Introduction to Probability Theory and its Applications
Prof. Prabha Sharma
Department of Mathematics and Statistics
Indian Institute of Technology, Kanpur

Lecture - 19
Inequalities and Bounds

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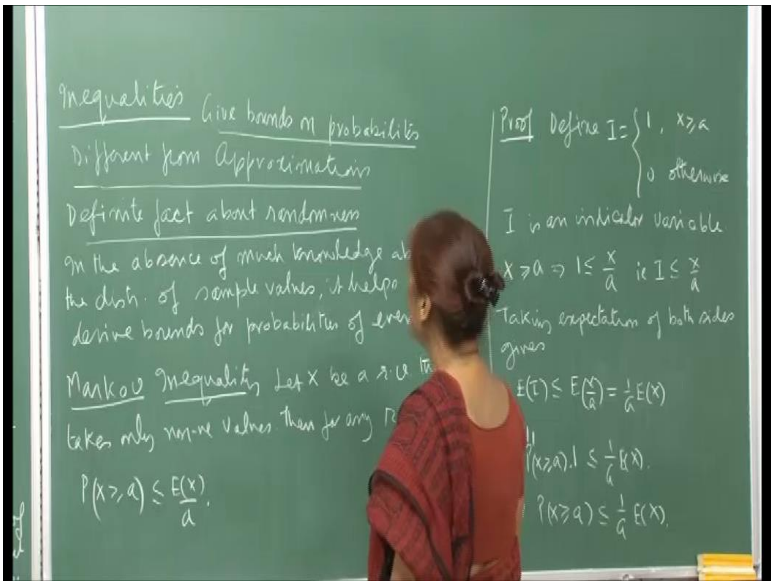

So, the next topic we want to talk about, is inequality, statistical inequalities. And of course, one we have already seen, which is the Cauchy-Schwartz inequality, but there some other important, once which we will talk about now. Now the role of inequalities, is that you get bounds on probabilities of certain events, and this is different from the approximation, because the inequalities make a very definite statement. So, it is a definite fact about randomness. You have some event, and you want to you are able to say that, the probability of this event, will be less than or equal to a definite number.

So, you give a bound, lower bound upper bound, whatever is possible. Mostly we will see that we talk about upper bounds. And of course, another thing is that, this is different, the inequalities give you information which is different from approximations, because approximation may be good bad, but here the inequalities trying to tell you, the probability of a certain event, is less than this particular number, which the approximation does not says. Approximation says that the probability may be this, and then depending on.

So, therefore, they have a very definite role to play; the inequality, in your statistical analysis of data and so on. So, see what happens is that when, in the absence of much knowledge about the distribution of sample values, you take different kinds of samples, and then you do not know much about the distribution of the population from which you are taking the samples. And then to derive bounds for probability of events, you know depending on the sample values, is is very helpful, and that is what we do through this inequalities. So. In fact, it may just happen that we may know, the mean or the variance of the population with the sample values are coming, and that is it, we may not know the nature of the distribution exactly. So, then it helps to, be able to compute bound on the probabilities of certain events.

So, the first one, simple one is Markov inequality, and this the statement, is that the x is the random variable, that takes only non-negative values. Then for any real number a greater than 0; probability x greater than or equal to a, is less than or equal to expected x upon a. And this is not difficult to prove, because if you are now define this indicate valuable where variable. So, which takes value 1, when x is greater or equal to a and 0 otherwise. so; that means, yeah. So, now, when you write x greater than or equal to a. This implies that 1 is less than or equal to x by a; that is and since i is equal to 1 whenever x is. So; that means, I can write that i is less than or equal to x by a. Now taking expectation of both sides. So, when I take expectation of both side, I get expectation i which is less than or equal to expectation x by a, but expectation x by a is 1 by a expectation x. So, this I am just relating the 2 random variable i and x. So, this is 1 by a this. And expectation i would be, because this is i is equal to 1 with probability x greater than or equal to a.

So, expectation would be 1 into probability x greater than or equal to a plus x equal to i equal to 0 into probability that x is less than a, but since i is 0, so that is no contribution. So, expectation i is actually equal to probability x greater or equal to a into 1. And where did we use, the fact that it’s non negative variable. I used it from here, saying that if I is less than or equal to x by a. then the expectation will also, when I take the expectation of both sides, the inequality will remain intact. And therefore, this is less than or equal to 1 by a E x, or probability x greater than or equal to a is less than or equal to 1 upon a into expected x. So, you see if for this event, just knowing the expectation of the random variable, I can compute a bound for the probability of this event, which is given by 1 by a into E x.

