Introduction to Probability Theory and its Applications
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Lecture - 12
Jointly Distributed Random Variables Independent R.V. and Their Sums

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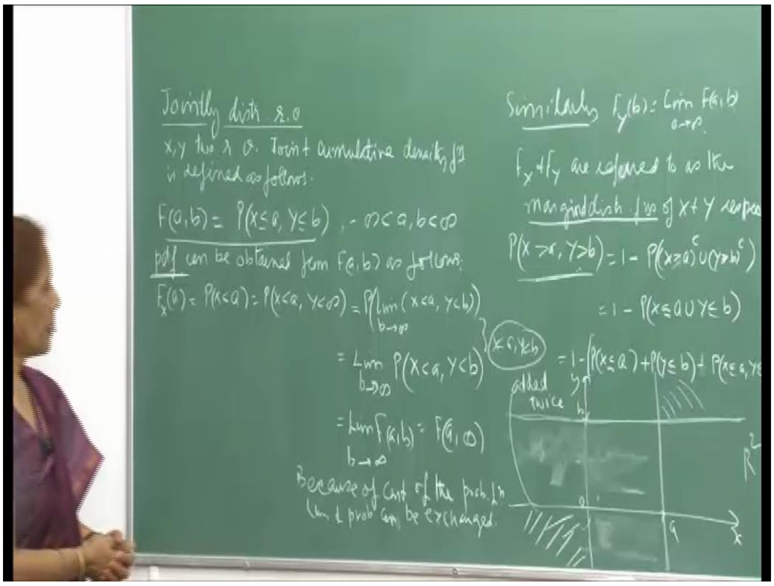

Now, we going to talk about Jointly Distributed Random Variables. See they may be events, which need to be describe by more than one random variable, so they may be the you know random phenomena concerned with the event or more than one dimension. And therefore we need to be able to of course, then we from two dimensions we go to multi dimensions also, but right now to keep it simple, we will first talk about two dimensional random variables and then may be a extend the notion two, more than two.

So, if x and y are two random variables, we define their joint cumulative distribution function as follows, so probability F of x less than or equal to a comma y less than or equal to b is actually the probability that x is less than or equal to a and y less than or equal to b, a and b two real numbers between minus infinity and infinity. So, essentially in this diagram, if this is b, then you are asking for y less than or equal to b and then x less than or, so it will be this whole region, if you can see this line and this line, so whole of r 2 extending from here to this end that will be a, so we are talking of the probability over this region. Then the moment did you find the joint CDF, a Cumulative Distribution

Function, then you can talk of the marginal CDF of a, so example for example, marginal CDF of x can be obtained from F (a, b) as follows, sorry this should be (a, b).

The probability part comes here x less than or equal to a, y less than or equal to b, so therefore, from F (a, b) we can obtain the marginal CDF of x as follows, so this is the F x (a), which is probability x less than or equal to a. So, this will be actually therefore, that means, this means that y is allowed to take any possible value, so therefore y probability x less than or equal to a and y less than infinity. So, now let me just define it in a proper way in the sense that, this is also the same as b going to infinity.

So, limit that means, if I take the event x less than or equal to a, y less than or equal to b and then I take the limit, so you see what will happen as b goes to infinity, so x less than or equal to a, y less than or equal to b, you are talking of this, then when say b 1. When you talk of this y less than or equal to b 2, where b 2 is greater than b 1, then this is a bigger event. this event is contained. So, therefore, now you have a sequence of these events which are increasing, so increasing sequence of events b is goes to infinity.

And if you remember my definition of probability as a continuous function, continuous set function, so then in that case I told you that we can in that case extended, when you have a increasing sequence of sets, then when you are computing the probability you want to take the limit, then the probability of the limit is the limit of the probability. So, therefore, I can take limit outside and this will be limit is b goes to infinity of the this event x less than or equal to a, y less than or equal to b.

So, we take the probability here and therefore, this will be limit and this is now your F (a, b) by definition, so this limit b going to infinity of F (a, b) and this will be then F (a, infinity), so F a and F (a, infinity) are the same. Similarly, if you want to compute the marginal of y, then will take the limit F (a, b) a go to infinity which will be F (infinity, b). So, exactly in the same way will argue out that the b remains the same and then a keeps increasing, so again you have increasing sequence of sets and so when you take the probability I can exchange the limit and the probability and get the answer.

So, now similarly you all to compute and then of course, you see the properties where we have defined for a cumulative distribution function must have for a single variable. So, the same will apply and you can apply them to F x and F y, so through those you can get the property for the joint. Because of property that F x and F y possess combine them, when you can, then put together the properties that you are joint cumulative distribution function must have.

Now, suppose you want to compute the probability of x greater than a and y greater b, so this we can write as 1 minus probability of the compliment of this event, which is x greater than a, y greater than b a compliment and then, since x and y are independent, in the sense that I can write this as. Now, this can be written as x less than or equal a union, y less than or equal to b, if you recall your De Morgan's law as so on. So, then this will be 1 minus probability x less than or equal to a minus probability y less than or to b and then, you add probability x less than or equal to a, y less than or equal to p.

And this diagrammatically also you can immediately see that, this how from here we have to get here, because you are computing the probability x less than or equal to a and y less than or equal to b. So, this will be you see x less than or equal to a would give you ((Refer Time: 06:23)) this region, and y less than or equal to b will give you this region. So, you see the region extending from here x less than a, y less than b is this region which you are have subtract it twice, because once when you do it for x less than a, probability x less than or equal to a.

So, it is this and then this region is coming into it, then when you subtract probability y less than or equal to b then again this whole region is coming and therefore, you add this again, so in words of your I should have written this that means, your. So, therefore, I simply write this probability in terms of 1 minus F (x a) minus F (y b), so this will be equal to 1 minus F (x a) minus F (y b) plus F (a b). So, therefore, to make the equation correct I add this once and then, because I need to subtract only c I need this region, so the valid region that I require is this.

