Introduction to Probability Theory and its Applications

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Lecture - 2
Sample Space Events Axioms of Probability

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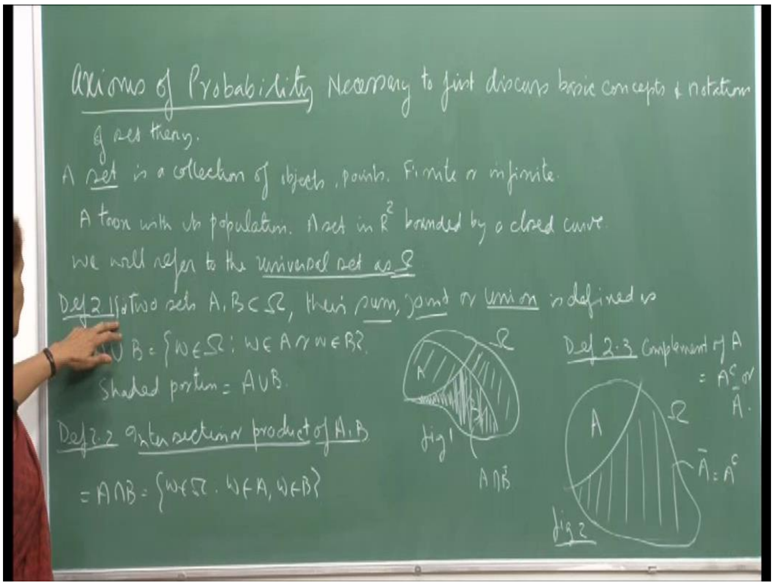

So, today I will talk about axioms of probability. So, I start talking to you about developing the theory of probability, and here to begin with this, first it is very necessary to discuss basic concepts and notations of set theory. So, I will begin with the concepts of set theory and give you some notations, and then after that, it will be possible to define the axioms and then build the probability theory based on these axioms, okay. Now, of course, a set you have already been using this word, and so I will just formally write down that a set is a collection of objects, points, whatever you may consider, whatever the collection, and this can be finite or infinite; the number of elements of the set may be finite or infinite.

So, enumerable examples, now if you have to consider a town with its population, then, obviously, the people living in the town would be a finite number. So, that will be a finite set, and if you consider a set in r 2 which is bounded by a closed curve and then if you consider all the points in this set and they will finite; in fact, they are not even countable, right. So, a set in r 2 bounded by. So, this will be infinite set and so on. Now we will always refer to when I develop the concepts of set theory, there will be a universal set which I will refer to as omega and then we will take subsets of omega and define all the possible; whatever the operations we can do on this set of subsets of omega, right.

Now definition 2 point 1; for two sets a and b subsets of omega; that means contained in omega, we define sum joint or union referred to by two different names. So, basically, the union of two subsets a b, then we define it as a union b and this is equal to all elements of omega such that omega belongs to a or omega belongs to b, right. And if you look at it, diagram was why so I have this because set as my set omega, this is a, this is b, then you see If you consider all this portion; this will be a union b, because the element which is either in a or in can be or which is in both a or b, then that constituted your union of the two subsets a b, okay.

Then, similarly, intersection or product is other two names given to of a b, and the notation is a intersection b and this is here for all omega in capital omega such that omega belongs to a and omega belongs to b. So, the common base, and here in the diagram, you see that the intersection would be this, this portion, right, which I have shaded differently. So, this portion will denote the intersection of the two sets and it is not coming properly, fine. Then, similarly, you would define complement of a set a which means and we denote it by a c or a bar. So, complement means that, okay, I did not write down the definition; that means this is a compliment is all omega belonging to this or that omega does not belongs to a; this is the idea, right. So, if this is your a and this whole is omega, then all elements which are not in a from the set a complement, okay.

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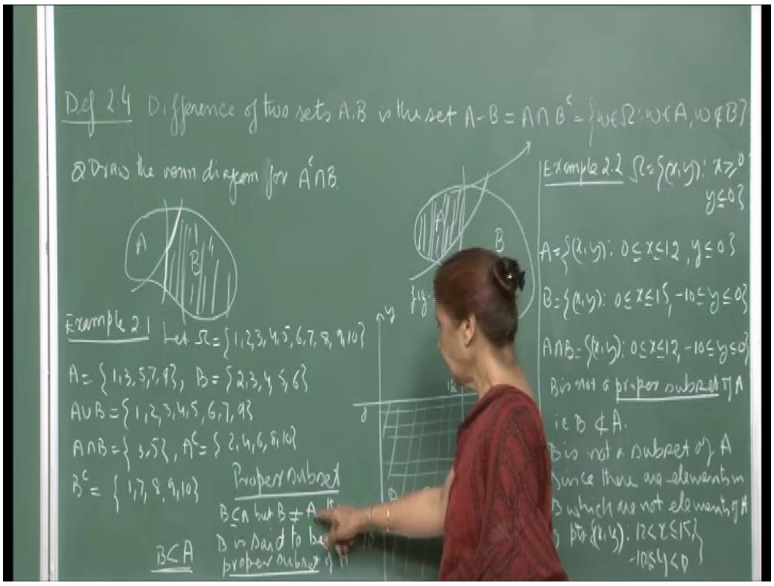

Definition 2 point 4 is difference of two sets a b is the set a minus b; they refer to it as a minus b or a intersection b compliment. So, that is all omega the universal set such that omega belongs to a, but omega does not belong to b because it is intersection b complement. So, if you use the definition of the product, then the elements here must belong to both a and b complement; that means the elements which are belonging to a but not belonging to b. So, in that case, I have drawn the figure; figure three should be so a and this is b, no, so I have to say b compliment, okay. This is not shaded correctly.

