**Linear Programming and its Extensions**

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**Module No. # 01**

**Lecture No. # 24**

**Mini-cost Flow Problem-Transportation Problem**

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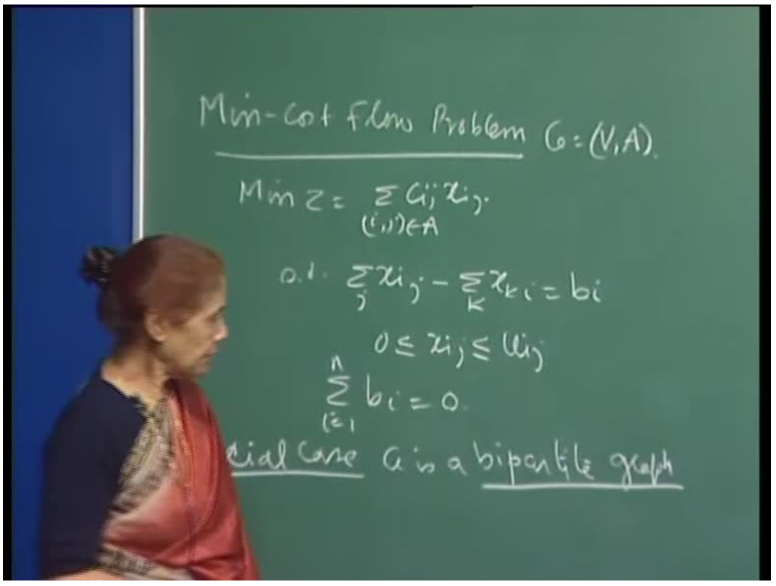


We had started discussion on mini-cost flow problem. Let, me just stated for you again, this was minimizing, so given a directed graph that's understood, this is over a directed graph G V A. I have minimized z equal to summation c i j, x i j, i j belonging to A; subject to summation x i j, summation of over j minus summation x k I, summation over k is equal to b i and we said 0 less than or equal to x i j, less than or equal to u i j; and for balance this thing you had that i varying from 1 to n, in this case is 0. So, this is a balanced situation where the supplies in the demands are all add up to 0.

So, whatever is available is also demanded, and we said that this is the necessary condition for the problem to have a feasible solution, and we will show that this is also a sufficient condition little later on I will discuss the special cases, so then I started discussing with you a special case of this problem, I had given you some definitions, so a special case.

Special case and the special case comes, because your graph G is bipartite graph, a bipartite means that you can divide the graph into 2 node sets where the connections are from one node set to the other.

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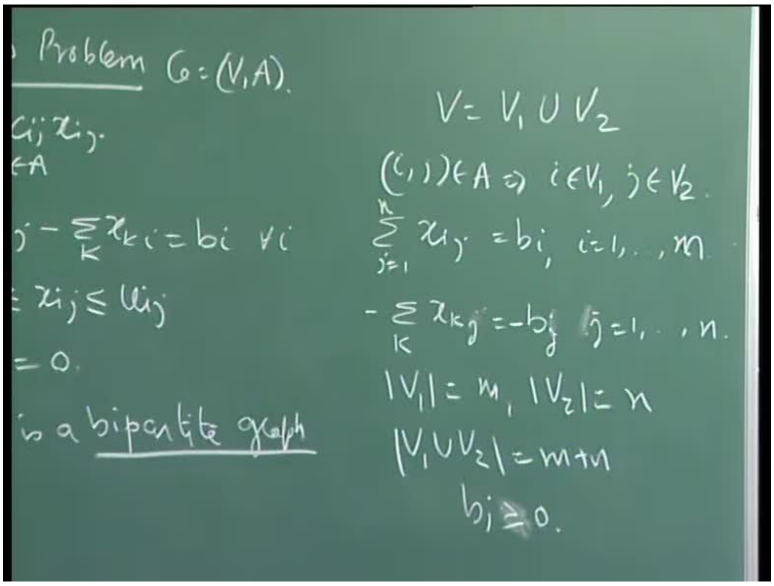


In other words, you can write your V as union of V 1union V 2 and i j belonging to A implies i belongs to V 1and j belongs to V 2. So, you have only arcs going from nodes in V 1to nodes in V 2, if you have this special case and just see look at the constraints, and I had told you that we will take this supplies as, you can leave them as b i's, but if the node i and this of course for all i this constraint, so consider a node in V 1; so that is we will treat that is a supply point, because arcs are going from V 1 to V 2; so supplies are available here and demands are here, so the nodes arcs go from V 1 to V 2, and you supply the amounts that are required at nodes in V 2.

So, if node i belongs to V 1 when you see these arcs are absent, and so this constraints will reduce to say summation x i j, summation over j varying from 1 to n, and here this is equal to b i.

We are assuming that i varies from 1to m, and then similarly when it is a demand point that means a node is in V 2 then there are no arcs going out of V 2, the arcs only going from V 1 to V 2, so this part will be 0. And therefore this will reduce to minus summation x k I, summation over k, this is equal to b i in this case or maybe I should write this as j just to make distinguish between, so j varies from 1to n.

So, that means I am assume that the cardinality of this is m and cardinality of this is n, so the cardinality of V 1 U to V 2 is m plus n, so the total number of nodes is m plus n and you have this. So, here this is a fine, because your b j's will be negative, the demand quantities are treat them as negative or if you want to be more specific just write b j here, so we are assuming that b j are less than 0. So, b j's are all greater than 0, so minus b j is negative. So, these constraints will reduce from this special structure.

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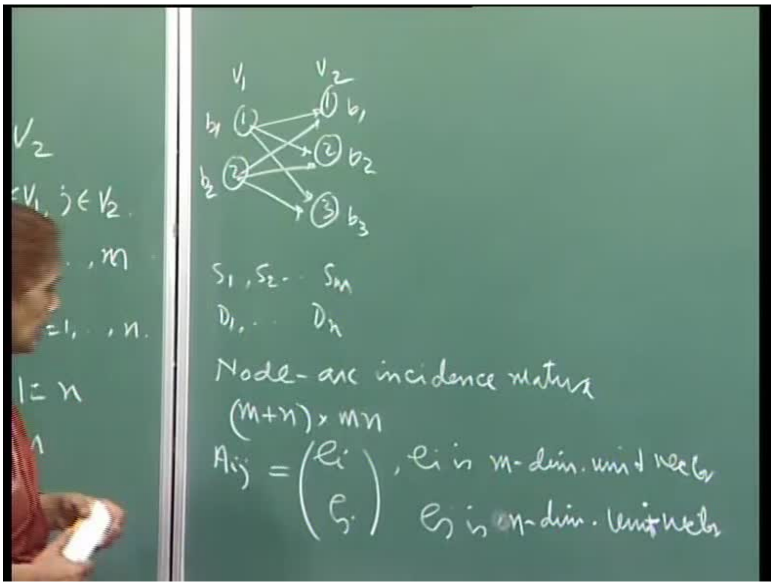


You can take situation where you have 1, 2 and then you have 1, 2 and 3, so this is your V 1 this is your V 2; so we are saying that the arcs are going from here to here and this is this and this, I have right now assuming that you have arcs going from all the markets, similarly from 2 also your arcs going to all the markets, and we will later on consider the more general situation when every supply point is not necessarily connected to every demand point.