And therefore, from 1 I subtract this whole and so this got subtracted twice, therefore I add it once to make it proper, so this will be your... That means, now one can compute whatever probabilities you want related to these two random variables, it can be done once we have made this definition.

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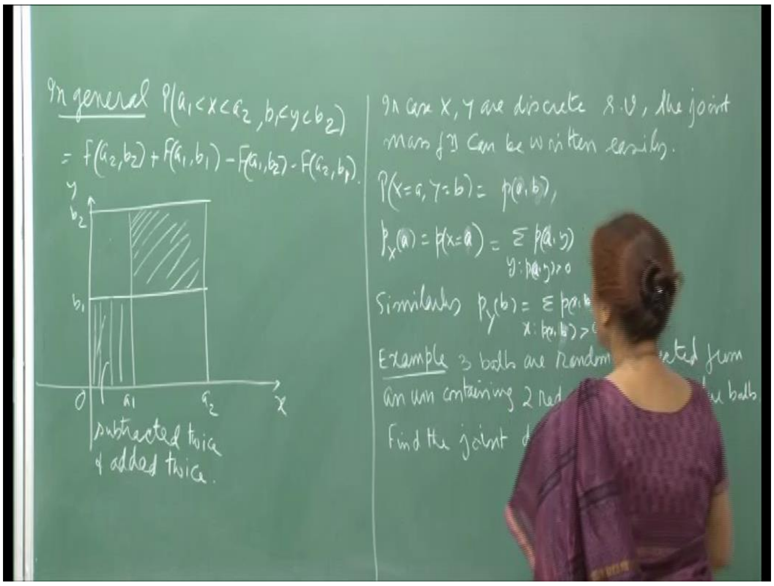

Now, in general if x is lying in the interval a 1, a 2 and y in the interval b 1, b 2, then this can be written as F (a 2, b 2) plus F (a 1, b 1) minus F (a 1, b 2) minus F (a 2, b 1), so here again we will just simply diagrammatically look at the equality that we have here. So, x is varying between a 1 and a 2 and y is varying between b 1 and b 2, then F a 2, b 2 is this whole area, in fact if it is only non negative variable then it is this region. So, just assume for argument sake that this is both the variables are non negative, but otherwise it will extend to infinity and this will full extend to infinity.

So, anyway because this is F a 2, b 2 is probability x less than a 2 and y less than b 2, so in the case of non negative variables this is the total region, then F (a 1, b 1) is this ((Refer Time: 09:12)) region, let me this here. Then F (a 1, b 2) is this whole thing and F (a 2, b 1) is this region here, so you see that here you are subtracting this region twice and in this a 2, b 2 you are adding it and then you are adding this, so that gets cancelled out and this region also gets cancelled out. So, from the whole of F (a 2, b 2) you are finally, left with this particular region, this is the idea I am equating the probability with the area in a sense and that is what I am trying to explain. So, if you this picture in mind and you can always make the right competition.

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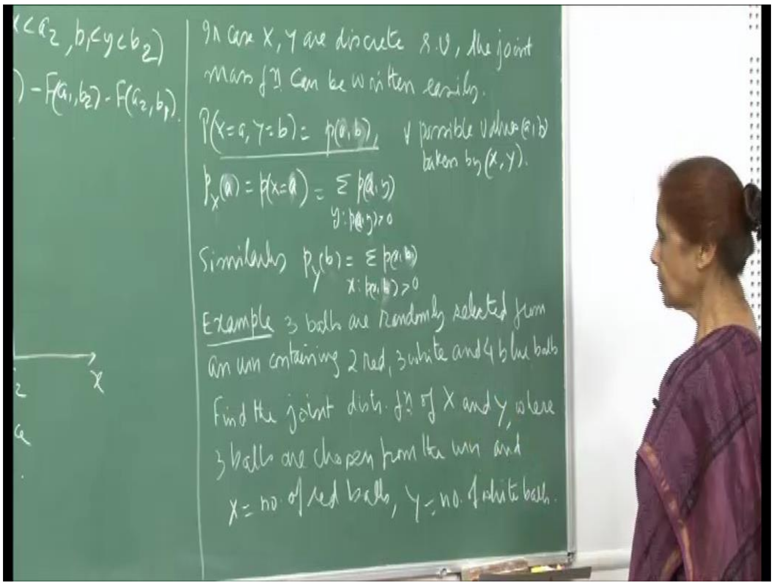

Now, in case x and y are discrete random variables, they joint mass function can be written easily. because here you are simply wanting to compute these probabilities. probability x equal to a y equal to b is p (a, b) this is how we will define. So, if we can compute this probability, then that is the probability a, b and this for all possible values, if the appears a, b taken by x, y. And then, we can also compute the marginal probability mass function of x, which will be simply probability x equal to a and this will be your summing these probabilities over, so fix the x at a and then summing it over all possible values of y.

So, your summing up p (a, y) for all possible values of y that will give you the probability that x attains the value a, because here the value poison is not is a material, therefore you submit of over all possible values that y will take, similarly p y (b) will be summing up this probabilities x, b when p x (b) is positive. So, in this case you want to sum up overall possible values of x to get the probability that y will take the value b, and we will go through an example here. So, let us consider this 3 balls are randomly selected from an earn containing 2 red, 3 white, 4 blue balls, so the total number of balls is 9.

And now you want to find the joint distribution function of x and y, when where 3 balls are chosen from the earn and x represents number of red balls and y number of... So, I pick up the 3 balls from the earn, then I note the number of red balls present in those 3 balls earn and the number of white balls, so now we want to write down the joint mass distribution function for x and y.