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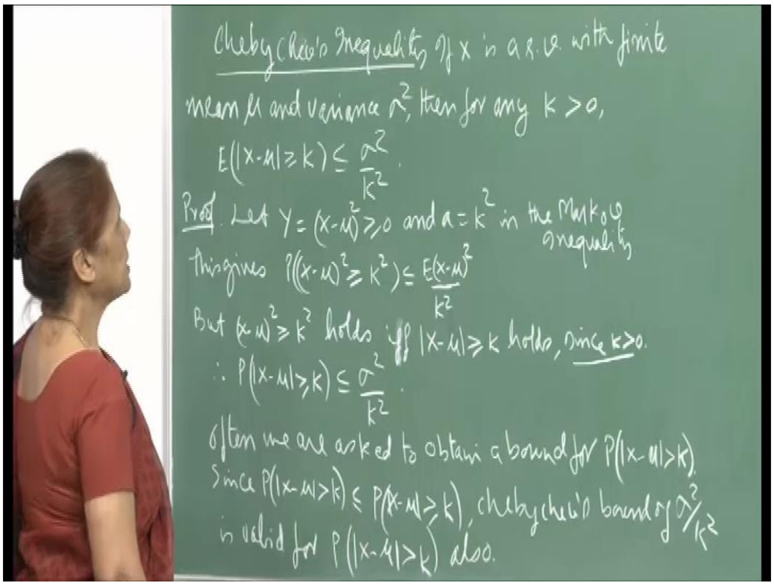

Let us look at the Chebyschev's inequality, which says that if x is the random variable, with finite mean u and variance sigma square. Then for any k positive, expectation of x minus mu, absolute value greater or equal to k. it should not be expect probability. Chebyschev's inequality is for giving an upper bound on a certain probability, which is probability absolute value of x minus mu greater than or equal to k, is less than or equal to sigma square upon k square. So, mean excess of mu and its mean. So, when you take. So, actually this can also be written as expectation of x minus mu whole square and then divided by k square. So, that is the variance. And of course, I do not need the absolute sign once I have taking square.

So, it is this is the actually expectation of, whatever the function here. So, you were taking x minus mu. So, expectation of this whole square divided by k square. So, we are defining this for mu as the mean of x and k some positive number. So, this is the inequality. So; that means, this gives you a bound. The inequality gives a bound on this probability, which has to less than or equal to sigma square by k square. Now we will use Markov inequality to probe this, we just this Markov inequality. So, now, let me define y as x minus mu whole square, and this will be therefore non negative, and the Markov inequality requires, is define for, is valid for random variable which takes only non negative values, which is greater than or equal to 0, and will take a to be k square.

So, then the Markov inequality will give us, that probability x minus mu whole square, greater than or equal to k square, is less than or equal to expectation of x minus mu whole square by k square, expectation of this divided by k square; E x. So, this is E x by a, where this is a and this is your x. So, then I am writing expectation my x is x minus mu whole square. So, expectation of x minus mu whole square by k square. So, by markov inequality this is this and then you are saying because my a k square and my x is x minus mu whole square. Now, just see that this event x minus mu whole square greater than equal to k square, holds if and only if this holds. Since, k is give taken to be positive, so if this is true than this is true, if this is true then this must be true. So, if the other same events and therefore, I can replace this probability by the probability, absolute value of x minus mu greater or equal to k. This is less than or equal to sigma square by k square. So, the inequality is established, and then we will through examples and so, we will the various ways in which this simple inequality can be used.

Now, the thing another, just want to make a note here that, often we are asked to obtain bound for a probability like this, which has a strict inequality. So, this is probability of x minus mu absolute value of x minus mu greater than k. But then we know that this probability is less than or equal to probability of absolute value of x minus mu greater than or equal to k, because you are taking a bigger subset here. So, this probability is bigger than this probability, and hence and since we have a bound for this, through Chebyschev's inequality.

So, the bound is also valid for this probability. So, that mean we can say that probability mod x minus mu greater than k, is also less than or equal to sigma square by k square, this is a whole idea. And through when we discuss many examples, it will turn out that we have to actually compute, this bound for this probability. So, we will see that, this can be again estimated by, not I should not use the estimate because this probability is less than or equal to this, and Chebyschev's inequality gives as a bound for this. So, therefore, the same bound is valid for this probability also.

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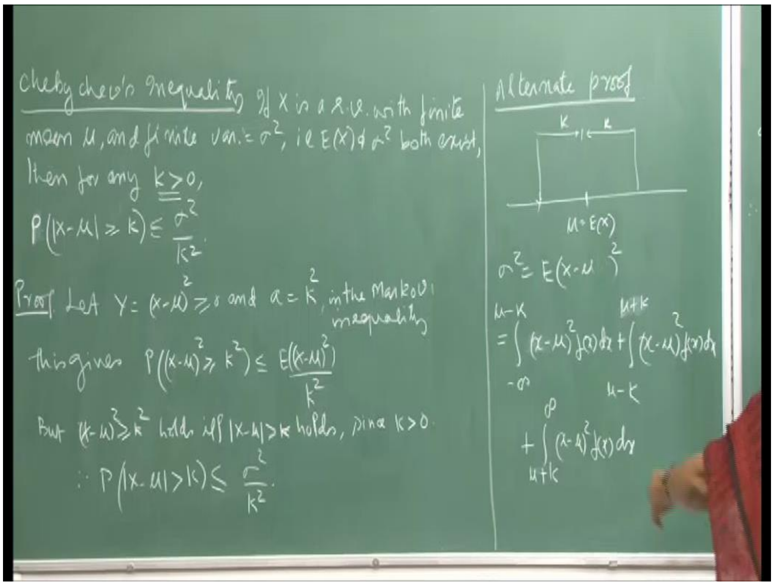

