Linear Programming and its Extensions

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Module No. # 01

Lecture No. # 08

Development of the Simplex Algorithm Unboundedness Simplex Tableau

In the last lecture, I had told you that if the linear programming problem as we have been defining the standard version, I have been referring to it has LPP 3. If it has optimal solution, then it will have, then a basic feasible solution will also be an optimal solution. Then I also showed you that if b is a basis for the same linear programming problem, then I can always find a vector, cos vector c such that b is the optimal solution for that corresponding to that cost function.

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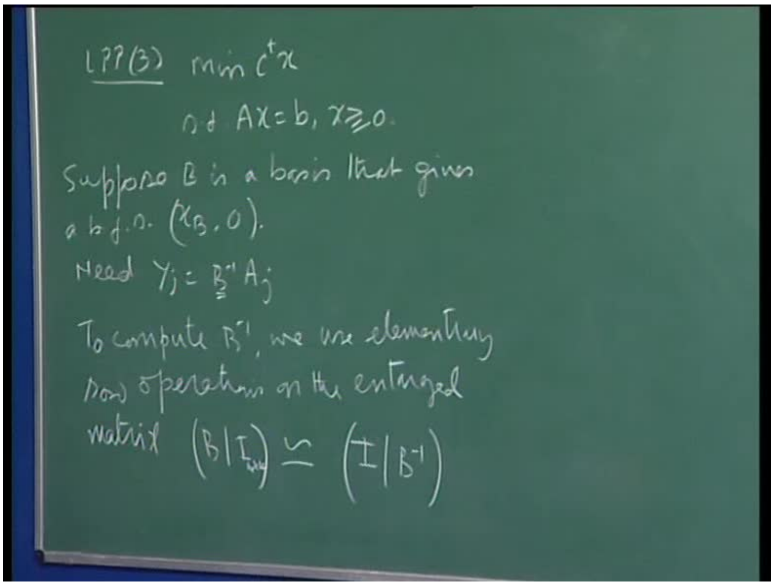

So, after that, we now start developing the simplex algorithm and let me now start building up and computing the quantities that we need to proceed with a simplex algorithm. So, the first thing is you see, let, so, we always have this LPP which I am calling as 3 is minimize c transpose x subject to A x equal to b x greater than or equal to

0. So, for this LPP, suppose B is a basis, that gives a feasible solution, that gives a basic feasible solution x B 0, and here, again, I am using this notation that I have renumbered the variables and the corresponding columns of a so that the first m variables are your basic variables and the others are non-basic variables and they are 0.

So, now, I need to first have the quantities; so, need Y j as B inverse A j, because then only I can pivot on element here corresponding to, so, I had already develop these formulae for you earlier. Anyways, now, this is B inverse A j. So, I need to compute this. Once I have this, then I can compute my Y j. So, I will try to show you an iterative method for computing B inverse, which requires, which makes up the, which uses up the calculations made earlier for 1 B, and then, when we change the basis, how do we update our basis inverse. So, this is the whole idea.

So, let me show you. So, the idea here is that to compute B inverse we use elementary row operations. So, everybody is familiar with elementary row operations. We use on the enlarged matrix B I - where I is of the same size as B, that is, it is m cross m. So, I do this and I reduce this to I, then what you get here is your B inverse, because by elementary row operations, I start making a first column as 1 0 0; second column as 0 1 0 0 and so on, and I do this, what happens is that you get this on the right hand side. So, this is how you compute the d inverse.

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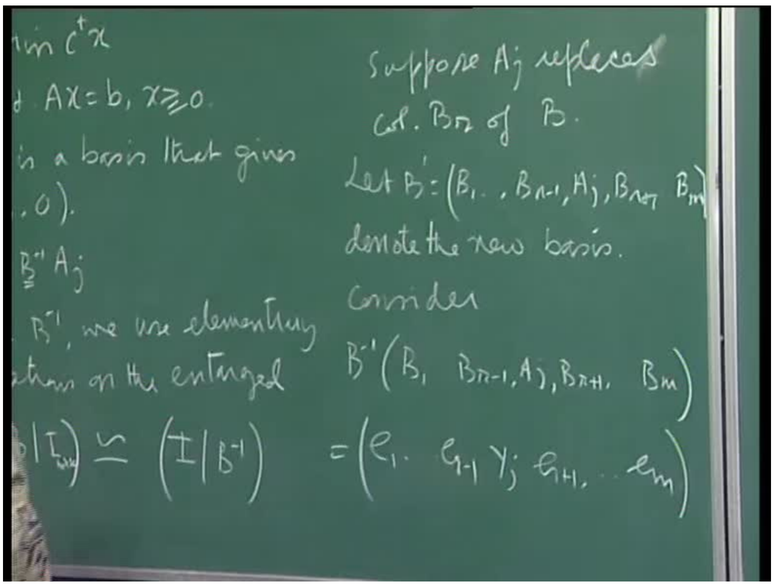

Now, suppose, suppose A j replaces column B r of B and I told you the rule for this so that you get a basic feasible solution. We will revisit at again, but anyway, so, suppose it replaces this. So, I need and let B prime equal to B 1 B r minus 1 A j B r plus 1 B m; this is the new basis. At this, denote the new basics.

So, now, I would need to compute the inverse of B prime also, and if I have to do it from (( )), it will take lot of time, and so, I want to show you. So, the thing is that if suppose, I carried my column A j along with when I was doing these elementary row operations, in other words, if you multiply, consider B inverse of B 1 B r minus 1 A j B r plus 1 B m. Now, since B inverse is the inverse of B 1 B 2 B r B m, you see this will reduce, this will this is actually e 1 to e r minus 1. These are unit vectors with this is 1 in the first place; this as one in the r minus 1th place and this will B inverse A j is your Y j by our definition; this is Y j and these are again all e r plus 1 e m.

So, in other words, I already have this. And if I have Y j with me, then in order to compute the B prime inverse, I simply have to reduce Y j to the corresponding e r by pivot by elementary row operations. Is it ok?

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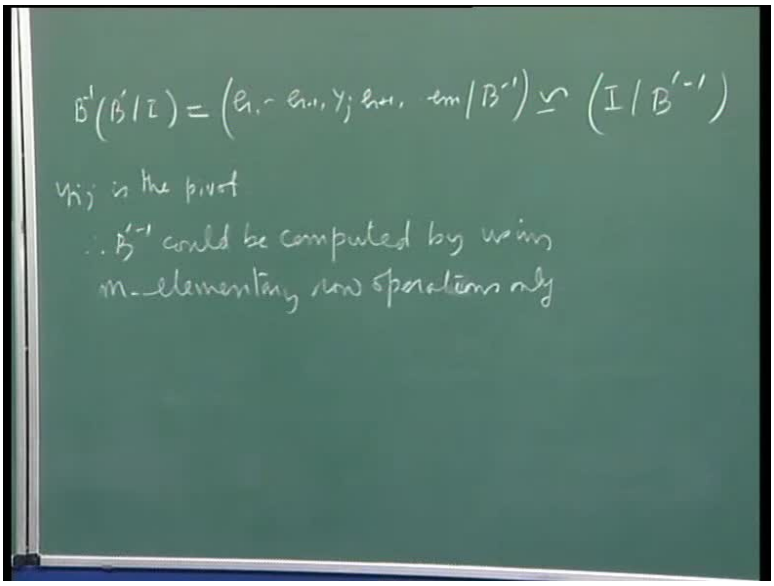

So, the in other words, what I am saying is that you start with you want to compute the inverse of B prime, so, we would have to reduce this to identity matrix. If you have already multiplied it by B, then what you have here is e 1 e r minus 1 Y j e r plus 1 e m, and here, you have your B inverse; this is B inverse. So, I have pre multiplied the whole thing.

