Linear Programming and its Extensions

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Lecture No. # 22

Parametric Cost Vector LPP

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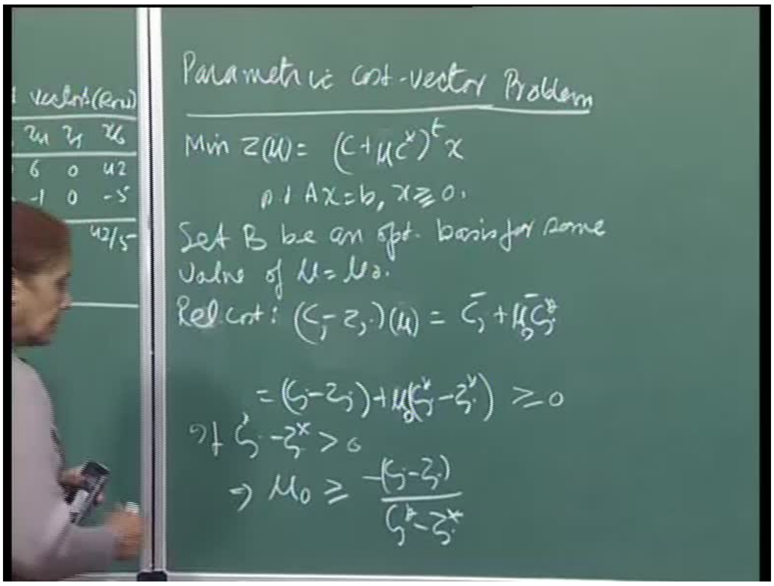

I will now talk about the parametric cost vector. So, I told you that this is nothing but the dual of the parametric right hand side problem, but we will handle it independently; so, the problem would be minimize Z lambda equal to C plus lambda C star transpose x subject to A x equal to b x greater than or equal to 0. And there can be many economic situations which would probably modeling to this kind of thing, because they sometimes the price vector is also under controlled by the government, and they might announce the different policies from time to time; so, you may want to keep this parametrized through some C star which probably is known or has been told; and then you want to be able to make sure that the moment the policies announced, you have excess to an optimal solution right.

So, let us first begin by saying that, suppose, so let B be an optimal basis for some value of mu equal to mu naught; so, suppose for some fixed value of mu naught and most often mu naught is chosen to be 0, and then you obtain an optimal basis by so many different methods that you have learned so far, so B is an optimal basis for some value of this.

So, then it is also feasible, it is optimal for mu equal to mu naught, and you want to now say that therefore the relative cost here is equal the relative cost, for example, for Z j lambda minus I am doing it the other way, see we are using the notation some text books use it as Z lambda minus C lambda, but we do it, so C minus C j minus Z j, and this as a function of lambda.

So, this would be in our case C j plus lambda C j star bar right, which we write as C j minus Z j plus lambda C j star minus Z j star right; because here the Z j star is computed with respect to the C b star, and this is computed with respect to C b C b be inverse A j, and this will be C b bar C b star B inverse, so the same this will be computed with C b star b inverse into A j, so this is the thing right. So, then you want this to be greater than or equal to 0, and why am I writing lambda, it should be I am using the parameter mu here just to distinguish from the right hand side case, so mu equal to mu naught, so this will be then mu, and so here also this will be mu.

So, in fact right now we are having this as a mu naught, and this is as mu naught, because for some fixed value of mu naught this optimality criteria is satisfied. And now, let us define, so let the same way we will define an upper value for mu naught and a lower value, so the same basis since its feasible, let us say for what value is of mu other than mu naught the optimality criteria will be satisfied. And so, we say that if C j star minus Z j star is greater than 0, then this implies that your mu naught is greater than or equal to minus C j minus Z j upon C j star minus Z j star right with C j minus Z j star greater than 0.