Yeah, so if I am saying that this is whole of a and this whole is b I am referring to, right, because the elements here are in a and not in b. So, this is the right diagram for a intersection b compliment, right, which is a minus b. So, all components all elements which are in a but not in b will be referred to by this one here, okay. Now let us just so what we are asking you [FL]. So, I probably drew the okay. So, this was the figure for this, right, okay. So, we will do it here. So, now it says that question is draw the event diagram for a complement intersection b. So, here if you have this and if this is a and this is b, then if I want a complement intersection b, then an element which is in b but not in a. so, then it will be this; this is what I had drawn, okay.

So, the elements here, they are not in a but in b; that is a by intersection b, okay. Again I will take some more examples. Omega let us say the universal let contains the numbers 1 to 10, okay, and then if I define a as a set of all odd numbers and b contains 2, 3, 4, 5, 6, then a union b would contain elements which are here as well as in b. And so, the union will be 1, 2, 3, is here 4, 5, 4 is here. So, it will get added here, 5 is here, then 6 is in b. So, that gets added, and then 7 and 9. So, this is your a union b, right, and a intersection b, you have to look for the common numbers in a and b.

So, 3 and 5; 3 and 5 are the common numbers. So, that forms your set a intersection b. And, similarly, you can write down all the others; for example, a compliment would be all even numbers, right, because this is all odd. So, a compliment would be all even numbers, then similarly b compliment you can write down the numbers which are not present in b. And then whatever other operations we have defined, you can work out the examples. Now I will take the case of the infinite term set. So, here is you take omega to be all collection of x comma y; that means points in r two. So, these are the points in r 2 such that x is non-negative y is less than or equal to 0.

So, if you consider this as a whole plane r 2, then your omega is on this; that means here y is less than 0 and x is positive. So, whole of this quadrant last quadrant; in arc 2, this is the first, second, third and fourth quadrant. So, we are in the fourth quadrant omega, obviously, then is a infinite set, okay. Now if I take a to be the set consisting of all points for which the x coordinate is greater than 0 less than 12, and y of course is less than or equal to 0, then of course the picture is not; if this is my number 12 on the x axis, then you can see that the whole of this we can say set. Again it is an infinite set where y is less than 0 and x coordinates are between 0 and 12 that forms a, right, and b is all x between 0 to 15 and then y is minus 0 and 0. So, minus 10 and 0, okay; so, this is minus 10 and 0, fine.

So, that means if I draw a line here, then my b extends like this and x is up to 15. So, now in this case your b is this set. So, b is this portion, right. The x is expending up to 15 and y is from 0 to minus 10. So, that is your b. Now you see that if you want to write a intersection b, then this will be all points x y such that x because you see in a, x is between 0 and 12; in b it is between 0 and 15. So, when you want to take all common points in a and b, then it will be x between 0 and 12, and here your y is less than or equal to zero. So, y in a was extending up to infinity. So, in this case, the common thing would be that y is between minus 10 and 0. So, that will give you the intersection, right.

And of course I have already used the concept that a is a subset of b or that means here a and b are subsets of omega. So, here you can see that b is not a proper subset of a, because you can find points which are in b but which are not in a. So, this is the other concept that you use. So, here I have said that b is not a proper subset of a; that is I showed you that there are points in b which are not in a, alright, and that you can see because the set of elements this we have already seen that when you take points for which the x coordinate is between 12 and 15 and the y coordinate is between minus 10 and less than 0. Her of course x is greater than 12, because a contain x less than or equal to 12.

So, if the point is not in a, then the corresponding x coordinate should be greater than 12, alright, and this is less than or equal to 15. So, these are the points which are in b but not in a. So, b is not a proper subset of a. So, now let me just also give you a formal definition of a proper subset; what do we mean by a proper subset? That is if b is a subset of a but b is not equal to a, then we say that b is said to be a proper subset; that means when b is a subset of a and b is not equal to a, it means that there are points in a which are not in b, whereas because b is a subset of a. So, all points of b are all elements of b are also elements of a but since b is not equal to a; that means there is at least one element of a which is not an element of b.

And so then in that case, we say that b is a proper subset of a, and we write it as without the equality sign here. So, b is a proper outset of a; this is the notation for. So, therefore, now at least whatever words I used the word not a proper subset. So, now you know what is a proper subset?

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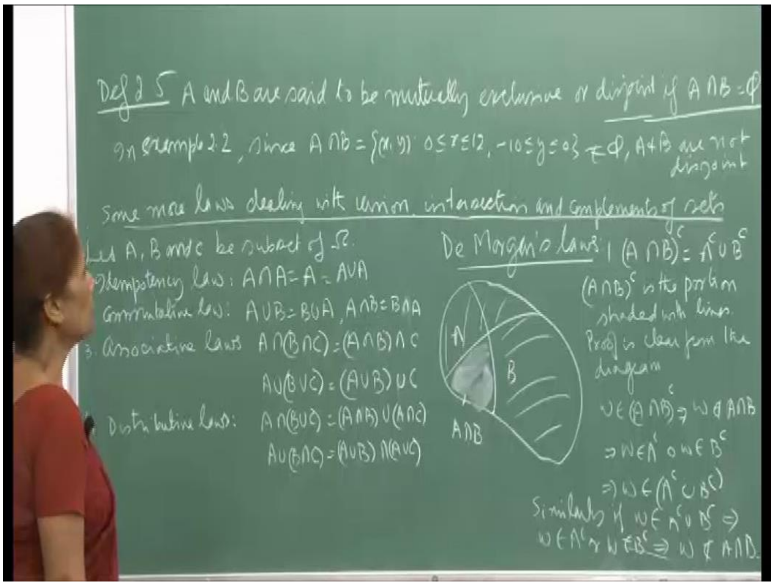