Now, one can also obtain, I mean 1 can there are more than 1 ways of obtaining these inequalities. So, I will just give you the alternate proof, which says that if and. So, I will start with the expression for sigma square which is, expectation of x minus mu whole square. So, you are taking k on either side. So, now, I can and this integral will be minus infinity to infinity of x minus mu whole square f x d x. So, this integral I break up into. So, minus infinity to mu minus k; that means, this point, and then it will be mu minus k 2 mu plus k and then mu plus k 2 infinity. So, I break up the integral. So, the total integral minus infinity to infinity, I break it up to these three.

Now, if you look at this expression for example, here x is greater than mu plus k. It is going up to infinity, and remember k is positive number. So, x greater than mu plus k implies that your x minus mu is greater than k. So, it actually this is implied. So, if this wherever x is greater than mu plus k, it means at x minus is greater than k, and similarly here your x is less than mu minus k. So, for x less than mu minus k, it implies that your x minus mu is less than minus k. And this is a, because again this is, you know x minus mu whole square you are integrating x minus mu whole square f x d x from mu minus k 2 mu plus k. So, this must be a non negative quantity. It can or be negative, because this is non-negative, this is none negative. So, therefore, the equality if I remove this, then the inequality changes to inequality.

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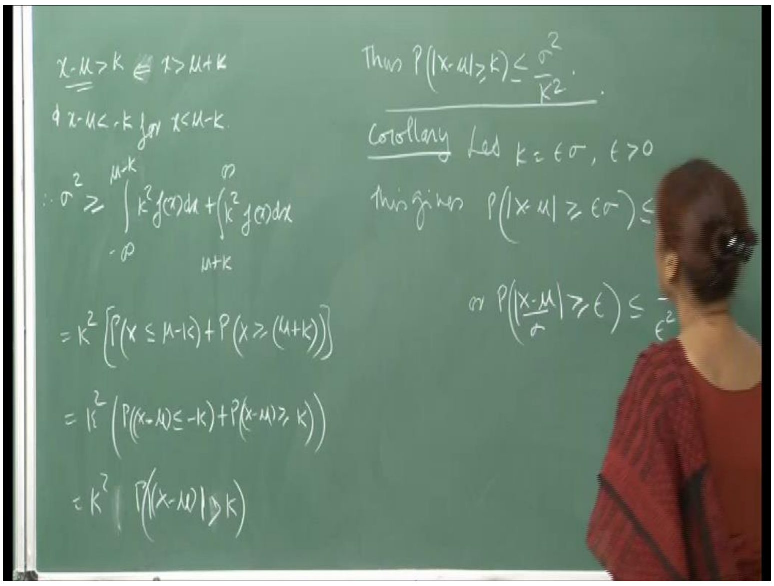

Secondly, as we said that in this interval mu plus k 2 infinity, x minus mu is greater than k. So, here again f x is a non negative function. So, if I replace this by k square then I am taking a lower value of the whole integral. So, again the inequality gets strengthened, and similarly here; your x minus mu is less than k in this interval also minus infinity to mu minus k, your x minus mu is less than minus k. So, square would be, what will happen to square x minus mu whole square will become greater than k square. In this interval again, because x minus mu is less than minus k. So, when I square up x minus mu whole square, it will greater than k square. So, then in both these integrals, I replace x minus mu whole square by k square, I am taking the lower value, underestimate of the integral, and therefore, sigma square is greater than or equal to k square of f x d x, this is minus infinity to mu minus k plus integral mu plus k 2 infinity k square f x d x.

But, this is nothing but k square of is a constant. So, this is probability x less than or equal mu minus k plus this is probability x greater than or equal mu plus k mu plus k 2 infinity, but then if you bring mu to this side, this will be probability x minus mu less than or equal to minus k. Here this will be probability x minus mu greater than equal to k. So, probability of absolute value of x minus mu greater than or equal to k. So, what is written is this strict inequality, but it should actually be, probability of mod x minus mu greater than or equal to k. So, we have the inequality sigma square greater than or equal to k square probability of absolute value of x minus mu greater than k, and so that gives us the inequality, if you wanted to prove. So, this is Chebyschev's inequality. This again gives you a upper bound on the probability mod x minus mu greater or equal to k, once we know the variance x.

So, this proof can be imitated for the discrete case also; that is when x is a discrete random variable with finite variance, and there will be a probability mass function also defines with it, and immediate corollary is that, if you put k equal to epsilon sigma, where of course, epsilon sub non negative number. Then the Chebyschev's inequality becomes greater or equal to epsilon sigma; this is less than or equal sigma square upon epsilon sigma square which is 1 by epsilon square, and if I divide here by sigma, sigma being a non positive number. So, inequality does not change, and probability x minus mu by sigma. So, this is, you can say standardized variable, you standardize variable x. So, this greater than or equal to epsilon, is less than or equal to 1 upon epsilon square; simpler version of the Chebyschev's inequality. So, we would like to now work out some examples, to see how these bounds can be used.