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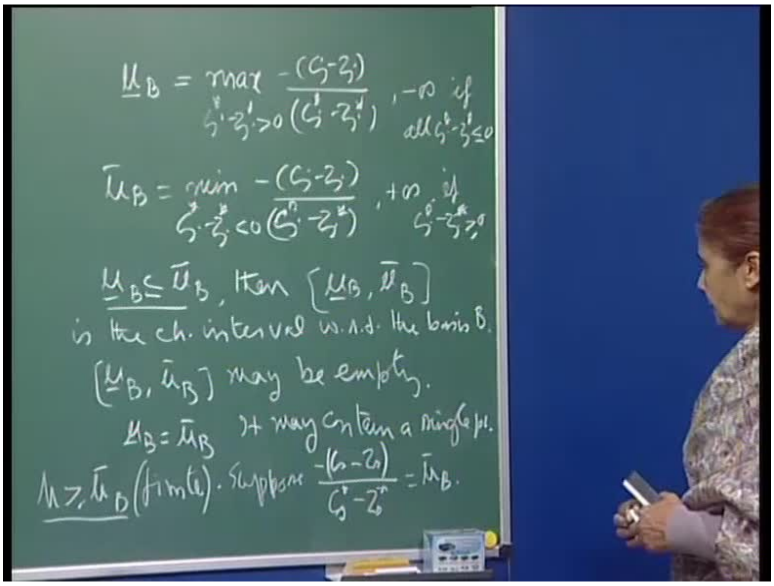

So, in that case we can immediately…, that means, if I take mu to be lower this, let me again find it as mu greater than so this lower bar with respect to B, I take the value as max right maximum of all such for which C j star minus Z j star is positive max of minus C j minus Z j upon C j star minus Z j star such that C j star minus Z j star is positive.

So, I defined mu bar, the because as long as this thing is positive for all values of mu greater than mu b bar, this number is the inequality the optimality criteria would be satisfied; similarly, you will define your mu upper bar as minimum of minus C j minus Z j upon C j star minus Z j star and your C j star minus Z j star is less than 0 in this case, so accordingly….

So, therefore, we will now define…, now see the thing is that, you expect that mu lower bar is less than or equal to mu upper bar; if that does not happen, that means, the basis is not optimal, it will never be optimal, because the interval is empty for the basis B, but here we have started with B as an optimal basis for mu equal to mu naught; so, in our case this will this is true, and in fact mu naught is a element of this interval which we call as the characteristic interval.

So, this happens, then we say that mu B lower bar, mu B upper bar is the characteristic interval with respect to the basis B, this is the characteristic interval; and as we saw it can be empty, but in that case there would be no optimal basis B; therefore, if I just want to consider for some basis a characteristic interval, it is possible that the characteristic interval for any basis may be may need not be non-empty right, so that is one thing, and secondly or so this is the interval.

So, now, we are saying that this may be empty right or it may contain a single point, which means that lower bar is equal to the upper bar right, it may contain that means this will be upper bar right, otherwise it is a proper interval that has more than one number; and so, B is optimal for all values of mu in this interval right. Now, yes so what else can we say about it, yes, you see that the moment you choose a value of mu outside this interval consider the case mu greater than mu upper bar B, then what will happen that this number for whichever j this minima occurs for that index the a corresponding relative cost value will become negative.

The moment you choose a value of mu bigger than mu upper bar B or mu less than mu lower bar B, then the corresponding this thing here for whichever the maxima occurs, so that index the relative cost price will become negative; and similarly, here for mu greater than mu upper bar B the corresponding relative price for that particular variable will become negative; so, you will lose optimality for basis B right, for values of mu outside this B is no longer an optimal basis, and therefore here the proof is very simple that the particular basis will never be repeated.

Because see it will not be optimal for values of mu outside this, and so as you proceed with the higher higher or lower and lower values of mu, that means, the moment you leave this interval optimality of B is gone right, and so B will never come back, because you are not going to come back to a value of mu again, you are going to change the values of mu; so, therefore, here it is very simple to say that the algorithm will be finite in number right, I mean the number of iterations for the algorithm will be finite in number. And also the same argument you can just discuss since to show that the overlapping intervals will not be there, the characteristic intervals will not overlap, why? Now it may contain a single point or otherwise it is a proper interval so fine.

