by 9 plus theta and then 0 0 x 0 y 0 that is this which is 1 by 9 right. So, therefore, this adds up to 1 by 3 minus theta and theta cancel out. So, this is 3 upon 9, which is 1 by 3 similarly x plus y equal to 1 so x equal to 0 y equal to 1 so again 0 1 1 0 and so on. So, 0 1 0 1 will be 1 by 9 plus theta 1 0 will be 1 by 9 minus theta and that is it. Because the sum to be equal to 1 these are the only 2 possible pairs of values that x and y can take. So, 1 by 9 plus theta plus 1 by 9 minus theta is 2 by 9.

So, this also matches with this and this matches with this 1 by 3. And then similarly you can see that x plus y equal to 2 simply be the pair x equal to 1 y equal to 1, which is 1 by 9 x equal to 1 and y equal to 1, 1 by 9 which also matches with this 1 right. And then x plus y equal to minus 2 this is x equal to minus 1 and y equal to minus 1, x equal to minus this is 1 by 9 and here also we got it as 1 by 9. So, we have checked almost for all yes well not exactly may be x plus y equal to minus 1 that is left out, but we have otherwise checked for x plus 0 1 2 2 2 n minus 2.

So, that 1 was 0, but you can easily verify that for all values; that means, the probability maths function of x plus y matches with the probabilities obtained by convolution for all theta less than or equal to 1 by 9. So, therefore, the two things match, but we know that x and y are not independent why, because you just take one pair see I just have to show that for one pair of values this does not hold that this probability x equal to minus 1 and y equal to 0 that is from again from the table is 1 by 9 plus theta, but this is not equal to sorry, yeah what I am saying is that this is not equal to probability x equal to minus 1 into probability y equal to 0. So, x equal to minus 1 from the marginals is this 1 by 3 and probability y equal to 0 this is y equal to 0, this is 1 by 3 right. So, that is 1 by 9.

And therefore, the 2 are not equal as long as theta some positive number of course, satisfying the condition that absolute of theta is less than or equal to 1 by 9. So, for any for a positive theta satisfying this, these two will not be equal and therefore, x and y will be independent only when theta is 0. So, this is an interesting example and must have taken them lot of time to construct such an example. Now, if you try to do it for when x takes only two different values, x and y takes two different values it will not be possible. Then you can think of no trying situation, where x and y take four values each then you have to make sure that you will have to write down this probabilities in such a way that the marginals are independent of theta right.

And then you can have a chance to show that to get construct such an example that the probability maths function of x plus y can be obtained by convolution, but the variables x and y are not independent. So, it might be a very interesting project, but I am not sure if it possible, but here it definitely lot of effort went into it to show that convolution does not imply independence on that time.

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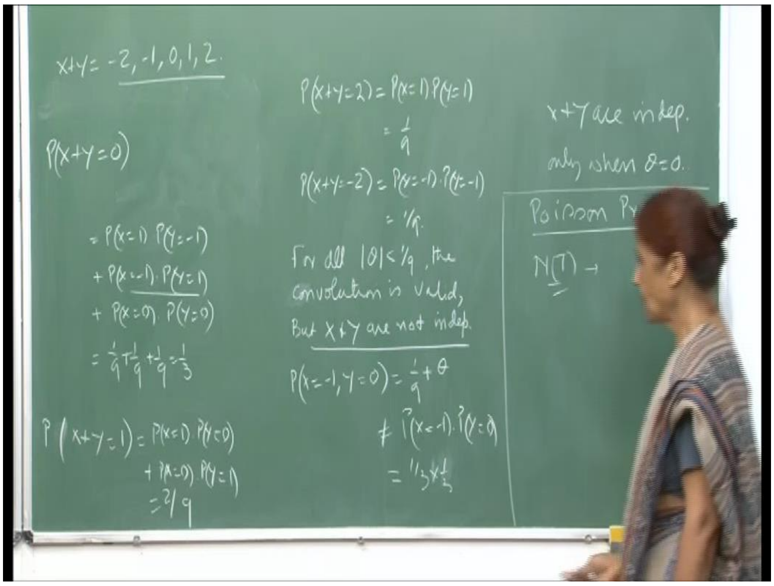

So, you see according to me we have developed enough machinery to able to now show you some more interesting application complex in applications of the tools that we have of the machinery that we have developed. So, far of probability theory and I would like to now devote the rest of the course on stochastic processes are only simple basic once.

