**Linear Programming and its Extensions**
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**Lecture No. # 39**
**Module No. # 01**
**Simplex Algorithm is not polynomial Time - An example**

So, today I would like to talk to you about, why linear why the Simplex Algorithm is not Polynomial Time; that means, we so the idea here is that you know we we talk of when we want to talk of a complexity of an algorithm, that means we want to know have an estimate of how much time it can take, computed time you can say where we we say that one unit of computation could be you know addition, subtraction or multiplication, division also may be but, so we say that basic a basic operation of calculation, we give it one unit of time and then you want to count the total number of computations that you will be that you will be required for the for the algorithm to come to a stop and this is the kind, so when we want to measure the complexity we actually talks.

First let me say why so, let me talk today why simplex algorithm is not polynomial time this is the idea and let us try to develop. So, that what we would you will show is so normally when you talk of the complexity of an algorithm you do the worst case analysis. So, idea here is see for example, in the simplex algorithm take the instants because, we are not specifying the, we are not specifying rule for or the choice of the incoming variable is not uniquely specified that means, it is incompletely specified right, it is not specified at all.

So, you can you can either like Dantzig said you go for c j minus z j maximum, well in our case it will be so you want to maximum improvement you go for that, but then maximum improvement as I told you is a more complicated a computation. So, you just go back the values of c j minus z j that will be one rule, so and or you could go for pickup any c j minus z j which is positive and so on.

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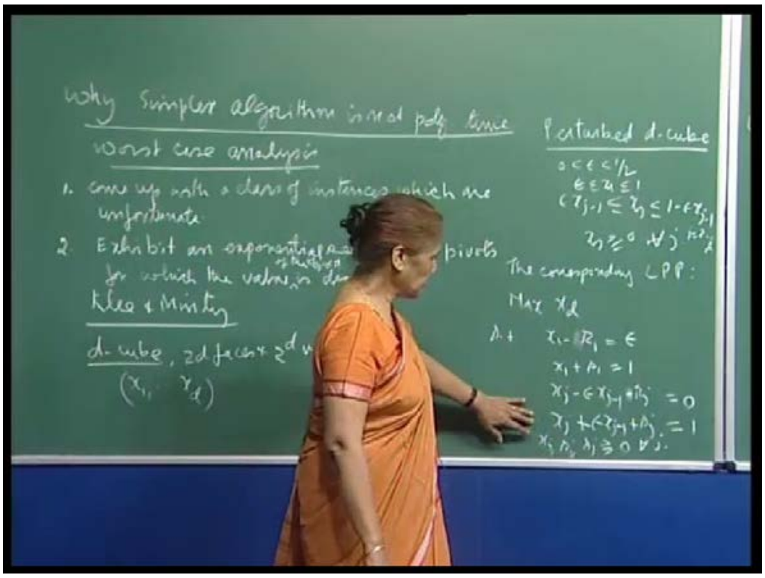


So, the entry if the incoming variable, the choice of the incoming variable is not specified completely then the idea would to, if you want to do to the worst case analysis here, you would have to see to two things you have to come up with with a class of instances, you will have to come up with the class of instances which I have really unfortunate that means, the algorithm does not perform well on such in switch are unfortunate and then the second thing will be that you will have to exhibit. So, here you will have to Exhibit along sequence an exponential exponential sequence of pivots for which the value pivots, for which the value is decreasing constantly right.

So, therefore the algorithm will not stop exponential, I should say exponential sequence of pivots for which the value is decreasing in the minimization case, and in the maximization case it will be as long sequence of pivots.

From which the value of the objective function keeps increasing, which the value value of the objective function is decreasing in this case then the maximization it is increasing.

So, this is the two things that we have to do and that is what Klee and Minty set down to do they constructed, klee and minty because this was a question which is bothering lot of people and specially computer scientist. And so on, to determine whether the simplex algorithm.

There was polynomial time or not and this was before Kashia came up with his ellipsoid algorithm. So, then the klee and minty came up with the class of instances and let me start building up there, how they constructed this set of problems?

So, a d cube if you have you have a concept of a 3 cube right which of course, I have shown you here, from the the dark lines show you, give you a three cube right. And the vertices here would be 1, 0, 0 and this for example would be 0, 0, 0 and so on 1, 1, 0, right so that means, so this would be a 3 cube and so if I just extend the thing to d d dimension d. So, d cube so this will have a 2 d faces and 2 raised to d vertices and if you take the coordinates of any point is x d is d dimensional then the idle and this has because this has 2 d 2 d subsets and for each subset what you will do is we will put the corresponding x is to ones and the remaining ones to 0. So, that will give you 2 d vertices of this polite out of this d cube right.

Now, let us just extend so, the idea is that you just pull pull two corners of the of this d cube from here and here diagonally opposite, just pull them by small distance and then what you get, will be a perturbed d cube it will not be a d cube essentially perturbed part of the polytope and you see that if as epsilon goes to 0 all these vertices of the perturbed d cube will collapse into the original vertices. So, this is the idea you perturbed the d cube by small amount and the description of the of the perturbed, so perturbed d cube.

Here let us see, the idea is that you take 0 less than epsilon less than half. So, it could be any value of epsilon which is between 0 and half and what the way will describe is that yeah 1 minus epsilon x j minus 1 less than or equal to x j less than or equal to 1 minus epsilon x j minus 1, I thing this should be the thing right and of course your x j are all greater than or equal to 0 for all j so this is the thing.