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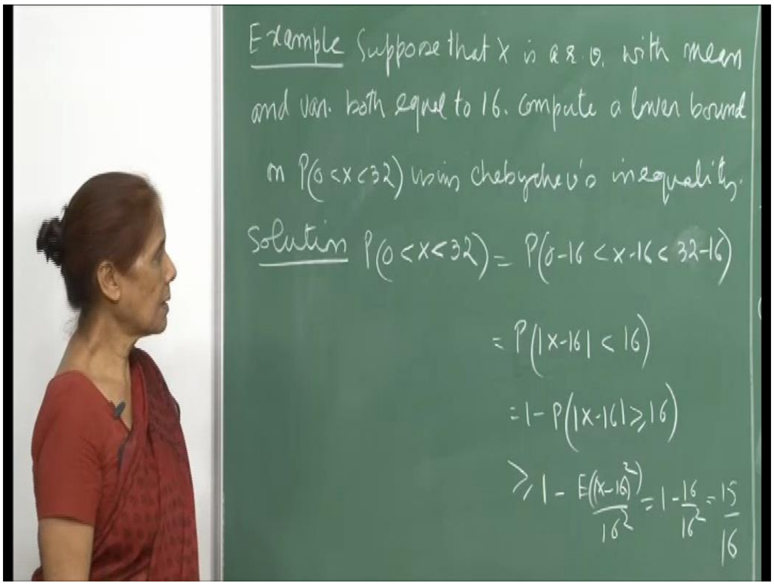

So, let us consider a few examples on these inequalities that we have just discussed. This example says that is a random variable, with mean and variance, both equal to 16. So, compute a lower bound on probability that x lies between 0 and 32, using Chebyschev's inequality, because here we cannot use Markov inequality. It is a 2 sided thing. and yes and again I am fine ya. So, therefore, we will use the Chebyschev's inequality here. The solution says that you convert this probability to probability of 0 minus 6. So, just subtract 16 from both the sides. So, it will be probability 0 minus 16 less than x minus 16 which is less than 32 minus 16. So, the 2 events are the same, and then this reduces to probability of absolute x minus 16 is less than 16. This is minus 16 and this is 16, so absolute. So, now, this is in the form of. Well this is the opposite of the, compute the lower bound for greater. So, now here I will have to write this is 1 minus probability absolute x minus 16 greater than or equal to 16. We had strict inequality here. So, this was strict, and therefore the opposite event would be. The compliment event would be absolute x minus 16 greater than or equal to 16. Since Chebyschev's inequality gives an upper bound. So, when I replace this by its upper bound minus of that will become. So, the equality here will change to greater kind.

Because I am writing; for this I am writing a bigger number, but the minus will become a smaller number, and therefore, this probability will be greater than or equal to 1 minus expectation of x minus 16 whole square divided by 16 square, but this we know is the variance of x which is 16. So, therefore, this is 1 minus 16 upon 16 square, and so this is 15 by 16, and you can see, that is a fairly loose bound. So, 15 by 16 is a number close to 1. Yes and there will be, but anyways. So, I am trying to show you that, these bounds that you get are rather loose. They are not very tight, but some situation they are quite helpful also. Now another example is, from fast from past experience; a professor knows, that the test score of a student, taking her final exam, is a random variable with mean 65; that is not bad. if the mean is 65 out of 100 then students are good. So, let us see, given upper bound for the probability that the students test score will exceed 85. So, you need a upper bound for the probability. So, therefore, we just know that the mean is 65, and that is it, and using that we compute.

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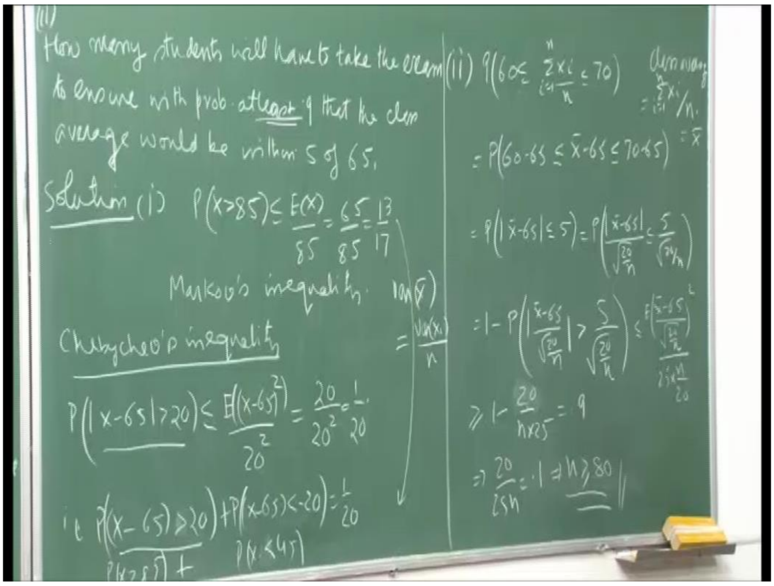

So, here you will to answer this, you will simply use Markov inequality, and that will give you probability x greater than 85. So, your a is 85. So, this will be less than or equal to expectation x upon 85, which will be 65 upon 85, so 13 by 7. Now, Markov inequality says, that probability x greater or equal to a, is less than or equal to expected x by a, but we can also use this Markov inequality for computing an upper bound, for the event x, for the probability of the event x greater than a. Since probability x greater than a, is less than or equal to probability x greater than or equal to a. So, therefore, probability x greater than a will be less than or equal to expected value of x divided by a. So, therefore, when to compute the upper bound for the probability x greater than 85, I could use the number E x divided by 85, and this is what we will use for future computation also that follows. So, this is the answer that you get from Markov inequality.