Now, in order to reduce this, so, once I reduce this to e r, then and I do all the elementary row operations for the whole row together. Then, what I will have here would be may be prime inverse and this is the whole idea. So, in other words, I do not have to reduce my B prime from the beginning to an identity matrix. I already have done that much work. So, these m minus 1 columns are already reduce to identity form. I need to do this. And y ij, so, y ij is the pivot element. Remember, we said that y ij has to be positive and the ratio has to be minimum for the new solution, for the new basis to correspond to a feasible solution. So, we have discussed on that. So, y rj is the pivot element. So, when I reduce this, finally to identity here, what I have will be B prime inverse. So, this is the crucial point here. All I am saying is that iteratively I am able to compute my new basis inverse at each iteration.

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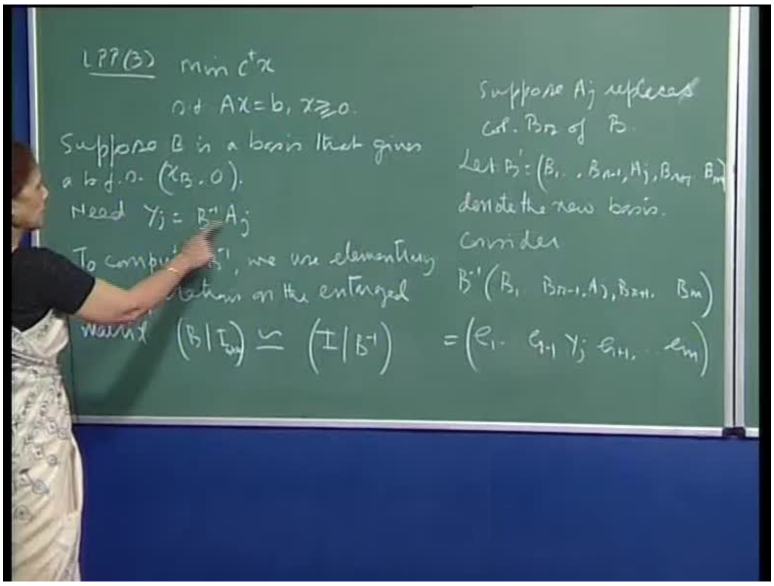

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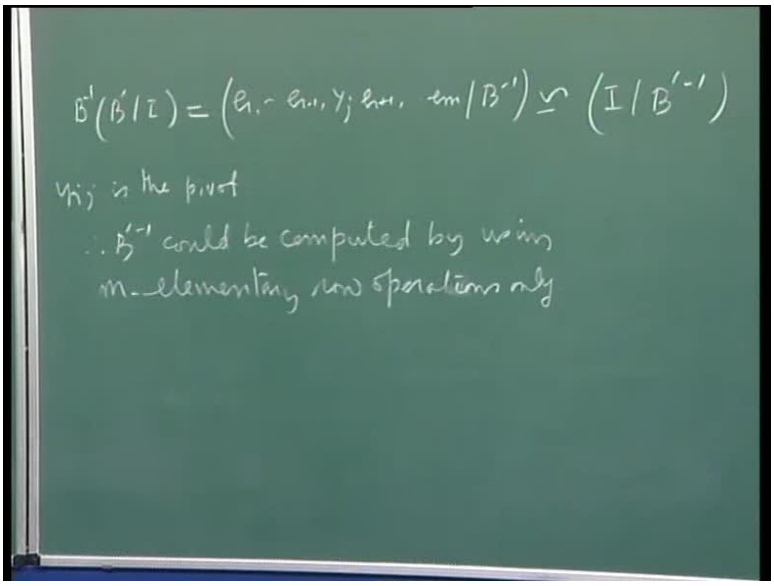

So, once I have computed this, then I can just do pivoting on one column - the column that I have brought into the basis, and if I already have these quantities with me, then I do not need to multiply B inverse A j. Remember, each operation cause something in the computer and the idea is that you try to make your computations as minimal as possible and as fast as possible. That is the idea. So, here, I have able to use my earlier computations for B inverse, for computing B inverse, and then, by simply doing m more operations is the pivot and so either. So, therefore, B prime inverse could be computed by using m elementary row operations only. So, instead of doing it for the whole this thing, and of course, we do it for the corresponding columns of B prime here, whatever B inverse here also to get this thing.

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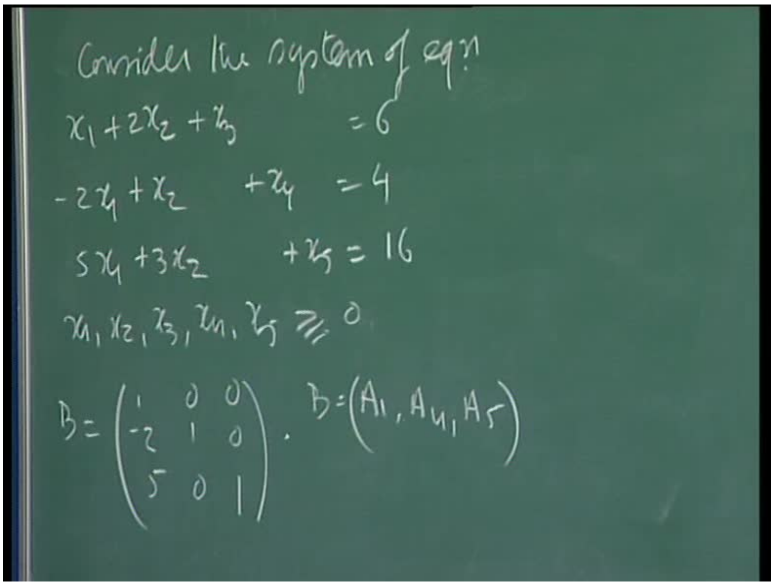

So then, I have shown you how to go prompt one basis inverse to in other, but then, we need few more computations. So, anyway, let me then take up an example here and show you the how we, suppose you have this system of equations. Consider the system of equations x 1 plus 2x 2 plus x 3 is equal to 6 minus 2x 1 plus x 2 plus x 4 is equal to 4 5x 1 plus 3x 2 plus x 5 is equal to 16 and your variables x 1, x 2, x 3, x 4, x 5 are all non-negative.

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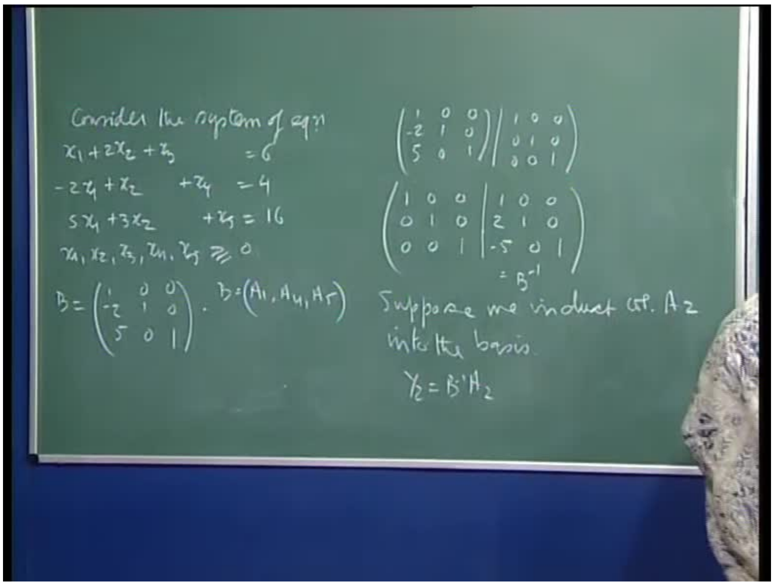

So, let me start with B, with my basis B as consisting of 1 minus 2 5 0 1 0 0 0 1 and that means my basis B consists of 1 minus 2 5 which is A 1, A 4, A 5, these are the column. I am not renumbering them here, because now, I will be actually computing it for you. So, therefore, we can see what is happening, fine. So, we said that we will use elementary row operations of 1 minus 2 5 0 1 0 0 0 1 and 1 0 0 0 1 0.