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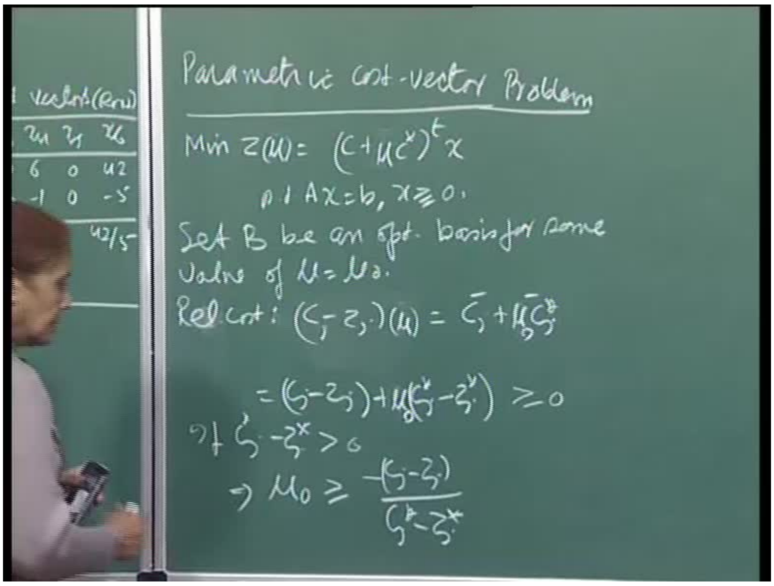

So, once you have this then you may want to proceed so, for example, if you want to take mu greater than or B upper bar, if you want to do this, so then in that case of course if this is finite, you can see from here also that, yes, I should have written this it is not complete yet, because if C j star minus Z j star, if there is one single j for which at least one j for which this is positive then you can compute this.

But then otherwise it will be what mu B lower bar will be minus infinity if all C j star minus Z j star or less than or equal to 0; yes, you can see from here from this inequality, if all these are less than or equal to 0 hold on, yes, when mu can be made as small as you wish and the inequality will continue to be satisfied; this is negative for all j then you can choose your mu to be as small as you want and the value will continue the…, this relative cost will continue to the less than 0 right.

So, in other words the mu can take values up to minus infinity if C j star minus Z j stars are whole negative; similarly, here this will be plus infinity the upper of this if C j star minus Z j star are greater than 0 greater than or equal to 0, because then it is only the yes, because I can make this very large no matter what the sign of C j minus Z j is if I take mu large enough, then this inequality will be satisfied provided this is all greater than or equal to 0.

So, if all these are non-negative then I can choose mu to be very large and irrespective of the sign of C j minus Z j, this inequality will continue to be satisfied. So, that means, in that case the value of mu will go up to plus infinity if this condition holds right, so this is important; therefore, here we are saying that you want to proceed with a value of mu greater than mu upper bar B provided, this number is finite; if its infinite we do not go beyond that then I will look at the values of mu less than mu lower bar B right.

So, if this is this; then you choose, so what will happen in this case, this is the thing here is, so suppose minus C s minus Z s I hope you can…, let me write it carefully C s minus Z s divided by C s star minus Z s star is equal to mu upper bar B, that means, this minima occurs for the index S for the variable s right.

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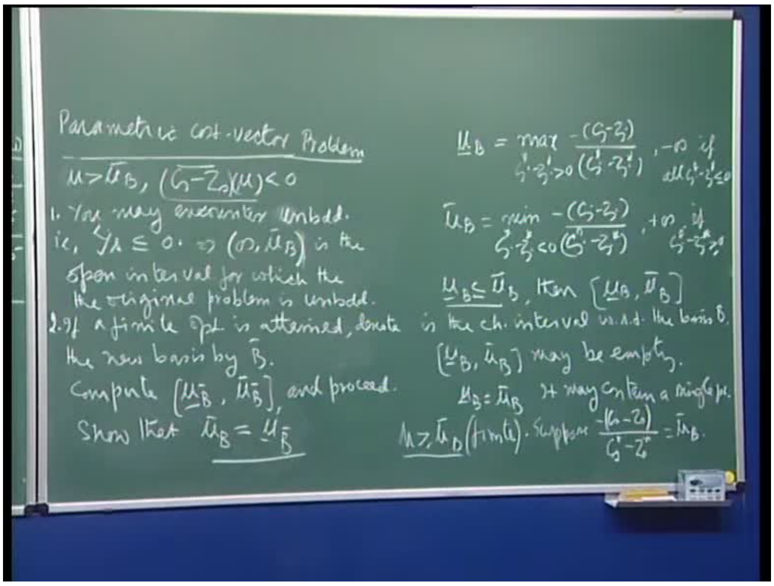