So, this would be your transportation problem and here your decision variables, so for example x 1 1, x 1 2, x 1 3 these are the decision variables connected to your first supply point and x 2 1, x 2 2, x 2 3 would be the decision variables related to second supply point, and you can decide how much to said from where to where, given that you have a this is b 1, b 2 and this will be b 1,, b 2 b 3; so the confusion will not arise, because you know when you formulate the problem, it will be clear which is supply and which is demand, though some of the text books are also write these as a supply points, the supply amounts and d 1, so the names are suggested these are the m supply quantities and these are the n demand quantity. So, you can also formulate the problem with s i's and d j's.

So, let us see now the simplifications just come through, and because we are looking at the structures, special structure, so here also we said that this condition holds; because that is a necessary condition even when we take the special case the necessary condition holds. And in this case, I will show you for the transportation problem right now and then in general that this is also a sufficient condition for a feasible solution, for the problem to have a feasible solution.

So, at least for network flow problems feasibility is not a big concerned, and once you put this condition then you are show that there will be feasible solution always. So, we do not have to worry about working out phase 1 and so on.

Now, let me show you that the node arc incidents matrix in this case what will it be, so your constraint node arc incident matrix is actually m plus n by m n, because your number of variables are m n, m supply points n demand points; so if you have connections from every supply point to every demand point, this will be m n. So, m n arcs and m plus n nodes, so they node arc incidence matrix will be this.

And you see that, if you look at the column A i j that means the a column for the arc connecting node i in the supply point to node j in the demand set, and A i j will actually look like e i and e j, where e i is m dimensional unit vector and you know that this indicates that it is a unit vector with 1 in the ith place all others are zeros, and similarly e j is a n dimensional unit vector, this is unit vector. So, you have m plus n elements here, because m plus n rows and m n columns to this thing, so you have this is very simple structure and you will see how your calculation of the simplex quantities becomes so simple.

Now, just one more thing here, see if you add up as we did for the flow problem general if I add up these constraints, so this will be respective to i and I sum up these respective j

So you see this quantities will be the same, because here you are summing up all the x i j’s, here also you are summing up all the x i j’s; and sigma b i, so what will it be.

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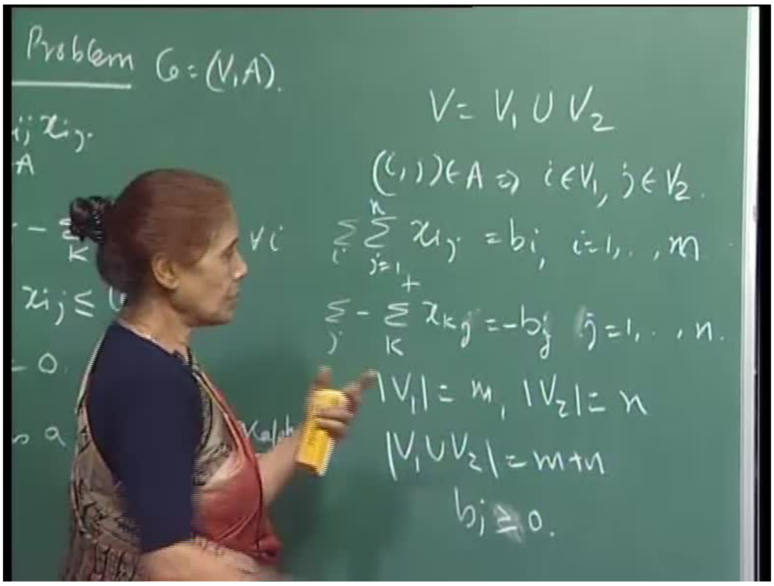

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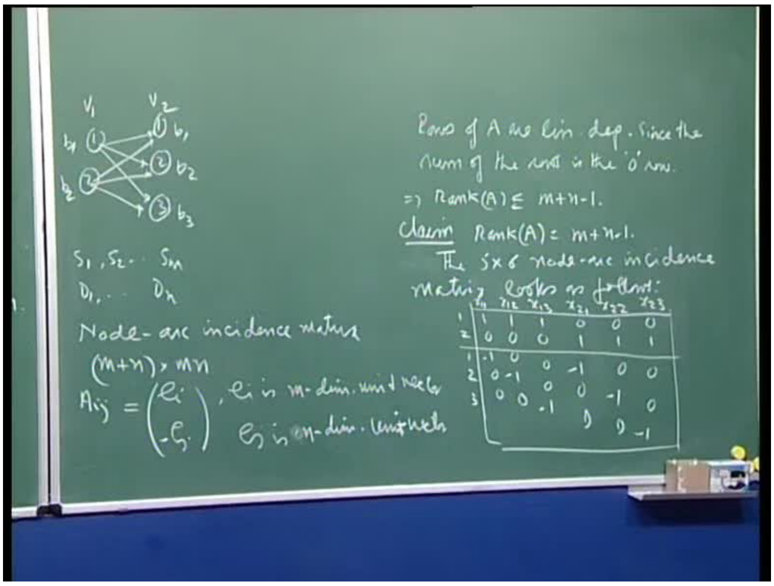


So, when you add up these things, I sum up this I add up these with respective i, I add up these respective j and when you get 0 on the left hand side and also 0 on the right hand side. So, what does it show you, this tells you that the rows of A are linearly dependent, because since the sum of the rows is the 0 row, its understood you know what I mean all its component, so therefore this implies that rank A is less than or equal to m plus n minus 1, because it has m plus n rows and I have shown you that all the rows add up to the 0 row, therefore the rank has to be at most m plus n minus 1.

Now, What I will show you is that claim is rank A is actually m plus n minus 1, so take the general situation here and as i showed you that these are the columns, so what I am suggesting here is, if you want to first take look at an example; so for example for this matrix for this problem 2 by 3 where 2 supply points and 3 demand points, so first of all for the 2 plus 3 5 for the 5 by 6 node arc incidence matrix.

So, I am taking a particular example and then show you that the proof for the general case matrix, I want to say that the 5 by 6 node arc incidence matrix will looks as follows, we said the then the first one will be e 1, so that will be 1, 0 this is 2 dimensional then 1, 0, 0 that is the first corresponding to x 1 1. Then, this will be again e 1 and e 2, so this will be 1, 0, 0, 1, 0 then e 1, 3 at e 1 and e 3. So, here numbering the columns as x 1 1, x 1 2, x 1 3 then you will have x 2 1, x 2 2 and x 2 3.