So, you want to simply reduce these two identity to e 1; which means that twice this you add here and 5 times the first row you subtract. So, I will do both the operations and one go and that will give you 1 0 0. So, the top row does not change here 0 0. Then twice this you adding here, so, that makes it is 0 here; this is 0, and twice this you are adding's with 2 1 0, then 5 times we are subtracting the first row from the last row which makes this is 0 0 1 and 5 times. So, this is minus 5 0 1. So, you can verify that this is now B inverse. So, elementary row operations helped us to obtain them inverse of this given matrix B.

Now, suppose I induct, we induct column A 2 into the basis, and so, I will need. So, the idea is that if I already have this quantity with me, the scalars that I need, see it is clear that what I would need Y 2 is B inverse A 2; that means these are the scalars that I need for expressing the column A 2 as a linear combination of the basic columns.

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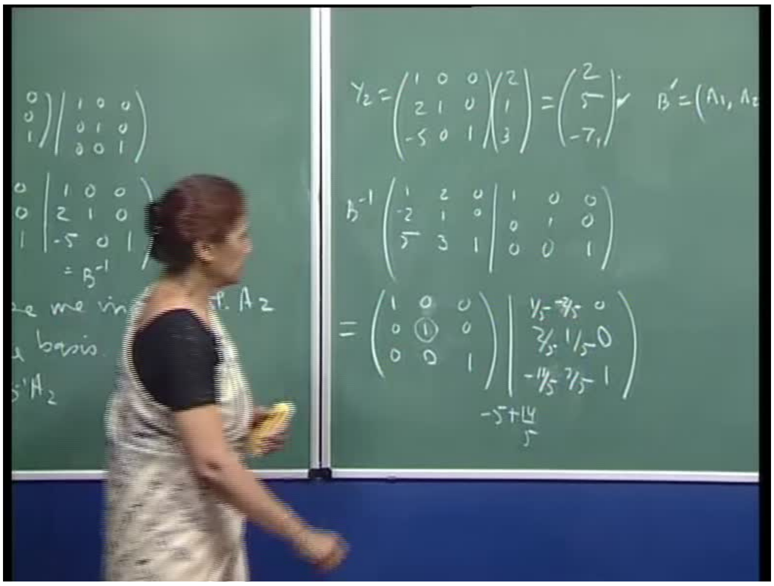

So, what is my Y 2? Y 2 would be B inverse; so, B inverse now is 1 0 0 2 1 0 minus 5 0 1 times the second column which is 2 1 3, so, that is, 2 1 3. So, that gives me 2 and this 4 plus 1 5 and then it is minus 10 plus 3, which is minus 7.

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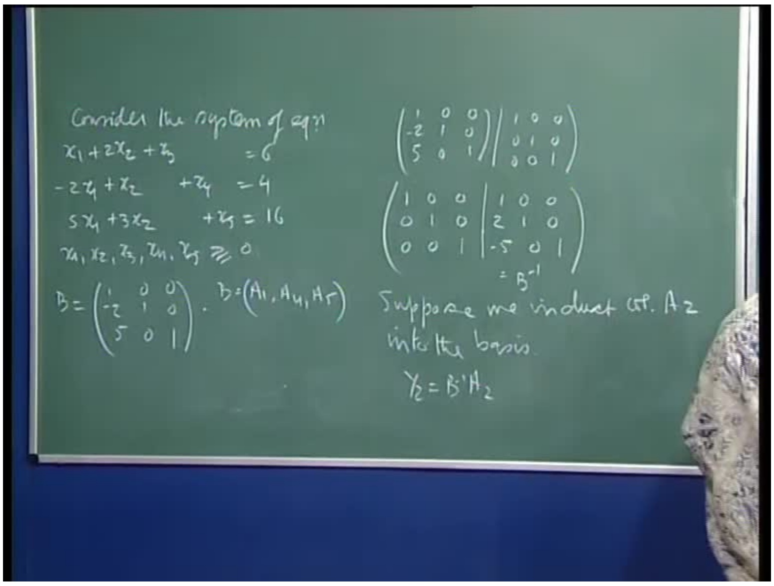

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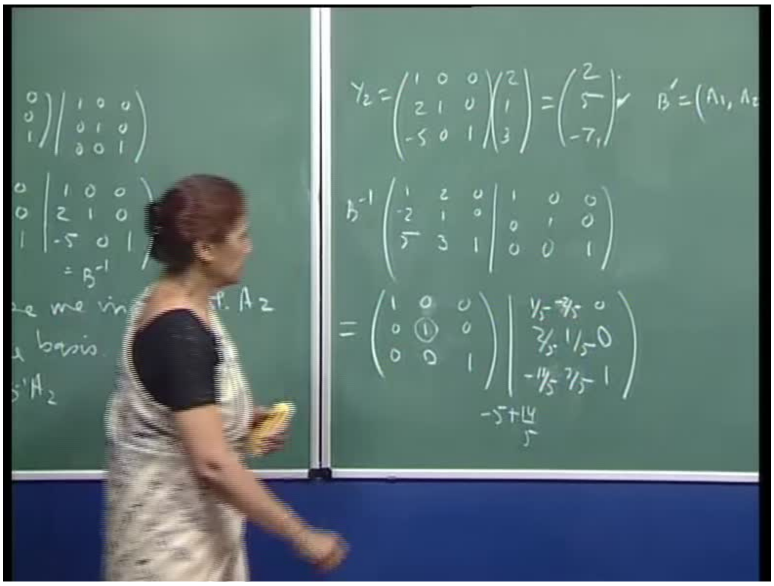

So, I need to now compute. Now, a B prime - a new basis inverse; so, B prime right now is I do not know, yeah, fine, sorry. So, all the components here are non-zero; so, I could replace A 2 by any of the basic columns, but since I want to a basic feasible solution, this one is out. I can either replace B 1 by A 2 or B 2 by A 2. So, let us choose this one.

Suppose I choose this one; that means now a B prime will be, B prime is will be your A 1 A 2 and A 5.

The second column here which was earlier A 4, A 4 I am replacing it by A 2. And now, I need B prime inverse. So, to compute B prime inverse, that is what I was saying that, here, you have already see in this thing, I start with say if you consider the matrix A 1 is 1 minus 2 5, now, A 2 is 2 2 1 and 3 and this is 0 0 1. So, I need to compute the idea is 1 0 0 0 1 0 0 0 1. I will do this, but if I multiply this by B inverse, then what do I have? This will reduce to identity matrix already, so, identity vector. So, this will be 1 0 0 and B inverse A 2 I already know is 2 5 and minus 7, so, 2 5 and minus 7 and this is 0 0 1, and so, on this side, it will be B inverse which is 1 0 0 2 1 0 minus 5 0 1, sorry, so, here, B inverse, this is B inverse.