So, now let me continue with the definitions and with the basic concepts of set theory. If a and b are said to be mutually exclusive or disjoint, if a intersection b is empty; that means there are no common elements in a and b, fine. So, for example, in the example 2 point 2 that we just considered, you know the coordinate in the subset of r 2, you see a intersection b is actually this set which has so many points and so it is not empty. Therefore, a and b are not disjoint or they are not mutually exclusive and so as we go along and see so many examples when a and b have sum points in common, then they will not be considered disjoint if they do not have any not points in common, they will be considered as disjoint or mutually exclusive.

Now some more concepts dealing with laws and this thing dealing with union, intersection and complements of sets; so, here if now I am talking of a, b and c, there are three subsets of your universal set omega, then idem potency law says that you know when you operate a with itself, then a intersection a is a or a union a will also be a. So, that means the union and intersection are either the operations we have defined for the sets these two are idem potent, okay. Then commutative law says that it does not matter whether you add a to b or you add b to a. So, therefore a union b is equal to b union a. Similarly, a intersection b is b intersection a.

So, the order is not important which is that you write first. Associative laws here if you are saying you are taking intersection of a with intersection of b intersection c, then you can also write this as you first take the intersection of a and b and then take the intersection with c. So, it does not matter. So, the associative law holds here. Similarly, with union also; with respect to union also the associative law holds; that is whether you add a to the union of b and c or you add, first take the union of a and b and then union with c; it does not matter, they are the same.

Now the distributive law is with respect to both the operations, intersection and union. So, when you take a intersection b union c, you can write this as a intersection b and then union with a intersection c. So, a gets distributed, alright. And, similarly, a union b intersection c will give you a union b, then intersection a union c. So, these laws hold, and just for completeness sake, I have written them down here. Then De Morgan’s laws are also important and we will be using them throughout, and therefore, it is better to talk about them right now. And so the first laws say that an intersection b complement is equal to a complement union b complement, fine.

And if you see diagrammatically of this set is omega and if this is a and this is b, then a intersection b is this, alright, the darkly shaded portion. And so its complement will be the portion which is shaded by the lines, and you can see that the portion shaded with the lines is nothing but the union of a complement and b complement, alright. Because this portion gives you b complement and this portion gives you a complement. So, the union of both the two, so the two is equal. But you can also prove it otherwise. See what we are saying is that if omega belongs to a intersection b complement, this implies that omega does not belong to a intersection b, right, which means that omega should not belong to at least one of them.

Because if omega belongs to both a and b, then omega will belong to a intersection b. So, since we are saying omega does not belong to a intersection b, this implies that omega does not belong to a complement or omega belongs to b compliment, aright. If you do not what omega to belong to both a and b, then it must belong to either a complement or b complement which implies by our definition of union that omega must belong to a complement union b complement. Now remember, okay, I did not say this out in the beginning but I should have said that whenever you are wanting to say the two sets are equal, then what does it mean?

Yeah, maybe I should spell it out; a equal to b implies, yes, component wise that if omega belongs to a, this implies omega belongs to b. And if omega belongs to b, it should imply that omega belongs to a, fine; this is the way of saying that the two sets are the same. So, here when I want to show that these two sets are the same; right now I have shown you that if omega belongs to an intersection b complement, then it must belong here, but I must show the other way also. That is if omega belongs here, then it should belong to this and which I am showing.

Similarly, if omega belongs to a complement union b complement that means if it is here, then this implies that omega either belongs to a complement or omega belongs to b complement which implies that omega does not belong to a intersection b, alright, because either omega belongs to a complement which means that it does not belong to a or it does not belong to b which means that omega does not belong to a intersection b, and so it belongs to a intersection b complement. So, I have shown you both ways, and therefore the two sets are equivalent.

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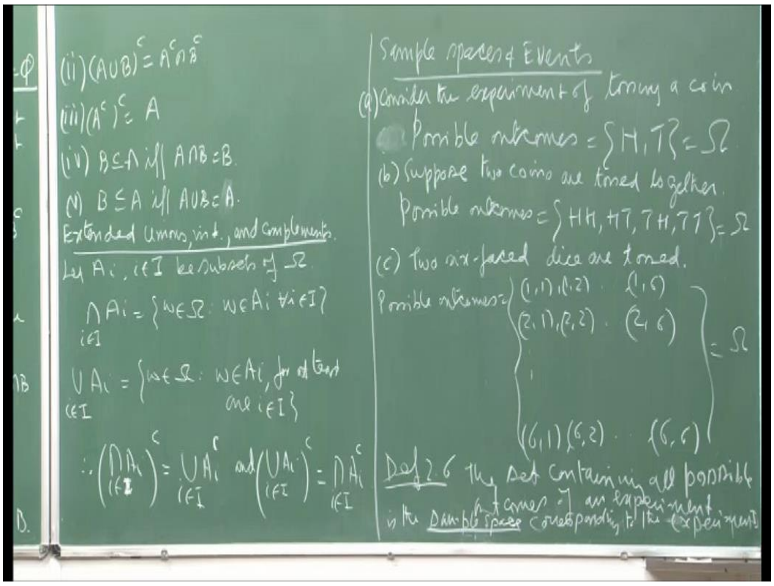