So, these are your columns, their correspond arcs and these are the nodes 1, 2 and these are the 1, 2, 3 for the supply; so it is something like this, so x 1, 3 would be 1, 0 and this is 0, 0, 1. Similarly, for 2 1, 2 1 will be 0, 1 then 1, 0, 0; because it will be e 2 and e 1, similarly this will be e 2, e 2; so 0, 1, 0, 1, 0 then e 2 3 would be 0, 1, 0, 0, 1. So, this is what your matrix will look like, the node arc incidence matrix, and by the way I should have written minus sign later on we will revert to(())

So, write now I am writing the constraint to the minus sign, but since you see both side its negative sign, so we can do away with the minus sign, but right now showing you that the rank is actually m plus n minus 1, it will be easier.

So, let us keep the minus signs and then later on will remove them, so here you see what we were saying is that if you add up these two rows you get 1 1 1 here and if you add up the last three rows you again get minus 1 minus 1 in all the 6 places, and therefore the sum of these 2 rows is minus time, so sum of the last three rows and they add up to 0.

So, therefore the rows cannot be linearly independent, so the rank has to be less than or equal to m plus n minus 1 this is what we are saying. And now to prove that the rank is actually m plus n minus 1, I will demonstrate to you a basis, if I can show you that there are always m plus n minus 1 columns present in the node arc incidence matrix which are linearly independent I am done.

So, we can also we can have a proof which uses method of contradiction or if uses the method of construction, so that is what we are doing we are going to construct for you a basis, so let us show you the proof is simple.

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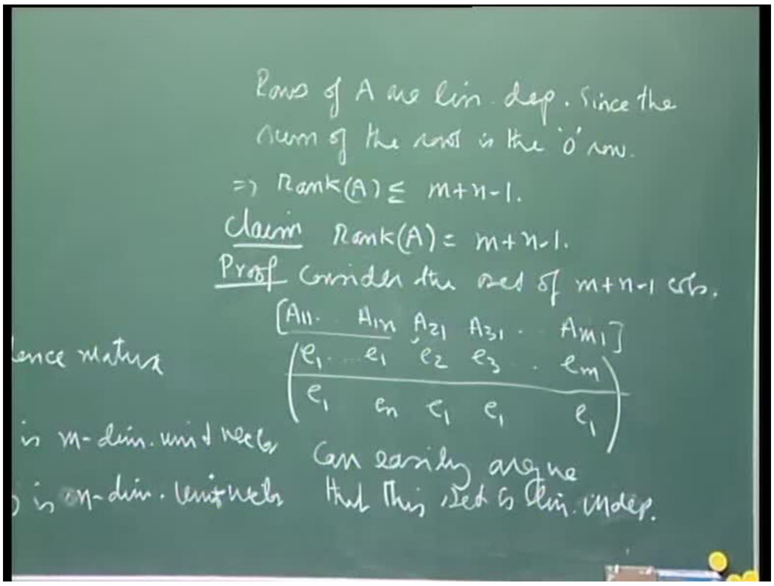


So, now you have this picture in mind as to how the node arc incidence matrix looks, so claim is this and I am saying that for the general matrix proof consider the set of columns, set of m plus n minus 1 columns as follows.

I am saying, consider A 1 1 to A 1 n, so the first n columns fine A 1 1 to A 1 1 n then I want you to choose select a column which would be A to 1; so see actually you saw that here in this case corresponding to each supply point you have a block of rows columns; so like x 1 1, x 1 2, x 1 3 then here you had x 2 1, x 2 2, x 2 3; similarly if you another supply point, it will be 3 1, 3 2, 3 3; so for each supplied point you have a block of columns.

So, then from the first block I am selecting all the n columns, then from the remaining m minus 1 columns I will select 1 column, first column of each; so A 2 1, A 3 1 and finally A m 1; so how many you have m minus 1 here and n here, so the total number of columns is m plus n minus 1, therefore what does it look like, see I can write it out e 1, e 1 and so on e 1, e n is of this then this would be e 2, e 1, e 3, e 1, e m, e 1 and so on.

And by the way, what you should do is sit down and find at least more than two different sets of such column, which are linearly independent. I have chosen one for you, so see now here there is A 1 here, but they once in the remaining part is a different place. So, for example if you choose the column e 2; so e 2 has A 1 in the second place, but there all these, see your brake up like this. So, these are the supply constraint rows corresponding to the supplied points and these are the rows corresponding to the demand points, so here e 2 second place there is no 1 here, and similarly there is no 1 here.

So, this column cannot be expressed as the linear combination of these columns, because all other columns in the matrix have 0 in the second place. Similarly, here e 3 all the remaining columns have a 0 in the third place and so on, so none of these and similarly here any none of these columns can also be expressed, because they have 1 in the first place, none of these columns have 1 in the first place. And here, you see for these columns you may say that once in the first place, but then here this is the identity matrix, because this is 1 in the first place and 1 in the second place and so on, and this will be one in the nth place.

So, this set is consider a set of this, therefore can easily argue. So, I am not writing the argument here can easily argue that this set is linearly independent, at this set is legibly is linearly independent.

So, therefore I have shown you that the rank, so please sit down and 1 value going through this material, we should able to write down many more sets of columns m plus n minus 1 columns which are linearly independent, so the rank is m plus n minus 1.

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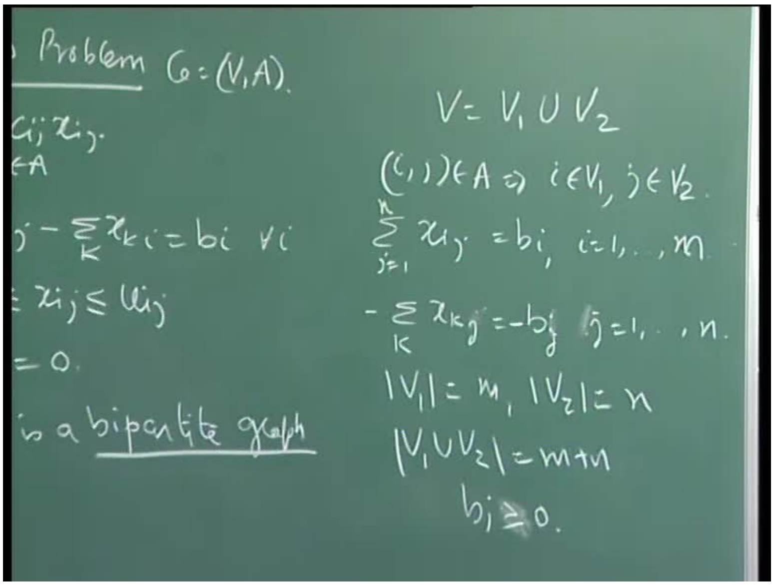

And so what we can do is, therefore now I can get rid of the minus signs and I will start with the transportation problem. So, we can now start looking at the how the simplex algorithm can be simplified and we need to define a few more, because I have to translate the concepts of these simplex algorithm to the transportation problem.