So, because I have multiplied the whole tableau by, so, this is equal to this. So, here, you have, now, this is the thing. You need to reduce this two - the identity vector which is e 2 here, the second basic a basis vector. So, I will reduce it to e 2 and do some elementary operations. And in the process, what I get here would be then my new basis inverse. So, since I have to pivot on this one, so, let us do the calculations here. I divide the whole thing by 5, so, that makes this 5 and this makes this 1 by 5 and this becomes 1.

Now, twice you subtract from here, so then, that this becomes 0. Twice I subtract from here, so, 1 minus 4 by 5 will be 1 by 5 and this will be minus 2 by 5 and this is 0. Then 7 times this row I have to add here; so, 7 times you do it, this will become 0 and 7 times I am adding here. So, this is minus 5 plus 14 by 5, which is minus 25 plus 14. So, that will minus 11 by 5 7 by 7 times this you are adding. So, that becomes 7 by 5 and that is it. So, I have new basis inverse.

So, this is the whole idea. So, just for one column, I had to reduce one column and then I could get, and using my computations for B inverse, I could now compute my B prime inverse because B prime inverse differs from B; B prime differs from B in exactly 1 column. So, why do the calculations from the beginning and we can make use of this?

Now, comes the next step; so, that means I have sort of given you a way, not a very organize thing. Right now I will give you the simplex tableau and then give you the organize way of, but any way, we know how to go from one basic feasible solution to another. In fact, right now I have given you the formula to compute the basis inverse from one basis inverse to another basis inverse. How to do that? But then, we need a few more things and this is we want to go to a basic feasible solution, which improves the value of the objective function. See the idea for the, when we progress with the simplex algorithm is to be able to move from one basic feasible solution to another, and in the process, I also improve the value of the objective function, and in our case, the objective function is a minimization function. Therefore, I need to reduce the value of the objective function.

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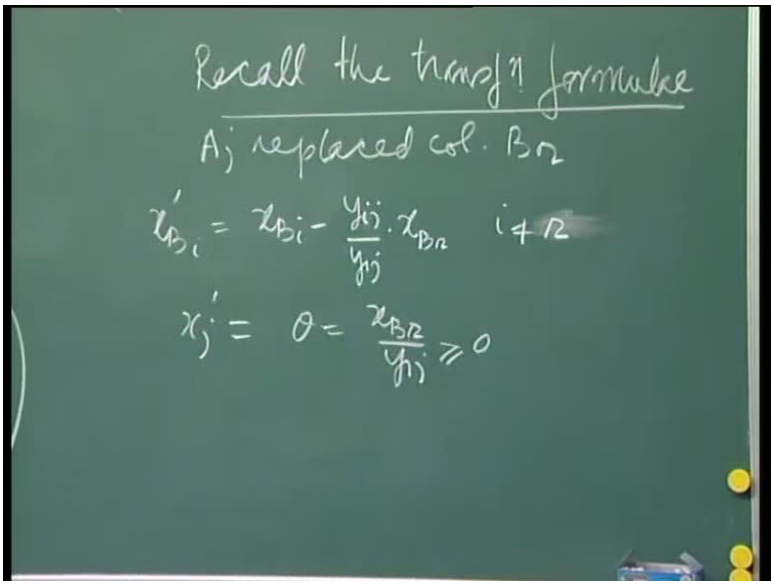

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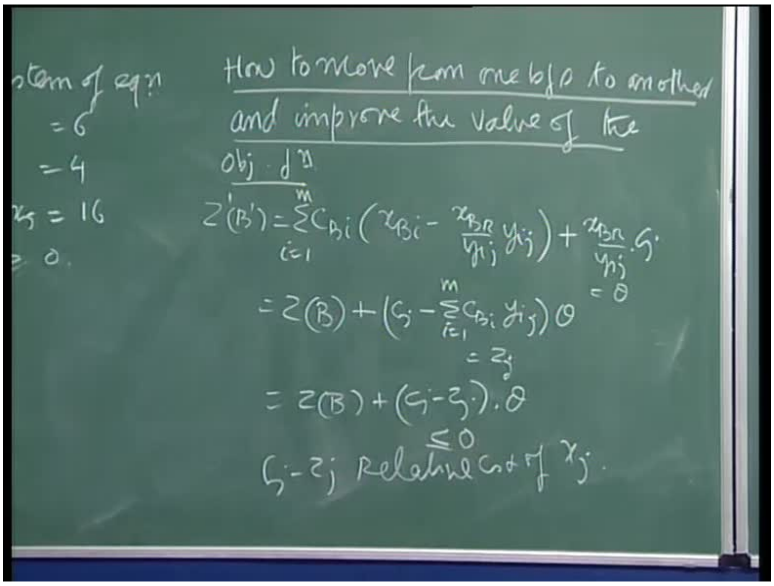

So, how do we do that? Now, let me just recall the transformation formula. So, I will recall the transformation formula and what did we say that when A j replaced column B r, the notation is all clear. Then our new x Bi prime was given by x Bi minus y rj into x. This would be the transformation formulae. This is your x Bi minus y ij. This is y ij y rj. This is would be, sorry, x BR when i is not equal to r. So, when I is not equal to r, because for I equal to r, so, this become 0. Remember, x BR has been made 0 in a new basis the rth. The old r basic variable has been made 0 been non-basic and you have your x j prime equal to theta which is your x BR upon y or j. And this is greater than 0 because y rj is positive. Remember, that is why we pivot on y rj. So, y rj is something which is strictly positive and then this is your new basic feasible. So, this is your transformation formula.

And using this transformation formula, let us see what would be, so, I will keep that in tacked. So, what I am now going to say is to, how to move from one basic feasible solution to another and improve the value of the objective function? So, this is our ultimate aim. We want to go and improving the value till we can know further improve on it and that gives us the…

So, let us see the value. So, your Z prime corresponding to B prime is what. Since that is your basic feasible solution, new basic this is C Bi into x Bi minus x BR upon y rj into y ij, this summation i varying from 1 to m. See I will include i equal to r in this summation, because as we have seen for i equal to r, this number becomes 0. So, therefore, the contribution to this sum will be 0. So, therefore, I will just keep it like this because that will help us to plus your x BR upon y ij into C j, because the jth variable has now become basic variable. This we also refer to as theta fine. Now, you see, when you sum C Bi x Bi is your old objective function value, summation I will, that is why I kept this summation from 1 to m, I am not excluding i equal to R, and then, you have plus. See here if you write C j x BR upon y rj we as writing as theta and this is C Bi, so, minus summation C Bi y ij, i varying from 1 to m, this times theta.

Now, we get this quantity as special name because this is Z j. Let me call it Z j corresponding to the column A j. So, this is equal to Z B plus C j minus Z j into theta; theta is positive. So, now, you see, if you want the value Z prime B prime to be less than Z B, then obviously you want this to be less than 0, less than or equal to 0. If C j minus Z j is positive, then by including A j into the basis, your value of the objective function is going to go up. So, this is what we, therefore, I need to compute these quantities also, and so, you see that for this, I need my y ij's was and your value theta. So, when we write down the simplex table, I will show you how we get compute all these quantities, let us (( )) at the same time. So, C j minus Z j if it is less than 0 and the value will definitely prove equal to 0, the value would remain the same.

So, therefore, and we call C j minus Z j as the relative prices relative cost of x j. So, relative cost because this is the rate of increase. You can there, the, many interpretations to C j minus Z j that one can give. This as you see that for 1 unit increase in the value of x j that the theta is 1, then the value of the objective function changes by C j minus Z j, and remember, because we have assume linearity axioms proportionality and so on. When there is increase of theta in the value of x j, the total increase in the value of the objective function is C j minus Z j into theta.