So, similarly, I am writing out all the other laws here De Morgan’s laws which you can sit down and work out yourself. So, there you will have to show that you first start with an element here and show that it belongs here, then you will have to pick up any element from here and show that it belongs to set on the left. So, that exercise you should now sit down and do for the remaining De Morgan’s laws but we can just look at them. So, this is in a way a complement of this says that a union b complement is equal to a complement intersection b complement.

Then if you take the complement of the complement, you get back to the set a, alright; that of course in words you can immediately say it out. And one way to classify correct wise is that one set is the subset of another. So, b is a subset of a if and only if when you take the intersection, you will get the set b, fine. If b is a subset of a set a, then when you take the intersections, the only element all the common elements must belong to b; similarly, union you can characterize that if b is the subset of a, then a union b is equal to a, because b is not adding any new elements to the set a union b. And therefore, a union b remains equal to a.

Now so far I had defined union intersection complements for two or three sets. But now we can extend this notion to any number of sets, and here I am just taking i to be the index set where this index set can be I can be having finite element of finite indices or infinite indices. And so this holds for whatever the status of I is. So, now if all these are subsets of omega i is in the index at i, then you can say that the intersection of all the subsets such that i belongs to i is omega belonging to the universal set such that omega belongs to a i for all I, alright. So, if an element is present in all the a i, so then it will be present in the intersection, and union of course would imply that omega belonging to a I for at least one i, right.

And omega belongs to the union if omega belongs to at least one of a i's, okay; it van belong to more than one, but it must belong to at least one of the a i's, right. And then, similarly, using the concepts here the definitions here, you can define now that this is not written very nicely, okay. So, i belongs to I if you are taking the intersection of all a i's so that I belongs to i. And then you take the complement. So, then by this law, you immediately write it down as union a i complement, yeah, and again in words you can say it out the same thing. And, similarly, here this is the equivalent of this when you are taking more than three sets or two sets; this is union a i, i belonging to i. The complement of this will be the intersection of the complements of all the a i's, right, okay.

So, given this now let us get started with taking about how we go about defining probability of an event and so on. So, before that I will simply spell out what we mean by sample space and then events and then we talk of how we go about estimating the probability of events, okay. So, again I will begin with the examples to give you an idea what we mean by sample spaces. So, for example, if you consider the experiment of tossing a coin; so, a single coin is tossed. And so what are the possible outcomes? Either I will get a head or a tail; these are the two possible outcomes.

So, you just collect all the possible outcomes of an experiment and that will be our concept of a sample space and again I will refer to it as for that particular experiment this is my universal set. Similarly, if I toss two coins together, now the thing is that you are tossing two coins and so let us just name one coin as number one and the other as number two. In that case, you see when both of them show heads and this will be h h. Now here if the first coin shows the head and the second coin shows the tail, then it will be h t. And if the first coin shows a tail and the second coin shows a head, then the outcome would be the t h, alright, and in case of both of them show t t tails, then this will be t t.

So, this will be the set of all possible outcomes when you tossing two coins together, okay. And so this will be your sample space; this would be the sample space. If you are now tossing 2 6 faced die. So, here again what we will do is we will say that we will number one die as number 1 and the other die will be number 2. And so, therefore, the outcomes will be recorded as whatever the numbers shows for the first die and whatever the number shows for the second die. And in that case, it will be s pair of numbers. So, 1 to 6, and therefore, the possible outcomes will be 1 1, 1 2 and 1, 6; that means the first die shows 1 and the second dice shows 1, 2 or 6. And, similarly, you can then say that it is 2 1, 2 2, 2 6.

So, therefore, it is important when you are throwing two die, then you have to number them as 1 and 2, so that because what shows the arrangements because here this is an ordered arrangement, and therefore, there is a difference between 1 2 and 2 1, okay. And so, we have to record all possible outcomes, and therefore, here this will be the collection of all 36 outcomes, and that will constitute your sample space, okay. So, therefore, I am just trying to show you that when you consider an experiment, then you have to be very careful in recording the outcomes. And so here when you are tossing two coins, then we will have to number one as first coin and one as second coin, and then we can record the outcomes.

Similarly, when you are two die, then you have to you know locate one die as number 1 and the other one as number 2 to be able to record all possible outcomes in a proper way. Then the possible outcomes I have not listed but this table shows that you know it can be the first one is the first die is showing one and then the second die is also showing 1, then it could be 2 up to 6. Similarly, it could be 2 1, 2 2, 2 6 and 3 1, 3 2, 3 6 and so on up to this. So, here the total number of possible outcomes you see will be 36, right, and again I will refer to this as a sample space. So, now formal definition of a sample space can be immediately said and that is if the set containing all possible outcomes of an experiment is the sample space corresponding to that experiment, okay.