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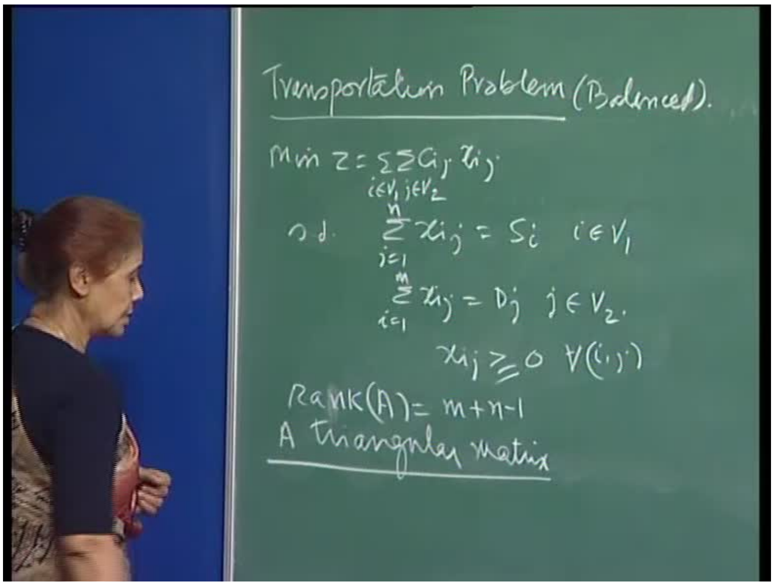


So, now this is our transportation problem and if you want to distinguish I mean people, so this is balanced 1. Balanced transportation problem, because you can have a situation where the sigma b i's are not 0, or if I remove the plus minus sign then the sigma b i's respective to the supplies. The total supplies are equal to the total demand we say it is a balance transportation problem.

So, we are writing this as minimize z equal to summation c i j, x i j; and you need to write i belongs to V 1and j belongings to V 2, subject to summation x i j, j varying from 1 to n.

So, let us write s i, i belonging to V 1, and summation x i j summation over i varying from 1 to m is D j, j belongings to V 2 or 1 to n, and we are not putting any x i j greater than or equal to 0 for all i j; that means the upper bound constraints I am write now ignoring, I am saying there is no upper bound constraints later on.

So, this is really as special case of the general mini-cast flow problem that I have formulated for you, and then we will come back to the upper bound constraints also the upper bounded version of the transportation problem, we will visit that after some time, but now I want to develop the simplex algorithm for this and show you the simplifications, and then one can always bring in some more complications.

So, this is about transportation problem and we have said that a rank of the coefficient matrix A is m plus n minus 1. Let, me show you now that this basis matrix for the transportation problem has a special structure. So, let me define this for you first a triangular matrix. So, what do we mean by a triangular matrix, this is a definition. so triangular matrix.

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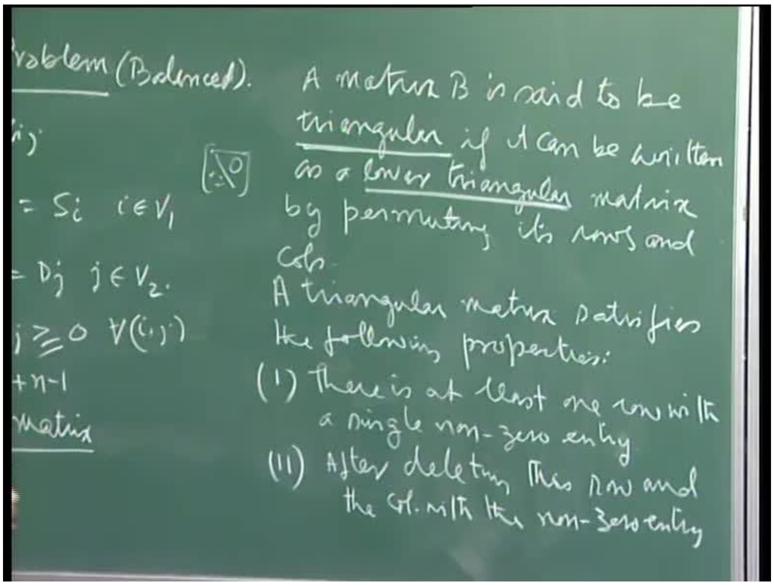


So, A matrix B is said to be triangular if it can be written as a lower triangular matrix, it is not necessary, but see if it is a lower triangular then you can again transpose it and get it. If, it is upper triangular you can transpose it and they are make it lower triangular. So, there is no restriction, it can be written as a lower triangular matrix by permuting its rows and columns.

So, this is what a matrix is said to be triangular, so if it can be written as a lower triangular matrix by permuting its rows and columns. And, so what would that mean that here, it has the following properties. The triangular matrix satisfies the following properties, what are the properties, 1 there is at least 1 row with the single entry with the single non-zero entry. See, when you say lower triangular it means essentially you see, so this is 0 and you can have entries here.

So, that means you see for example, here the first row, first row will have only 1 entry all others are zeros. So, there is at least 1 row with single non-zero entry, I will explain, because again this may sound like a restriction, there is at least 1 row with the single non-zero entry and then after deleting the row, this row and the column with the non-zero entry, the reduced matrix which will be m plus n minus 2 by m plus n minus 2 also did the same property, which has property 1, that means again you can find a row with 1 single non-zero entry.

Here, by the way we are we are defining this for a basis matrix, because I am saying non-zero. Let, me write it down a 0 matrix is also lower triangular by the way, because all entries are 0; so its lower triangular, so we are actually here defining basis matrix, because I am saying non-zero entry, and it should be clear to you why.

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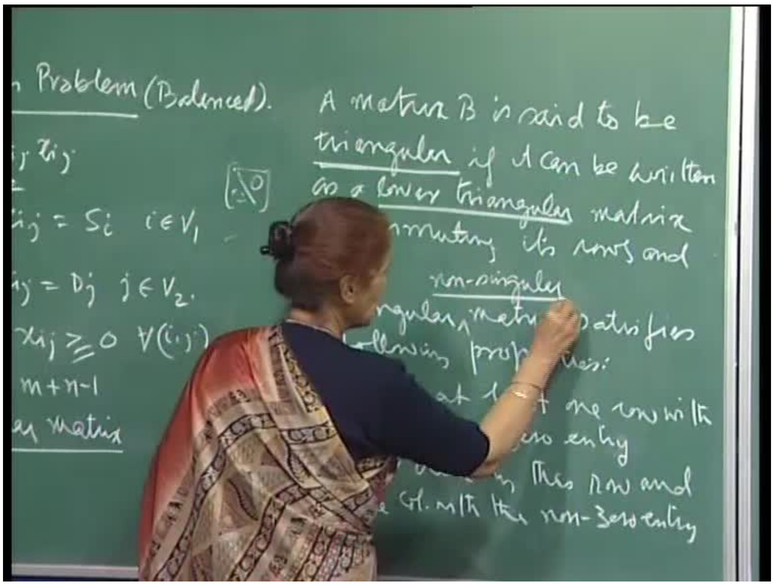


So, may be some where I should say that a triangular matrix or triangular non-singular matrix, this is important and non-singular and triangular, so understood it has to be square matrix, a triangular non-singular matrix satisfies the following properties. So since it is non-singular the determinant cannot be 0.

