Linear Programming and its Extensions

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Lecture No. # 09

Simplex Tableau and Algorithm Cycling Bland's Anti-Cycling rules Phase I and Phase II

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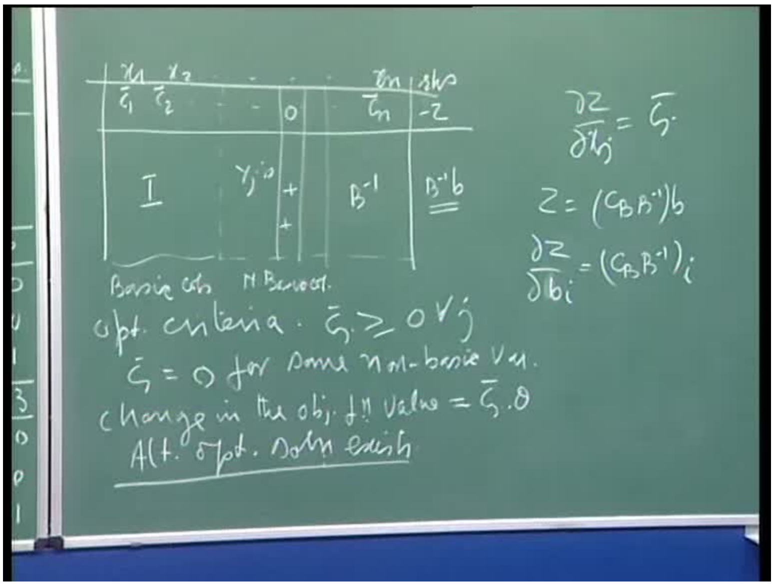

So, let us have another look at the simplex tableau. Let us see what is where. See, here, these are the variables x 2 to x n; this is your right hand side, the b vector, and then, it always give you the basic feasible solution. So, in the top row, we decided to write c 1 bar c 2 bar, the relative prices, and this is your value of the objective function. This is minus you have; this value always reflects your minus z. Then, let us then assume that the first m columns are basic columns. So, you always have the identity matrix here.

Then, here, you have your y j(s), your, for example, these are the non-basic columns, and this is your basic feasible solution - B inverse B. Then, if, see, as we saw, if you started with identity matrix here, and so, you could read your initial; that means your, mostly if you have all slack variables added to all the constraints, then you will have identity matrix and then you can read your basic feasible solution right away if you are back to b also non-negative. So, that will always give you B inverse, but this is not necessary always, but what I am saying is that if in the beginning, you could start with an identity matrix, as your basic feasible solutions as your basis matrix, then in because you started with an I, we are always premultiplied by B inverse.

So, therefore, these columns will always give you the B inverse, and this can come in handy; otherwise, we will, in any case, when we get this starting basic feasible solution, we will have B inverse with us and then we can store it somewhere when needed, but basically, I am trying to show you that this is your this thing.

Now, remember, our optimality criteria was that your c j bar should be all non-negative for all j and we saw that the c j bars for the basic variables are all zeros, and for non-basic variables, they should be all non-negative. Now, it is possible that c j bar is 0 for some non-basic variable.

Therefore, if you have a 0 here for a non-basic column, non-basic variable, your c j bar is 0. Then, it is possible. If you can find, because this value is 0, I can enter this column also into the basis. The value of the objective function will not change, because remember, the change in the objective function value is given by c j bar into theta - a level at which the variable x j will enter the basis - and this could be a rate which your objective function value changes.

So, this is the total. So, if c j bar is 0, the value of the objective function is not going to change and that is what should happen, because we are saying that we are at optimal solutions; so, we have the best value of the objective function possible among all the feasible solutions. So, when c j bar is 0 for some non-basic variable, there is the possibility of an alternate optimal solution and that is what we are investing here. So, the values of the objective function not change. So, all I would need is some positive entry here, and if I have more than one positive entry, then obviously I will use the same primitive rule, that is, I will take the ratios enter the minimum one. Let me, I am just replacing one basic column by another basic column and which I can do here provided I have some positive entries here and then I will have another basic feasible solution, which will give me the same objective function value. So, I have an alternate optimal solution.

And some time ago, I prove to you this lemma that if you have more than one optimal solution to your linear programming problem, then you have infinity of them, because every convex combination of the two optimal solutions, two basic feasible solutions will give you another optimal solution. So, this is analysis. So, therefore, you can have. So, optimality criteria, this is, say this is 0 for sum this, then alternate optimal solution exists. And once you, there is more than one optimal solution, then there more than, there are infinity of them in the optimal solution. This is one.

So, now, this should become very clear to you; this should sort of map into your minds, because we can then do lot of things with the when later on post optimality analysis. Even developing some other alternate methods of solving l p p, you would need this. Then another thing is that, yes. So, as I told you that c j bars are nothing but the rate of change; therefore, you can say that delta j by delta x j is c j bar.

So, when, for basic variables, this is 0, and so, what we are saying is that in increase in the basic variables, will not change the objective function value, whereas, this is the… Therefore, when this c j bars are all positive or non-negative, we say that we have an optimal solution, because the value of the objective function will only go up and not come down. So, this is this thing. Then also you can see from here that your basic feasible solutions if you want to, so, yours value of Z C B B inverse B.

So, this is the constant. Therefore, if you want to look at delta Z by delta b i, that means the rate of change of the objective function of the side number is increased or changed. So, this the, this will give you this and that will be C B B inverse i; so, that means this vector also has a meaning, and after a while, I will be able to give you a very interesting interpretations of this quantity, and so, this is the ith component here of this vector will give the rate of change of the objective function where right hand side, and much more sensitivity analysis, we will do later on. So, this just to give you an idea of the simplex tableau, how it looks like.

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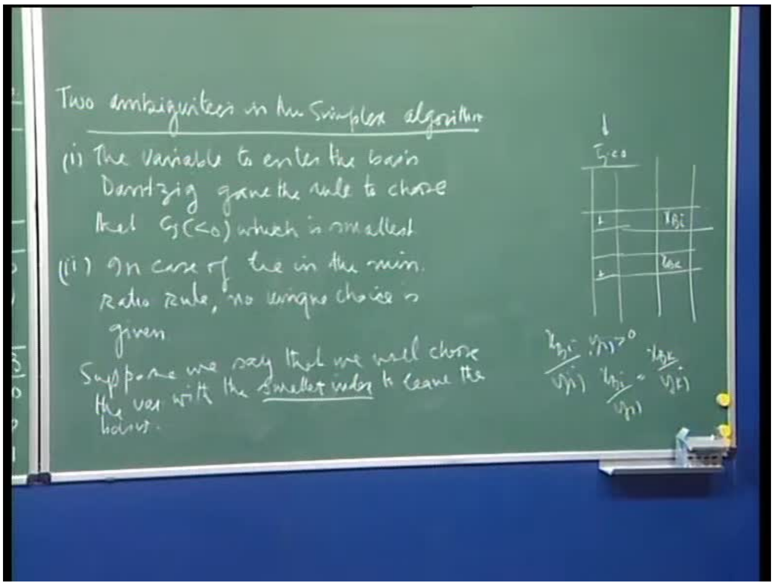

Now, let us come to another important aspect of this algorithm. See, in the simplex algorithm, we, we said that as long as your c j bar is less than 0, there is the scope for improving the value of the objective function, but the algorithm as such does not specify. So, let me write it out what are the uncertainties or ambiguities in the algorithm? So, one is the uncertainty about the variable to enter the basis. If you have more than one choice, then, so, two ambiguities in the algorithm, in the simplex algorithm. So, this is the variable to enter the basis.

Because as long as c j bar, so, here, because I told you that Dantzig gave the rule to choose that c j, that c j, less than 0 which is smallest. Suppose you fix this ambiguity and say that you will go for that variable to enter the basis for which the c j bar is a smallest negative.

For the second one is - in case of tie in the minimum ratio rule, remember, no unique choice is, choice is, given algorithm. The algorithm is silent about which one to choose. You know you understand what I am saying. See, here, when you have a column, suppose this is the one c j bar is less than 0, you are wanting to enter this column, and on this side, you have your x d i's, then we said that we will look at the positive entries here and then take the ratios; so, that means we were looking at the ratios x B i upon y i j.

In this column, so, this is for all y i j positive; maybe I can say y i j greater than 0. Now, suppose it turns out that, your smallest one x B i upon y i j is equal to x B k upon y k j.

These are the minimum ratios and, they, two of them are equal; so, that means this one here and these ratios here. So, you have your x B k. So, the ratios here and here are the same; they are the smallest. Now, which one should leave the basis? Anyone could leave the basis.

So, I want to show you that even if you make this choice, if you even if you specify a rule for this with the current choice, are given to you in the algorithm. It is possible that the algorithm will cycle and I will try to explain what is mean by the word cycle in through an example. So, in case of tie in the minimum ratio rule, no unique choice is given.

So, suppose, suppose we say that we will choose the variable with this smaller index to leave the basis. So, now, I have fixed the, both the ambiguities unique choice for the variable to enter and a unique choice for the variable to leave the basis; so, that means here, which ever, see when I say index, one has to be little careful a this smallest index. What we mean is that see, I remember there is a order for the variables - first basic variable, second basic variable and so on.