It is not very clearly written but you can just read it; that means the set containing all possible outcomes of an experiment is the sample space corresponding to that experiment. So, this will be our concept, and here you see that in this case of course when do any experiment, you except the outcomes to be finite. And so, my sample space in this case would be the number of I will just list the number of possible outcomes and that collection of possible outcomes. So, you can see that the sample space can have any kind of structure.

So, here in case, for example, it was h and t means these are the possible outcomes; in this case, it was h h, h t, t h, t t. And now in this case it is pair of numbers where the numbers can differ from 1 to 6 for wither this thing and so collection of those 36 pairs of numbers are the elements of sample space omega and that will be. So, now I will go on defining the concept of events and then we will try to estimate the probability of events.

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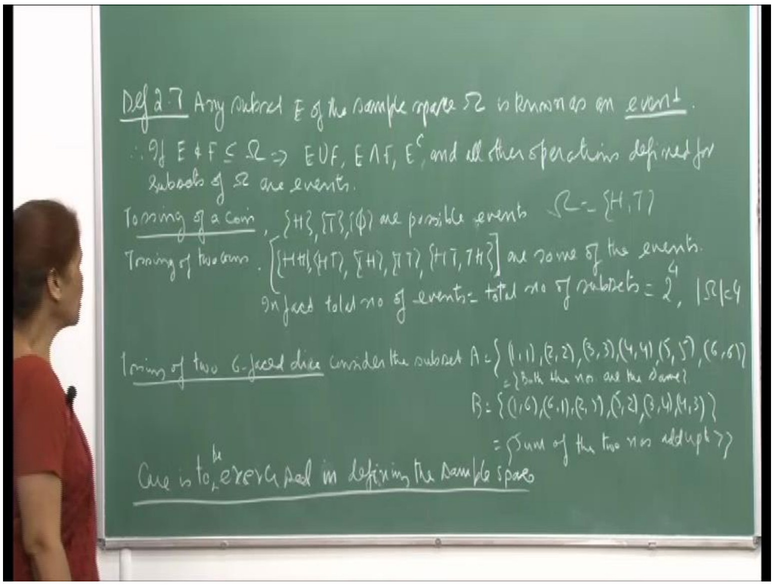

Now I will defined what we mean by an event. So, any subset E of the sample space idea, first idea of a sample space is clear; that means all collection of all possible outcomes. Now any subset of the sample space is known as an event; we will define this way. And we have seen that, if for example, two subsets are there e and f is subsets of omega; they both are events. Then whatever operations we have done so far we have discussed on the subsets, then e union f, e intersection f, e complement and whatever. We discussed all those various operations on the subsets, then all those will again be subsets of omega, and therefore, by our definition they will again be events.

So, in fact, when I told you the extended concept of union and intersection where you took number of subsets whether finite or infinite and again if you take their intersection and union and complements and so on, they will again all be subsets of omega. So, therefore, you can see that the collection of events becomes very very large and in fact, it is formalized which I am not actually taking to you about; I will just mention the name that it is known as the sigma algebra of events. So, essentially what we have to remember is that, however, way we can generate subsets of omega, then those which can be obtained as through union, intersection and complement. Through these three operations whatever subsets of omega we can obtain, they will all the considered as events for the corresponding experiment, okay.

Now examples are there; when you toss a single coin, we saw that the sample space omega just contained these two possible outcomes. And now the subsets can be if the single turns h t and of course we always include consider pi also as the subset. So, therefore, these will be possible subsets or the events in this case. When you are tossing two coins, then you see your possible outcomes were four, in fact, h h, h t, t h, and t t. Now you can start forming subsets; I have just written down a few. You can take this each outcome of each element of the sample space omega as subsets, and so it is an event. You can put two together; that means either head or tail or tail or head, and this will again also form an event and you can then perform.

In fact, in the last lecture I have shown you that if set has whatever the number of elements, then the possible number of total number of subsets of that set would be 2 raise to 4; remember it was an application of the binomial theorem and so on. So, the number of subsets here, in fact, would be 2 raise to 4 which will be 16. So, you can list out all the possible events that can be formed for this particular experiment of tossing two coins. Now again as an example when you are tossing two 6-faced die, then consider the subset which has both the numbers that means both the phases show the same number, alright. So, both the numbers are the same and so here the components in a would be 1 1, 2 2 pairs, 3 3, 4 4, 5 5 and 6 6.

So, this will be one event; that means you can also describe it in words that both the phases both the numbers are the same. And b for example I have listed pairs which were the two numbers add up to 7. So, the sum of the two numbers adds up to 7, and this way you can go on continuing. In fact, here the number of subsets will be very large because omega itself contains 36 pairs, and so the number of subsets would become this thing, right. Yeah, but now one word of caution and that is when we are defending the sample space, we have to be careful as to what our experiment is.

And here, for example, you have to; in fact, I have to say that there are two possible cases. One is with replacement wherever this is relevant, and the other possibility can be without replacement, okay. So, we have to be careful on how the experiment is being conducted and then you can define your sample space that means the possible outcomes that can be there of the experiment.