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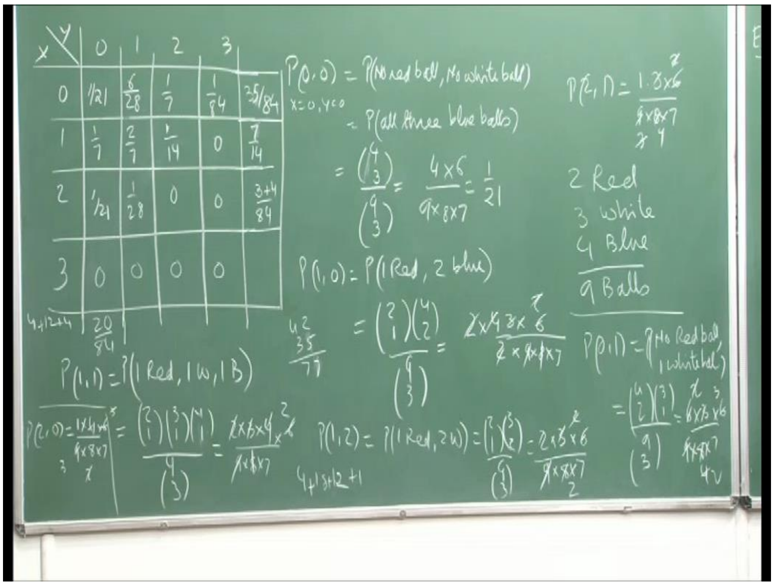

So, let us now continue with the example with which they were the earn contains 2 red balls, 3 white and 4 blue, total number of balls is 9. So you want to compute the cumulative mass function for the variables x, y, where x is the number of red balls and y is the number of white balls, when you pick up 3 balls from the earn. So, I will just compute a few words, then you can try to complete the table I think maybe I have completed it already.

So, when you want to compute the probability of (0, 0) that means, x is equal to 0 and y 0, so which is no red ball and no white ball, so that means all the 3 balls that you pick up from the earn must be blue. So, the probability of all 3 balls being blue is 4 c 3 and total number of ways in which you can pick up 3 balls from 9 balls is 9 choose 3 and so this number comes out to be 1 upon 21, which is here; then I should have computed p (0, 1) and I think I have written it here as, so it is this number is actually, so it is got mixed up.

So, this is 1 by 7, so that is probability 1, 0 is 1 by 7, so if you want to compute have I done it somewhere here, no so therefore let us just quickly compute this number p (0, 1). So, (0, 1) is a probability no red ball and 1 white ball, so this will be no red ball that means, 1 pi white balls in 2 blue balls, so blue is 4 c 2 and 1 white out of 3, so this is 3 choose 1 divided by 9 choose 3. So, let us quickly compute this should be 6 3 and then, from here you will get a 6 and this will be 9 into 8 into 7, so 6 3's are 18 and this will be twice, twice 4, then 2 and 3, so this is 3 by 14, so 3 by 14 is this I have, so 3 by 14 is this number.

And this way I have it some computation where 0, 2 you have to compute and so on, so I will shown you some computations here p (1, 2), then p (1, 0), p (1, 1) we have computed and so you go on. So, now the whole ideas is that once you have completed this, then as I was saying that if you want the marginal distribution for x then, so for example if you are looking for the probability let me just take it from here.

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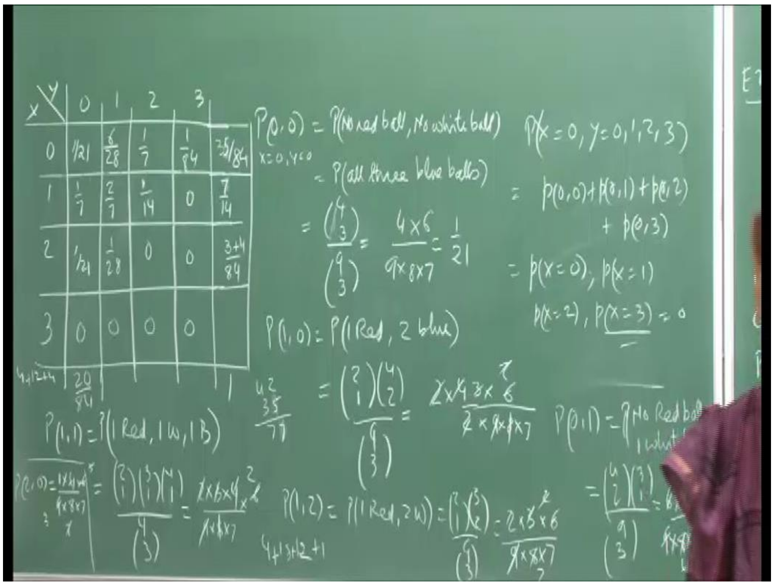

So, if you add up the numbers here, this is what probability x equal to 0 when you are giving values to 0, 1, 2 and 3, in other words you are adding up probability, let me write it as (0, 0) plus probability (0, 1) plus probability (0, 2) plus probability (0, 3), so they can be 3 white balls, because total number of white balls is 3. So, all this will give you the probability at x is equal to 0, because when you pick up the 3 balls if it does not contain any red balls, then those 3 balls will either contain 1 white ball, 2 white balls or 3 white balls.

So, all these probabilities add up to the probability x equal to 0 and so this will be quite similarly, when you add up the second row that will give you probability x equal to 1, said third row will give you probability x equal to 2 and this be probability x in to 3, but since this is all 0, because there are no then number of red balls is only 2, so you cannot have number of red balls in the sample that you pick up as equal to 3. So, therefore, this is all zeros, similarly here you cannot have the combination (3, 3), because you are picking up only 3 balls and similarly (3, 2) (3, 1) is also not possible from here (2, 2) and (2, 3) is also not possible.

So, these are the only numbers and you see this now finally, because this is probability x equal to 0, probability x equal to 1, probability x equal to 2, which are all the possible values that x can take in your sample of 3 balls, because there only at most 2 white balls that can appear in a sample. So, this must add up to 1 and similarly, see when I add up these probabilities this will be give me the probability of y is equal to 0 that means, there is no white ball in the sample, when you add up these numbers this will give you the probability that y is equal to 1 and similarly this.