And for a triangular matrix you know, for a triangular matrix what are the determinant. So, this determinant is the product of the diagonal entries, so if any of the diagonal entries is 0 the product is 0, so the determinant is 0. Therefore, if since it is non- singular and it is triangular, therefore there is at least 1 row with the non single non-zero entry, and we will make it a diagonal element, because we will bring that row to the top then what we are saying is that you remove this row and the corresponding column, then what we you are left with these again a non-singular matrix, and since it is triangular it will again have a row with single entry, because this second row, now will have a only 1 non-zero entry, because this 1 has been struck off; therefore you can again bring that row into the second position and continue with it, so column with the non-zero entry.

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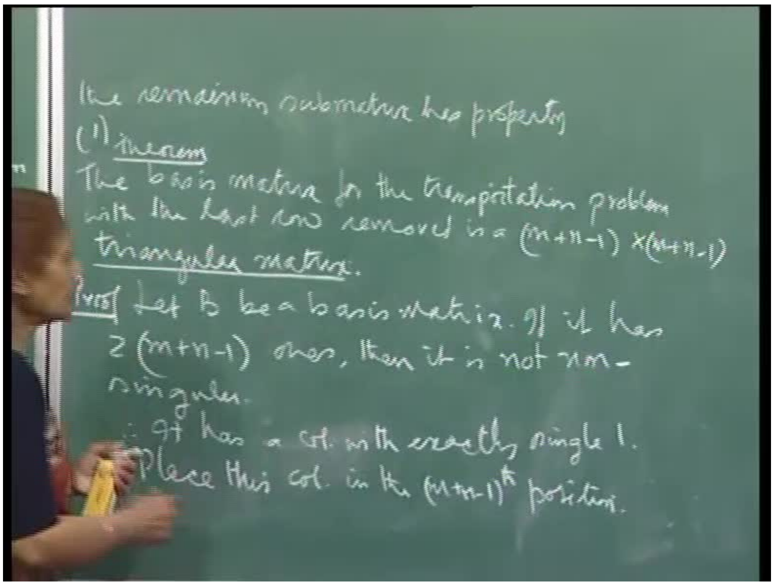


So, after deleting this row and the column with the non-zero entry, the remaining sub matrix has property 1. So, therefore a triangular matrix will always satisfy these 2 properties, and therefore we can always you know keep bringing the rows consecutive rows here in the top and then we will come to a triangular matrix.

So, the therefore now we say that the basis matrix for the transportation problem, so we remember we want a square matrix, so here as it is I am taking the column if you take the m plus n minus 1 columns, and they will be m plus n by this thing, so we will say that the basis matrix for the transportation problem with the last row removed is m plus n minus 1 by m plus n minus 1 triangular matrix.

So, when you remove the last row you have only m plus n minus 1 rows, therefore the matrix the basis matrix becomes m plus n minus 1 by m plus n minus 1, it is non-singular, because it is a basis matrix and we can show that it is a triangular matrix.

So, if you want you can write this as a theorem or whatever it is, so you need the proof. Now, this is the time we are going to explode this special structure, because let b be a basis matrix. If, it has twice m plus n minus 1 positive entries, so now I am referring to this situation. I have removed the minus sign. So, 2 m plus n once, then it is not non-singular why, because b is a basis matrix. So, if it has 2 twice m plus n minus 1 once and that means what, that means that every column has 2 entries, because they are only m plus n minus 1 columns and each column has only 2 entries.

So, if it has twice m plus n minus 1 once then every column must be having 2 entries, and we saw that you can add up the supply rows and the demand rows and the sum and show that the linear independence is not there, so then it is not non-singular.

Therefore it has only m plus n minus 1 what shall I say, therefore it has column with exactly single 1, so they must be a column. Since, it cannot have twice m plus n minus 1 once in it they must be a column which has only a single entry, which is the single 1, so it has a column with exactly single 1; place this column in the m plus n minus oneth position. I bring it to the last.

I should have point it out here that this definition we use the word row, but as I would remember, since you are saying that permutations of rows and columns are allowed; so even if this the same applies to column also.

So, please I should have appointed this out that the definition I simply wrote the word row, but it also can be for a column. So, if column wise also the matrix has the same property it satisfies these two then also its triangular, because then if you can write it as a upper triangular, you can again transpose and take the, so there is no problem, therefore the column with the single entry I will place in the last position. See, here this column, the last column has single entry, then you remove this row and that means now place this column in this thing.

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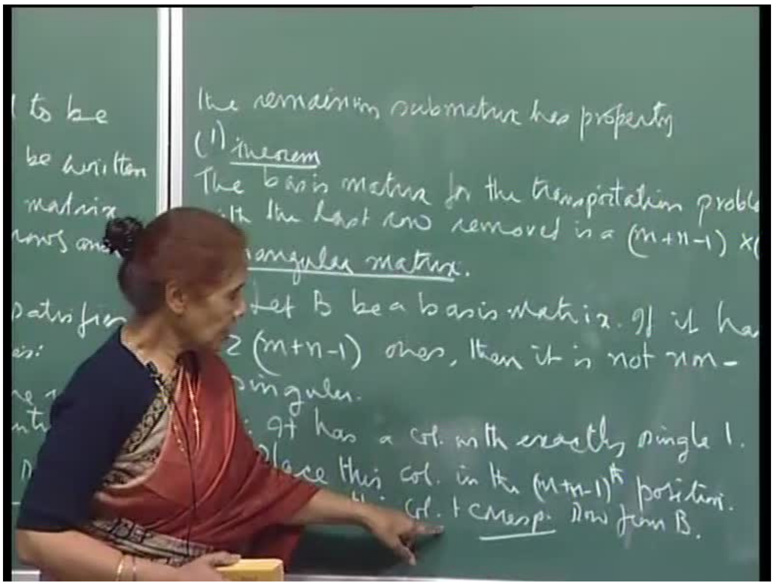


Now, remove this column and the corresponding means, the row having that positive single 1 this column and corresponding row from B and the remaining the sub matrix again, because we are using the property that if a matrix is nonsingular and they cannot be any square sub matrix of a basis matrix whose determinant is 0. So, therefore remove this column and corresponding the row from B.