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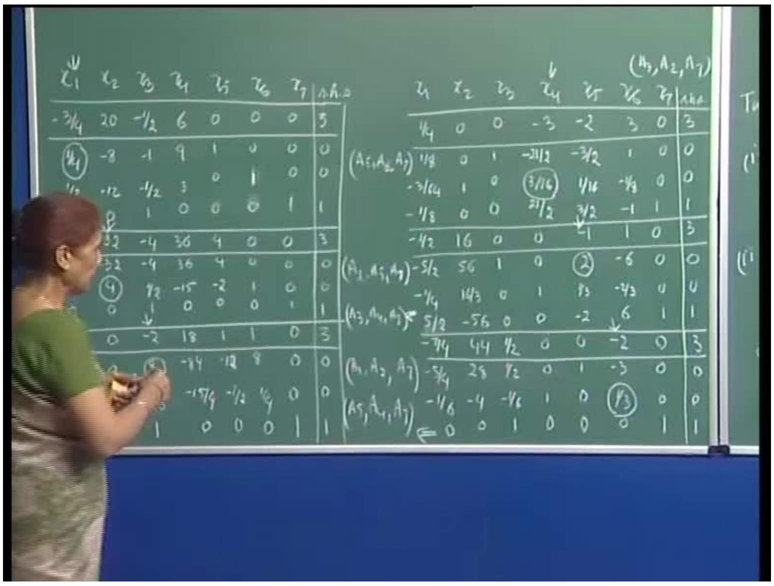

So, in this case, if this is the third basic variable and this corresponds to the second basic variable, then I will choose the second basic variable to leave the basis. So, the smaller index variable should leave the basis. Now, suppose I have these two rules, and now, let me show you what can happen? So, I, this is, this represents tableau of a simplex problem of a linear programming problem. Now, right now the starting basis, so, let me fix it up here, I will just write down this way A 5, A 6, A 7 is the starting basis here and we are going for the variable to enter the basis which has the smallest c j bar. So, this is your currently correspondence to enter the basis, and then, here, you see, again you have the ratios both the ratios are zeros; so, there is a tie, and we said that we will, so, this corresponds to the first basic variable; this corresponds to the second basic variable. So, we will choose according to our rule; we will choose this as be pivoting element.

So, I pivot on this and I reduce it to 0 1 0 0; the calculations are given here. So, the second table, by mistake first row got repeated as a top row. Now, please make the correction - the top row actually reads as 0 minus 4 minus 7 by 2 33 0 0 and 3; c j minus z js are in the top row. By mistake, while in the lecture, I wrote it as 1 minus 32 minus 4 36 4, 0 0. So, the first line had got repeated, please make the correction. This variable was entering the basis at 0 level. Therefore, the right hand side will not change; the value of the objective function also will not change, because the variables entering the basis at the 0 level.

So, this shows you the next tableau, and then, here, again we see that you have negative entries and it turns out that this is the one with the smallest c j bar value, this is, and here, you have only one choice, because this is only positive entry; so, this will be your pivoting element and your new basis will be… So, after this, your new basis came out to be A 1, A 5 and A 7. Now, we again pivot on this one; I make it 0 0 1 0. Then, this is the only negative entry; so, this will enter the basis, yes, and the ratio here again, I will choose this, because see, this is the first basic variable and this corresponds to the second basic variable; so, the smallest index is this. Therefore, I will choose to have x 1 or A 1 leave the basis; x 1 becomes the non-basic variable.

So, we do the pivoting here and I come to this. Therefore, now, your this thing will be A 1, A 2 and we always referring to the current basis; so, A 1, A 2 and A 7, this is your this thing. So, now, this is the tableau, and here, again this is the one which is this smallest c j bar. So, I enter this column into the basis, and the pivoting element here would be this again, because here, this is the only one. This is negative; so, I will be taking the ratios corresponding to these two. This is 0; this will be something positive. So, obviously, I choose that to be the pivoting element, and after the pivoting, we reduced it to this form and I have these entries here.

So, now, this is where I will be able to apply my rule, that is, see I have minus 1 by 2 and minus 1. The rule says that I should go for the most negative c j bar. So, therefore, this is the column which will now become a basic column and the corresponding variable x 5 becomes basic, and here, the, I am sort of space. So, here, what is the basis? Maybe we can write it somewhere here. So, corresponding to this, I will write the basis. What was your basis here? A 3, A 2 and A 7 corresponding to this, and remember, so, that is why you always write the first column, first basic column at this corresponding to 1 0 0, then 0 1 0 and 0 0 1.

So, pivoting on this, I get this, and here, when you do this, what is the corresponding basis here? The corresponding basis here is now A 3, A 4 and A 7. So, maybe I can this way A 3, A 4 and A 7. So, this is your new this thing, and now, again I have this. Now, here also you see minus 7 by 4 is greater than minus 2. Therefore, I will, by my rule, I will go for this column and this particular variable becomes basic.

So, therefore, when you do this, the pivoting will be on this, and here, we did not decide. So, here, again the pivoting will be this, yes, because 0 by 2 and 0 by 1 by 3, so, tie, and therefore, you go for the smaller index. This basic variable to leave the basis; so, you pivot here, and therefore, the corresponding basis here when you did that, your basis was A 5 and A 4 and A 7.

So, when you pivot on this one, that means now A 6 will again enter the basis, you see that, and from here, you will come back to this table, and this is what I am trying to show you that after all this, see because at each iteration, you had a degenerate solution. Nothing was changing; your basic feasible solution was not changing; your value of objective function was not changing, but you had different basis representations and this long ago when we were discussing representing 1 1 correspondence between basic feasible solution and an extreme point, I told you that they can be more than one basis corresponding to the same basic feasible solution, and in that case, your basic feasible solution will be a degenerate one. So, this is what is this phenomena we are seeing here and, that is, that we continued changing the basis, and after we started with A 5, A 6, A 7 and after this tableau, I come back to because A 4, in this tableau, A 4 gets replace by A 6; so, I am back to A 5, A 6, A 7; I will get exactly the same tableau.

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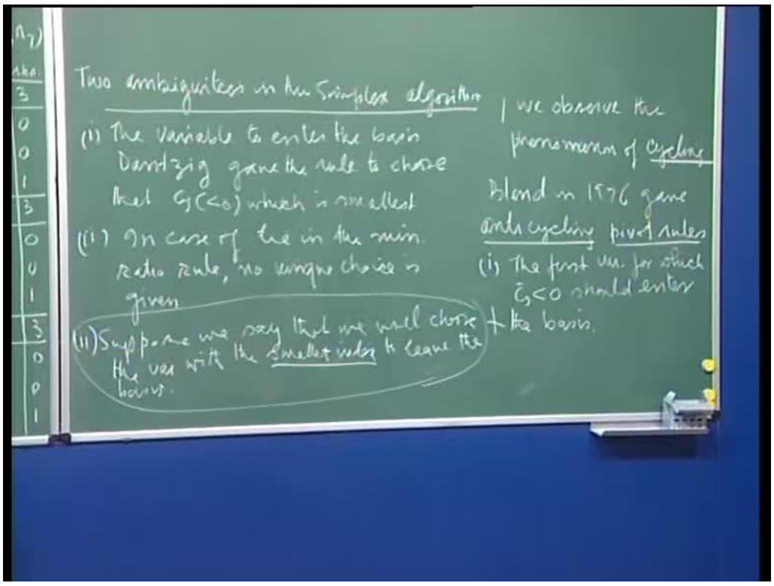

And you see, if because I am following these two rules, if I now continue, I will again repeat the same sequence of basic feasible solution and nothing will change. I will not prove anywhere; I will stay with the same basic feasible solution to same value of the objective function, but bases keep changing. So, this is what is known as cycle. So, we, the, observe this phenomena, so, this is a short coming in the simplex algorithm, and now, of course the advocates for the simplex algorithm is say, let this is a very rare case and it is not that cycling will occur most of the time, but again, if you are designing an algorithm, you have to make sure that all such pathological cases are also taken care of.

So, we observe the phenomena of cycle. So, in the simplex algorithm, and this was of course discovered a quite late that simplex algorithm can cycle and you saw that when you have degenerate basic feasible solutions, this is possible. So then, came bland; so, now, bland in 1976 gave anti-cycling pivot rules and not much needed to be change very simple this thing, but the proof required lot of effort. So, what was the change he made? He says that instead of going to, instead of a choosing, the variable that has c j bar the smallest just choose. So, he change this thing to variable enter the basis. I will write down.