So, what are the things possible, I have to come back here, so you find out this thing, and you can immediately see that your relative price so for mu greater than mu upper bar B your C s minus Z s mu right, this relative price is less than 0; so, as soon as mu is greater than mu upper bar B this number will become less than 0, so it will be a candidate for coming into the basis, you want to improve the value of the objective function right. So, what are the two things possible? So, you start with the simplex algorithm look for an optimal basis, because this one is going to enter into the variable x s is going to become a basic variable right.

So, then either, so in that case, see you may encounter so as you proceed with the simplex algorithm, you may encounter sorry unboundedness, yes, because a feasibility is not a problem since you will in the simplex algorithm you will go from one basic feasible solution to another. So, once you enter x you try to enter x s into you try to make access of basic variable then you may encounter unboundedness, which means that because this thing is any less than 0, which means, that is your Y s is less than or equal to 0.

So, in that case what will you conclude immediately, because for mu greater than mu upper bar B this is less than 0, and it will continue to be less than 0 for all mu greater than mu upper bar B, and if the corresponding column underneath is less than or equal to 0, this implies that for this interval sorry this will be opened because for upper bar B you have B as an optimal solution.

So, this implies that, this is the open interval for which the problem original problem is unbounded; and if you do not encounter the unboundedness criteria you will get a new basis by including A s into your basic set of columns, you will get a new basis and again you will find out, so you may encounter if a finite optimum so maybe I can write this as two steps here these are the two possible outcomes.

So, if a finite optimum optima is attained denote the new basis by B bar, and remember I have used they are not use the word adjacent very often but when describing the simplex algorithm first we defined adjacents here so on; so, any way here see B bar and B differ by one column right. You have replaced and you have entered this thing, the A s column here you have entered A s into the basis B bar and some column from B which by the minimum ratio rule you will find out will be taken out of the basis and A s will come into the basis, so this is your new basis right.

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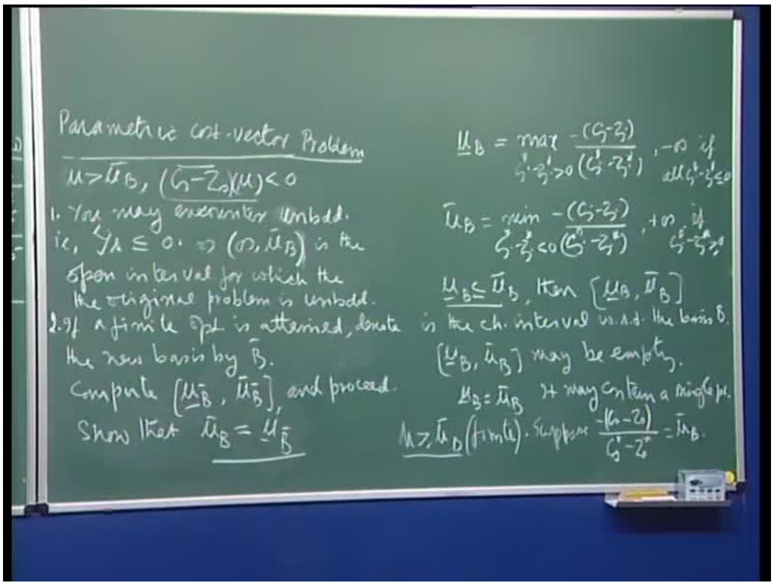

And then accordingly, so, then compute this and right I am putting it close provides you know with the assumption that this is finite, if it is not then you will have a…, I mean you know how to…, so you compute this and proceed right.