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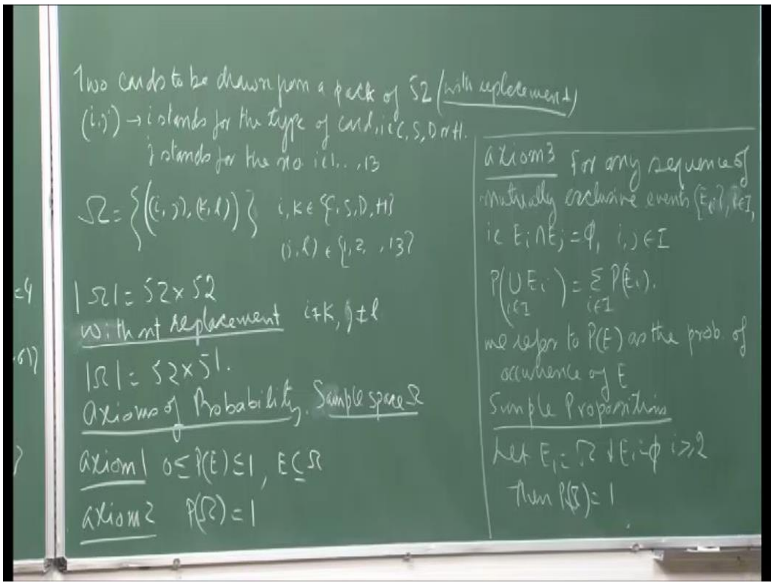

So, now consider the case that two cards are to be drawn from a pack of 52 cards, and if you are doing this drawing of the card; that means you draw a card. You look at it, note it down somewhere and then put it back in the pack and then again you draw the second part. If this is your experiment, then let us say that I comma j stand for. So, you note down when you take out a card, you look at the number; you look at the card. So, then the first index here tells you what card it is, what type; that means whether it is a club, spade, diamond or heart. So, I will refer to c s d or h; that means four possibilities and j will stand for the number from 1 to 13; that means whether it is an ace; that is number 2, number 3 or number 13, right.

So, then this pair will denote the type of the card that you got, and so your omega will be collection of all two pairs I comma j and k comma l where again k indicates the type of the card and l indicates the number. So, therefore, I and k can be anything from c s d h, because you are doing replacement and then j and l are the numbers 1 to 13. So, essentially, the way you can describe the experiment is that the first draw of the card, it can be any 52 cards. And, since, you have replaced it in the pack, again your choice of drawing a card is from among the 52 cards in the pack. So, therefore, the total number of outcomes that are there is 52 into 52, okay. And so this actually by the way denotes the cardinality of a set, which means that what is the total number of elements in the set.

So, here the sample space will consist of 52 into 52 such collection of pairs, okay. So, to indicate you what the cards you have got. But now if you are doing it without replacement, then you do not want I and k to be the same and j and l to be the same; that means once you have drawn a card, it is not there in a pack. So, it will not appear again, right; in that case, your choice will become that means in the first draw you have a choice from among the 52 cards, but in the second draw, it will be only out of 51, because one card has been replaced is not there; it is kept aside.

And, therefore, in this case you will have to be careful because your choice of the sample space will contain such pairs but there I will not be k and j will not be l. So, the whole idea is to say that we just do not write out you have to be careful when you spell out all the possible points that can be there in the sample space, okay. Now let us begin talking about the axioms of probability; that means how do we characterize a function or whatever we mean by a probability; I mean what type of characteristics it should have? So, here again I am now referring to the sample space omega and then e is some subset of omega which again is an event. So, then we say that the function p which will assign. So, essentially what we are saying is that p assigns, okay, I am writing capital P.

So, P is assigning E to a real number; that is it takes it to P E which belongs to the real number r, right. So, we can say that this is a mapping or an assignment. So, here for every event E for every subset of omega, I am assigning real number. And so, P E is a real number here, and P E must be between 0 and 1; that is the first requirement. Then for the whole sample space p omega should be 1, alright, and axiom three says that for any sequence of mutually exclusive events; that means you just take any collection of events a i that means subsets of omega such that they are mutually exclusive which means that e i intersection e j is empty for all I j in I, alright.

So, this is the collection which can be a finite, infinite; it does not matter, but we are referring it to as an index set I and then probability of union E I, i belonging to I should be equal to the sum of the probabilities. So, because the events are mutually exclusive, they are disjoint. So, therefore, when I add up the compute the probability of the union of these E I's, then it should be equal to the sum of the probabilities. So, these are the three characteristics that we associate with the probability function, and using this, you see that any function f p which satisfies these three axioms will define the probability. And this is actually what we say is that p E is probability of occurrence of e.

So, remember in the beginning I started saying that we would be trying to develop the theory of occurrence of an event. So, here we have now defined something which we refer to as the probability of occurrence of e. So, this is and we will now see that what things you can derive. So, basically using these 3 axioms, we will be able to build up the probability theory. And the first simple observation that we can make is that n this case for axiom three, if you choose your e I to be the whole set omega that is your sample space and all other e i's you choose as empty sets, right, for i greater than or equal to 2 because this axiom must hold.

So, here the union will become your set omega, right, because all other e i's are empty, E 1 is omega. So, union gives you omega and from here from axiom 2 p omega is 1. So, you get this. Okay, p omega I have to write that is secondary and that is not a big deal. I mean, okay, what I am saying is that this validates the axiom because we have already assumed that p omega is 1 and so here also if you put e 1 as omega and all other e i's as empty sets, then you get p omega is 1, alright. Now here in this case, if you look at this, you want to compute the probability of let us say h and t. So, here if I take what will be my subsets here this would be in the first example, my e 1 would be h, e 2 will be t, alright.

So, in that case from this axiom, what I will get is that p e 1 plus p e 2, alright, is equal to 1, because e 1 union e 2 is my whole space omega; p omega is 1. So, therefore, I get that p e 1 plus p e 2 is this but then I should be able to compute, okay. Then what I am trying to say is that from here, I should be able to show that p e 2 is equal; of course, from here it follows that p e 2 is 1 minus p e 1.