And because otherwise the, you know what subject on the stochastic process and things can get very complex, but so the first thing would be the first stochastic process that I would like to talk about would be poison process. And the idea you have already talked of poison random variable and then we have also looked at exponential and gamma random variables. And we will see that how these things get you know, we sought of use the different kinds of distribution that we have developed into answering questions to essentially the thing is that you know, you have it is a counting process. And you have a service that is a post office or railway booking counter well there still people will go and book at the railway counters not everybody does it online is not possible for everybody do it. So, whatever these services are there now you want to have an estimate us to how people are coming and what is the...

So, and therefore, then accordingly you can design the surveys. So, that people do not have to wait for a long time to be serviced how many counters should be there and so on. So, these are the kind of things we will talking about and so basically we will be defining n t as the number of people who are entering let us say post office up to time t. So, we will keep a count and will measure the number of people I mean we will count the number of people who enter the post office say starting from 0 time to t time and then we will answer lot of questions depending on that, but then the thing is that because we are calling it Poisson process. So, this four whether the certain conditions, which have to be satisfied by you know this process.

Because when you are modeling it then we have to have some basic conditions which are met by this particular counting process. And then we will develop the structure and try to answer few questions and the other one would be the other process that we talk about we be mark o process. And so they are both interesting and the basic and they we can sort of develop we will answer lot of questions through these, because we have develop the machinery to do that. So, this would be my you know next few lectures would be on Poisson process and then on mark o processes.

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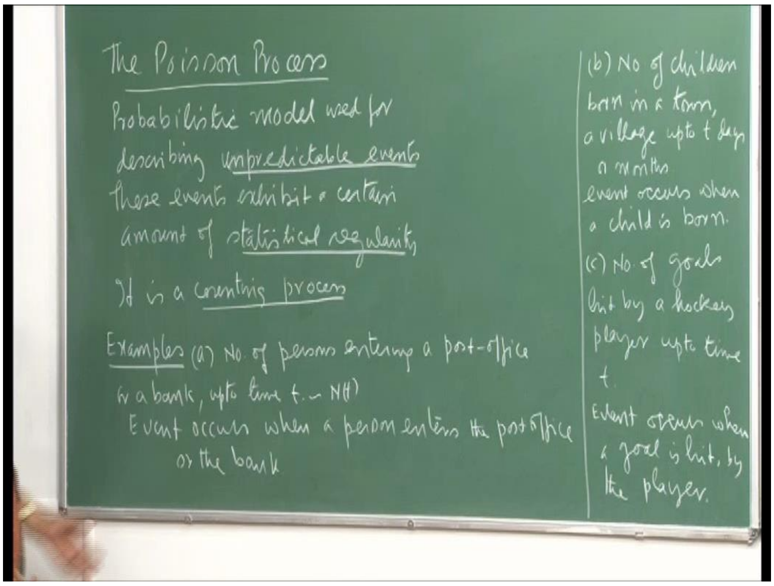

In my last lecture I had told you that we would now we talking about some stochastic processes. And in fact so and one of them is the Poisson process this is you can describe this as a probabilistic model used for describing unpredictable events right, because there is a chance element.

For example when and or the quick will happen or when it certain person walks into a post office these are unpredictable events. So, the Poisson process is a probabilistic model and settles even you model a situation then they are certain rules are certain and of course, so the idea here is that you are modeling a situation where the events exhibit a certain amount of statistical regularity. And that of course, this means that you know you can approximate the occurrences by probability density function. So, these events they exhibit or a approximately exhibit a statistical regularity and so essentially the poison process is a counting process. So; that means, essentially these unpredictable events they get counted by the model that we will create here and then of course, lot of discussion and all lot of conclusions can be based on the counting process.