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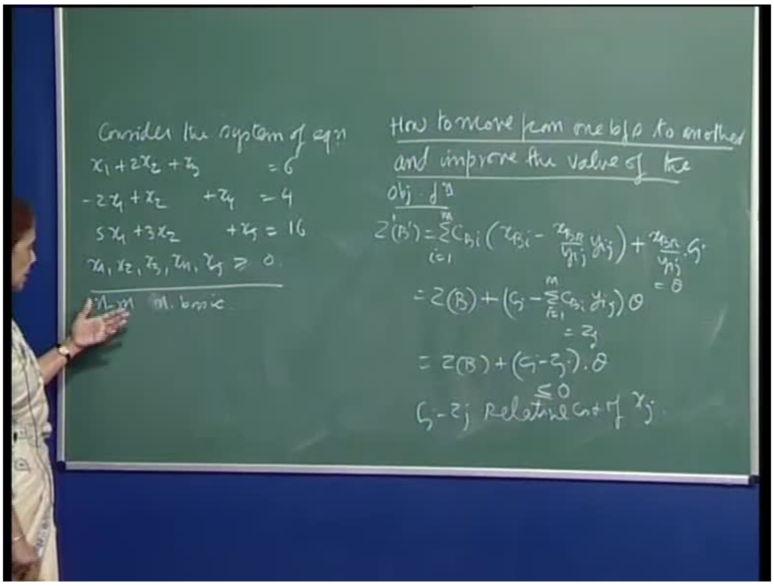

So, this will be your this thing, and now, we can, so therefore, you now with that means we know that we can go from one basic feasible solution to another, and in the process, improve the value of the objective function; that means now, let me just begin with this, so, you have n minus m non-basic variables; so, that means any of these n minus m columns can come into the basis depending on the condition being satisfied; that means when they come into the basis, the new solution should also the basic feasible and so on.

So, we have choices, so, we can do this, but then, that means I should have computed C j minus Z j for all these non-basic columns n minus m and then I choose the once for which there is an improvement in the value of the objective function, and as (( )) in his initial simplex algorithm specified that let us go for that column for which C j minus Z j is most negative, because then you improve, supposedly you improve the value of the objective function by a large amount but that is not correct, because the value of the objective function changes by C j minus Z j into theta. So, non-basic column for which variable for which C j minus Z j is very small; that means in magnitude, it is large. It is a negative number, but the corresponding theta may be small. So, that part is there, but in any case for all practical purposes, this was being used as a rule to enter a non-basic variable into the basis and then I will later on discuss the other ramifications of this rule, because it was shown to be and practical.

And therefore, now that means in a nutshell, the algorithm is that you start with the basic feasible solution. You compute these C j minus Z j’s. If all C j minus Z j's are greater than or equal to 0, you cannot improve the value of the objective function and I have already given you this optimality criteria; we discussed if some time ago. Therefore, if at a basic feasible solution, you have C j minus Z j’s are all non-negative. You will say that the current a solution is optimal, and if you have some C j minus Z j’s which are less than 0, then I will bring that column into the bases and then compute the new C j minus Z j’s check for optimality criteria and to proceed till I hit the optimality criteria.

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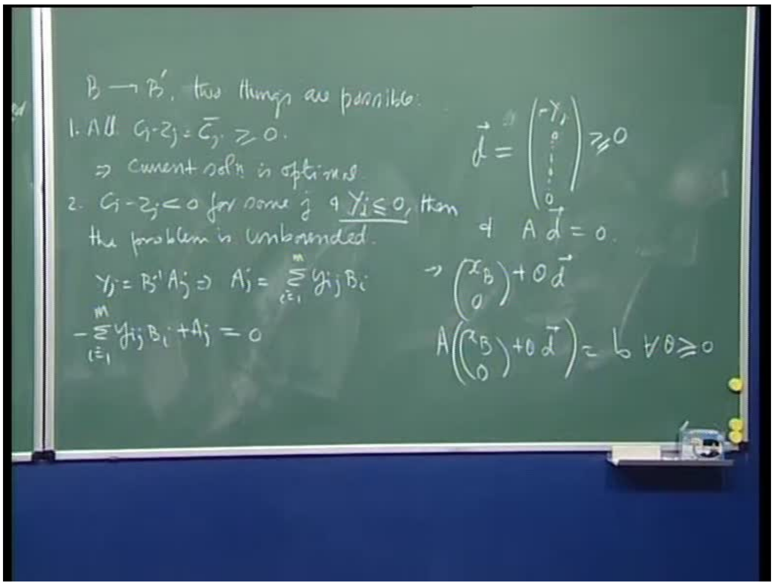

So, two things are possible. At this point, when we go from B 2, so, I move from B to B prime, now, two things are possible - one is or in other words, actually from B to B prime I proceed two things are possible all C j minus Z j, which we also refer to as C j bars is convenient. So, these are greater than or equal to 0. This would imply, this implies that the current solution is optimal because we have the optimal, optimality criteria or what will be second thing that C j minus Z j is less than 0 for some j, for some j, and the corresponding vector is all less than or equal to 0.

Remember, to move from one basic feasible solution to another, I needed the condition that my y rj in the element on which I want to pivot to be positive. Now, if I want to C j minus Z j is less than 0, so, I want to bring this into the corresponding column A j into the bases, but I cannot proceed with the simplex algorithm because all entries under the column A j or less than or equal to 0 and I need at least one pivot element which is positive. So, we will, we cannot proceed with the simplex algorithm here, but the.n, we can make certain conclusions about the problem.

So, let us see, how do we see, I want to show you that if C j minus Z j is less than 0 and for some j and Y j is all components of Y j are less than or equal to 0, then you can show that the problem is actually unbounded. So, if this happens for some j and this this, then the problem is unbounded, and if you remember our discussion earlier, I should you that unbounded; that means that your polyhedron is has a direction, and now, actually we will say that will show you that it is a direction along which the value of the objective function is decreasing.

So, if there is a direction in the polyhedron and along the direction, you, by moving along the direction, you can keep on improving the value of the function; that means it can be made as small as you wish. This is the whole idea.

So, let see. Now, remember, you are Y j is nothing but B inverse A j, or in other words, this can be, this implies that you can write A j as summation y ij B i i varying from 1 to m or I can write this as minus summation Y ij B i i varying from one to m plus A j is equal to 0. And since all Y j's are, the vector Y j has all components less than or equal to 0, it means that y i j's are all non-negative of 0.

So, minus this is all positive. So, therefore, you have here, this implies that minus Y j then 0 0 1 0 0, this is minus; this is a feasible solution. All components of this solution are non-negative and, sorry, no, it is not a feasible. I am sorry, this is not a feasible solution. So, this implies that this is non-negative. As a vector, it is non-negative; it is a non-negative vector, and from here, it follows, and your A of, let me call this as, because we have been giving it a names.

So, this is let me call this and A d is 0. So, d is the non-negative vector and such that A of d is 0. So, this implies, this implies that when you write x B 0 plus theta d, this is a, when, if we consider this, then A of x B 0 plus theta d is actually b, because A of d is 0 and this is already a basic feasible solution. So, this is equal to b and it is the components here as I have shown you are all non-negative, because components of x B are non-negative; components of d are non-negative and this is a true for all theta greater than 0.

Actually greater than or equal to 0, but so, therefore you see that the solution remains feasible, no matter what the value of theta. I can take the value of theta as because I wish and I will remain in the feasible regions. So, this is the direction. This is a direction which is present in your feasible set; that is the direction of the polygon with; so, this is what you have. Therefore, when you have Y j's all non-negative or 0, then I can show you the existence of a direction in the feasible set, and now, compute the value of the objective function.