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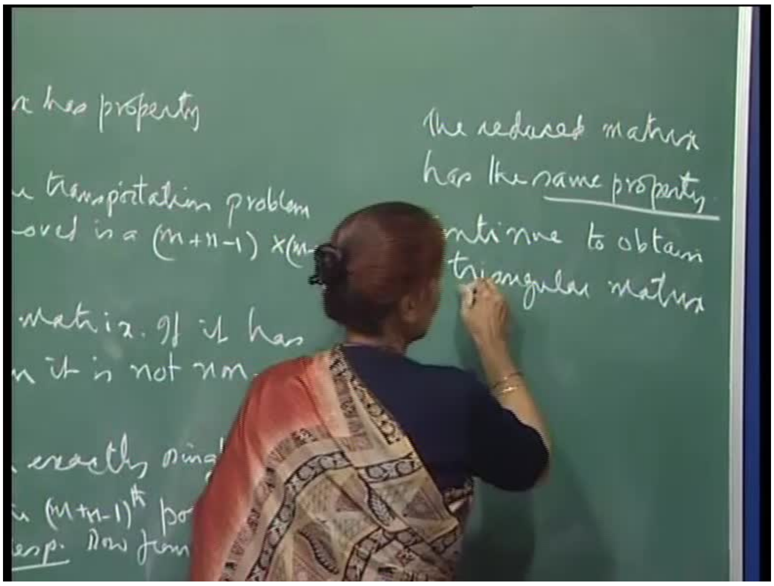

The reduced matrix has the same property that means it is triangular what do you mean by that it has the same property that you can again find, because the reduced matrix again will not have, see when you have removed the row and a column the reduced matrix will be of size m plus n minus 2. So, it cannot again have twice m plus n minus 2 once in it, and so you will be able to find another column with only 1 single non-zero entry which is 1, and then you can place it the last second and so on. So, therefore you continue to obtain a triangular.

So, we just discussed that any basis for this will be triangular matrix and what was the property we basically used, we said that at every while trying to transpose not a permute the columns, we can always make sure that, because it is a non-singular matrix there will be at least 1 column with a single entry, non-zero entry; and so I can place it as the last column then when I remove this column and the corresponding row, I will again be left with a sub matrix which is non-singular, and therefore it must have 1 column which has only a single non-zero entry, so I will place it here and so on.

So, basically I am exploiting the idea that any non-singular matrix here for this particular coefficient matrix will not have all columns having two once in it. Essentially, we are using that property and as I go on deleting a column with the single entry and the corresponding row, I will again be left with the sub matrix which has a column which has only 1 positive entry in it, which is a single one.

So, essentially we are using this and we will reformulate this in another way, so in any case when you have this, see what is the idea behind trying to show that the basis matrices triangular.

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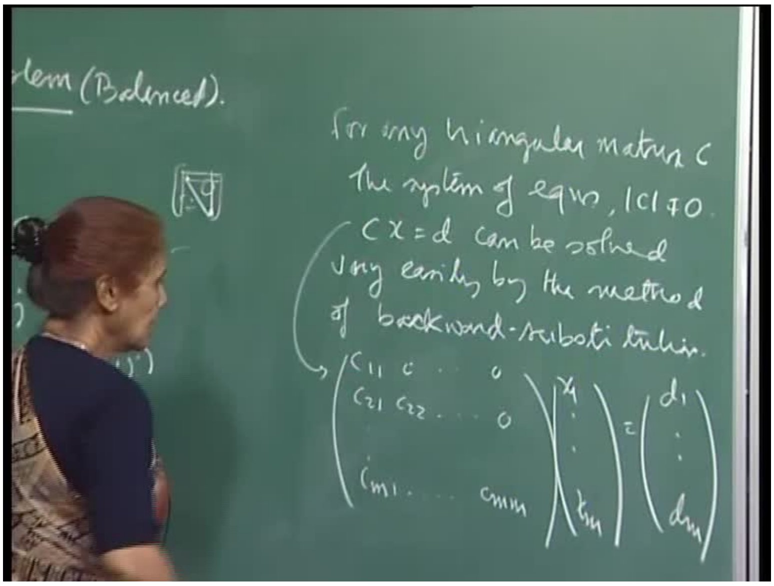


For any triangular matrix, say A what do you want to call it here, let us say c matrix c the system of equations c x equal to let us say d can be solved very easily by the method of backward substitution how, because see the first equation will look like this is c 11, 0 and it is a c 2 1, c 2 2 and 0 and I'm assuming that determinant c is not 0.

So, its triangular matrix determinant is not 0 that means all diagonal entries must be non-zero. This is how you have it, then this will be let us say c m 1 and so on, it will be c m m, so this times x 1, x m; suppose you have this system is equal to d 1 to d n. so, this is your system here.

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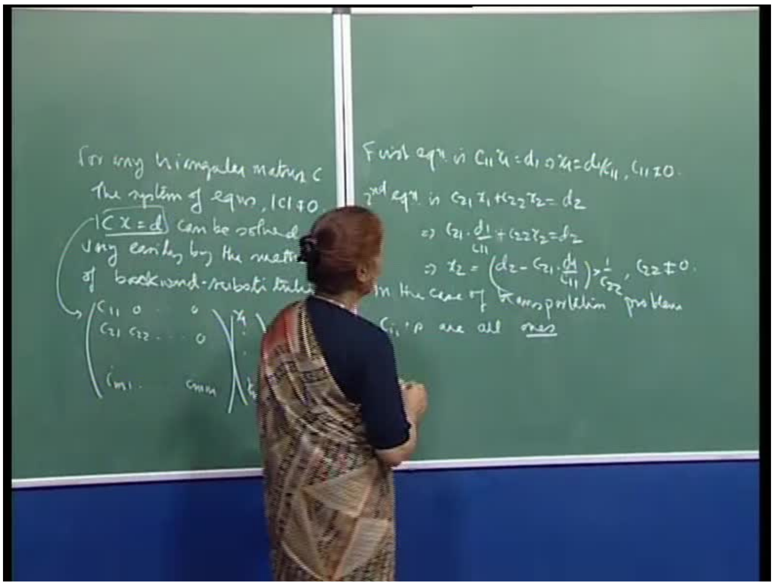

Now, you see because this is a singular entry here, this equation reduces to, so the first equation reduces to, first equation is c 1 1, x 1 is equal to d 1 which implies that x 1 is d 1 by c 1 1, since c 1 1 is not 0, all diagonal entries are non-zero.

So, once you have the value of x 1 the second equation is c 2 1, x 1 plus c 2 2, x 2 is equal to d 2. Since, I already have the value of x 1, this implies that c 2 1 into d 1 upon c 1 1 plus c 2 2, x 2 is d 2. I am just showing you the details, so that you get the idea in your mind get it fixed and therefore this implies that your x 2 is equal to d 2 minus c 2 1 into d 1, c 1 1 into 1 upon c 2 2, where again c 2 2 is not 0. So, by this backward substitution you can immediately get your solution to the system of equations.