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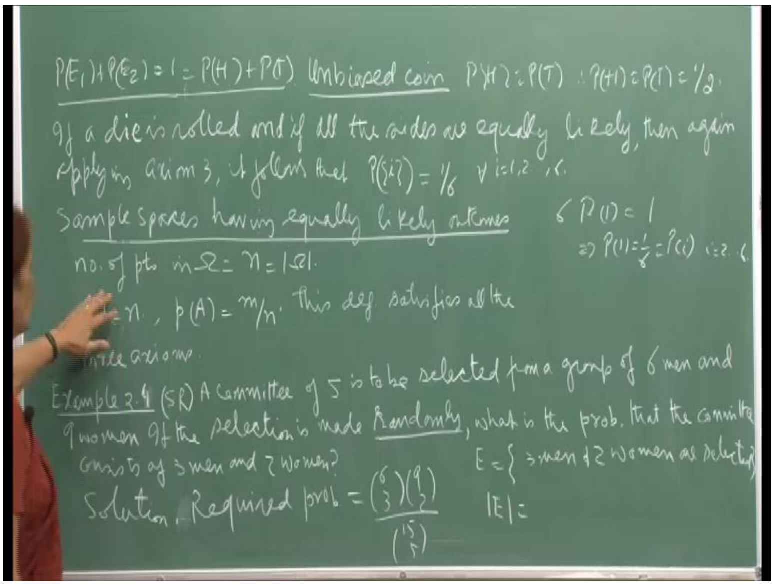

So, we got that using axiom three in the case when single dice rolled tossed and then we got that p e 1 if e 1 is the event that it is showing ahead and e 2 is the event that it is showing a t, then we got that p e 1 plus p e 2 is 1 which is equal to probability of h plus probability of t. Now we understand the concept of unbiased coin; that means it is equally likely whether a head shows or a tail shows. In that case, the two probabilities are equal; that is what we mean by an unbiased coin. And so, therefore, it will immediately follow from here that p h equal to p t is equal to half, because they both most add up to o1. So, therefore, what is what I am showing how you apply the axioms to arrive at the probabilities of the events. So, here of course that it is an unbiased coin.

Similarly, when a die is rolled and so you have the 6 number showing up, either of any of the six numbers can show up. So, here again applying axiom three, you will see that the probabilities of all numbers but that means p 1 plus probability of p 2 plus up to probability p 6 and that should be equal to 1. And since we are again saying that all size are equally likely which means that probability of p 1 is equal to probability of p 2 means it is equal to the probability of 2 that it is equal to probability 3 and so on. So, all six together; that means essentially what we are saying is that six any of them; this is equal to 1 which implies that p 1 is 1 by 6 which is equal to all other this thing, right, for I varying from 2 to 6.

So, we immediately conclude that probability of each phase showing up is the same which is 1 by 6. Now what happens is that most of the time in many, many situations, we now already because of the nature of the experiment and so on that the elements of the sample space have equally likely outcomes because that is how I showed you these two example that when you are throwing up a unbiased coin, then you know that whatever the outcome has the same probability that a head showing up or a tail showing up, they are the same. Similarly, as I said that if you are throwing up a die and you have no reason to say that it is loaded die or a biased die, then in that case we except that any of the six numbers have the same probability will show up the same.

So, we will see that and of course we have to be what I am trying to say is that I am giving you an alternative way of defining the probability of an event, but the basic assumption for this definition is that your sample space has this property that all the outcomes in the sample space have the same probability equal likely chance of occurring, right. If that is there if this is satisfied, then let us just start with this definition that suppose omega has n number of points; that means the cardinality of omega is n and the number of points in a where a is an event, okay. A is subset of omega, alright, and then we are saying that the number of points in a is n, then we define the probability of a as m divided by n.

That means the number of points available in a or you can also there is another way of saying it that the number of favorable cases; that means the number of points which actually are in a; that means for occurring of a those are that points which will occur. So, then the definition of the event a is number of favorable cases for a divided by the total number of cases; that means total possible outcomes and the outcomes which are part of your event a. So, then this is the definition, and now you can very quickly verify that the three axioms, because remember I said that any definition of probability must satisfy the three axioms. So, since a is subset of omega, this means cardinality of a is less than or equal to cardinality of omega, right, which implies that m is less than or equal to n, alright. Therefore, probability a is less than or equal to 1 and also by definition, probability of a is greater than or equal to 0, because m is either 0 or it is a positive number. So, therefore, this satisfies the first axiom, right.