And now consider, so corresponding to this polytope, we define the the corresponding. The corresponding LPP, so here I will say a maximize x d in the minus sign, normally that the finite as minimize minus x d, which at least maximize x d the minus sign can be outside, subject to yeah I should have said here 0 less than or equal to x 1 less than or equal to 1.

x 1 it is this holds for j varying from j varying from 2 to d the constrain is include. So, here subject to what we will have is that x 1 is greater than or equal to 0, so, here. So this will become now epsilon for the original d cube you have x x x x 1 are x j is between 0 and 1, so here it will be epsilon less than or equal to so. So, therefore, the first constraint will be x 1 minus R 1 is equal to epsilon.

So, I reduce it to the standard form this polytope, I reduce to the standard form and then x 1 plus because it is less than or equal to 1, so s 1 is equal to one this is the a set of constraints corresponding to x 1 and then here similarly for this one we will say that x j minus epsilon x j minus 1 plus r j is equal to 0 yeah.

And x j plus epsilon x j minus 1, epsilon x j minus 1 this should be minus r j, because the constraint is x j greater than or equal to epsilon j x j minus, so when I bring it it is greater than or equal to 0. So, make it to 0 I have to say minus r j. So, here it is less. So, that will be plus s j and this is equal to 1 and all the variables x j r j and s j are greater than or equal to 0 for all j.

So, this is the corresponding LPP and we will try to show you that in the absence of specific rule for an incoming variable into the basis I will exhibit along sequence of x pivots. So, the value in this case keeps on increasing, so the first thing that we notice about this is to the first lemma, yes Lemma 1. So, what it says is that every basic feasible solution for this linear programming problem will have the all x I s present in it right and so. So, I am I first describe the structure of the basic feasible solution.

So, every let me call this as problem two this was my description of the polytope and this is two right. So, the corresponding LPP, yes every basic feasible solution of 2 will have all the x j's present in it, in it and remember this is now 2 d the number of constraints for each x j you have two constraints right counting from x 1, so to the number of constraints is 2 d and you can show that the, a corresponding coefficient matrix is full rank because, you have two identity matrices here corresponding to r 1 and s 1.

And that part we will not worry about here right now, but it can be easily checked. So, have all the (( )) x j's present in it and only one of r j or s j right.

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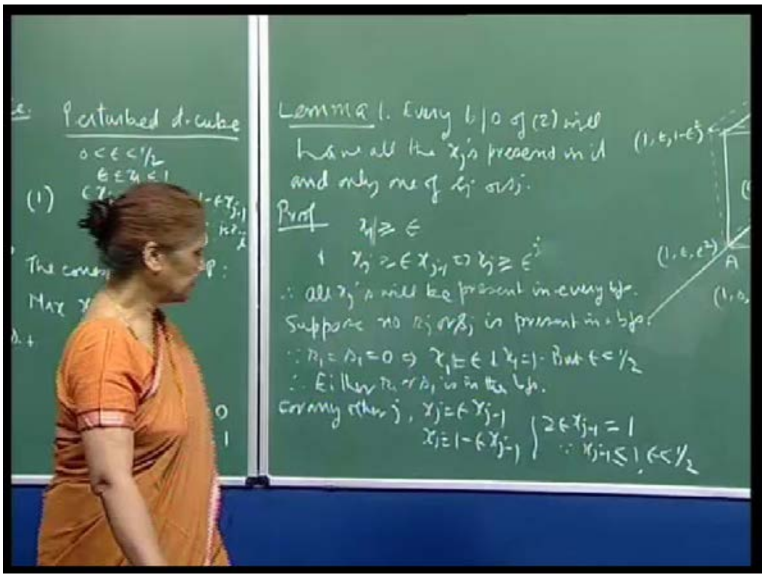


So, that means either r j will be present in the basic feasible solution or in s j. So, that means it makes it 2 d right, because all the d x j's are present and then either r j or s j presents for every j, so you have 2 d basic feasible a basic variable let us prove this simple. So, now let see, we are saying that yeah from here we have that x 1 is greater than or equal to epsilon in this constraint right.

And from this one x j is greater than or equal to epsilon x j minus 1 which implies that x j is greater than or equal to epsilon j, because yeah I think it will be j minus 1 because let say let see, then x 2 will be greater than or equal to epsilon x 1 which is epsilon square. So, this is this epsilon j fine, so therefore, you have shown that all x j's in any basic feasible solution, any feasible solution satisfying these constraints.

The x j's will all be greater than or equal to the corresponding I mean x j will be greater than or equal to epsilon j. So, therefore, so this this implies and therefore, all x j's will be present in every basic feasible solution.

Now, you want to show that, so we say suppose second part of the lemma suppose no r j or c j or s j is are present in a basic feasible solution. So, start with so since since r 1 is equal to s 1 is 0 this implies on the first two constraints; here we get this imply that x 1 is equal to epsilon.

And from here it says and x 1 is equal to 1, but epsilon is less than half therefore, it is not possible because epsilon cannot be equal to 1, but epsilon is less than half, so this is not possible right.

So, therefore, therefore, either r 1 or s 1 is in the basic feasible solution right. Now look for the other, so now now for any other j or any other j since r j is 0 you get from here that for another j, x j is equal to epsilon x j minus 1 right. And from the the second constraint you have because s j is 0 x j is 1 minus epsilon x j minus 1 right. So, if you equate the two these two together imply this equal that twice epsilon x j minus 1 is equal to 1, but remember x j minus 1 cannot be epsilon is less than half right, because we started the assumption that your epsilon is strictly less than half.