Now, for the triangular case, in the case of transportation problem the c i i's are all ones, because every column has only 2 ones in it and it is a non-singular matrix, so these have to be ones. Therefore, for example in this case what does it become, so see the thing is that is to simplify notation, because since we are permuted the rows and columns.

So something whatever was s 1will be something else and so on, so I am not going to it, but the basic you can just translate this concept to the concept here in the sense that if your matrix is triangular and you are solving the system of equations then the first equation will be what.

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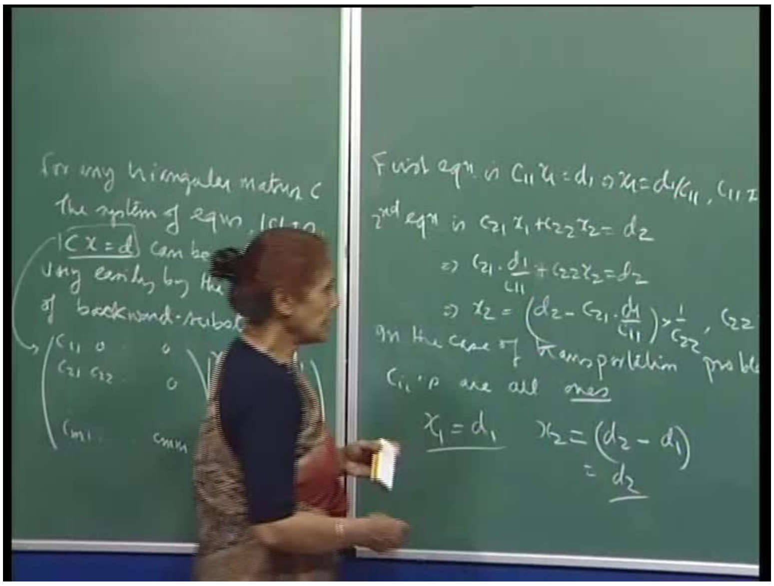

So, the first equation would be something like c 1 1 is 1, so I am calling x 1 here, but it will be some particular x i j; so x 1 will be simply d 1 and then similarly, x 2 would be what from here. Just put your c 1 1 and c 2 2 to ones, so this will be d 2 minus again c 2 1 will be plus 1, so this will be something like d 1.

Because, the entries in this matrix for the transportation matrix are all here going to be plus ones and zeros, so it is possible that this is either d 1 or this will be d 2 does not matter, so therefore you can see immediately that, because the basis is the triangular matrix the basic solution.

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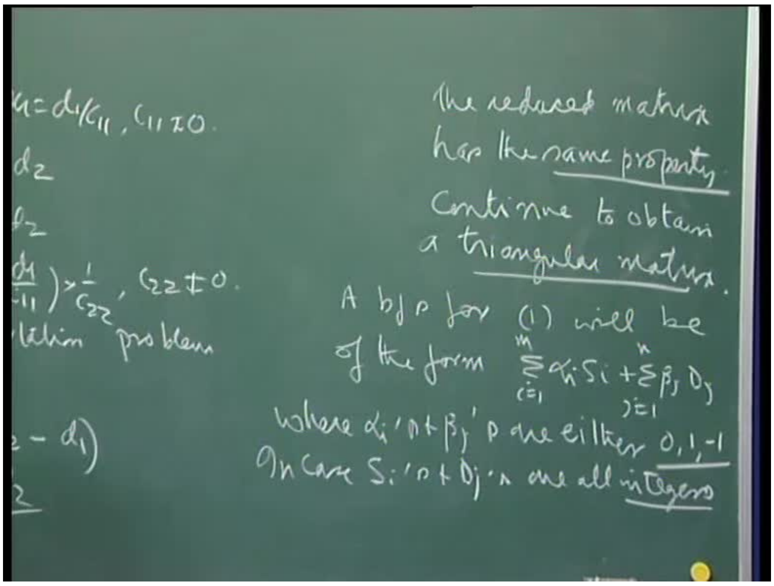


So, let me refer to this problem as 1 here, so a basic feasible solution for 1 will be of the form sigma you can say alpha i, s i, i varying from 1 to m plus sigma beta j, d j, j varying from 1 to n, where alpha i's and beta j's are either 0 plus 1 or minus 1; that means it will be some combination adding subtracting the s i's and d j's; if you just translate this, because might d ones or simply s i's and d j's and the coefficients are the diagonal entries are all ones, and these can also be only ones or zeros here the entries, because every column has only 2 entries.

So, obviously the question of all these being positive does not arise it just that you will have some ones here and some ones here, but diagonal entries will all be ones and you will be adding and subtracting the supplies and demands together, so this is this.

And, so immediately have this property that in case s i's and d j's are all integers, then your basic feasible solution will also be integers. Since, it will be simply adding or subtracting integers there is no division involved here.

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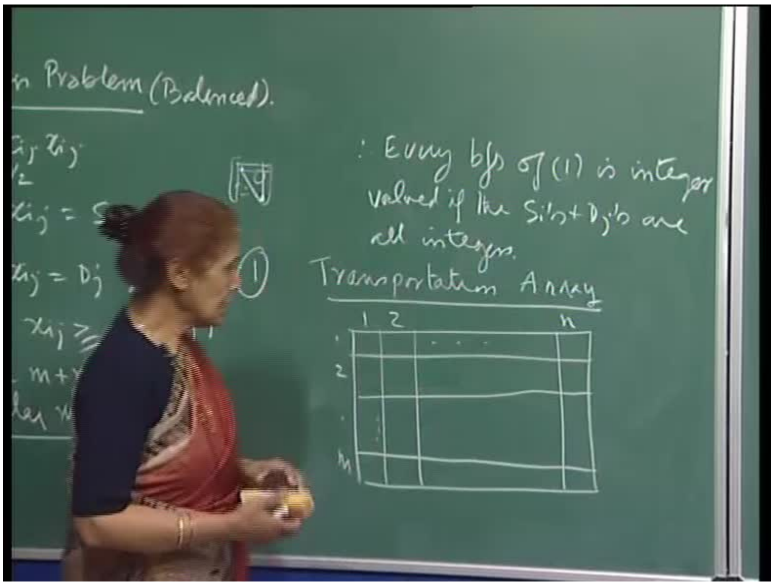


So, therefore every basic feasible solution of one is integer valued, if the s i's and d j's are all integers.

Now, this is a very nice and very good property of the transportation and that this has been possible we could show it, because we could exploit the structure of constraints, and the moment in a linear programming problem the moment you impose the condition of integrality that is you want to your variables to be integers, then the problem becomes very difficult in general problem.