So, all these four when you add up that should also add up to 1 and that will give you good checks, so again at means when we are defining the marginal PDF and so on, then we must continue to check the validity. And so make sure that your calculations are ok, because otherwise you will know that you have made a mistakes somewhere, so you can go back and check. So, all these numbers, so the numbers here will give you the marginal distribution of y, here the numbers will be marginal distribution of x and so on, so this how you write down the joint probability distribution of two random variables.

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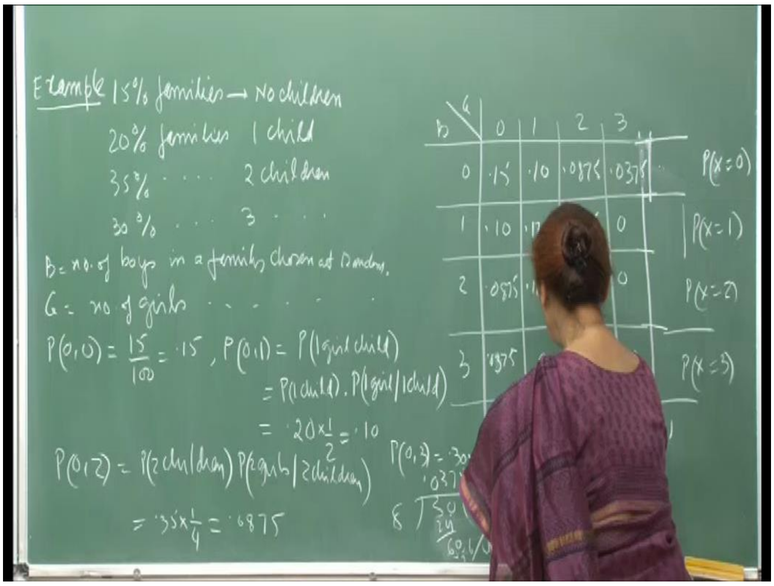

Now, take another example, so that I want to make sure that you understand how the calculations are being done. Now, this is an example in which there is a community, in which 15 percent of the families have no children, 20 percent of the families have 1 child, then 35 percent of the families have 2 children and 30 percent have 3 children. Now, the probability of they are the children being a boy or a girl is same, that means it is half half, so once you pick up a family at random.

So, the experiment that we are doing here is let you pick up a family from the at random that means, any family is equally likely, then you want to find out the number of boys and girls in that family. So, both of them are random variables that means, number of boys in the family and the number of girls in the family, this family that you have picked up at random. So, here again let us see I have made few calculations, so if you want to compute p (0, 0) then that means, no children, so of course this is straight forward you know that 15 percent of the families have no children, so this is simply 1.5, so which we write here.

Then when you want to compute p (0, 1) that means, no boy just 1 girl, so this is the event probability of 1 girl child, now I write this as a conditional probability is saying that probability 1 child into probability 1 girl given that there is 1 child in the family. So, that will come out to be 0.20 in to half, because there is only 1 child in the family, p (0, 1), so that means, only 1 girl and no boys, so this total number of children in the family is 1, so that probability is 0.20. Then it being a girl or a boy the probability is the same, therefore it will be in to half, so that is 0.10 that is a number you enter here.

And similarly, if you want to compute p (0, 2), then it will be 2 children and so I am writing the conditional straight away, so 2 children probability of that and then, 2 girls given their 2 children. So, this is 0.35 from here, families having 2 children that is 0.35 and then 2 girls, so 1 by 2 in to 1 by 2, so divide it by 4 and so that is 0.875. Then when you want to compute I have done it here, it is not very this thing, but any way it is coming maybe I can just show the calculation in a better way, so when you we want to have a (0, 3) that means, all the children are girls.

So, this again will be probability 3 children which is 0.30 when into 1 by 8, because all the 3 are girls, so 1 by 2 raise to 3. And therefore, you divide this by 8, so this is 0.0, then 3 will be 24 then 6, then 8 7' s are 56 and then, 40 and 8 5' s 40, so 0.0375. So, when you add up this numbers this will here will be the probability that x is 0 that means, no probability when you pick up a family at random from the community, there are no boys in that family. So, that number will come out to be this you can add it up, similarly this will give the probability at x is equal to 1 that means, 1 boy in the family, probability x equal to 2 and this will be probability x equal to 3.

I am here also you can see that this these combinations and of course, because the boys and girls are equally likely, so the tables once you have computed this part, then this is symmetric, because whether having a 1 1 when you have 2 children, so it is whether girl or boy is same, therefore this will be the same. Then 1 2 see this one we are having, ((Refer Time: 23:06)) this is 2 0, I am sorry 0 1 and 1 0, so that was a number, then this number for example, 2 1 and 1 2, so this two numbers.

So, this is symmetric, because boys and girls are equally likely, similarly here whether all the 3 children in the family are boys or all the 3 children or girls, the probability must be the, so I am sorry this should be 0.375, so make that corrections, so this will 0.0375. And then again as we had said when you add up all these probabilities, they should add up to 1 and here also you will get the marginal’s, so this will be probability y equal to 0, probability y equal to 1, probability y equal to 2 and probability y equal to 3.

So, I would like you to complete the tables, I have made some computations for you this so this is the same, now you have to make just these two computations and then you can complete the table. So, this gives should give you an ideas to how to go about computing joint, if I when the variables are discrete, how to compute the joint distribution function for discrete random variables.