So in that case, this is where do we assume that x j's, are x j is less than or equal to 1 minus epsilon x j minus 1, so x j minus 1 are non-negative epsilon is positive, so this x j is less than or equal to 1 right. So, this and since x j minus 1 is less than 1, less than or equal to 1 and epsilon is less than half, this cannot be satisfied right because, the product has to be equal to 1.

So, therefore, either r j or s j has to be in the basis for all j varying from one to t. So, this proves this lemma and yeah, and now another simple lemma I want to show you, before I go to the main theorem. So, today I have try to, now introduce you to a little bit of complexity theory and where I am telling you what it means to you know talk about the time taken by an algorithm and normally you would do it for the worst case. So, that your lot of giving upper bound and the time for that algorithm cannot go beyond the worst case analysis that we do right ok.

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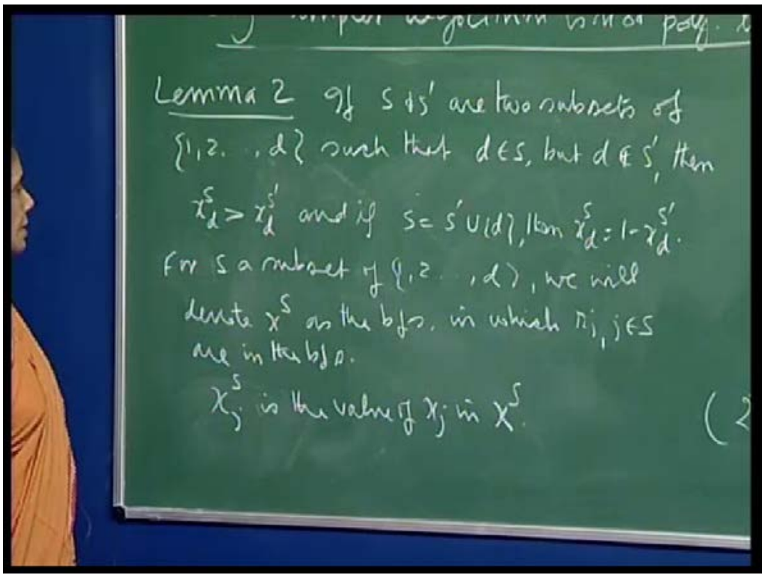


Now, the second lemma, is lemma 2 if s, now like to give you a definition, so I will do it after if s and s prime are two subsets of the set of 1, 2, d are (( )) such that d belongs to s, but d does not belong to s prime. Then and I will, I will give the definition I should have given them earlier; then we will say that x s d is greater than x s prime d and if s is equal to s prime union d that means, d index was not present in s prime and s and s prime differ only in the index d, then then x s d is equal to 1 minus x s prime d.

Now, let me explain first, so in the before the proof see what we are saying is we will for for s a subset of 1, 2 to d we will we will define, we will denote x s as the basic feasible solution in which r j, j belonging to s or part of the basis or in the basic feasible solution or in the or in the basic feasible solution. Let me explain, so what we are saying is that, you see as I this lemma says that either r j's or s j's have to be in the basis, in in a basic feasible solution.

So, what I am saying is now, that you take some subset of these d numbers and then you decide you say that those the indices which are present in s will correspond to the r j's which are in the basis in a basic feasible solution right, so r j, j belonging to s right. And then and we will say that the corresponding basic feasible solution is denoted by x s; that means in x s for example, if I simply say x 1 then this means that this is a basic feasible solution to the system in which r 1, is the r 1 is in the basic feasible solution; that means, for the remaining 2 to d variable indices s 1, s s j is present right.

And so r 2, r 3, r d are all 0s it will consider the basic feasible solution x 1 right similarly, if you consider the set x 2 then that means, r 1 and r 2 are in the basic feasible solutions and then s three onwards are in the basic feasible solution, so this is the idea ok.

So, subset of this, so now what we are saying is here that if the lemma says that if s and s prime are two subsets of this such that a d is present in s, but d is not present in s prime then the corresponding x n, so x s d is the value of and so x j s is the this needs to be need to go slowly so that is the value of x j in x s.

So, this is the, so 6 with this definitions the lemma is not clear that if s and s prime are 2 subsets here such that d it belongs to s, but d does not belongs to s prime then the corresponding x s d value in a basic feasible solution x s to the value of the dth component is greater than x s prime d. So, this prove is not difficult let me just quickly show you yeah I will like keep this here, so we will prove.

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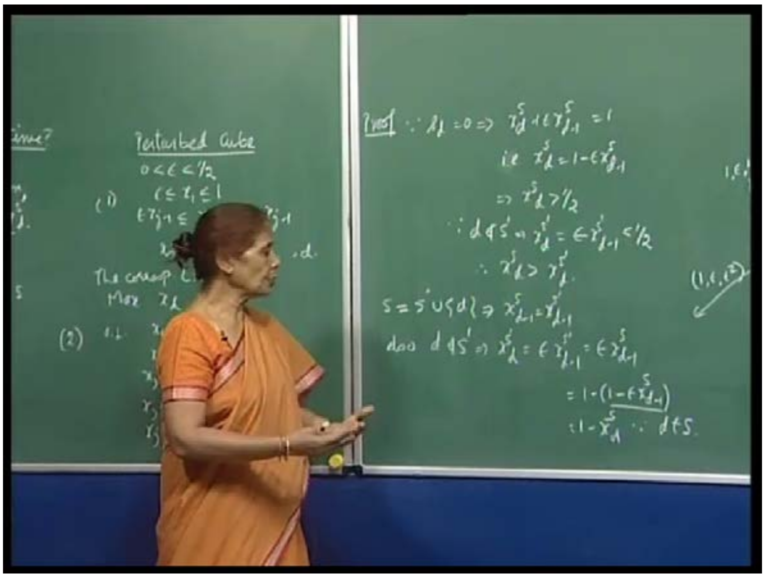