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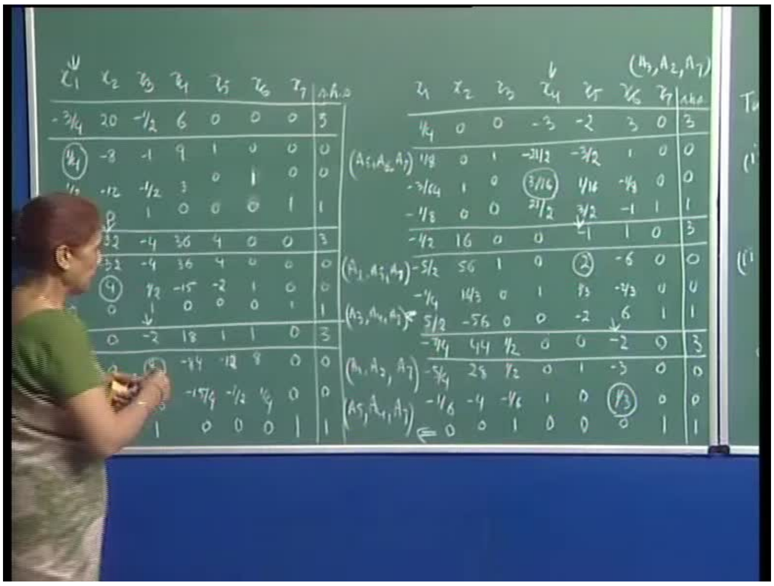

Now, the anti-cycling rule maybe I will write it here. The first variable for which c j bar is ensured enter the basis. This is a change he made. Do not go for the most negative c j bar, just the first variable that you encounter have. First in the sense that send that when you scan this row, top row looking for whether there is some c j bar which is negative or not, then you stop at the first c j bar which is negative and you enter; that means it will be the smallest index non-basic variable. Smallest index non-basic variable which should enter the basis if the correspondence c j bar is less than 0. So, it should enter the basis and the second rule is this. So, second rule is this, so, this plus this. (Refer Slide Time: 24:05)

So, this plus this, these two together certain plan gave this these two anti-cycling pivot rooms and he could prove a proof. Right now for a course is little complicated, and when usually do not give it in the first course on linear programming. So, any way, you just take my word for it that he could, and you could see that here, in fact, what I have suggest is that as an exercise, see what did we do. This was the place where we have to use these two places. See, if here, we had chosen this instead of this as a entering this thing, then and then after that if again we would have maintain blands rule.

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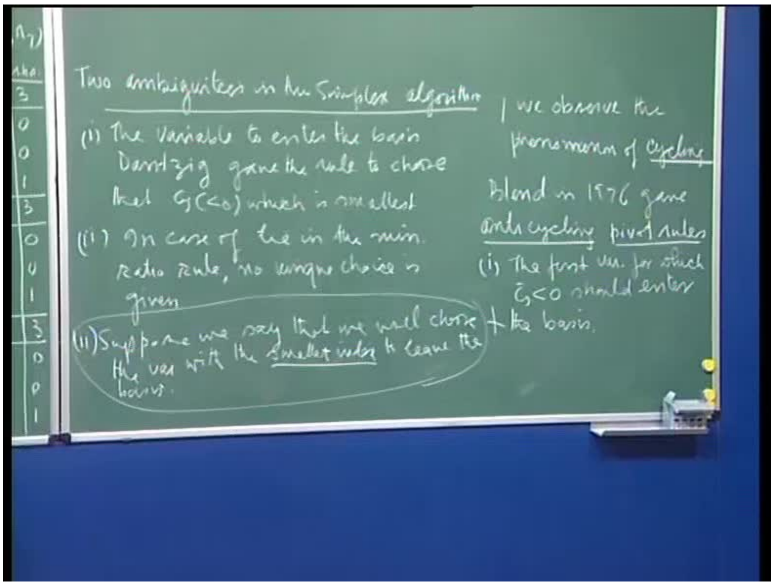

When you see, you would not have cycle; you would not have come back to the same basis. So, please sit down, and when you have time, this course try to do the pivoting; that means now you will enter this instead of this when you (( )) that you will not cycle. So, this was blands anti-cycling rule and it prove very effective, because now, all the programs that are written for solving linear programming problem on the computers incorporate blands rule, because it make sure that your, you will not cycle, and therefore, at the end of the algorithm, either the, either you will come to, you will obtain a optimal solution or the algorithm will tell you that there is a direction present and the problem is unbounded. So, this is the understanding that we started with the feasible solution. So, therefore, our linear programming problem was a feasible one, and then, continuing with this, algorithm we will either reach an optimal solution or we will discover that the problem is unbounded, and so, we can stop there. So, this is about the pivot rule.

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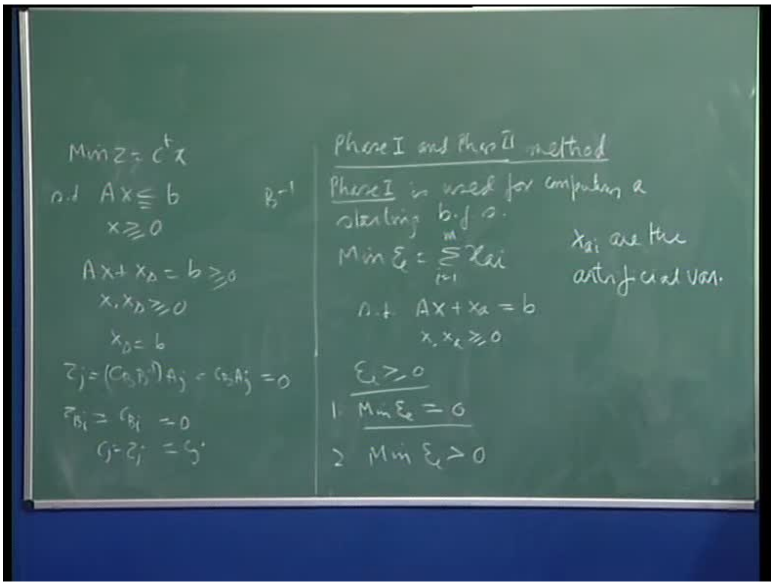

Now, the second issue that we have is that you may not always be very lucky to get a starting feasible solution by inspection. So, let me start on that now. So, once we make sure that we have a definite way of getting a starting feasible solution, the algorithm is all set and we also have to take care of another assumption that we made in the beginning that our l p p is feasible; that means our feasible set is non-empty, I started building up the theory based on the fact that our feasible set is non-empty, but now, I will show you that we can do away with that constraint also over that assumption.

So, see, what I was saying is that in case, suppose you have A X less than or equal to b, suppose you have a linear programming problem in this form, then I will add this and say that. So, if all components of the are non-negative, I can immediately read my feasible solution as basic feasible solution as this equal to b, because then, my basis matrix is identity, and in that case, remember my z j quantity is C B B inverse A j. So, since b inverse is, yes, so, since B inverse is identity matrix because B is identity. Therefore, this reduces to simply C B A j, and here, for basic variable, this will simply be C B i, Z B i. So, you have to write C j minus Z j; so, that means we will need just one single. So, what will it be? I shall have say C B A j and yes. Now, what are your C Bs? For the, for the slack variables, your C Bs are all zeros; so, this is 0 and for this is 0. So, therefore, my C j minus Z j will simply be C j, and for the basic variables, anyway they are all slack variables, so, the top row; so, that means the top row is as we want and the right hand side gives you the basic feasible solution. The tableau is also because B is identity matrix, premultiplication by B inverse does not change anything. So, I can just write the tableau as it is and start working with the simplex algorithm.

But you may not always be so lucky; your constraints may be greater less kind or equality kind. As, we, I told you that we will mostly study the standard form of the l p p. So, we need to have a mechanism for getting a starting basic feasible solution. So, I will give you first one this thing, basic thing here this is phase 1 and phase 2 method. So, phase 1 essentially gives you phase 1 is used for is used for computing a starting basic feasible solution.

So, what we do is we say that transform the problem to minimize psi equal to summation x a i i bearing from 1 to m subject to A X plus x a equal to b; so, that means whatever constraints, whatever kind of constraints you have, I first reduce it to quality form as I discuss with you in the beginning that by adding slack suppress variables, you can always reduce the constraints to inequality constraints to equality constraints, and so, I have an equality system. Now, I want to, I add artificial variables; so, these are all non-negative. So, this is the problem. I will first…

Now, what are the possibilities here? When I apply the simplex algorithm to this problem, we know that psi is greater than or equal to 0 because all variables are non-negative. So, the, this problem is actually bounded below. So, there is no question of phase 1 l p p ever incurring the unboundedness criteria, because the objective function value is bounded below by from 0; so, it cannot be made as small as we wish psi greater than it is (( )). Two things can happen - first minimum psi is equal to 0; that means I am able to get when I end up with the, with the simplex algorithm.

I will either have minimum psi equal to 0 2 minimum psi greater than 0, these are the two possibilities. What happens when minimum psi is equal to 0? I should have set X a i are the artificial variables and the name is very appropriate because this is artificially we are obtaining a starting feasible solution, and you can see from here also that if suppose an any feasible solution to any feasible solution to this system, if an X with corresponding, if one of the artificial variables is positive, then what does it mean that the regular variables will not satisfy that particular constraint as an equality constraint.

So, therefore, as long as in any feasible solution here, some artificial variables are at positive level; then that solution cannot be feasible; that means, if I drop the artificial variables and try to see, if I have a feasible solution for among the regular variables that will not happen. So, therefore, and so, we add here. What we are trying to say is that in case, there is a feasible solution here from among the X is variables, then that means because I have minimizing this and the minimum value will be 0 provided no artificial variable is at a positive level. So, in case there is a feasible solution here to the original system A X equal to b, then the minimum value here will not be positive; this is the idea.