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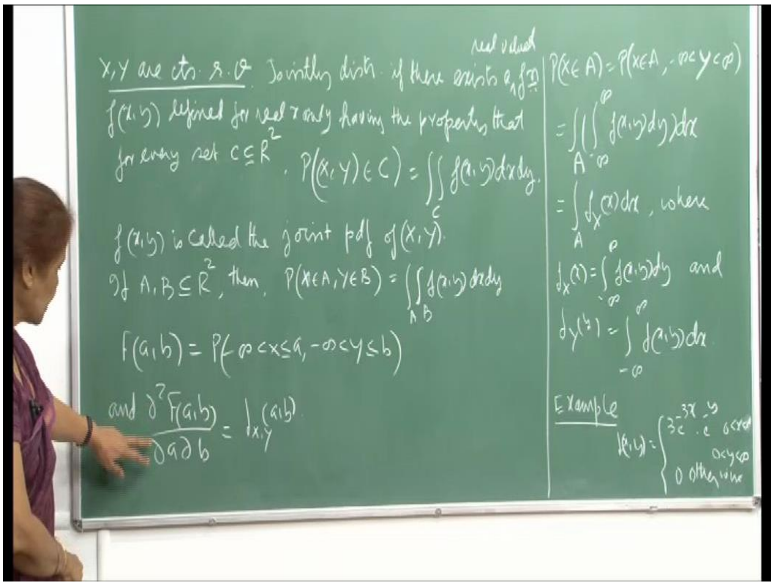

So, let us continue with the jointly distributed random variables, suppose x and y are continuous, so I will now define in other way in the sense at now I will define this through your joint PDF and so. We are saying that if there exist a function F (x, y) real valued function, which is defined for real x and y having the property that for every set C a subset of R 2 that means, all the pairs of values, which are there in C, then (x, y) belonging to C that means, you see the convention is this a pair of co-ordinates, then the first value is for x, and the second value is for y.

So, then probability (x, y) belongs to C is this, so now this is different from the way we define the cumulative distribution function right in the beginning, so of course two will must to the same thing. So we have the probability should be f (x, y) d x d y for it continuously distributed, a jointly distributed continuous random variable. So, f (x, y) is called the joint PDF of x, y and if A and B are two subsets in R square, then probability that x belongs to A and y belongs to B will be integration over a with respect to d x and actually see one should make a convention that this must refer to the later one here.

So, I should have said this is d y and d x, because just a convention, so therefore when you writing the limits, range, then it is nice to remember that this one refers to the second integral and this is the first, so the order has to be maintained. So, therefore, this is A B, so I should written d y d x of f (x, y), so the same thing is being on the sort of repeated and therefore, if your F (a, b) as we defined earlier is now x lying between minus infinity in a, y lying between minus infinity and b; then the partial derivatives here delta square delta a delta b f (a, b) will be f (x, y) (a, b).

So, this is the relationship between the cumulative joint cumulative distribution function and joint PDF. And then of course, the marginal distributions here, so this will be x belonging to A and y is from minus infinity to infinity. So, in that case your integrating with respect to y from minus infinity to infinity and this integral, when you integrating with respect to y will be the marginal. Just as I showed you in the discrete case, that you add up for all the possible values of y to get the marginal and the probability of x for a certain value.

So, same here, the marginal respect to x will be you integrate the joint PDF from minus infinity to infinity and for the marginal of y the PDF of marginal y, this would be integrating respect to x from minus infinity to infinity. Then of course, depending on whatever the region of actual definition else, now let us just take up this example. So, if you have this function define here, which is define for all values of x is from 0 to infinity and for all values of y from 0 to infinity, and it is 0 otherwise, so it is in the first quadrant that the function is defined.

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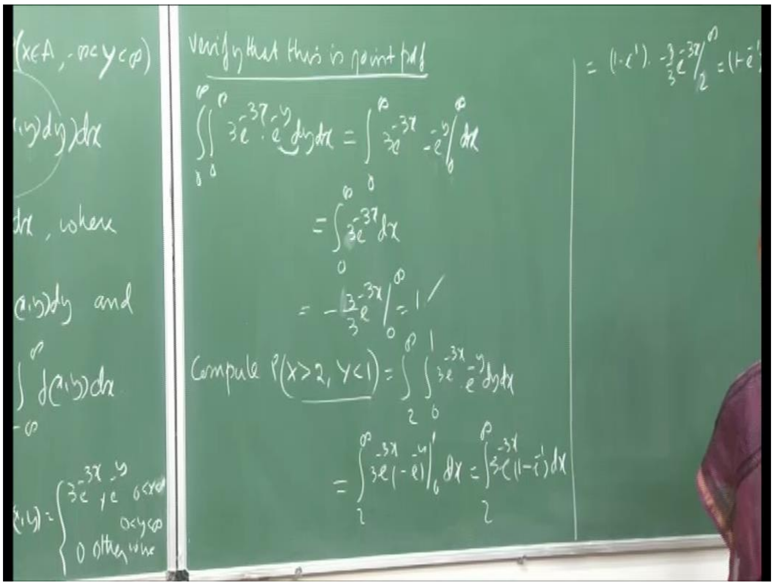

Then verify that this is joint PDF, so therefore the total integral from 0 to infinity and this should be 1, so suppose I just integrate respect to y first, so the integral is minus e just to minus y from 0 to infinity that gives you 1, because at infinity this is 0, at 0 this is plus 1. So then this is this now you integrate this respect to x, so x is minus 3 upon 3 e just to minus 3 x going from 0 to infinity, which is equal to 1 now if you want to compute this particular probability, then your limits will be...

So, this time have taken care of the order, so this is 2 to infinity for x and for y it is from 0 to 1 and so again the same integral you do it respect to y first, take the limits and then this is 1 minus e just to minus 1, then you integrate this respect to x and this will be, so the final answer will be this ((Refer Time: 29:08)), and so you can go on. So, this is nothing new except that your dimension has increased and the same concepts are there, the same axiom will be followed. And so we will now continue developing this theory and then, talk of independent random variables, jointly distributed independent random variables, then we will talk of some of random variables. So, of course the thing is let this concept can be extended to more than two and expect that the writing part is in little TDS, but the same thing will follow.