So, we are saying that d is in x s, but d is not n is an s, but d is not in s prime. So,therefore, what do we have from here; that means, for the d for j equal to d s d is 0. So,since, s d is 0 this implies in the last constraint that x d minus epsilon a plus sorry, plus epsilon x d minus 1 is equal to 1. So, which that is x d is equal to 1 minus epsilon x d minus 1 right.

And this quantity is less than half because, I have been showing you already right because, epsilon is less than half x d minus 1 is also less than or equal to 1. So, this quantity, so minus will become greater than, so which will imply that x d is greater than half, I should continue writing this otherwise if you confusing.

So, that means; so I have shown you that the value of x s d when d is present in the set corresponding set s is going to be greater than half. Then I want to show that this value is greater than x s prime d, so now since, d does not belong to s prime this implies that your x s prime d is equal to epsilon x d s prime minus 1, which is less than half right.

You are always saying that, because epsilon is less than half, so this is less than half. So, therefore, you immediately get this implies that x s d is greater than x s prime d.

So, first of all I just want to point out that when I stated this lemma I did not actually mention that this will be valid only when s prime is not empty. So, we should not try to see if it should it if, I mean we should not applied if in case s prime is empty now to prove the second part that when s is equal to s prime union d, then x s d is exactly equal to 1 minus x s prime d we want to prove this.

So, I am using the, see in since s and s prime differ only in the element d therefore, all other elements are the same. So, therefore, the values of the variables will also remain the same up to d minus 1, because they differ only in the dth component, so x s d minus1 is x s prime d minus1 right.

So, now and since d does not belong to s prime that means r d is 0, so here you see that r d is 0, so you get that x s prime d is equal to epsilon x s prime d minus 1 right from here, because r d is 0 since d is not in s prime and, but we have just seen that x s prime d minus 1 is the same as x s d minus 1, so this becomes epsilon x s d minus 1 right.

Now this number, I write as 1 minus so I add and subtract 1. So, this is 1 minus of 1 minus epsilon x s d minus 1, but then this is this is because d belongs to s, d belongs to s therefore, s d is 0 right, and so from here, you will get that x s d is 1 minus epsilon x s d minus 1. So, x s d is equal to 1 minus epsilon x s d minus 1, so I write x s d here, for this number, so this becomes 1 minus x s d right.

So, therefore x s prime d is this which you can write either way, so you can bring x s d on this side then it will be x s d is equal to 1 minus x s prime d, so this is a relationship.And, so you see that when I should prove the lemma three then you will see the connection that, you will actually be able to show that you can, as you move from 1extreme point to another the value of the objective function; that means, a dth component is increasing right. And therefore the, you will follow a monotone path and that the the length of that path will be exponential.

So, let me continue with the proof of showing why a simplex algorithm is not polynomial time. So, now lemma three that the, subsets of one to d be a numerated in such a way, that you can order the value of the dth component, so s 1, s 2, s 2 d because d a subset having a, set having d elements will have two raise to d subsets, we all know that right, including the empty set right.

So, it will have two raise to d subsets and let me call them s 1, s 2, s 2 raise to d, so this two raise to d is the suffix here, this is right. So, then we have ordered them in such a way respect to the value of the last component is the corresponding basic feasible solution right.

So, given a subset here as I told you, how we we can construct basic feasible solution. And so in the basic feasible solutions x 1, x s 1 this is the last component the dth component to the basic variable and so you have this. Suppose you have this numeration then the inequalities are strict. In fact, see the values will keep on increasing strictly and the basic feasible solution x s j and x s j plus 1 are adjacent right, these two bases are adjacent the way you order and so this happens for up to do two raise to d minus 1 this is the lemma. So, let us prove and will prove this by induction.

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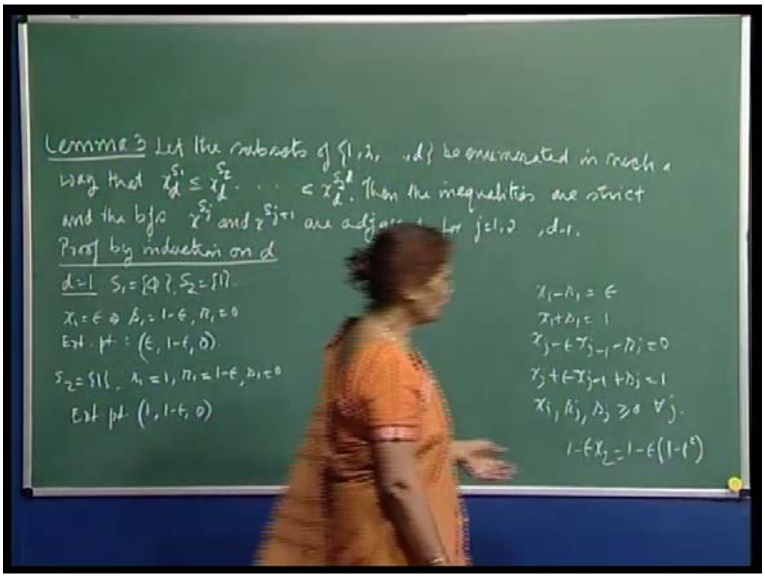

So, the proof by induction on d, so I will may be, I will need the formulations suppose d is equal to 1 right, then see remember your constraints I will just rewrite them again, so that you might yeah. So, this was a, x 1 minus r 1 equal to epsilon x 2 plus s 1 equal to 1 right, when we had x j minus epsilon x j minus 1, minus r 1 was equal to 0 right right and then you had a 1minus, so and x j plus epsilon x j minus 1 plus s j, s j equal to 1 right, and all x j's, r j's and s j's, s j greater than equal to 0 for all j these for your constraints.