Now, if there is a additional information, that the professor knows that the variance of a student’s test score is equal to 20. So, if you have knowledge of the variance, then you can use Chebyschev's inequality, and then you will say that probability absolute x minus 65, is greater than 20. So, this probability will be less than or equal to expectation of x minus 65 whole square upon 20 square, because this is your k. So, that is 20 square, and therefore, since we know that the variance of x is, x obviously, is the test score. So, then that is 20, so 20 upon 20 square this is 1 by 20. Of course, the number are little contrived, does not matter. See this event, is actually probability x minus 65 greater than 20. Either this is greater than 20, or this is less than minus 20. So, and you were looking for this probability, but in any case, this is probability x minus 6 x is less than 45. So, I could write plus here, because the 2 events are disjoint. So, in other words if you want a bound for probability x minus 65 x x yeah this. This will, your probability x greater than 85 plus probability x less than 45, which is 1 by 20.

So, in other words, your probability x greater than 85, this is less than or equal to 1 by 20, because this will be something positive. So, you will subtract from 1 by 20. So, therefore, this bound is definitely, I mean there is a dramatic difference between 13 by 7 and the number which is 1 by 20. So, you see the moment you have more information about the distribution, about the random variable, you can get better bounds for the the bounds are tighter. So, this was just, I think I just sat down and contrive these numbers and though, so they may not be look realistic, because 20 is a probably high number for the variance. So, therefore, there is a dramatic change, because you see this this had no knowledge of the variance. So, I just computed this. So, this number we computed knowing the expected value. So, if here the variance is much smaller, then; obviously, this probability will also be, this bound will be higher.

So, this is just to show you that the difference between the two bounds. Now the second part of the problem is, that how many students will have to take the exam, to ensure with probability at least 0.9, that the class average would be within 5 of 65. So, how many students. So suppose, we assume that there are n students, were taking the exam, and then the class average would be given by. So, class average would be a summation x i, i varying from 1 to n divided by n, which in our notation we can also write is x bar. So, that will be your class average. So, what they saying is, that the class should be within 5 of 65. So, it is either 5 less than 65, or a 5 more than 65.

So, therefore, this class have average should be within 60 and 70. So, this is the probability that you have to. You are told that this probability should be at least 0.9, and then you want to know how many students should take the exam, so that this probability is at least 0.9; that means, its greater than or equal to 0.9. So, here again I am trying to standardize, or cut it in the form of the Chebyschev's inequality. So, x bar minus 65, because mu of x bar is, the expected value of x bar is also 65, since each exercise. So, therefore, the 60 minus 65 less than or equal to x bar minus 65, less than or equal to 70 minus 65, and then I can also here divide by. So, what is the variance of. Here variance x bar will be variance of, any of the x i the same divided by n.

So, because variance x i divided by n square, so this becomes this, so under root of this. So, that will be under root of 20 by n. So, if I divide both sides by under root of 20 by n, the event does not change. So, this is what I have now, and this is 1 minus probability x bar minus 65 upon under root 20 by n. So, this gives me; that mean this is my, I mean I am applying Chebyschev's inequality here; this is greater than 5 by under root 20 by n. So, this would be again, because I am writing 1 minus of this. So, if you have less here; that means, this is less than some number by Chebyschev's inequality. So, minus that will become greater, and so this is 20 by n into 25, because what is the; have I written it correctly here.

So, you want to Chebyschev's inequality this will be what. This is this number, if I just write this number, this is less than or equal to expectation of x bar minus 65 upon 20 by n this whole square divided by, you can say 25 n by 20, but this has variance 1. So, actually it should be yes. So, see this has variance 1, because I have standardize it. So, expectation x bar minus 65 upon under root 20 by n whole square has variance 1. So, the number is 20 upon n 25. So, this should be, and this is greater than or equal to. So, my probability and therefore, if I put this equal to 0.9, then my at least part is satisfied. So, the value of n that gives me this quantity equal to 0.9 will satisfy, because the probability that I have written on the left hand side, is greater than or equal to this.

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$$\frac{20}{25n} = 0.1 \rightarrow n = 8 \text{ (and not 80 as written).}$$

$\therefore$ For $n \geq 8$, the class average will be within 5 of 65.