So but then we want to show you that there are cases of the a linear programming problems which have this special structure, and therefore you can show using the structure that a basic feasible solution will be integer value and that really helps a lot, because such situations when you have if you are transporting you know barrels of oil and so on then, because these supplies will be integers, the demands will also be in terms of number of barrels, you are sure that your linear programming solution will not give you in half a barrel or 1 fourth of a barrel or something like that. The basic feasible solutions are all integers valued, so one of them will turn out to be optimal and hence, you would not have the embarrassment of having to you know make 1 tenth of a barrel or something like that as an optimal solution. So, this is that.

So, you can immediately have this property from the triangular ness of the basis and you want me to take up an example here. So, now what we want to show you is that how do I progress with the simplex algorithm, so I have showed you that a basic feasible solution will correspond to this a triangular basis.

So we will not actually work with the simplex tableau, but again there is a simplification here and I will talk about the transportation array. So, let me say this how do I actually represent this, so this will be a transportation array and that means the idea is simple. See, what I will do is I will write it as a collection of these cells, so here this is; so m rows corresponding to each supply point and then you have 1, 2 and so on, and these are your demand points.

So, for example here you have these m n cells, so you obviously you can immediately connect that a cell corresponds to a column here or to an arc here, we will have these m n cells here. So, for example the i j cell, we will not really need to compute and update all these columns and so on the nice way, I mean of course you need the updating of the incoming column and so on in the basis, but it will all be we will able to show it through this. So, now here we have to translate the concept of a basis, because you have these m n cells.

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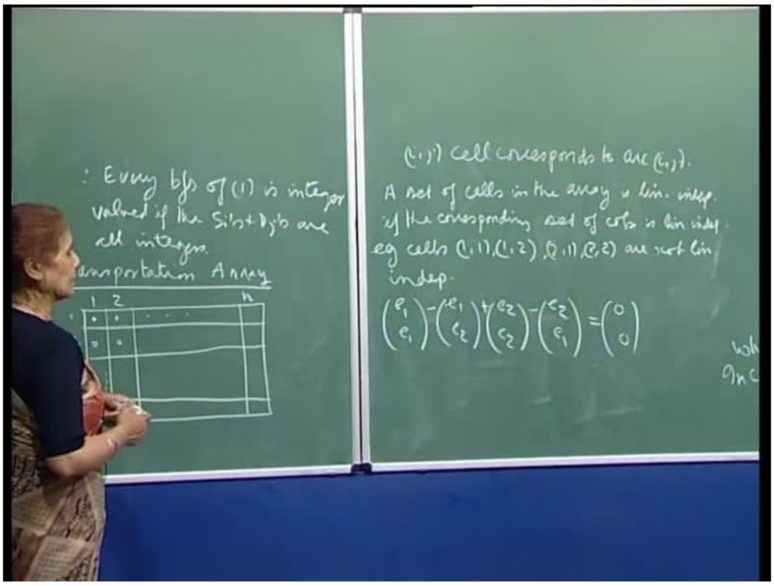

So, for example your i j cell corresponds to arc i j, therefore the corresponding column for the cell is you are A i j column which is made up of e i and e j. So, how will I translate the concept of independence of a set of m plus n minus 1 columns to this representation of this transportation array.

So, we will say that a set of cells in the array is linearly independent if the corresponding set of columns is linearly independent, so for example, you cannot have these four cells as linearly independent why; because you see if you write down what would this correspond to, say for example cells 1 1, 1 2 and 2 1 and 2 2 are not linearly independent.

So, see for example I will write 1 1 as e 1 and e 1 then what I can do is 1 2, so I will write this is minus e 1, e 2 then I will come here I will say plus e 2, e 2 and then minus e 2, e 1; so just look at this. Alternatively, I have this 4 cells and alternatively, because I am able to go from here to here, first row, first row and then you come here second column, then from here I move downwards along the column; so this becomes 2 2 and then from here I go to the first column; so this is 2 1 and then this, therefore you have this here.

You see this one will cancel with this, because 1 in the first place, 1 in the first place; so this entry goes away, then you have 1 in the second set and you have 1 here, so plus and minus 0, then here also this is minus e 2 plus e 2 and this is e 2 minus e 2; so this is 0 0. Now, here these are vectors, this is m-dimensional vector and this is n-dimensional vector so this set.

So, therefore the same concept that means and what is happening is that each of the columns here you can see, each of the columns has 2 ones, this has 2 ones, this has 2 ones, this has 2 ones and this has 2 ones. So, the columns are linearly independent.

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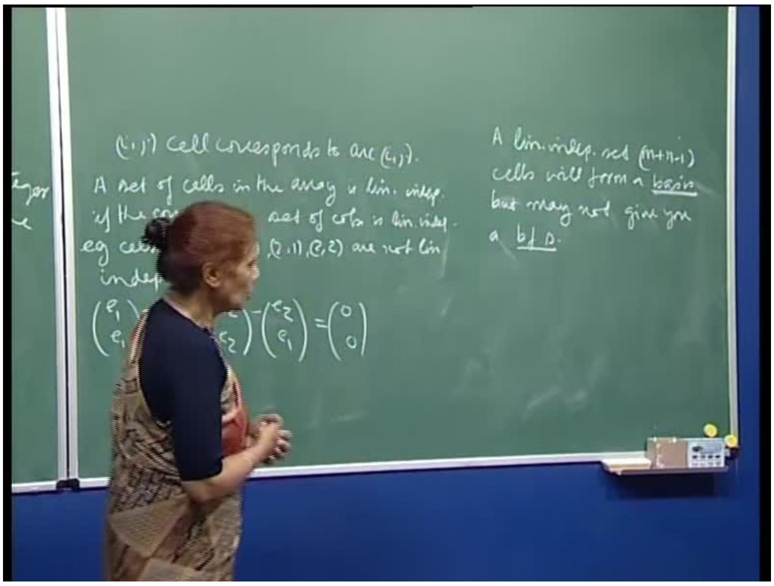


So, therefore the cells will be linearly independent if the corresponding columns are linearly independent, and so how do I now obtain, so again just to give you an example, if you have a set of basics that means a linearly independent set of m plus n minus 1 cells will form a basis, but whether it will lead to a basic feasible solution or not has to be seen and here again let me take up an example.

So, let me just tell you a linearly independent cell, a set of cells will form a basis, but may not give you basic feasible solution, just as for the simplex algorithm you have a basis, but that does not necessarily mean that it will lead to you a basic feasible solution.

So, the idea is that how do we make sure that how do we obtain a basic feasible solution, given a set of m plus n minus 1 basic cells or how to obtain, may be given any basis how do I go about obtaining a basic feasible solution.

So, in the next lecture we will consider it on, essentially building up the simplex algorithm for this special case for the special structured problem and you will see the simplifications that you know you can, it is amazing that by exploiting the structure you can make life so much easier.

So, this is what we will attempt how to obtained that means the things that we have to now look at is, how do I get a starting feasible solution, basic feasible solution, once I have it then how I want to improve or go to a better basic feasible solution. There, will be a simplification and finally, you can continue with the simplex algorithm for solving the problem, so that should be in the next lecture.