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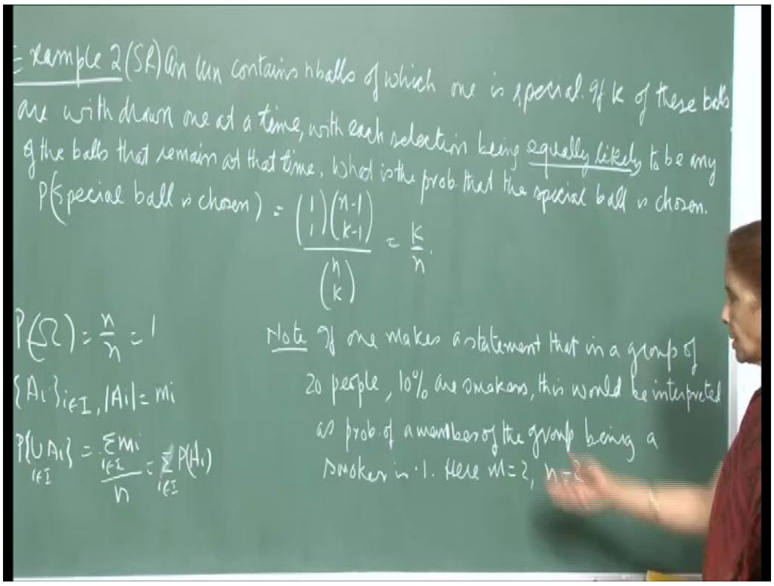

Then, similarly, probability omega would be n by n which is 1, alright, because omega has n points. So, therefore, n upon by verifying the definition; so, this probability is 1. So, this axiom is also satisfied. Now if I take a set of events a i's in an index set I capital I, and here, since, we are taking the omega as the finite space. So, I is also a finite index set and then cardinality of each a i is m I; that means the number of points of the sample space in a I is m I, alright. Then we want to look at the probability of union a I, and here the a I's are disjoint, because, remember, I am verifying the axioms 3 of probability. So, here a I's are disjoint; that means the elements of the sample space which are in one a I are not in any other.

So, all these a i's are disjoint meaning that the points in one a I the elements of sigma in one a I are not in any other a I, alright. So, in that case, now if you want to compute the probability of union a I I belonging to I, then this will simply be because the number of elements of omega sigma of omega in union a I will be sigma mi. Since, the elements in each a I are different from all others; so, therefore, we will just add up the total number of points which are in the union here will be sigma m i. And so by our definition, this will again be when you are computing the probability for this union, it will be sigma m i I belonging to capital I divided by n.

And now here the sigma I can write as individual sigma's that means I can write this as summation of m i by n I varying from 1 to n and each m I by n. So, for a particular I, the m I by n is the probability of a i and so this is the sum of the probabilities. So, axioms 3 are also satisfied here. So, that means this particular definition when you assume that the outcomes, the occurrences or the elements of sample space have equally likely chance of being of occurring, then I take the definition of the probability of an event as the number of elements in that events number of elements of the sample space in that event divided by the total number of elements in the sample space.

So, m by n if I take that as the probability here, then, since, it satisfies all the three axioms; so, this is a very convenient way of computing the probability provided of course you can assure that the outcomes in the sample space are all equally likely. And, yeah, if you can assume that if you can assure that, then we can use this way of computing the probability of any event where we can just look at the number of favorable cases for a particular event and then count them up and then divide by the total number of outcomes in the sample space, and you get this anyway. Now we just want to make another comment here note here and this is you know we sometimes refer to the probability in terms of percentage.

So, for example, if you make a statement that in a group of 20 people, ten percent are smokers; sometimes he also refer to be probability in this way. So, this would be interpreted as probability of a member of the group being a smoker is 0.1. So, 10 percent; so, 10 upon 100, okay, and here if you want to interpret it in terms of your m by n, then you see your n is 20 and the ten percent of 20 is 2. So, m is 2; so, therefore, 2 by 20 which is 1 by 10 which again is equal to 0.1. So, please remember that as the course develops, we will often be referring to the probabilities in terms of percentages and so the interpretation is simple, alright, okay. You just take the fraction which is so 10 percent means 10 upon 100, and that gives you 0.1. So, that will be your probability.

So, exactly the same way you are counting as you are doing here m by n. So, therefore, all the conditions all the axioms are satisfied. So, this is also a proper definition, but of course this is valid when you can assure that the all outcomes in the sample space are equally likely, alright, okay. So, I will quickly take up this example. Now here a committee of 5 is to be selected from the group of 6 men and 9 women, alright. Now if the selection is made randomly, so this is important. Since, the selection is made randomly; that means that the choice of any of the men or any of the women is equally likely.

Under this assumption, we will say that what is the probability that the committee consists of 3 men and 2 women? So, if you want to compute this, I will apply this definition because the example here the experiment here satisfies this condition. And therefore, you see now here I will apply this thing multinomial thing. So, you want to select 3 men out of 6, and 2 women out of 9 women. So, this gives you the total number of points in your set a, right, or the event E or if you define the event E as 3 men and 2 women are selected, then this cardinality is given by 6 3 9 2, alright.

And the total number of ways in which you can select 5 people from the set of 15 is 15 choose 5, and therefore, the number of favorable cases is this, total number of cases is this. So, the probability of selecting a committee of 3 men and 2 women is given by this ratio. And, similarly, here an urn contains n balls of which one is special. If k of these balls are withdrawn one at a time with each selection being equally likely; so, here again the same thing is stated. So, if each selection is being equally likely; that means whatever the balls are left, choosing the ball from there is equally likely, then what is the probability that the special ball is chosen. So, here again the probability of the special ball is chosen. So, here you see you have to choose the special ball, so 1 upon 1; the probability of choosing that is 1 upon 1.

Then from the remaining n minus 1 you want to choose k minus 1. So, this gives you the number of favorable cases. The number of cases in which the special ball will get selected and then the total number of ways of selecting k balls from n balls is n c k. So, that number when you compute comes out to be k by n. So, the whole idea is that things become much easier if you have this condition being satisfied, then this is very convenient way of determining the probability of an event.