So, for d equal to 1 you see only this is x 1, so you see what are the possibilities, because you have only element your set has only one. So, what are the two possible subsets either? So, s 1 is s 1 is the phi, phi set right and s 2 will be just to containing one these are the two possible subsets when your element when your set has only one element right ok.

So, so corresponding to this what will you have, because r 1 is 0 and so x 1 is ahs 1 will not be 0, r 1 is 0, so x 1 x 1 is equal to epsilon this will imply that your that your of course, r 1 is 0 and yours 1 will be from here 1 minus epsilon, 1 minus epsilon.

So, the corresponding extreme point, extreme point is (( )), extreme point will be what; epsilon then 0 and 1 minus epsilon right. And for s 2 equal to 1, r 1 is there s 1 is 0, so then x 1 becomes 1 right. So, x 1 is 1 and that gives you r 1 as, r 1 as 1 minus epsilon and s 1 is 0 right.

So, the corresponding extreme point here is 1, 1 minus epsilon and 0. See you see here, the value is increasing remember the lemma lemma 2 we said that see because d is 1, so s 2 contains d and s 1 does not contain d. So, the component the value of x 1 will be higher for this 1 and this 1 which you can see that here, x 1 is higher than 1 is greater than epsilon right.

So, I have still after obtaining these two extreme points when d is equal to 1. I want to show you that the lemma is valid, because you see we said that the values must keep on increasing. So, here you see this one is epsilon, and this one this first component is one here, so this is satisfied and since lemma two is not valid when s 1 is empty.

So, therefore, the point that this should be equal to 1 minus of this will is not valid right because, your starting set s 1 is empty. So, that part will not be satisfied by this lemma,because the conditions are to met but otherwise the values are increasing. So, now let us go to for for d equal to 3 for example, and this is a perturbed cube that we have been looking at ok.

So, here I just want, but before that I want to show you that two extreme points are adjacent, that part we have to show still right. Because then the inequalities are strict and the basic feasible solutions x s j and x s j plus 1 are adjacent for j varying from 1 to d minus 1 and so that part I want to show you.

So, the two extreme points are adjacent why because, let us just first defined this, a notation that a j's are the columns corresponding to the x j's in your constraint matrix, d j's are the columns corresponding to r j's and c j's are the columns corresponding to s j's.

So, for the extreme point epsilon 0 1 minus epsilon when s 1 is empty then r 1 is 0 right, so, in that case your therefore, your solution comes out to be x 1 is epsilon and from here s 1 is 1 minus epsilon and r 1 is 0, so this is the extreme points.

So, the corresponding basis is a 1 comma c 1, because you have two constraints when d is equal to 1, so this is at and for the second extreme point that we obtained see here, r 1 is, r 1 is present in the basis. So, therefore s 1 is 0, so x 1 is 1 right and x 1, then your r 1 becomes well 1 minus epsilon right, r 1 becomes 1 minus epsilon.

So, that is 1 minus epsilon and 0 and so this changes because, now r 1 into the basis so you have the column b 1 according to our notation. So, these are the two basis and there adjacent right only c 1 has been replace by b 1. So, I want to stress this point and therefore, the two extreme points are adjacent to the two basis are also adjacent right.

Now for d equal to 3, if you go for d equal to 3 then I just picked up these two points and extreme points and you can then verify the theorem for other points also. So, here for example, the point c is epsilon 1 minus epsilon and you know that this point is actually 0 1 0. So, I will take r 2 to be; that means, r 2 will be in the basis my s, the corresponding set s I will take a 0 1 0. So, then r 2 is in the basis and r 1 and r three are 0.

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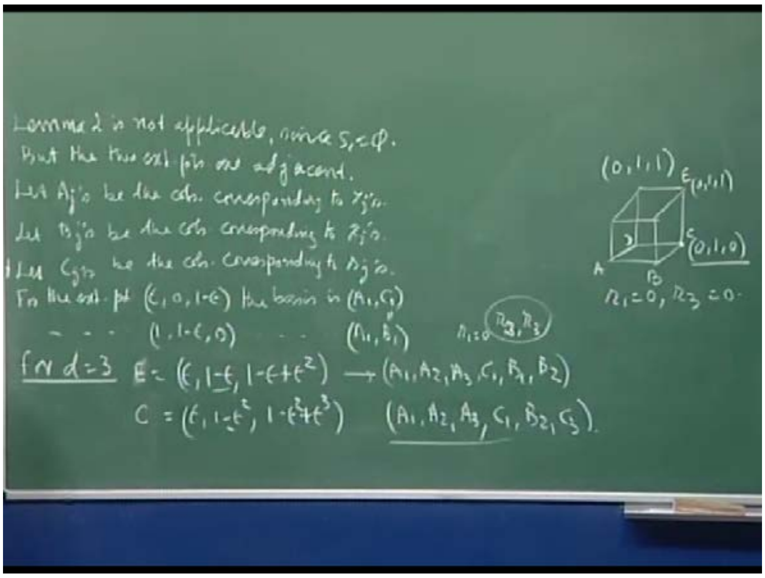


So, if r 1 and r 3 are 0 then you have x 1 equal to epsilon right and then from these two again you can just solve and we have done this exercise already. So, this will be the corresponding, because I am just showing you the three coordinates not the r 1 that of course, you can fill up.