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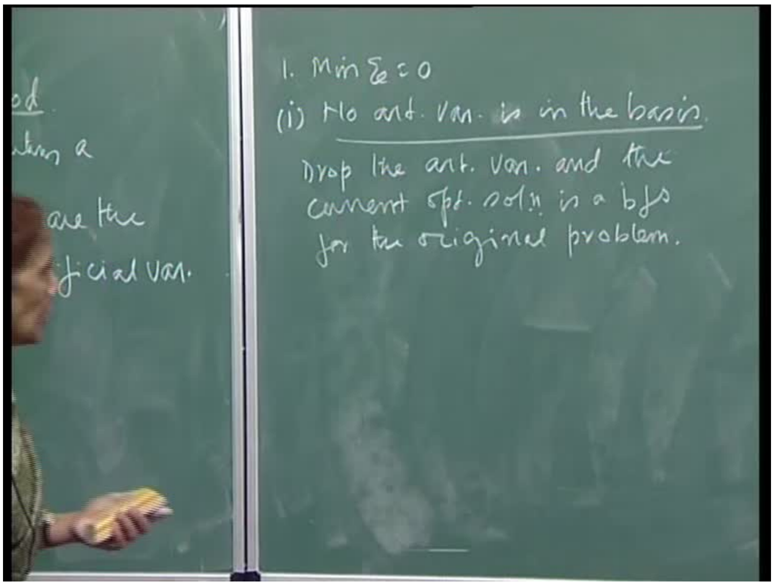

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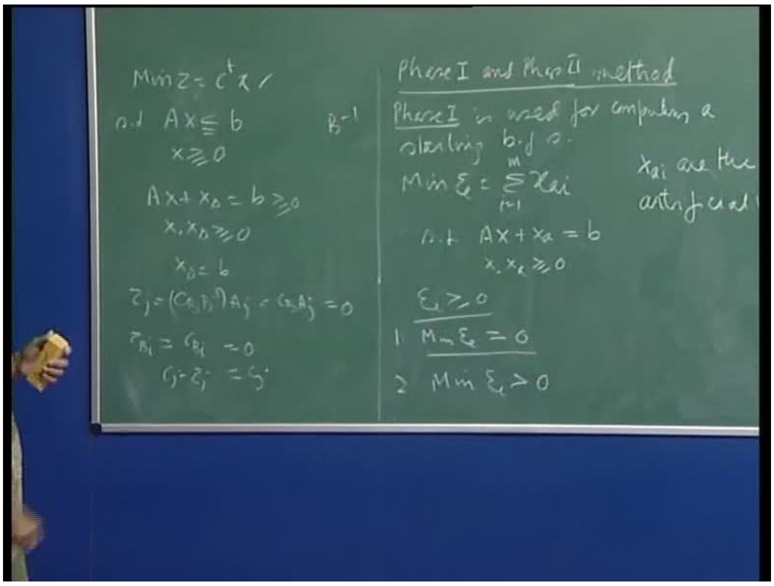

So, you are forcing, you are using the simplex algorithm to find out a feasible solution to the original system by it making a proper choice of the objective function and these are the many other things that one has to learn to be able to see how many ways you can use your simplex algorithm. So, let us see what can happen when psi is 0, when the minimum value of psi is 0. So, two things can happen. This is minimum psi is 0. Now, one no artificial variable is in the basis; so, no artificial variable is in the basis. Then, drop the artificial variables and the current optimal solution is a basic feasible solution for the original problem. So, once I have a basic feasible solution, I can then switch off to the original problem.

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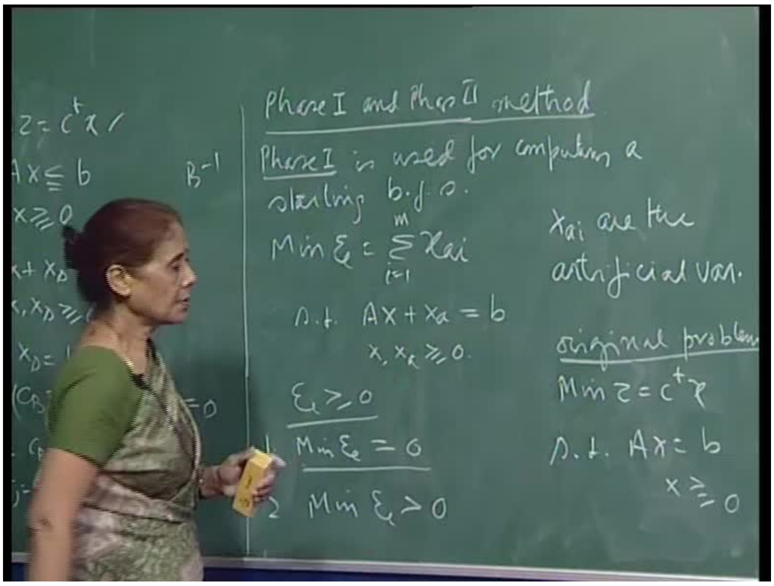

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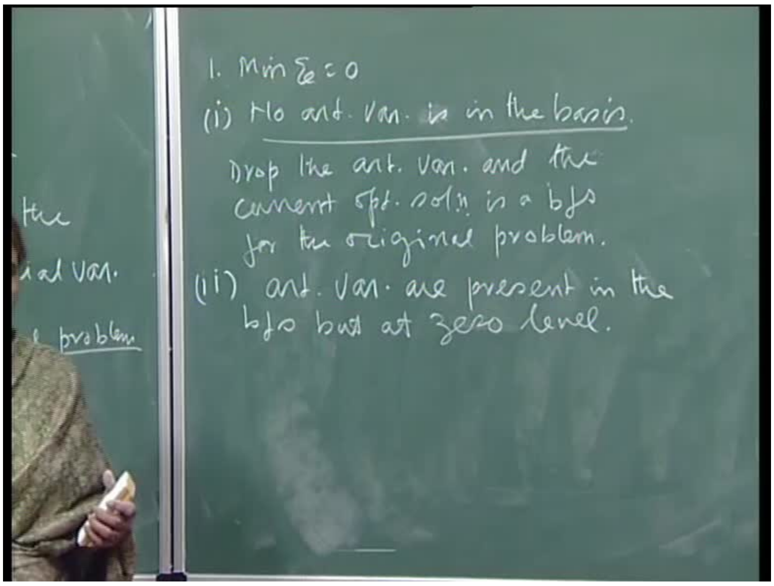

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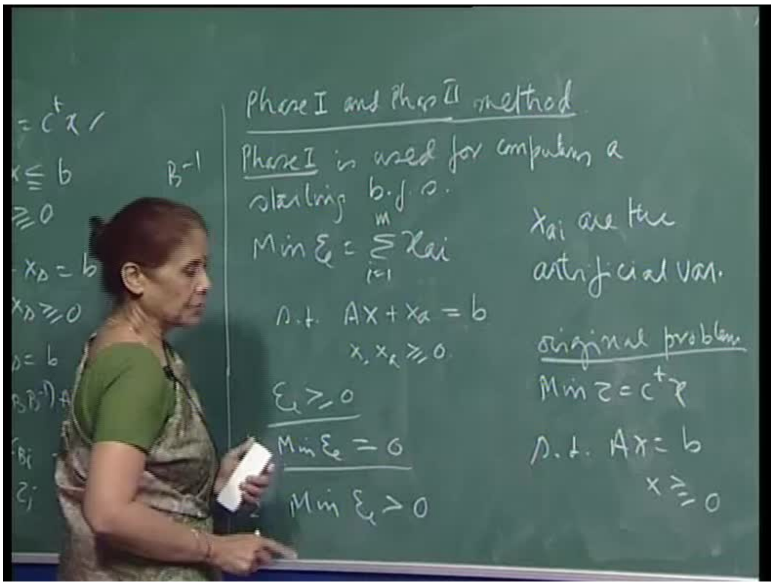

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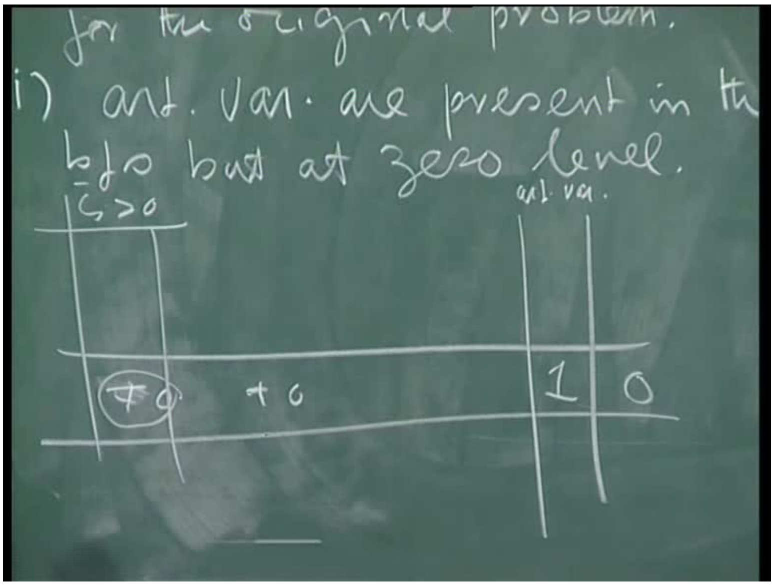

Well, maybe I should state somewhere the original problem. I should have stated it this is minimize Z c transpose x subject to A X equal to b. So, once all your artificial variables are 0, I have a feasible solution for A X equal to b and I can start with my simplex algorithm by replacing this objective function by the original objective function. What else possible? Artificial variables are present, are present in the basis, present in the basic feasible solution but at 0 level. So, artificial that means you have a degenerate basic feasible solution, and since the artificial variables are at 0 level, therefore, new became 0; the minimum value became 0, but I still do not have basic feasible solution consisting of the original variables. Remember, the idea is to have a basic feasible solution from among the x variables; so, I will basic at 0 level.