20 upon 25 n equal to 0.1 implies n is equal to 8 and not 80 as written. Therefore, for n greater than or equal to 8, the class average will be within 5 or 65. So, you know lots of different kinds of probabilities you can obtain via, you know using the Chebyschev's inequality you can get the bound, you can get estimates of the numbers and so on.

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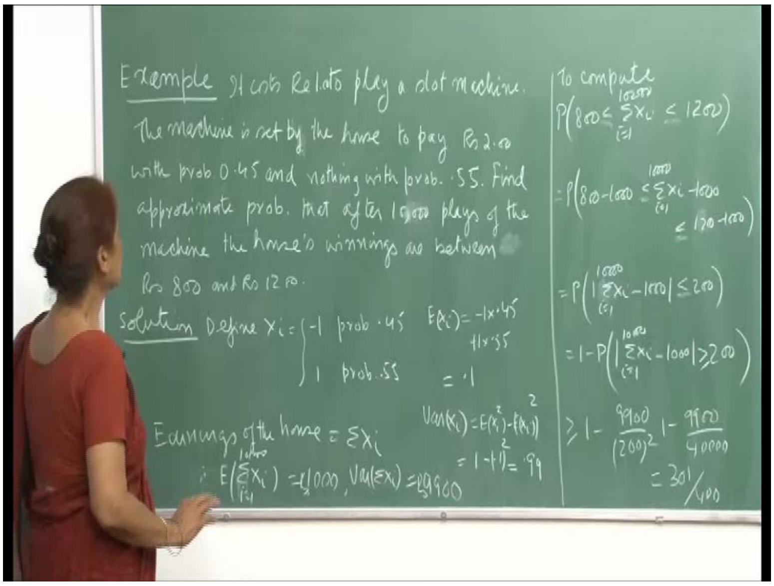

So, let us consider this example; it says that, it cost rupees 1 to play a slot machine. See now some of you have an idea that you put in 1 rupee, and then if you are lucky, some money comes out, otherwise nothing comes out. So, the machine is set by the house. So, wherever your this slot machine is put, the machine is set to pay rupees 2, with probability 0.4 5 and nothing with probability 0.55. So, you see you put in a rupee and then with probability 0.45, you expect to get 2 rupees. Otherwise with the probability 0.55 you do not expect to get anything. So, you lose that rupee fine. So, fine a proximate probability that after 10,000 plays of the machine, the houses winnings are between rupees 800 and rupees 1200.

So, you see a winning means that when the house has to pay, to the player then its loosing. Since when the player puts in 1 rupee, and the house has to pay 2 rupees, then that is a loss for the house. So, therefore, x I is equal minus 1 is with probability 0.45, and x i is equal to 1 probability point. So, this represents the earning of the machine, in the i th play of the game, when the some slot machine is being played for the i th time. So, x i is equal to minus 1 with probability 0.45, and this is 1 with probability 0.55. And of course, you can see that E x i, the expected value of x i will be minus 1 into 0.45 plus 1 into 55. So, this is 0.1 as you expect, because otherwise why would people or house want to invest money in a slot machine, if the expected earning is not positive. So, this is 0.1 per game of the slot machine.

And similarly, the variance x i would be 0.99, because it will be expectation x i square. So, once you put x i square, then this will become plus 1 into 0.45 plus 1 into 0. 55 which will be 1. So, expectation x i square is 1, and then expectation x i whole square will be 0.1 square, so this becomes 0.99. So, the variance of each x i is 0.99, and earnings of the house are represented, total earnings are 1 to 10,000.

So, sigma x i summation from 1 to 10,000 gives you the earning of the house, in which you know losses and incomes are included. The net net income the net earnings of the house is sigma x i i varying from 1 to 10,000 and. So, expected value of the earnings is rupees 1000, because this is 0.1 multiplied by 10,000 which gives you rupees 1000, and the variance sigma x i is 9900. Now I have not standardized this here. It is ok to carry on the computations with these values. So now we have to compute the probability, that total earnings of the house lie between rupees 800 and 1200. So, here again we will do the same thing, we will try to standardize this probability, and I will subtract the expected value of sigma x i which is 1000 rupees on either side and so that gives me that absolute of sigma x i i varying from 1 to 10,000 minus 1000 is less than 200.

So, again we have it in the right form, in the sense that. Now I will write this as 1 minus probability sigma x i i varying from 1 to 10,000 minus 1000 should be greater than or equal to 2. So, it saying between 800 and 1200, so I am taking strict inequality here. So, therefore, the compliment event will have greater or equal to 200. So, I have set it right for the use of Chebyschev's inequality. So, again the same reasoning, that this thing is less than I get a upper bounds, so therefore, minus of that will give me the lower bound, so minus of that. So, therefore, this will be, this is less than or equal to variance of sigma x i divided by 200 square. So, variance of sigma x i is, remember this is 9900. So, therefore, this is greater than or equal to 1 minus 9900 upon 200 square, which is equal to 1 minus 9900 upon 40,000, so this becomes this. So, again; that means, this close to 0.75, the probability. So, you would expect that, because 10,000 place, and you know the machine the expected value from each play of the machine is point 1, expected earnings. So, therefore, this is probably not a very bad bound. So, at least. So, here of course, the probability is at least 301 upon 400, it can be more. So, it is understood that the Chebyschev's bound are not tight, but it gives you an idea. It gives you a feeling about the probability of the event that you are trying to estimate.