So, then the corresponding basis will be A 1, A 2, A 3 and then since your see here, C 1, B 1, B 2 and this is C 1 is, r 1 is 0 oh what what point I am taking here, this is this is 0 1 0. So, I think what I have done is I am taking the point because B 1, B 2 means that R 2 and R 3 are in the basis, so that means, and R 1 is 0, so this is, this not the same point as that 1 may be I can just remove this, may be you should not take this (( )).

Yeah, just without the figure, do it without the figure. So, anyway this is R 1 is 0 and R 2 and R 3 are in the basis, basic feasible solution.

So, then correspondingly what will be the point when R 1 is 0 we are saying x 1 is epsilon and when R 2 and R 3 are there. So, for example for x 2 the component x 2 this is 0, so it will be 1 minus epsilon right and then again since s 3 is 0 it will be 1 minus, it will be 1 minus epsilon or x 2 which is 1 minus epsilon of 1 minus epsilon, so 1 minus epsilon plus epsilon square. So, this is the thing, so I did not have the right figure there anyway, so this this is the basis right.

So, I said that heap the point which corresponds to 0 1 1 right, so I had R 2, R 1 as 0, R 1 as 0 and then s 2 and s 3 are 0. So, R 1 0 gives me immediately x 1 equal to epsilon which I got here and then you can immediately see that, because s 2 and s three are 0. So, 1 minus epsilon square you know it will be yeah, this will be 1 minus epsilon 0 1 1 yeah.

So, this is epsilon and then this will be 1 minus epsilon will come when you have, so I am writing the point 0 1 1, so this will be when R 1 is 0 you have x 1 epsilon. So, then x 2 would be and because s 2 is 0. R 2 is there in the basis, so s 2 is 0 therefore this is 1 minus epsilon square, this 1 minus epsilon square and then the third when s 3 is also 0. So, it will be yes because s 3 is 0 and so yeah for this one for this one only r 1 is 0 and then you have s 2 equal to s 3 is 0 right for this point.

So, therefore, this will be then 1 minus of epsilon epsilon times 1 minus epsilon square. So, this will become. So, let us just go through these points c, for c you see, you have R 2 is there in the basis and R 1 and R 3 are 0; R 1 and R 3 are 0 which means that then s 1 and s 3 will be there, if you remember the lemma. So, R 1 is 0 see from here you immediately see that R 1 is 0 then x 1 is epsilon. So, I am going for the point c right now right.

And then what you have is that s 2 is 0, so from here you see x 2 will be equal to 1 minus of epsilon x 1 and x 1 is epsilon, so 1 minus epsilon square right. And then again, what you have is R 3 is 0 so if R 3 is 0 you get x 3 equal to epsilon of x 2 which is epsilon of on 1 minus epsilon square, so epsilon minus epsilon cube right. And so the corresponding basis is this right because, R 1 and R 3 as are not there in the basis.

Then for the point e I am trying to show you here again you see c 1. So, R 1 is 0, R 1 is 0 therefore, x 1 is epsilon then you again have R 2 and R 3 are there in the basis. So, s 2 and s 3 are 0. So, if s 2 is 0 you immediately get 1 minus epsilon square which is and then again you get 1 minus. So, this is 1 minus epsilon square and since s 3 is 0 you get that x 3 is 1 minus epsilon of 1 minus epsilon square. So, this is 1 minus epsilon plus epsilon cube this is the calculation.

And so this is the basis and you can see that immediately they differ only in. So, this is b 2 and this they differ in b 1 and c 3, because you have yeah, you should write b 2 and this is b 3 sorry. So, this is b 2 and b 3 so b 2 is there and b 3 is c 3. So, they differ only in one column and therefore there adjacent. So, I just thought that I will through in example I will show you this part of the that the adjacent, but of course you can exactly in the same way do it for any j and you will be able to show that these, two basis will be adjacent basis.

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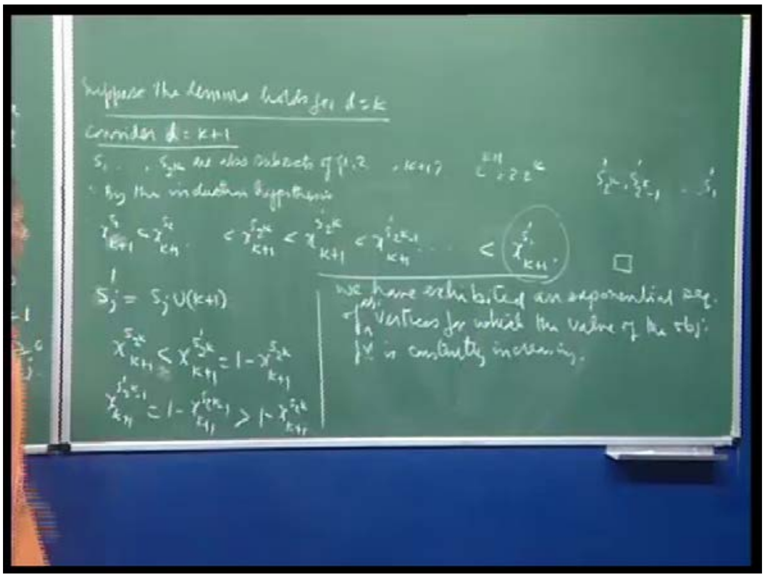