So, in this case, we would want to replace the artificial variables from the basis and get a basic feasible solution from the regular variables. So, that is simple because what is happening is I again just use a diagrammatic this thing. See the idea here is, so, this corresponds to a basic feasible to a artificial variable. So, this is, this is an artificial variable, and now, see if I have as long as there is a non-zero entry in this row, I can do my pivoting on it, because remember, now the sign of this number is not important since this number is 0. So, the minimum ratio will anywhere be 0; so, I do not have to worry about the sign. As long as there is a nonzero entry here, I can pivot on that and also I know that the C j bar is greater than 0, but that is also of not, no concerned because the entering variable will be at 0 level and I showed you some time ago that if the variable is entering at a 0 level, there is no change in the value of the objective function.

So, the value of the objective function will not change, it will remain optimal. So, therefore, all I need is in this row, I should have at least some non-zero entry. So, as long as I have a non-zero entry, I replace the corresponding artificial variable; I get a basis from among the regular columns. If there is no entry - non-zero entry – here, that means all are zeros. If all are zeros, that means in the original equation, original system, that particular row, that particular constraint is redundant, because all zeros are, all the, in the reduce system, one row become zeros; that means that row can be express is a linear combination of the remaining. So, that means now the constraints are not all linearly independent which was our initial assumption, but see through this system, we will come to know if there are any redundant constraints, we can drop them and start with a reduce system. So, we can always replace the artificial variables and get the starting basic feasible solution from among the regular variables.

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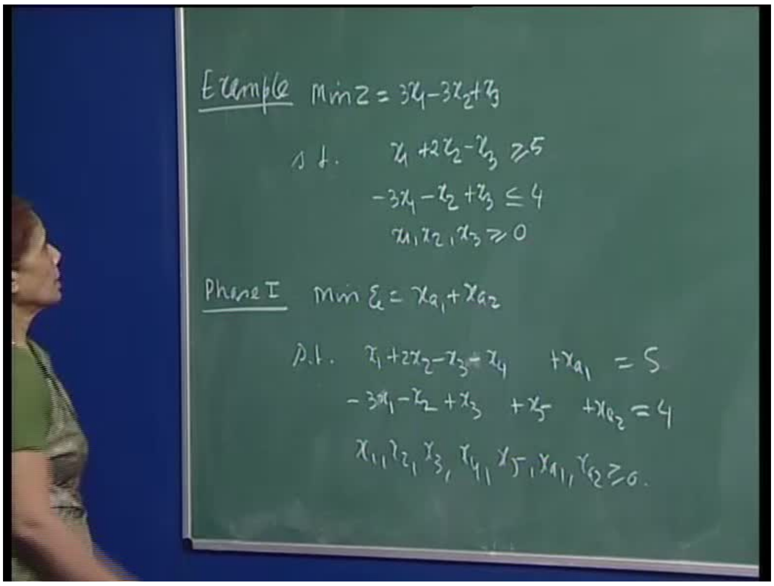

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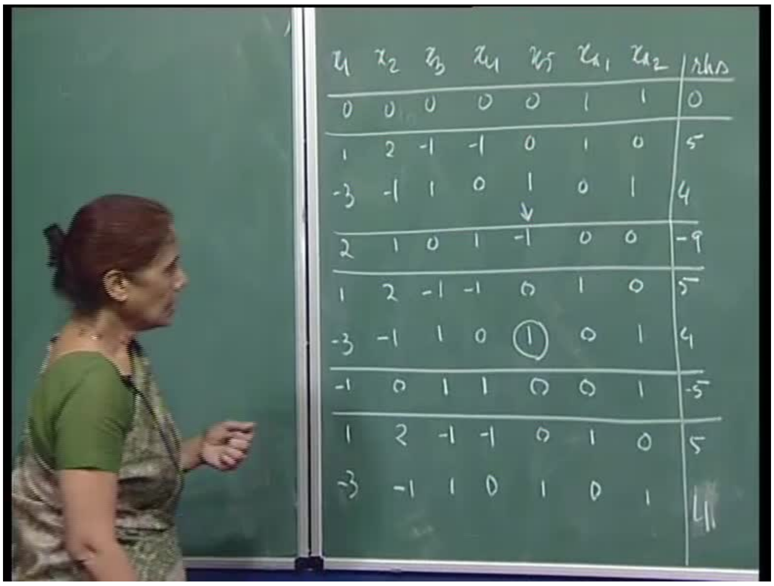

So, now, let us demonstrate the phase 1 phase 2 method through this example. So, suppose you have this linear programming problem to solve. This is greater than inequality, less than inequality. So, I add here a surplus variable minus x 4, slack variable x 5, then I get the equality constraints. I add x a 1, x a 2, because by inspection, I am not able to find a starting feasible solution from this system. So, therefore, I just add the artificial variables x a 1 and x a 2. My phase one objective function is x a to minimize x a 1 plus x a 2 and all these.

So, now, I can immediately because 5 and 4 are non-negative, I can immediately read. In case, suppose one of these numbers was negative, I would multiplied by minus sign, because it is important that my right hand side should be non-negative, and then, because I will add a slack of surplus variable accordingly and add a artificial variable if necessary. So then, my starting feasible solution, and so, let us see this is the first tableau which I have it note here. So, this is my basic feasible solution, basic matrix; this is my basic feasible solution. Now, the top row as to give me relative prices which are not right now available, because I have to make these zeros. Once I have make these zeros, then what I have here would be the C j bars. Therefore, to do that, simply add the, these two rows and subtract the sum from here that gives me zeros here. The value of the objective function is 9, because if your x a 1 is 5 x a 2 is 4, the value is 9 and that would there minus of the objective functions.

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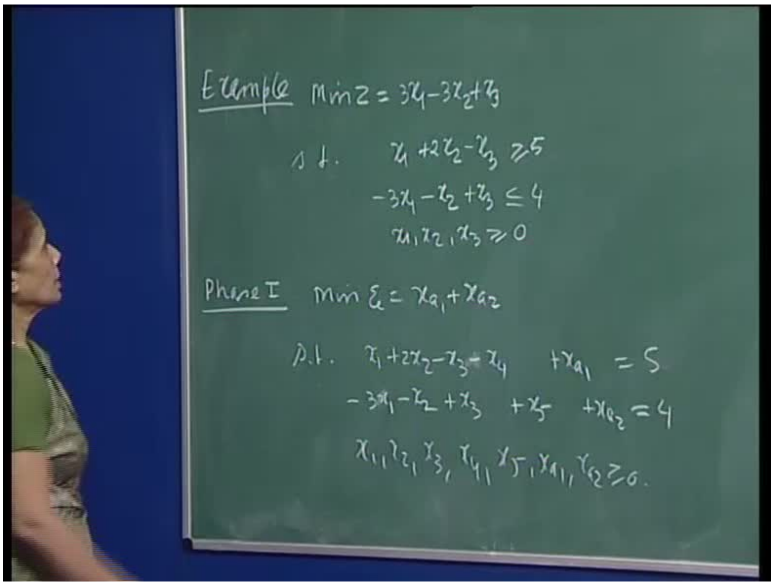

So, this is your c j bar row and I will enter. This is the only negative entry here, so, we will enter this variable into the basis and this will be my privet element. So, when you see that by making this as 0 0 1 0 0 1, I do the pi. This is what I get as my next basic feasible solution. The value, because this is 1, I do not, because this is already 0 1, I do not do any, in fact, I simply make a 0 here, I do not need to do anything here which reminds me that we could have, we could have use the fact that this is the unit vector. So, I should have added only artificial, but anyway, but I wanted to show you the phase 1 method.

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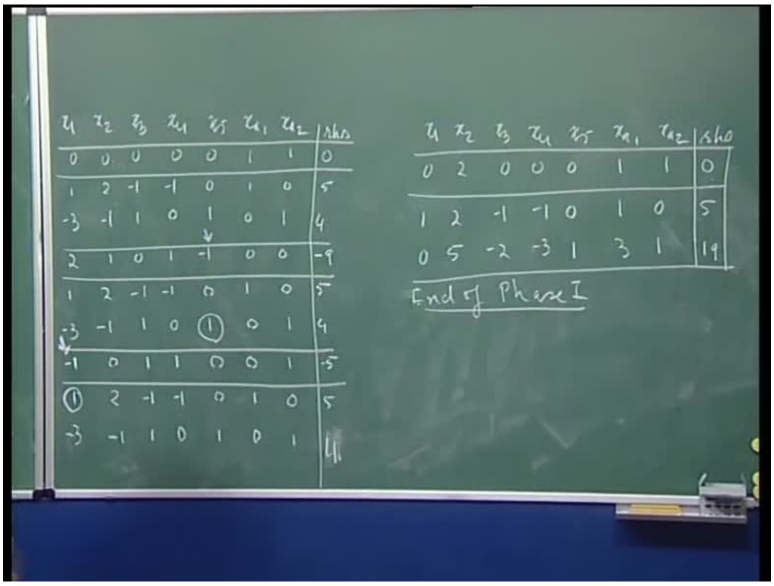

So, therefore, we added artificial variables to each of the equality constraints if I try. So, here, I do not have to do any previting except that add this row to this row to make a 0. So, that gives you the new C j minus Z j or the C j bars, and again, this is the only negative entry. So, this is the column which will the, enter the basis and your x 1 becomes a basic variable. So then, here, of course the pivoting would be this is only positive entry; so, this is the privet element here. Next tableau, when I enter a one into the basis, so, this is your basic feasible solution and you are value of the objective function is become 0; that means your min psi is 0 now.