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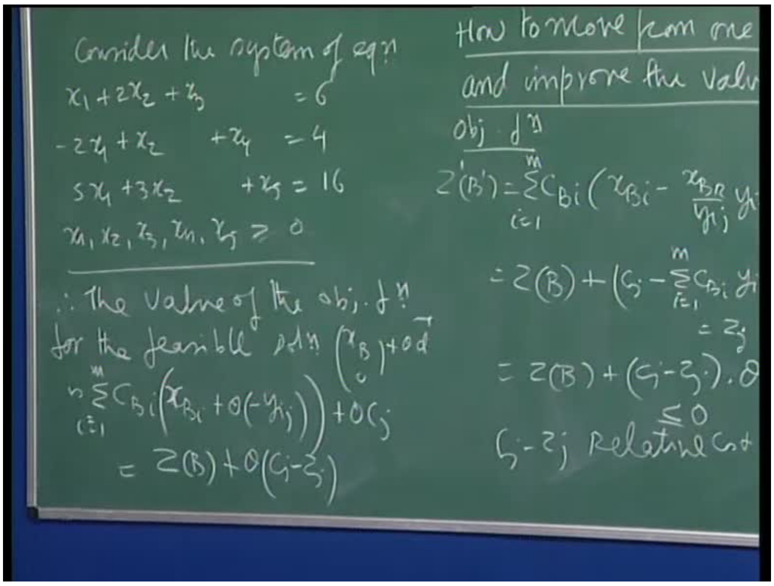

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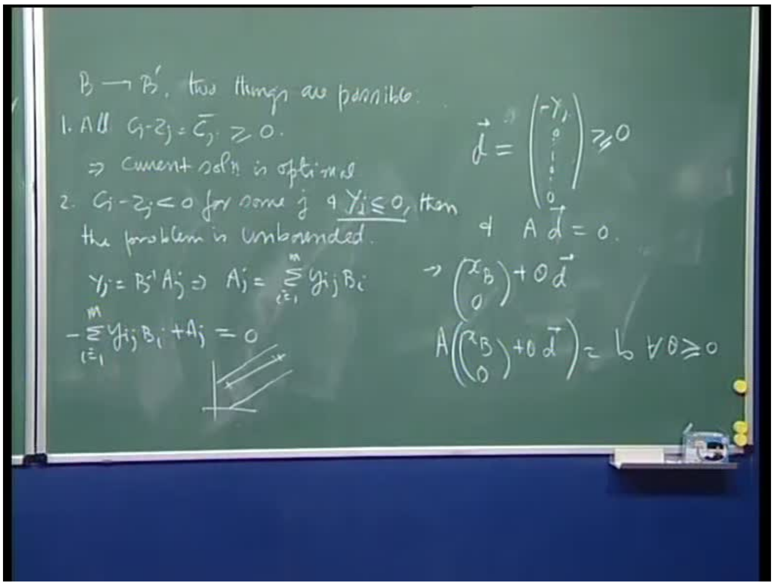

So, from here, you see if you do this, let me all, I will compute. Therefore, the values of the objective function for the feasible solution. Remember, it is not a basic feasible solution because it has m plus 1 component. This feasible solution has this, this, vector d has m plus 1 components non-negative and this as m component. So, the sum is has m plus 1; so, this only a feasible solution, and if you recall your this thing, I had showed you that you have this. Then there is a direction present, then you can go it move along this direction and remain feasible.

So, therefore, when you move along this direction, you are no longer at a basic feasible solution, because you are not you are in the interior of the feasible region so that what has to be taking it. Therefore, the value of the objective function for the feasible solution x B 0 plus theta d is, see remember C Bi, so, this will be summation C Bi x Bi. Just be patient and will be able to write down. From here, you will get plus theta minus y ij. So, this whole thing summation i baring from 1 to m, the same expression actually plus theta C j.

So, exactly what you get from here only thing is here. Therefore, this becomes Z B plus theta times C j minus Z j, the same expression. The only thing is that now here C j minus Z j is less than 0 and theta can take any positive value. So, you see that this can be made to go minus infinity. I can go on increasing theta and that is the idea that I am moving along this direction. My value of the objective function is decreasing as I move along this direction, and so, I can make the value of the objective function as small as I wish.

So, this shows the, that the problem is unbounded. So, the movement you encounter this situation you can stop because your problem is unlimited. So, we will not be able to obtain a finite optimal solution.

Now, let us demonstrate this of the, I will show you these steps of the simplex algorithm through an example may be will take this example only and also give you the organization of the tableau, because then, we can get all the information that you need from the tableau.

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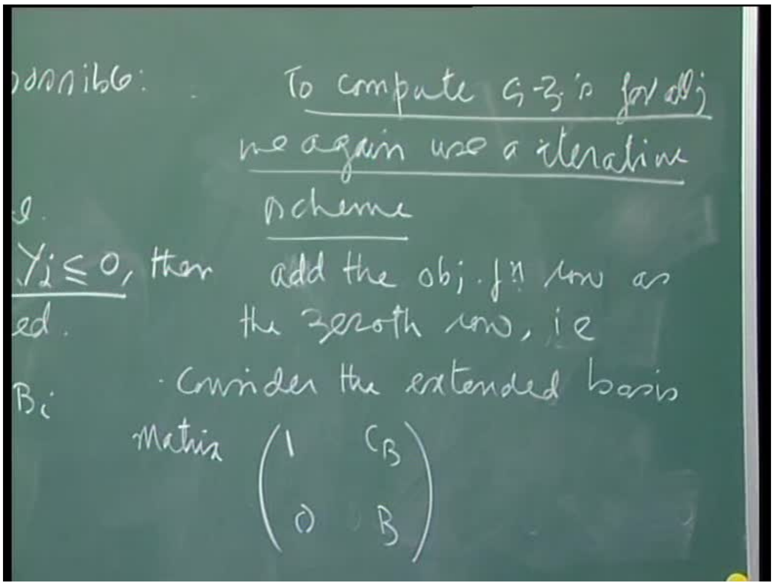

And yes, but before that, so as I said that we need the quantities C j minus Z j also. So, to compute C j minus Z j's for all j, we again use iterative scheme. So, the idea is that, so, the same iterative scheme I will try to use except in extended way. So, consider add the objective function row as the 0 through and where. So, essentially I am saying is, that is, consider the extended basis matrix 1 C B 0 B. So, I have added the objective function also as a part of the constraints.

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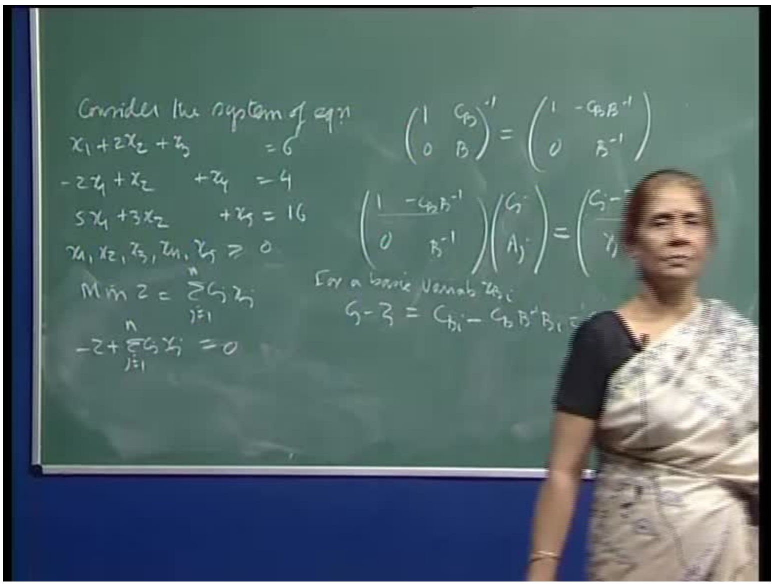

In other words, what we are saying is that I will write down since you have minimization of z equal to summation C j x j j baring from 1 to n. What I am suggesting is that we write this as, we just consider this as an equation that will be minus z plus summation C j x j j baring from 1 to n as 0. So, to the, already given constraints A x equal to B, I am going to add this as also equation. And therefore, my basis matrix will then become m plus 1 by m plus 1.