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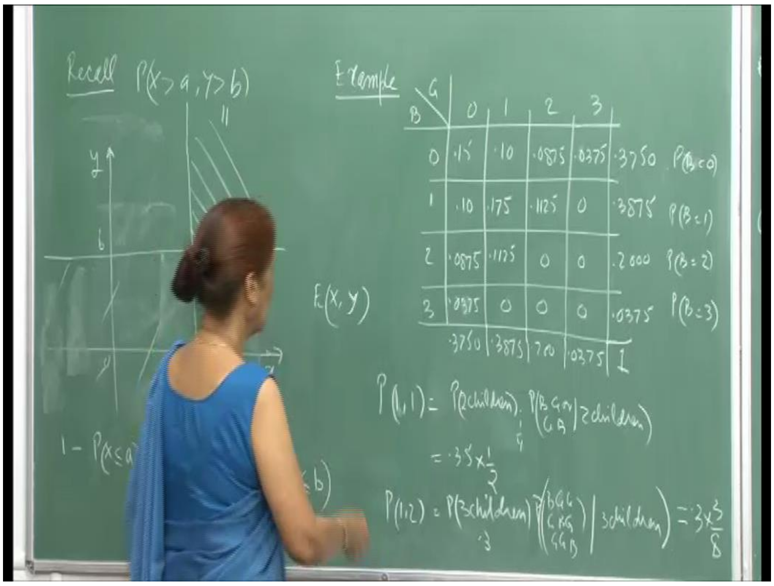

So, I will just revisit this, I had shown you this probability try to computing this probability, I had drawn the diagrams, so let me just make it more clear, because I had a feeling that I did not do a very good job last time. So, let us see probability x greater than a and y greater than b, then if this is my origin this is x equal to a, this is y equal to b, then this is the area that you want to compute the probability on, your region x greater than a, y greater than b is this event represented by this area.

So, now we said we will write it as x less than or equal to a, so x less than or equal to a is this whole area, extending on this side the whole area and then, y less than or equal to b will be this area. So, therefore, all this and all this gets covered and you see this area the area that is x less than a, y less than b this is, so this portion because when you are covering x less than a, then it is all of this y less than b then it is all this. So, therefore, x less than a, y less than b this particular area gets covered twice, so you and here and therefore, you get this.

So, therefore, 1 minus of all this, that means if I write a probability x less than or a plus probability y less than or equal to b minus this, then I will get the region corresponding to this and therefore, so 1 minus of that and that will be the probability for this. So, just wanted to revisit this thing and here also in the other example that we had taken, we would talking of a community in which there are families and they were probabilities associated with families having no children, 1 child, 2 children and 3 children.

So, I thought that though I asked you to compute add compute it is some probabilities for you and I asked you to compute the remaining one, but I realize that probably you need a little more working out and the so I thought I will give you the hints here. For example, when you computing (1, 1), then it is the probability 2 children into a conditional probability that it is either a boy girl or girl boy. You see when you are saying 1, 1 so then it can be, the first child is boy and the second is girl or the first child is a girl and the second child is a boy.

So, therefore, this will be 0.35 into, because a probability of families having 2 children is 0.35, then into actually boy girl will be 1 by 4, because each of them are equally likely what since they had two possible cases, so 2 into 1 by 4 therefore this is half, and so this number comes out to be 1 1, this is 0.175. Now, for 1 2 when you want 1 boy and 2 girls, then that means the 3 children, so the probability of having 3 children is 0.3, 30 percent of the families have 3 children, so this is 0.3.

And the conditional probability of having a boy and 2 girls and then, with 3 children, having 3 children and then, 1 boy and 2 girls. So, now here again you need to say that see it could be the first child who is the boy other two are girls, then it is first girl, then boy, then girl and girl girl and boy, so these three possibilities are for there, therefore it become 3 by 8. Because, each of them is half, so then when you have these three possible cases favorable for your event, then this is 3 by 8, so 0.3 into 3 by 8 that comes out to be 0.1125 and at the rest we had computed.

And then these are the marginal, so that means this I told you is probability x equal to or B equal to 0 and probability B equal to 1, probability B equal to 2 and probability B equal to 3. Hence similarly here, this is the marginal for probability girl is 0, no girl in the family of course, so probability no there is 1 girl in the family, 2 girls and 3 girls and then, you see that this adds up to 1. Because, they are marginal PDF' s are probability mass functions and so so once you find out the joint, then you find out the marginal's and then you can do all other competitions that are required the expectation, variance and everything you can compute as we have done it for you.

Like even for the joint also you can do it, expectation (x, y) you will do it and so you will multiply that I think I have given you the expression p (x, y) this we worked it out this into your values that x and y takes imply y is positive and so on.

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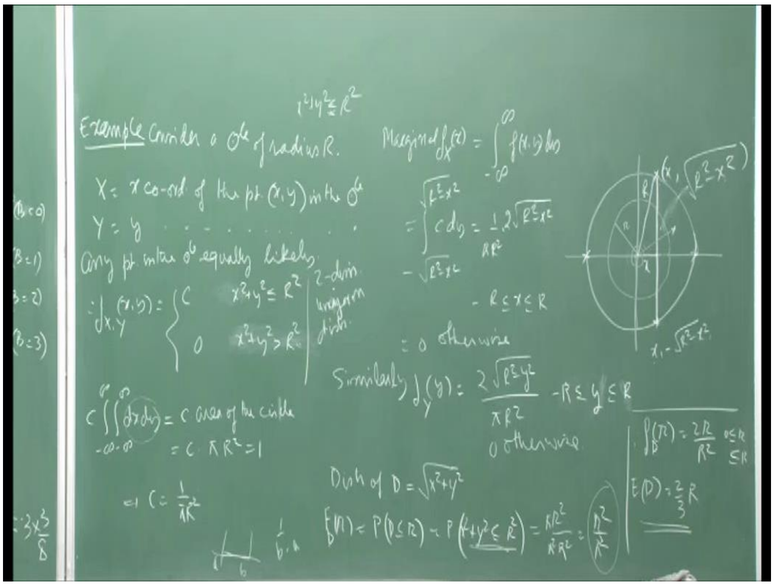

Now, I will continue with some more examples of the continuous random variable, so here let us see you take a circle of radius R and let us say it centered at the origin, so the equation of the circle is x square plus y square equal to R square. So, when you are taking the inside of this circle then it is less than or equal to R square, so this is the circle given to you, and we just pick a point and inside the circle, then that is a random because and we are saying that all points are equally likely to be picked up, inside the circle.