And therefore, I have arrived at the situation where and no, none of the, what are your basic variables are x 1 and x 5, basic variables are x 1 and x 5. So, there is no artificial variable in the basis, and so, I am fine my phase one ends here, so, end of phase one. This is end of phase one; this is end of phase 1 and I end by my phase 2; that means I can now solving the original problem. So, the basic feasible solution for the original problem is available.

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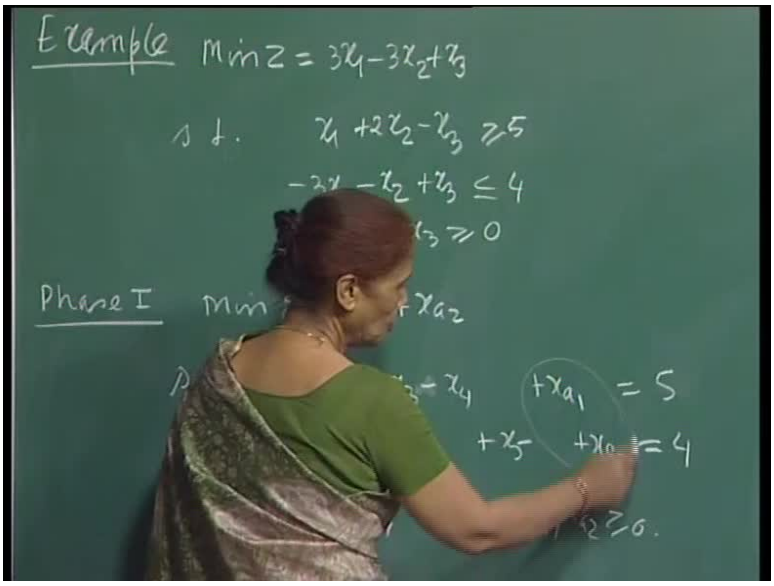

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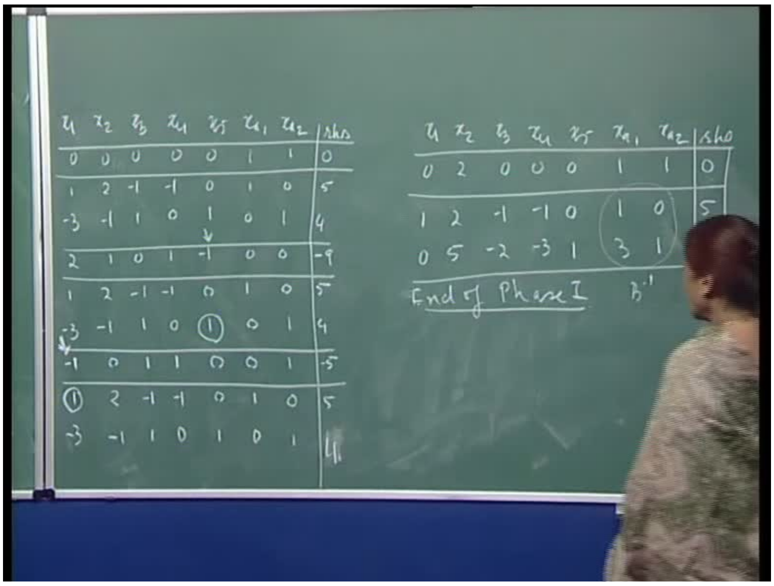

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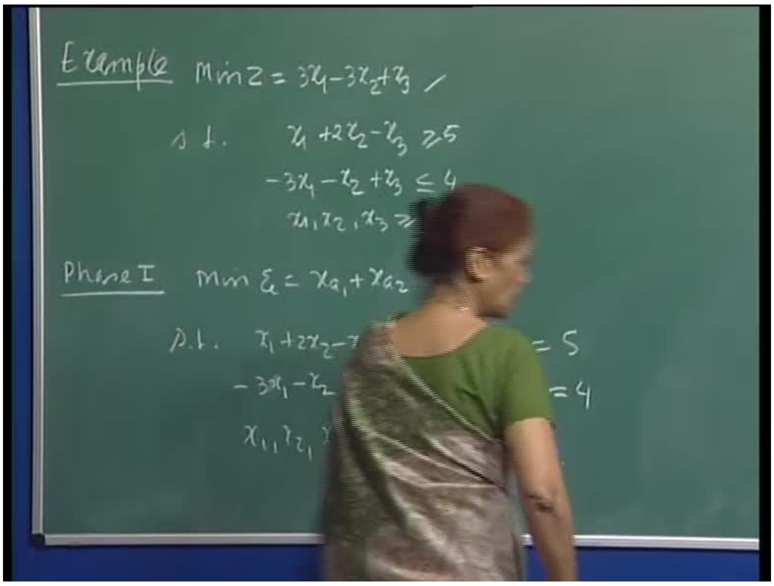

Now thing is that I will have to this is the, remember, in the beginning I show it showed to you that since this two columns made up my identity matrix at any iteration, at any iteration what I read in these two columns would always be my b inverse, because remember I am premultiplying by B inverse. So, this is my basis inverse, the current basis inverse, and to start the original problem, that means to start solving this original problem, I need my C j bar, and so, what is the definition?

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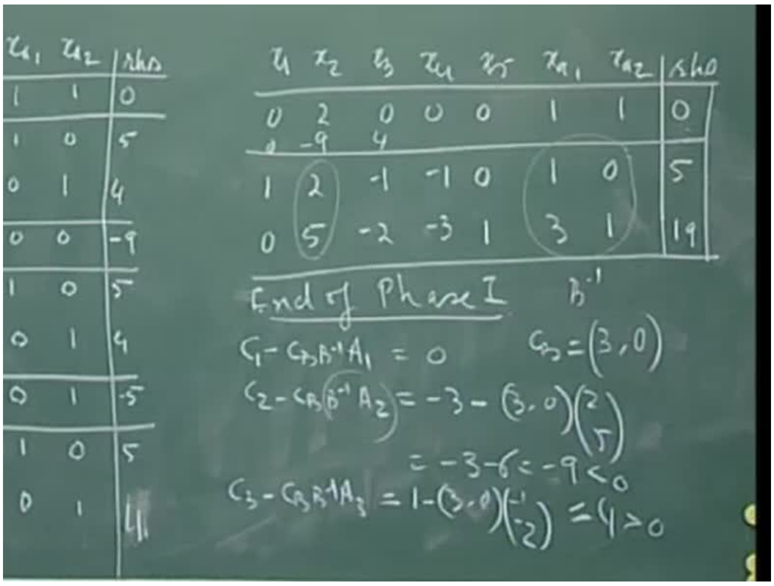

So, this will be, for example, so, C 1 minus C B B inverse A 1 which of course here. This is the basic variable; so, this is going to be 0. I know it, I do not have to, but so, if you want compute for C 2 minus C B B inverse A 2, you want to compute this C 2 bar and this will be your C 2 is what? You will go back to this thing here. This is minus 3 minus. Now, what is your C B? Your C B, because your basic variables are x 1 and x 5.

So, C B is what is the, see, this is three and x 5 was a slack variable; so, that was 0. So, this is your C B and B inverse A 2 you know, B inverse A 2 is this column y 2. So, minus 3 and this is 3 0 and 2 5, yes, I am showing you the calculations in detail. So, reputation, but anyway, it does not matter, because when you through, become clear in mind. So, this is minus 3 and minus 6, so, minus 9 which is less than 0; so, that means here this will be minus nine; this is 0 minus 9. Then, A 3 is another thing.

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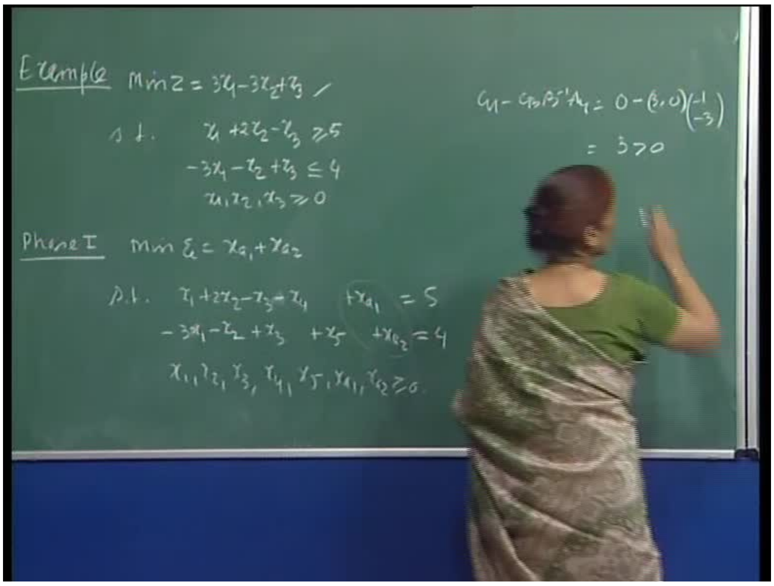

So, I need to compute C 3 minus C B B inverse A 3, which is, what is your C 3? C 3 is plus 1. So, 1 minus and this is 3 0 and B inverse A 3 is minus 1 minus 2. So, this is equal to, what will it be equal to? plus 3 because this is minus 3 plus so that will be 4 which is positive. So, here, this becomes 4. So, C 4 minus C B B inverse A 4; so, C 4 is 0 0 minus 3 0 and B inverse A 4 B inverse A 4 is minus 1 minus 3, minus 1 minus 3.