This is what we will see in the next couple of lectures. Now, the examples as I said are number of persons entering a post office or a bank up to time t. So, always the measure; that means, when I am saying n t is the time to start counting the events start at 0 and then up to time t. So, we count the number of events that take place and therefore, like for example, when you counting the number of persons entering a post office or a bank then this is. So, n event will occur when a person enters the post office right. So, therefore, n t will give the number of people who have entered the bank or the post office up to time t counting from 0, starting time 0, 0 to t number of people who have entered the bank or the post office. That examples could be number of children born in a town or a village up to t days or t months which ever you know time period you want fix the time frame work is decided.

And then you start the counting process of course, in this case event will occur when a child is born right. So, you keep counting these events. Now, number of goals hit by a hockey player up to time t. So, when the match starts and then after that up to time t may this could be the half time or whatever it is when you identify the particular hockey player in the team and then you say number of goals hit by a hockey player up to time t and here again the events will occur when a goal is hit by this particular player right, because you counting the number of goals hit by a particular hockey player. So, this situation then for example, also you can have if you want to pick up a country or a place where a volcano is and then you want to find out the number of times a volcano erupts. So, here of course, your time spend may be may be 5 years or 10 years and then you may because volcanoes luckily do not erupt very often.

So, therefore you would your time spend would be much more then for these particular events. So now, through these examples, we realize that whatever our counting process is and this n t must satisfy the following.

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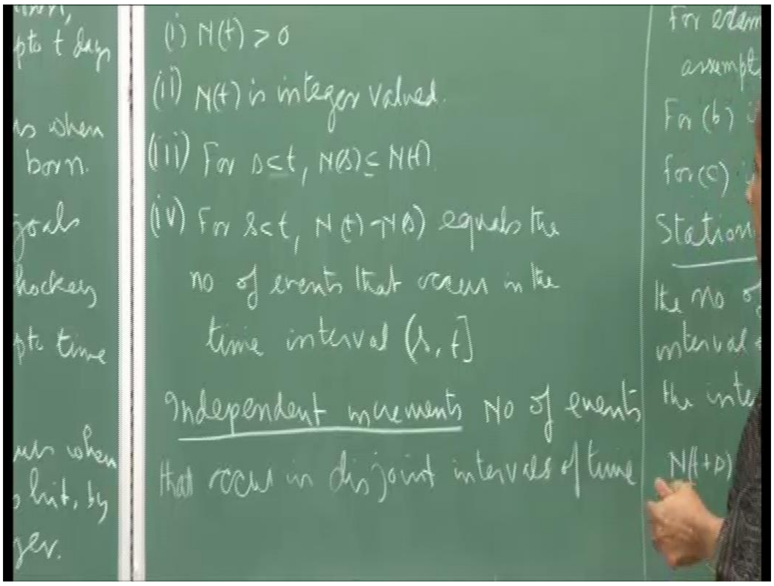

So, since it has to be 0, because when this counts the number of events that have taken place; obviously, this has to be 0 positive. Then n t is integer value, because we are counting the number of events and then for s less than t your n s must be less than or equal to n t. So, either no event occurs in the interval this or some event occurs after s, time s. So, therefore, n s must be less than or equal to n t.

So, this number counting process this satisfies this condition. Then for s less than t if you take the difference as so 3 and 4 are you can we club together also, n t minus n s is equal the number events that occur in the time interval s comma t. So, here of course, n s you know you have counted the events up to time s. So, after s you start counting the events at take place up to t. So, this will be open at this end this interval time interval and close at this end. So, s comma t. So, therefore, this is this, now other things that we want to impose, because remember we are thinking of modeling this situation where you want to count the number of events that take place, but certain conditions will have to be met.