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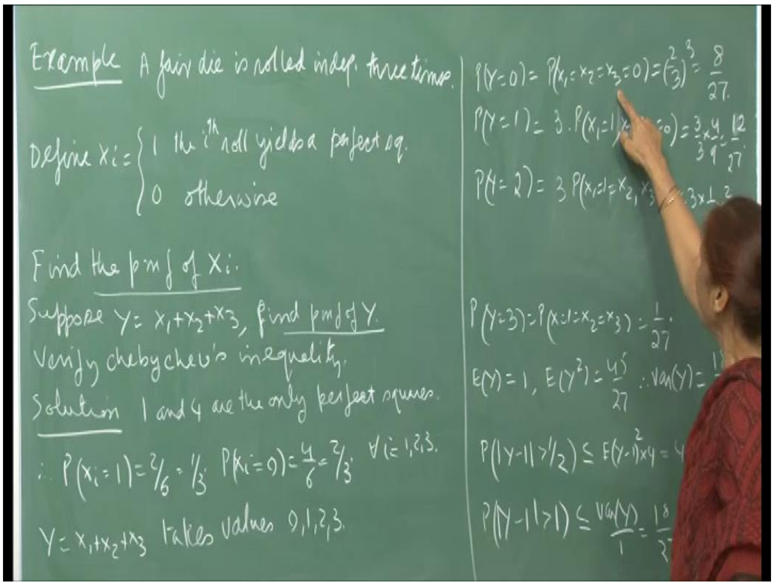

Another example, because I feel that the there are so many different situation, where you have to learn to how to compute, how you can apply Chebyschev's inequality, and therefore, I just have collected the lot of a different examples. So, let us look at another example here, which says that a fair die is rolled, and what you mean by independently 3 times, which means that there is not bias, so the outcome are independent. So, the outcomes of the dies roll 3 times, whatever the outcomes are independent outcomes. Now you define x i as 1 the i th roll yields a perfect square.

So, x i denotes the random variable, which is equal to 1 if the i th roll gives me a perfect square and 0 otherwise. So, first find the P m f of x i, and I need to do all this work before I apply the Chebyschev's inequality. So, the P m f of the x i, and then of y is equal to x 1 plus x 2 plus x 3, then you have to find the P m f of y, and then verify Chebyschev's inequality. So, we will compute the actual probability, and then also get the bound by the Chebyschev's inequality and compare. so; obviously, we expect that, since this gives us a upper bound so the actual probability that we compute, will be less than what we get by the Chebyschev's inequality this is whole idea, and I thought that we can if we you work out in detail, you will get a good feeling about a whole thing.

So, anyway. So, let us see the solution procedure. Now 1 and 4 are the only perfect squares, in the sixth numbers that come up when a die is rolled. So, 1 and 4 are the only perfect squares, and therefore, probability x i equal to 1 will be 2 by 6, which is 1 by 3 and probability x i equal to 0, will be 4 by 6 which is equal to 2 by 3 for all i, and i' s are all. So, and x i are independent. So, now, you take y to be x 1 plus x 2 plus x 3, and y takes values 0 1 2 and 3, because none of the die show a perfect square. So, then this values 0, one of them shows a perfect square, two of them show a perfect square, or all three show a perfect square. So, the values possible values of y are 0 1 2 and 3, and not difficult to compute the P m f of y, because y equal to 0 means that all 3 variables take 0 value, and since they are independent. Therefore, it will be product of probability x 1 equal to 0 x 2 equal to 0 and so, that x 3 equal to 0. So, it will be 2 by 3 square, which is a cube, which is 8 by 27.

Similarly, probability y equal to 1, will be. see now here, I just take this case that x 1 is 1 and x 2 and x 3 are 0, then they can be 3 such combinations; where 1 of them is 1 and the other 2 was zero. So, this will be 3 times, this particular probability, which is again using independents x 1 equal to 1 is 1 by 3. So, 3 times 1 by 3 into this is 2 by 3 raise to 2, which is 4 by 9. So, the probability 12 by 27, and y equal to 2 again the same case, that if I take this particular event x equal to 1 equal to x 2 and x 3 0, then again the 3 combinations which will give me the same value of y which is equal to 2. So, three times probability x 1 equal to 1 equal to x 2 and x 3 equal to 0. So, this will give me 3 into 1 by 9 into 2 by 3, which is 6 by 27, and probability y equal to 3 requires that all 3 must be equal to 1, and that probability will be 1 by 27. Now you can just make sure that, you have computed the right P m f, by adding up all these probabilities. So, 7 plus 6 13 13 plus 12. This is 1 and 6 7, 7 and 12 19 19 and 8 27. So, all these probability add up to 1. So, this is a right P m f.