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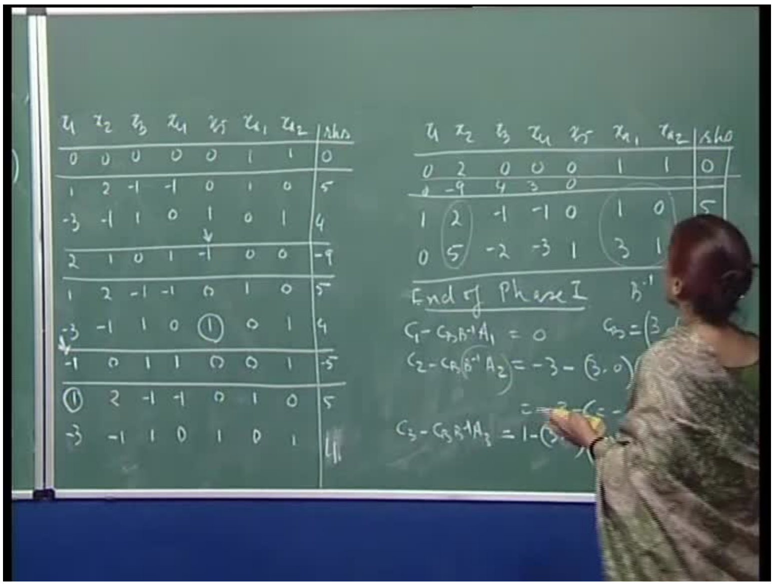

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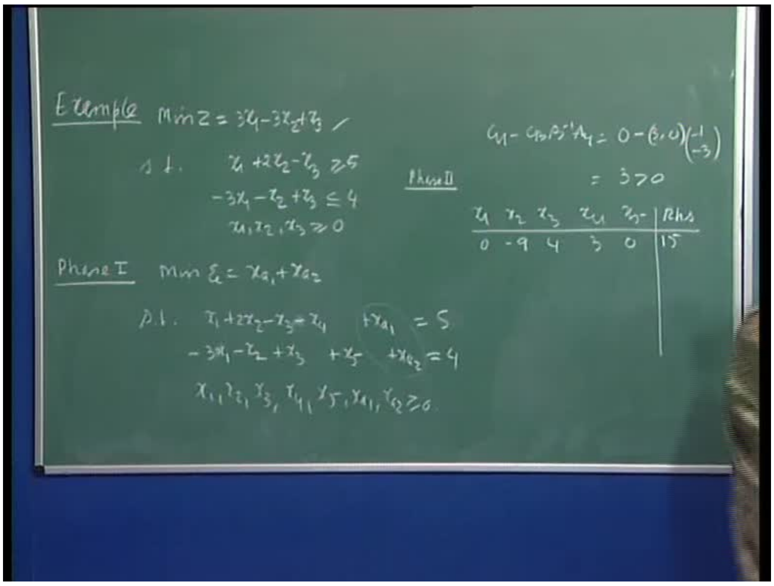

So, this is 3 which is greater than 0. So, this is 3 here, and finally, I do not need to do it for this x 5 is the basic variable which will also be 0 here. You can verify the computation, it will be 0. So, you see, now, I have new C j bars everything else is the same. So, I can just remove this; I can get rid of this, but I will leave it, leave it. So, therefore, you can start your phase 2 from here, may be let me just do it for you here. So, what will it be now x 1. So, this is your phase 2, phase 2, so, x 1, x 2, x 3, x 4, x 5, that is all I need. The artificial variables have done their job right hand side and your new this thing is 0 minus 9 4 3 0 and this value x 1 is 5.

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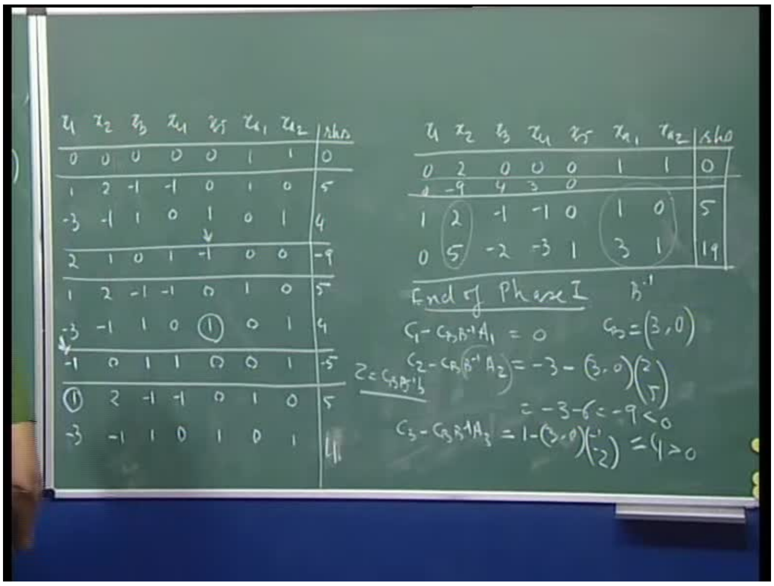

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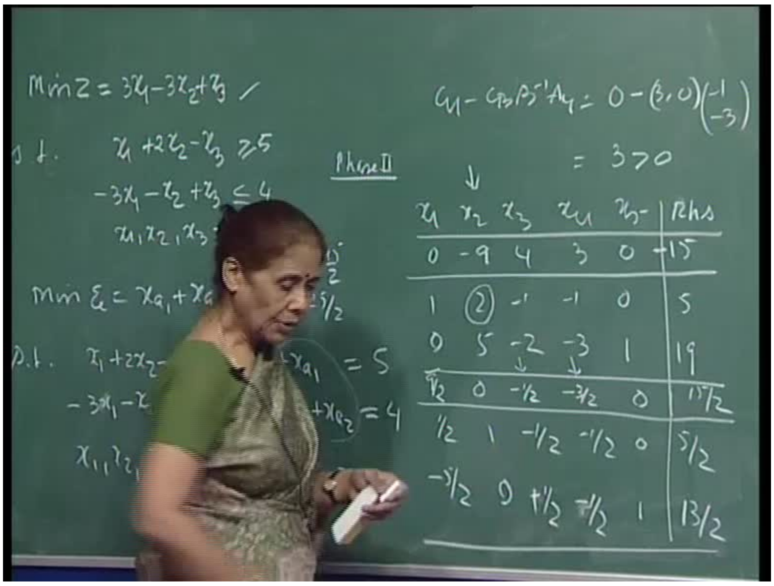

So, this is 15, this will be 15. You can verify that, you can again because your Z is C B B inverse B. So, just make the computation and see that this is 15, but I will show you in another way also. This is this and the rest of the as it is 1 0 2 5 minus 1 minus 2 minus 1 minus 3 and n 0 and your basic feasible solution is 5 and 19. So, this is your phase 2 starting tableau and we will now…

Since this is negative, the solution is still not optimal; we need to work for it. So, this will be the entering variable and 5 by 2. So, this will be your privet element. So, let us quickly do the computation here. This will be, let me that of 1 by 2 1 minus 1 by 2 minus 1 by 2 0 and this is 5 by 2.

Now, you want to make the 0 here. So, 9 times this I have to add; so, 9 times this will become 9 by 2 this is 0 9 times 4 minus 9 by 2 would be minus 1 by 2, then 9 times 3 minus 9 by 2 would be minus 3 by 2 and this is 0, because this is 0 here and then 9 times this you are adding, somebody should have corrected me, this is minus 15. So, 9 times this, so, you will need to compute minus 15 plus 45 by 2 which is 30 15 by 2, so, 15 by 2. And now, I want to make a 0 here. So, 5 times this, you have to subtract. So, this becomes minus 5 by 2 0 5 by 5 times you have to subtract, so, plus 5 by 2 and minus 2 minus 2 and plus 5 by 2, then plus 5 by 2 and minus 3.

So, that will be minus this is got and this remains as 1 and 5 by 2 you was multiplying by 5. So, 25 by 2 is subtracting from 19; so, 19 minus 25 by 2 38, so, 13 by 2. If there any error, you can make it. Now, this is our, this is our new tableau and what do you see here that corresponding to this. Of course, you might say that since we are applying Blands rule, I would enter this, and so, I would not look at this, but just to make the calculation short.

I agree, we can, we can, we would enter this, but the thing is that you see from here, this is negative; that means this column, corresponding column can enter the basis but the entries here are all negative. I cannot proceed with the simplex algorithm, and of course, this is your unboundedness criteria, but you, this is the objection would be valid in the sense that I may not come to this away. I will first go here, and when I enter this, this boundedness condition will persist because there is the direction present in the polyhedron in the feasible reason. So, we will detect it at some time.