Now, suppose we will assume that the, suppose the lemma holds holds for d equal to k, suppose the lemma holds for d equal to k which means that I can enumerate the subsets here 2 raise 2 k subsets with this property and the corresponding this things are adjacent and so on. So, suppose the lemma holds for this, so now consider d equal to k plus 1 right.

So, the thing is that s 1, s 2 raise to k are also subsets of 1, 2 to k plus 1 right there also subsets of this right and so this property holds, so up to 2 raise to k these the values will satisfied, this because there was same subset as they were there right. Now and so therefore, therefore, by by the by the induction hypothesis, by the induction hypothesis x 1 d plus d sorry I should say k plus 1 right because now that I have added k plus 1. So, that is the last component.

So, this for the subsets is less than x s 2 k plus 1 and less than x s 2 raise to k k plus 1. So, up to this is right by this thing why why because, you see the sets s 1, s 2, and s 2 raise to k when they are subsets here they do not contain, they do not contain k plus 1 right. And so then why should I (( )) able to say this, why should this will since k plus 1 does not belong to x s j, j varying from 1 to 2 raise to k therefore, x s j, k plus 1 is equal to epsilon x s j k yeah right.

So, against from here only because, the if you consider the these set of constraints for j equal to k plus 1, then for k plus 1 you see this is 0 because, all these subsets do not contain k plus 1 component and so x j is epsilon x j minus 1 right. So, that what I am saying x s j k plus 1 is epsilon x s j k right and since these are ordered. So, epsilon is a positive number. So, these will be also ordered same thing right, so this is fine ok.

Now, let us consider because we must have 2 raise 2 k more sets, see this 1 this has to the number of sets is k plus 1 which is twice 2 raise to k. So, I have already take in care of 2 raise to k subsets now you need more so; obviously, you are going to add k plus 1 to each 1 of them right. So, these 2 raise to k subsets you have and two each of them you will add k plus 1, so we will define we will say that in your set s j prime, let us call it is equal to s j union k plus 1.

So, that means, I am numbering them again 1 to 2 raise to k so, but there will be 2 raise to k in number. So, they together with these two raise to k subsets will form 2 two times 2 raise to k subsets right which was what we need, and what more? Yeah. So, now here I need to order these remaining the new 2 raise to k subset that I have form out of those, so I need to order.

Now see, we need to see this here that, yes if you look at x s 2 raise to k k plus 1 and you look at x s prime 2 raise to k k plus 1; then you can see, that this is less than this.

Why because, remember the lemma that I just did before this one what is, what is the difference between these two s 2 raise to k is here, and yes and x s prime 2 raise to k this contains k plus 1 and this does not contain k plus 1 therefore, this holds right.

And you also have and that this is equal to 1 minus x s 2 raise to k a k plus 1 by the same lemma right fine. So that means, in this here I can conclude this, so here that means; I am saying this is less than x s prime 2 raise to k k plus 1 fine, that part I have already shown you. Now, what is happening is that when you look at x s prime 2 k minus 1, see remember the numbering, the numbering of the sets is see you have s prime where I am writing the things ok.

At this is 2 raise to k the numbering is s prime 2 raise 2 k minus 1 and so on. And then you will have s prime 1, so this this have you have 2 raise to k new sets and we are just trying to order them here right. So, now if you look at x s prime 2 raise to k minus 1 then this will be equal to by same thing what we have d 1 is x s 2 raise to k minus 1 k plus 1 k plus 1.

See you just need to go slowly through the proof again and then you will have followed it. Now, see this is 2 raise to k minus 1 k plus 1 is less than by this ordering, see the x as raise to 2 k minus 1 k plus 1 is less than this. So, when you do 1 minus yeah this this will be greater than 1 minus x s 2 raise to k k plus 1, and so you have the ordering that this number is bigger than this. So, therefore, I will write this here x s prime 2 raise to k minus 1 k plus 1 and so on. And, so you will finally you reach to s prime 1 k plus 1 and that is your.

And the adjacent the part also you can immediately do, because see what is happening is that all these are adjacent, all these are already adjacent. Now in the prime if your adding d plus 1 the k plus 1 column; that means, R k plus 1 is the new variable that is part of this basis and so the adjacent (( )) continues. And So, the really 1 does not have to spend time on that, so you should be able to, so I think the lemma, so you have shown that you can if if you have the ordering for d equal to k, you have the ordering for d equal to k plus 1 and so.

You can you have demonstrated; that means, what we have d 1 is, so we have demonstrated essentially we have. So, we have exhibited I mean the lemma I according to me the proof of the lemma is over because, the adjacency part is already being shown.

So, I need need it really to show you that if the ordering of the subset is therefore, d equal to k then it is for d equal to k plus 1 and already I have shown you that, you can order them for d equal to 1. So, therefore, by induction hypothesis the lemma is over and we have exhibited, we have exhibited an exponential sequence of vertices for which the values of and of course, they are adjacent sequence of adjacent vertices I should say adjacent vertices for which the value of the objective function is constantly increasing.