Then let x represent the x co ordinate of the point that you are picked up y is the, capital Y represents the y coordinate, so now both x and y are random variables and since any point in the circle is equally likely, therefore it will be a constant the PDF of (x, y) that joint PDF of (x, y) will be a constant inside the circle and 0 outside. And this is an example of a two dimensional, two variable uniform distribution, so this represents uniform distribution, because any point in the region in on the circle is equally likely.

And then, now you want to find out the value of C, so we will have to say that this must integrate to 1, the whole thing and since this is simply minus infinity to infinity and minus infinity infinite d x d y, this is an element of area. So, when you integrate over the whole of this circle you will get the area of the circle by everybody, so this is you have already done it in your class 12 and so on. So, then C into pi R square is equal to 1, therefore your constant is 1 upon pi R square which is expected, one dimensional also I have told you that when you had uniform random variable this was the length of the...

So, if this was point a this was b, then the probability density function for this uniform random variable was 1 upon b minus a, which is 1 upon length of the interval in which the random variable is defined. Here the pair (x, y) is defined inside the circle when it is uniformly distributed this pale and that means, that the PDF must be 1 by pi R square, the area of the region, in this case it is the area of the circle. Now, you want to compute the marginal of x, and marginal PDF of x and marginal PDF of y, so for x you will integrate from minus infinity to infinity f (x, y) d y.

And this what I want to point out that is, so you are fixed value of x, if you are fixed the value of x, then how does a y vary, y varies along this card and the length of the card is because this length is x, this is R the radius of the circle, so this length is under root R square minus x square. So, therefore, this point has the co-ordinates x, under root R square minus x square and this point has co-ordinates x, minus under root R square minus x square. So, your y varies from minus under root R square minus x square, 2 under root R square minus x square, so this is it.

This reduce, because there is no other ((Refer Time: 38)) outside this, so this is C into d y and this C is 1 by pi R square, so this becomes twice under root R square minus x square. And this is defined for all x between minus R and R, because your x varies from this point to this point along this, and see your x various along this, your y varies along this, so the whole circle gets covered. Similarly, because the symmetry, so your marginal of y will be twice under root R square minus y square upon pi R square, where again y varies, this got next step, this is minus R less than or equal to y, less than or equal to R, 0 otherwise.

Now, suppose you want to find out the distribution of random variable d, which is the distance of the point from the origin, so this is under root x square plus y square. So, you take any point here and then, this is the distance length would be under root of x square plus y square. So, we find out the distribution function of d which will be F D (R), so that means, probability d less than or equal to R, now since everything is nonnegative, distance is a non negative number, R is non negative.

So, this event is the same as squaring up both the sides that means, x square plus y square less than or equal to R square, the two events are the same. And again by the same argument that for x square plus y square less than or r square, so all points inside the circle having radius R, all these points will satisfy this or define this event, every point inside the circle of radius small r. And therefore, the area here is pi R square small r square and divide it by the PDF, which is pie r square, so you can you know want you can do it from by integrating in everything.

But, this is a straight forward because everything is uniformly distributed, so therefore, the probability of a point lying inside the circle small r is pi R square. Just as we said that the you know; that means, this is the area, which is favorable to our event and then the density of the function is of the 2 pair is 1 up on pi square R square. So, therefore, the probability the distributional of function of D is R square up on capital R square, so now, if you want to find out the PDF of D would be did differentiating this respect to small r.

So, 2 R up on R square and small r varying from 0 to capital R because now D is a non negative random variable and the value of D will vary from 0 to R. Because, the point is here, then this way and this way, so you can go up to capital R and similarly we can find out the expectation of D, which will be 2 by 3 R. So, you now geometry helps, you draw pictures, then you can you know get a good idea is to how to go about solving a problem.

So, and the moment you have more than one, you know if you consider a pair of random variables. And you can see that the complexity will increase the moment to consider higher dimension you know; that means, 3 random variables together joint density function of three. But, and as I go long I will try to solve some more problems relating to two random variables joint distributions of two random variables.

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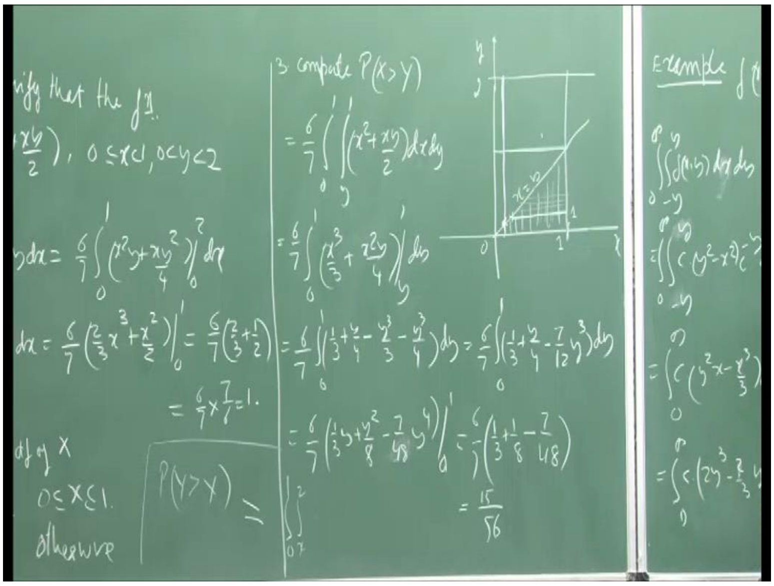

So, another example on joint distribution of random variables, suppose f x y is a function given like 6 by 7 x square plus x y by 2, x is between 0 and 1 and y is between 0 and 2. So, then you want to verify this represents a PDF of the variables x y, so I just integrate from as 0 to 1 and 0 to 2 and the working out shows that this x square y plus x y square by 2, if your differentiating with respect to y. Because, I have written the limits with respect to y first 0 to 2, so let us integrate respect to y first.