So, this is now m plus 1 by n plus 1, and if you look at the inverse of this, now, this, you can sit down and verify for yourself, would be actually 1 minus C B B inverse 0 B inverse. Remember, I have added one more variable and that is why this column. This is minus z; so, here, the corresponding column z does not appear in any of the equations. So, therefore, the corresponding column is minus 1. I will corresponding column is 1 0 0 because I need to have a m plus 1 by m plus 1 matrix; this is a whole idea. So, this inverse will be this.

Now, if you multiply 1 minus C B B inverse 0 B inverse with C j and A j, so, you pre-multiply. Remember, this is what we are doing because we need the quantities Y j's so that I will get from here B inverse A j, but now, I will get something more. When you multiply with this, that means the first row here with the corresponding column and you remember added the first row. So, this will become C j minus Z j, because C B B inverse A j and this will be your Y j.

So, this is really good because I heed these quantities also. And remember, before I proceed from, proceed with the simplex algorithm, that means from one basis to another, I need to know whether my optimality criteria is being satisfied or not. So, if I have all these C j minus Z j's for the non-basic variables at hand, then I can first scan, this, these numbers, and if there is any negative number, then I will proceed with the simplex algorithm, and of course, as you have seen that even if this held and then you discover this condition, that of course you will stop. So, I definitely need to have the quantity C j minus Z j's with me.

So, therefore, the ideas that you maintain this as a equation and you consider the expanded basis and then you keep computing the inverse, and since these elementary row operations, the premultiply, the do, we executing the elementary row operations is equivalent to premultiplying by the basis inverse, and in this case, it will be the extended basis inverse.

So, we can keep updating these numbers. Yes, may be there is a (( )) but we can keep going back to the calculations, and so, they should be no problem. In a sense that this is C j minus Z j by Y j and you see that the C j minus Z j for a basic variable, for a basic variable x Bi the C j minus Z j will be C Bi minus c B B inverse B i, because the Z B i would be C B B inverse B i, but B inverse B i is, what is B inverse B i? B inverse B i is e i. So, this will reduce to C Bi minus C Bi which is 0. So, for the basic variables, the corresponding relative cost which is only 0, and so, when I want to update, so, in other words, when I want to update this column and I am going to pivot on y rj, then I also make this number is 0 because A j is now with going to enter the basis.

So, it is become a basic column and the corresponding variable is basic variable. So, the new relative price for x j should be 0. So, therefore, in my pivoting since I making this column as e r, i make a 0 here and then I would have updated the whole tableau. So, let us hope that if I have list out on something or the order has been different, then when we do the computations, try to make it more streamlined.

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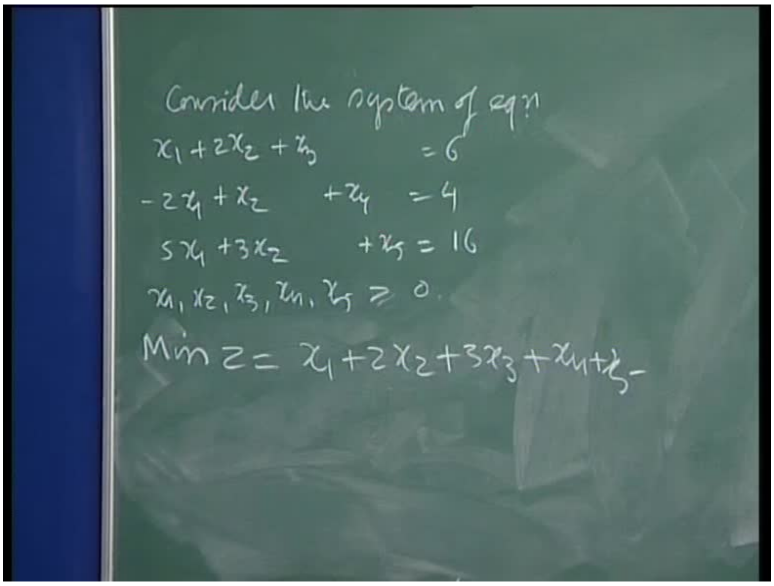

So, let us now take up this example. I will introduce A 1 objective function for you and go on explaining the computations. So, you can take, suppose I am saying that you minimize z equal to x 1 plus 2x 2 plus 3x 3 plus x 4 plus x 5.

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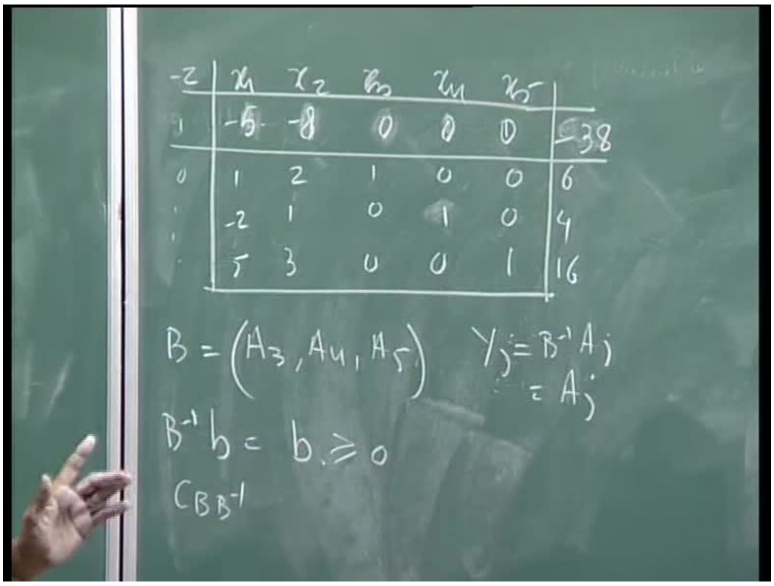

So, this is your new problem; simplex linear programming problem that we write down the tableau for you. So, you begin that tableau. This is x 1, x 2, x 3, x 4, x 5, and remember, we said that we write the objective function as the first row. I am not try, you can may be if you like you can say minus z here, then, the, but this column, we will not carry because this is always remain as part of the basis.

So, I will not write anything here, then this would be your 1 2 3 1 1. And remember, right now this value is 0. Then, you have your, so, let us just to separate it from the equality constraints. We now do this 1 minus 2 and 5 2 1 and 3 and this is a 3, x 4, x 5 are all unit. So, this is 1 0 0 0 0 0 1 0 0 0 1. Then you have here 6 4 and this is 16, 64 and 16. So, this is your…

Now, another point which some of you may have, I mean you may have out of this question is that see what I am starting with this system of equations and I am doing elementary row operation. So, I hope you all are aware of the fact that elementary row operations do not change a system of equations.

So, that means when I do elementary row operations, the new system will have the same set of solutions as the original one, and therefore, I can very freely go on doing these elementary row operations and we try to reduce the system of equations to get a basic feasible solution and go on to another basic feasible solution and so on. So, this is the thing.