So, the idea behind this was that you should get familiar with the kind of things that have that are surrounded with linear programming problems. So, once we have exhibited this we cannot claim that the simplex algorithm will be is a polynomial time algorithm, so that is one thing. And else we can so basically yes this was the idea now while going through this proof I came across I will give you the reference which I have already been referring to that book by linear programming.

Where they have transform the problem? See the corresponding linear programming problem here which was maximize x d right they have transform this into a form where you can actually show that the basis corresponding to this fine this; that means, your x s prime 1 actually the all the c j minus z j's will be non-negative which is your optimality criteria.

And the value is of course is maximum yeah, and another thing yeah just want to point out one thing more here, is that see you started with this if you remember the diagram had drawn you see that was your the d cube, so remember this was this, was the thing. So, we actually, so we actually started from here then went here and this here, but you see if and this was your maximum point.

So, actually you can show that if from here to this point they also are adjacent they differ only in one this thing because from x s 1 k plus 1 you can go to here; that means, from sorry what I mean is that from the basis x s 1 differ basic feasible solution differs from x s prime 1 only in r d in the case of course r k plus 1, but in the in the general case for the d cube it differs only in this.

So, I could have in one iteration gone from here to here I could have if I specified the maximum improvement in the objective function value which in the simplex algorithm we will do we do not do we simply go by c j minus z j, but you could have done it and could have avoided this long sequence of pivots. But then some people have also construct a example where they have shown that even if you was specify the rule that the that you go for the maximum improvement in the objective function and then enter the variable.

Even for those they have construct to the example to show that they would be a along sequence of exponential pivots. So, therefore, it has mean sort of establish that simplex algorithm is not polynomial time. And so that is one thing and then yeah, so I was talking about I will give you the reference they have transform this problem to show that, you can, then you can actually see the basis and you can see the change in the objective function value and you can show that your you know that c d minus z d is all is is less than 0 continues to be 0 less than 0 for because, that is the improvement thing if you this is less than 0 you will enter it.

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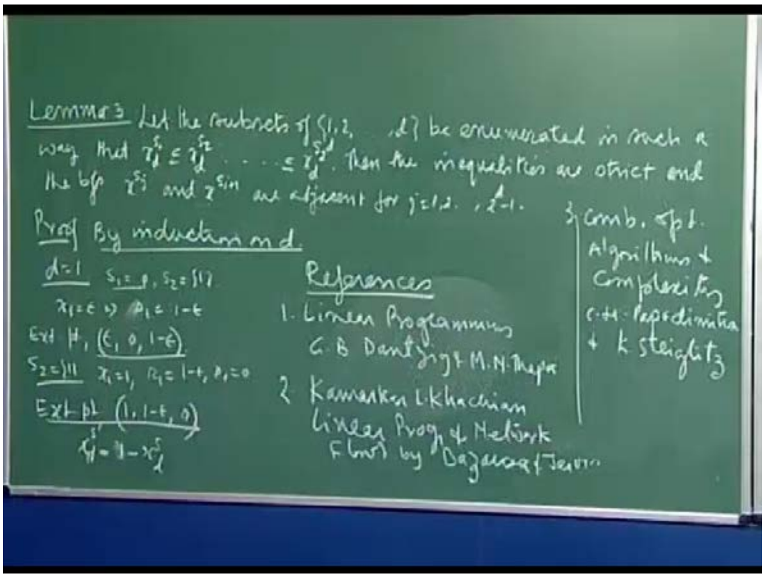


So, actually for all these basis it continues to be less than 0 well in this case here right, but then because we want to show that the simplex algorithm has not actually specified the choice of the entering variable. So, you can try to we know nasty and you can (( )) where use this along sequence of pivots and then arrive at the optimal solution.

So, that weights a quite interesting the example, that they took sometime Klee and Minty to construct this. So, let me just write down the references for you and for this particular material references. Now, so this is one is linear programming, this is a G B Dantzig I think for the interior point method, I gives this is basically for interior point method Dantzig and M.N. Thapa this is for interior point methods.

So, and this is a right, so this is for interior point and the other one that I want to give you for (( )) this is for Kamarkar L Khachian khachian linear programming in a (( )) this is the was are and jarvis for kamarkar and khachian I would like you to also go through this material has been done by they has try to make the authors have linear programming the book, Linear Programming and Network Flows by Bazaara and jarvis.

Jarvis and the third one is yeah then I thought that the in this particular a treatment why simplex algorithm is, why simplex algorithm yeah, so let me say bazaara and jarvis they describe Karmarkar and Khachian algorithm very well. And they also have a different as I told you by transforming the problem they show you why a simplex algorithm is non-polynomial ok.

So, both for both the topics this is good book you can go through them then finally, the treatment that I gave you here is from, so this is three Combinatorial Optimization that is a Title Algorithm and Complexity Algorithm and complexity. This is C.H Papadimitrou and K Steiglitz he was translated it from Papadimitrou rotate in Greek is been translated by this, and so here this treatment yeah and let me just quickly make the correction here for by this was in error. So, this is epsilon and one so why is it not holding it here the part that it should be equal less than 1 minus epsilon is 1.

So, any way this calculation is and fine. So, later on maybe I will add the correction if necessary, but finally I just want to say that, I hope that you will fine many how gone through out the lectures you will have good view of the linear programming theory its practices, its application and the other variations and related new developments in the area I hope enjoying the course, thank you.