And therefore, this is what you get and then you this you look at this, then integration respect to x 0 to 1. So, that will give you this function and finally, you get the integral to be equal to 1, so therefore, the function does represent because it is a non negative function, since x and y are both non negative. So, this is a non negative function, the integral in the defined the specified area integrates to 1, so it represents PDF, now to compute the marginal the PDF of x or the marginal of x, you see I have already done it here.

Because, you have to integrate this integral from 0 to 2 d y, so this integral gives me the marginal of x. So, therefore, this is 2 x square plus x 0 less than or equal to x less than or equal to 1 and you see that if I now integrate this from 0 to 1, it will give me the answer 1. And hence this is also a PDF it is non-negative, in the specified region and of course, and I should also say 0, otherwise it is very important that here definition of the PDF's must be complete in the sense that you must specify the region on which it is defined.

So, here it is in between x 0 and 1 this is the PDF is defined by this function and it is 0 otherwise, then you are asked to compute x greater than y. So, if you have to compute x greater than y, then see I have written down this the line with represents x equal to y, so this is that region over which you want to compute this probability because you have specify the event. So, then it will be see here, if you are integrating with respect to x then you fix a y; that means, when I am integrating respect to x by x is varying, so y is fixed.

So, when you fix a y in this region how will your x vary from y to 1 because this represents 1 here is this line. So; that means, you are integrating from here to here for a fixed y and as y changes from 0 to 1, you will cover the whole area like this, so therefore, range for a variable x is from y to 1 and y varies from 0 to 1. So, this is very important and again I will keep a repeating this that draw the diagram you can two dimensions you can always do it and then you get a feeling for how the what are the ranges of a integration and, so on.

So, integration respect to x gives me a x square x cube by 3 plus x square y by 4 from y to 1 and that is important because I must integrate respect to x first, since limits for x are in terms of y. So, then I will first integrate with respect to x and then that function as a function of y and then I integrate with respect to y, so this has to be the order you must keep this in mind. And so finally, this comes out to be 15 upon 56, so the arithmetic should be correct, now here what I was saying is that suppose you had to compute the event probability y greater than x.

So, in that case you see what will happen is it is this region, which you have to do it, so then you see you will have to break it up in to or what you can do is may be now it is not necessary. Because, then if y is greater than x, you will want to write the limits; that means, for a given x how will a y vary, that is no problem, so y will vary from here to here; that means, it will vary from x to 2. So, I will integrate respect to; that means, if y is varying from x to 2, so we will have to integrate respect to y here, so the limits would be x to 2 and then x varies from 0 to 1.

So, this will give you this probability, if you integrate first with respect to y x to 2 the same function and then you integrate with respect to x because for a given x. So, this how when you have the diagram you can immediately see that given value of x, the corresponding value of y can range from x to 2, then that is it you can compute this.

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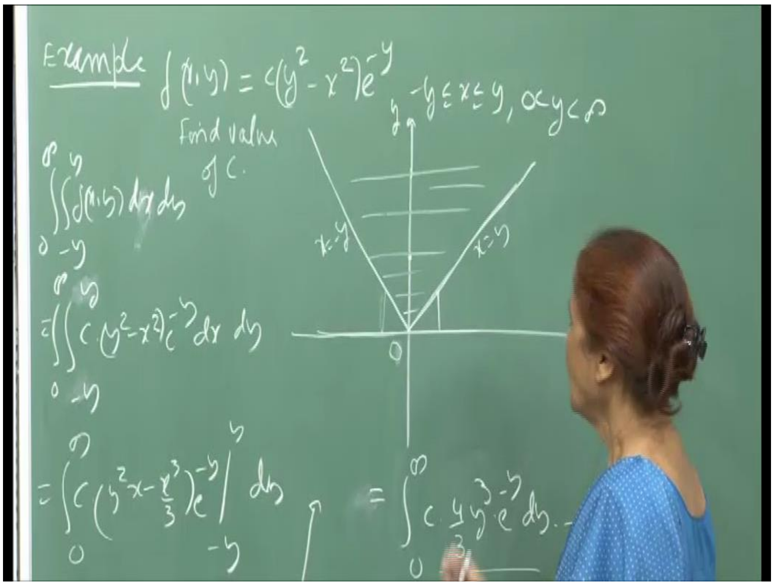

Now, just look at another example, and maybe we do not need to compute it fully, but I will give an idea here, this is f x y is defined as c into y square minus x square into e rise to minus y, now the limits for x are from minus y to y and y varies from 0 to infinity. So, therefore, to draw the diagrams see what I have said is that, when y is fixed then my x is varying from minus y to y. So, it has to be between the lines, this is x equal to y and this is x equal to minus y or minus x equal to y whatever whichever way.

So, this is this, and this is this line, so you are x is in between these two lines and your y varies from 0 to infinity. So, it is this region extending to infinity, this is the region and, so once you know this, then there is no problem because anyway you was first want to find out the value of c. So, find value of c sorry that this defines a PDF, so then you can integrate and you see with the what will happen is that you come up to here, now this is y cube a is to minus y.

So, you will have to you know do integration by parts and it will have to be done 3 times because you know you this will be a first function this is second. So, you will have to in the first iteration you will get y square, then you will have to do it again to get y and then get rid of the y. So, therefore, it will be 3 times you will have to repeatedly apply integration by parts to get the value of c, so that this integral finally, has to be equal to 1 and this...

So, once you have the diagram in front you, you cannot go wrong you, you can always find out the correct limits, and then decide how to you know like what I was doing is I was dividing the region in a certain way to do the integration, which you have already done in your while you would doing your calculus course.