And now, you see that these numbers are all non-negative. So, I have a readymade starting solution. I can chose my B to A 3, A 4, A 5 is a simplification, and of course, we will also have vocation to sometimes chose a basic feasible solution which is not as easily available. In this case, it just happens that I have a identity matrix. Therefore, my basic feasible solution is simply because otherwise your basic feasible solution is given by B inverse b, but B inverse is identity. So, in this case, it is simply b, and since b is all non-negative, so, I can say that I have a starting feasible solution basic feasible solution.

Now, this simplifies lot of things, because in this case, what is your… Now, you need to compute your C B B inverse, because since B inverse is identity, what about Y j's Y j is B inverse A j, which is also A j because B inverse b is I, so, B inverse is I, fine. So, now, the only thing is that I want these quantities that top row to read C j minus Z j, and for that the condition is that, these number should be zeroes. So, if I do that, then I would have extended basis inverse.

So, now, what happens is – here, you see that in order to make a 0 here, I simply multiply this by 3 and subtract. So, three times if you subtract here, what will you get here? So, this will be minus 2 3 2 minus 6 minus 4; this is 0; this would not change and three times this is minus 18. So, remember, the value is the, right now, what the objective function value, this is minus z. So, minus z is minus 18.

Then to make a 0 here, I simply, in fact you see that this is 1 1. So, at the last two rows and subtract. So, at the last two rows, this gives me three and subtract. So, minus 3 minus 5 then 4 and minus 4, this becomes minus 8 and this is 0 and this is 0 adding 20 subtracting minus 38.

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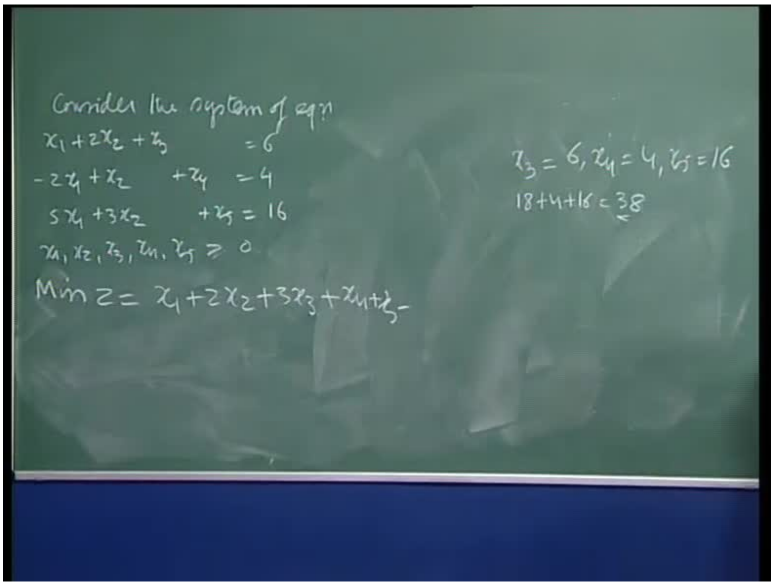

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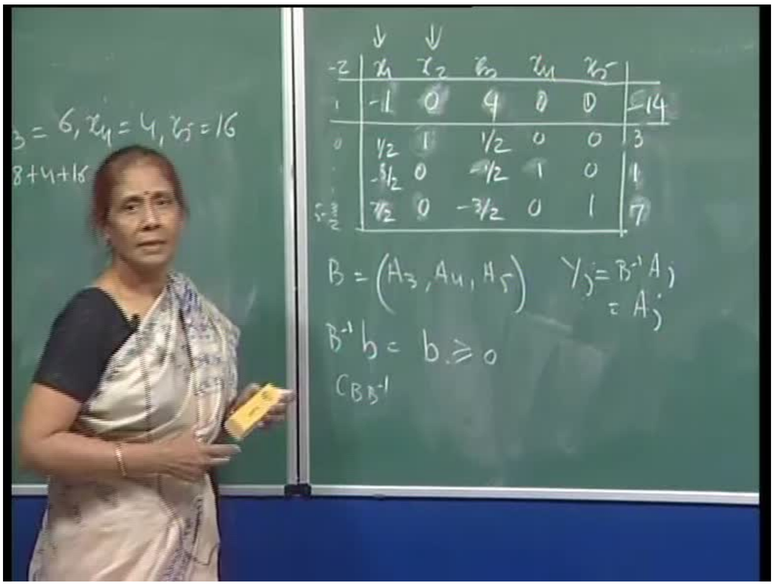

So, this tableau now gives you the C j minus Z j's at the top because you see this thing is 1 0 0. So, what is your basis? Your basis consists of this column and these three columns and this is an identity matrix of type 4 by 4, and so, I have that means multiplied the whole tableau by the extended basis inverse, and therefore, and that these quantities therefore give me the Y j's and these give me the C j minus Z j.

And you can verified that since your solution here is basic feasible x 3 is 6, x 4 is 4, x 5 is 16, you can actually verify the value of the objective function 3x 3 is 18 plus 4 plus 16 which is 38. So, remember always that this value here reads the minus of the objective function, because I have minus z plus this equal to 0 and I made the objective function. I wrote this as, actually this equation I converted as minus z plus this equal to 0.

So, when you do operations, you get a number here; your minus z is equal to that number. So, z will be equal to the minus of this number; so, this gives you this. And you see that here, the optimality criteria, so, therefore, in the simplex algorithm, you have this tableau. Representation of the whole problem, this gives you the information that what is your current basic feasible solution? What are the relative prices corresponding to the given basis, and then, since the optimality criteria is not satisfied, I can just to follow Dantzig's rule. I will enter; I will say that this is a candidate for coming into the basis, because C j minus Z j is negative. So then, this is minus 8, and now, I want to decide what would be the pivot element. So then, remember, we take the positive entries; all the entries are positive here. So, the ratio corresponding ratio will be 6 by 2 which is 3; in a ratio here would be 4 by 1 which is 4 and the ratio here is 16 by 3 which is 5 1 by 3. So, obviously, this corresponds to the minimum ratio, and therefore, my pivot element is this.

So therefore, I have to now make convert this column into 0 1 0 0. So, when quickly do this and that is the idea here. So, if I, that means I divide this by 2, so, this makes this; this is this and you see that this becomes 3. And now, you have to make a 0 here, 0 here and 0 here. So, when you do it 8 times, you add 8 times, you add here. If there any mistakes, you can correct them because this four; so, this becomes 1; this becomes 0; this is 4 and 8 3s are 24, 24 you add here that becomes what? 14 I think. Just check the calculations - minus 14. So, you see that from the, from 38, the value has now come down to 14 of the objective function; it is reducing, and therefore, this happening. Now, I just subtract this here. So, let us quickly do this 0 minus 5 by 2 minus half this will be 1 0. That is all why I want to ensure that your new basic feasible solution, that the new solution that you obtain will be basic feasible, then 3 times to subtract from here quickly. So, 5 minus 3 by 2 will be how much? 10. This will be or you are multiplying it by 3. So, 5 minus 3 by 2 10 7 by 2, I think 0 minus 3 by 2 and 9. So this will be 7.

So, now, this is your new basic feasible solutions. If the tableau again, you get this identity matrix of 4 by 4 size the solution is feasible here; the value has improved here and your optimality, so, now the top row gives the new C j minus Z j's. You scan them;

you find out that there is still some negative entry here; that means this will be the next candidate to become a basic variable in the corresponding column will come into the basis and then we can proceed with the simplex algorithm.

So, I will do some more pathological cases and try to show you unboundedness case. I will try to take up an example to show you how you encounter unboundedness.