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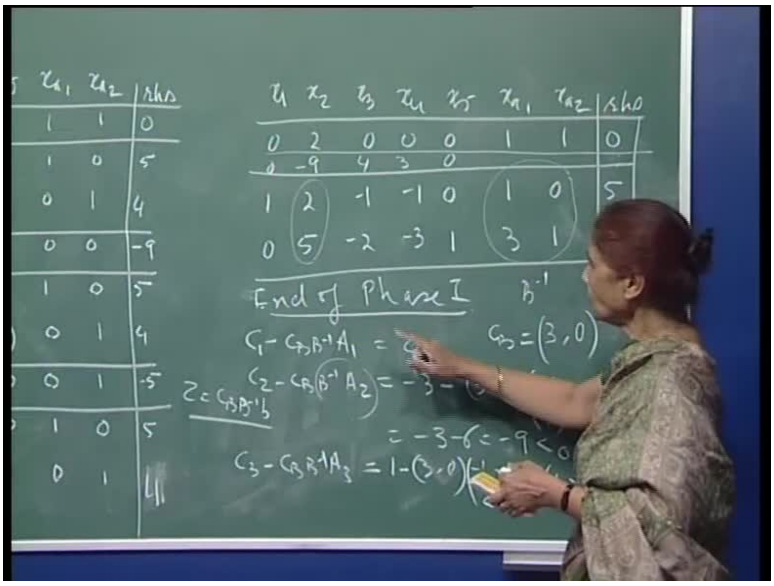

Right now I just look at the argument chart. I am saying that you see under this column, you have this is the profitable column but the entries are all negative. Therefore, the problem is unbounded; so, we can stop here. Now, what I want to say here is one, just one improvement on this method, is that see you we need the after phase 1, I need the C j bars corresponding to the new object to the corresponding to the original objective function.

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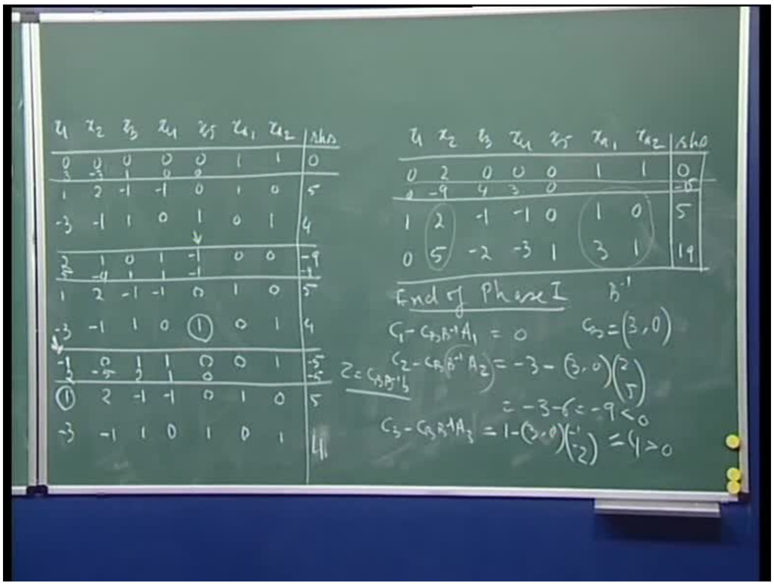

So, why do not we just carry? So, let me this quickly show you that if I carried my, so, this will be 3 minus 3 and 1; this is 0 0. If I carried this objective function and I did the same operations, that means when I, so, here, this was, so then, what we did we do? We added the last the last two rows and subtracted from this. So, let us do it for this also; that means when you add these two rows, this is what equal to minus 2.

When you add, this becomes minus 2, and you subtract, so, this becomes 5. Then plus 1, so, that becomes minus 4. This is 0, so, this does not change. This is minus 1, and so, that becomes 1, and that is 1, so, that is minus 1, and then, the objective function value here would be also minus 9, would it be because I am adding that this thing. So, this is what is happening. That is what I am saying. So, whatever pivoting you doing, you do with this carry the second objective function so that when you are ending this phase one, you can simply remove the top row and continue with that this thing.

So, let us just verify that, the, what I am saying make sense. Now, when you are previting here on this one, I will make a 0 here also because I made a 0 here, and this, so, therefore, you again add this row to this row. So, what will happen here? This entry will become two readable; so, you are simply adding this row to, the, this one. So, that becomes 2. Then you adding here, so, this is minus 5; this is 2 and this is 1 and this is 0. So, the last row you added to this one, so, that becames minus 5, because you added the last row to, the, this one, and therefore, that is also minus 5. Then, what did we do? We pivoted on; we pivoted on this 1, because this was minus 1. So, here, again I will make a 0. So, do, to do that, I will have to add twice this multiply this by 2 and subtract. So, just do that. Twice if you do that, what will you get? Twice 2 minus 2 0, you get 0. Then 4, so, minus 4 minus 5, you get minus 9. Then minus 2 and you subtract, so, therefore, becomes plus 2 at 4 and similarly 3 and the value here would be what we got was what was the value? Minus 15, because twice ten you subtract, you already had minus 5, so, minus 15. So, this is here. So then, you can make your algorithm because you do not have to do these extra calculations.

So, we carry both the objective functions making sure that this corresponds to phase 1 and this corresponds to phase 2. once we have done with phase 1, we can remove the top row drop the artificial variables and continue with the phase 2 algorithm. Another point which again since we are talking of C 1, we have an algorithm; you do not want to leave any loose ends in the sense that what other happen.

So, let me now conclude about phase 1 and phase 2. As I had been pointing out in the beginning also that when problems are very large, you have no way of just by looking at the problem and saying whether it has a feasible solution or not, what is the rank of the coefficient matrix. So, mostly all the software that are designed for solving linear programming problems have phase 1 and phase 2 built into them. Phase 1 as we said, obtains for you a feasible solution, a starting basic feasible solution. A base one in the if there is one available.

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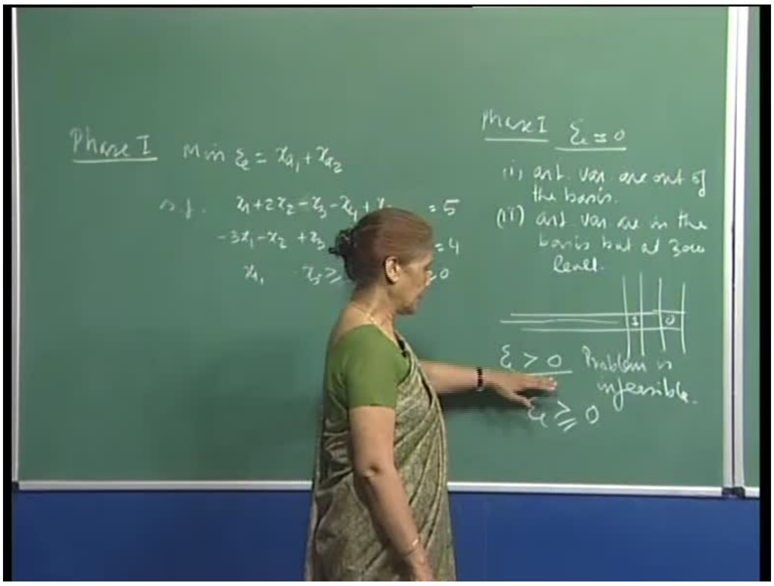

So, we saw that the two cases were there. In fact, so, phase 1 we saw that, if psi is 0, then that the problem is feasible, because all the artificial variables are out of the basis. Then, there were again two cases possible here; that are some artificial. So, case one would be that artificial variables are out of the basis. So, in that case, I have a basic feasible solution from among the regular variables, and so, I will continue with phase 2 or the other case would be that artificial variables are in the basis but at 0 level, because the value of the objective function is 0.

So, the artificial variables are in the basis at 0 level. Then, I showed you how you can try to replace the artificial variables by the regular variables by doing the pivoting, linear simplex pivoting, and you should be able to replace all the artificial variables. There can be a case when you have corresponding to an artificial variable, which is at 0 level in the basis, and then, there are no, see, remember to replace an artificial variable at 0 level, you just need some non-zero entry in the, this row corresponding to the regular variables so that you can do the pivoting and get rid of this artificial variable from the basis, but in case, there are no non-zero entries here, then of course you can conclude that this constraint, because if you removed this column and all the elements in the row, in the, in the transformed row are zeros, and therefore, you know, from your linear algebra theory that this row is, this constraint is a redundant, because you can express this row in terms of the remaining rows of the matrix, and so, this constraint is not required; it is redundant.

So, therefore, now you see phase 1 at the end will either give you a basic feasible solution from among the regular variables or it will tell you that there is a constraint which is redundant or more than one constraint redundant, and of course, the third case is when you have psi greater than 0, then that the problem is infeasible, and we also said that since psi is required to be non-negative. Since all the artificial variables are non-negative, so, their sum will be always non-negative. So, the unboundedness criteria will never be satisfied.

So, that means now whatever we had started with the assumptions, we had made in the beginning to build up the simplex program, simplex theory. We can now, we do not need any of those assumptions, because our phase 1 and phase 2 will tell you whether the problem is feasible or not, and if it is feasible, are there any redundant constraints.

We can remove those redundant constrains; that means, we do not anymore need that the rank of the coefficient matrix must be equal to m. We will know which are the redundant constraints. We can drop them, and then, so, in the, in the reduce matrix, will again be a full rank. All you will know that the problem is in feasible. So, the unboundedness criteria will not be satisfied. Then, at, then we start with phase 2 and then phase 2 will either give us a finite optimum solution or it will tell us that the problem is unbounded, because then, phase 2 we are back to the original problem, and so, you will know at the end whether the problem has a finite optimal solution or it is unbounded. So, in a sense, now, they are no loose ends left out. Whatever assumptions we had made, we have done away with them, and, we, by running this software used for simplex programming which are phase 1 and phase 2 already included in the program will give you all the answers. What, whatever answer is possible, it will give you that answer. So, I think that sort of things the simplex algorithm to an end.