Now, therefore, we can immediately compute expected value of y, which is 1. I mean I leave that to you, because now you have the probabilities, multiply by the corresponding value of y, and you add up and get this this. Similarly expected value of y square is 45 by 27. So, the variance comes out to be 18 by 27. So, now for example, probability y minus 1 greater than half, if you want an estimate for this upper bound, then this is less than or equal to expected value of y 1 whole square y minus 1 whole square into 4 1 by 2 square, the denominator will become 4. Now, this certainly there is no verification is needed, because the actual probability cannot be more than 1, whereas, this number coming out to be more than 1, because you have put a number 1 by 2 here. Are you with this, because 4 into 87 by 27, this number is greater than 1. So, here I do not need any verification. So, more meaningful verification will be; for example, I want to say that probability y minus 1 absolute value greater than 1, is less than or equal to. So, this will be variance y upon 1 which is 18 by 27. So, now we will actually compute this probability, and show that it is less than 18 by 27, to make sure that this actually gives you an upper bound.

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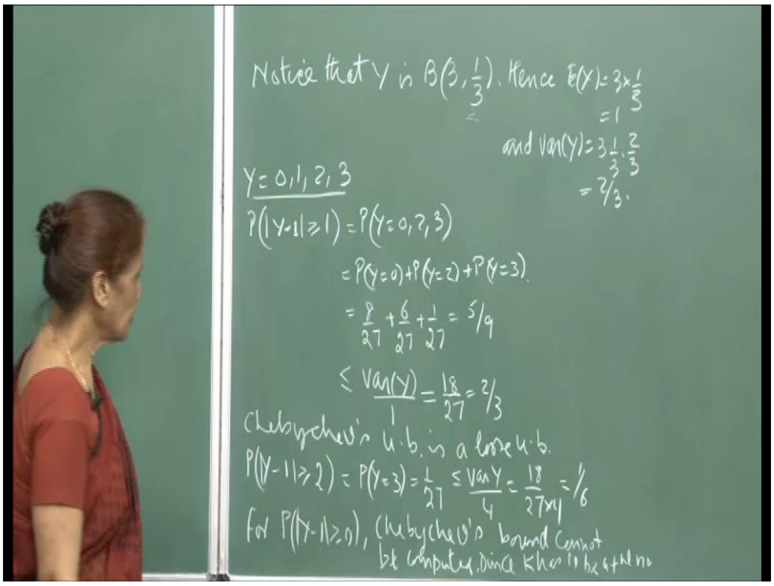

If you observe that b is y is also distributed has a binomial distribution with the parameters 3, 1, 3. So, the probability of success is given by 1 by 3, and number of rolls is 3. And therefore, immediately you know that hence expected value of y will be n p. So, 3 into 1 by 3 is 1, and the variance of y will be 3 into 1 by 3 into 2 by 3 and P q which is 2 by 3, but we computed this independently as 18 by 27 which is also 2 by 3. Now just want to we are we are testing the, we are comparing the actual computation of probabilities with the Chebyschev's bound. So, if you want to look at this probability, absolute value of y minus 1 greater than or equal to 1. So, you see that here; y minus 1 greater than or equal to 1, implies that y can take the value 0 2 and 3, because y is 0 then absolute value of minus 1 is 1 which is equal to 1. So, remember the event, is greater than or equal to. Therefore, y can be equal to 0 2 and 3, and in that case this probability will be equal to probability y equal to 0 plus probability y equal to 2 plus probability y equal to 3.

So, you make this computation 8 by 27 plus 6 by 27 plus 1 by 27; which is equal to 5 by 9, but if you compute the Chebyschev's bound, then this will be variance y upon 1 1 square is 1, and this is variance y which is 18 by 27 or 2 by 3. So, you see when you compare these two numbers, you will say that Chebyschev's bound is a loose upper bound, because 5 by 9 is less than 2 by 3; say 15 is less than 18, when you compare by 9 is less than 2 by 3. And then let us see take another event. So, this will be probability absolute value of y minus 1 greater than or equal to 2, which is simply probability y equal to 3. So, we can use the binomial probabilities here and. So, y equal to 3 would be simply 1 upon 27, 1 by 3 raise to 3. So, this by the Chebyschev's inequality will be variance y upon 4, because your k is 2. So, this 18 by 27 into 1 by 4, which comes out to be 1 by 6. So, when you compare 1 by 27 with 1 by 6, the gap widens. This is loose upper bound, by the Chebyschev's inequality.

And now if you want to look at the event probability absolute value of y minus 1 greater than or equal to 0, then we cannot apply the Chebyschev's bound, because this number has to be positive; remember k greater than 0, so that is required. So, therefore, I cannot compute a bound for this. So, you may compute the actual probability here, but Chebyschev's bound cannot be computed. What I have tried to show you, is of course, through various examples how to get the probability required to compute the lower bound by Chebyschev's inequality. So, doing that, and then also try to give you feeling, that the bound we are computing are, loose bounds they are not tight ones. And thirdly, now what I would like to show you in the next lecture is that, apart from computing bounds, it has also proved a very useful tool for showing. We will be talking of conversion theorems in the next lecture, and that is where I will show you, how useful a tool the Chebyschev's inequality is. So, that will be the next.