Now, I have already told you wise, but anyway you write it down, so show that, so this is a question, show that mu upper bar B is equal to mu lower bar B bar, that means, the intervals the characteristic intervals that you obtain will not be overlapping and also we already showed you that the number of intervals will be finite, because the number of basis is finite since a basis will not be repeated for…, since you are changing the values of mu at each iteration.

So, the number of intervals is finite, and there will be open intervals on two ends of the…, so when your you are partitioning the real life, so there will be closed intervals and some open intervals, so the open intervals will always figure at the two ends of the real life right. So, that shows you how to continue with this. Now, of course, the question is, so we still have to answer the question as to how to we begin with the…, how to get a starting; see, this was all done with the assumption that, B, you already have a B for some fixed value of mu which is an optimal basis; and then I know how to proceed; so, one more aspect of this algorithm that needs to be fixed is, how do you start with this, how do you begin the algorithm right basically, so let us do that.

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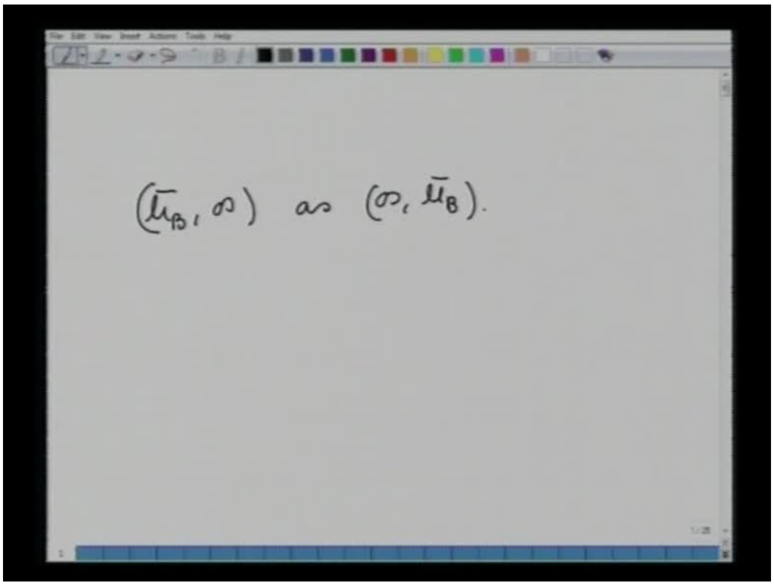

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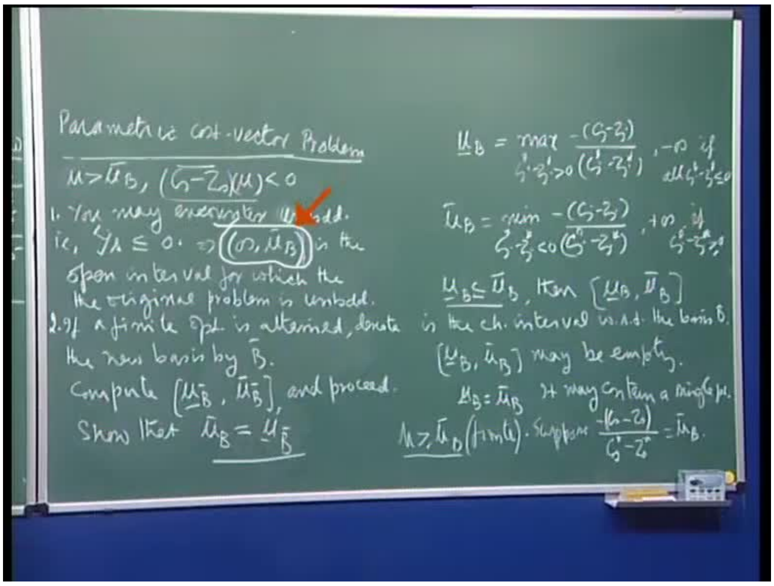

This is to point out that, instead of writing the interval as mu B bar infinity I wrote the interval as infinity mu B bar, so please while going through the lecture kindly replace infinity, mu B bar by the interval mu B bar, infinity. Thank you.

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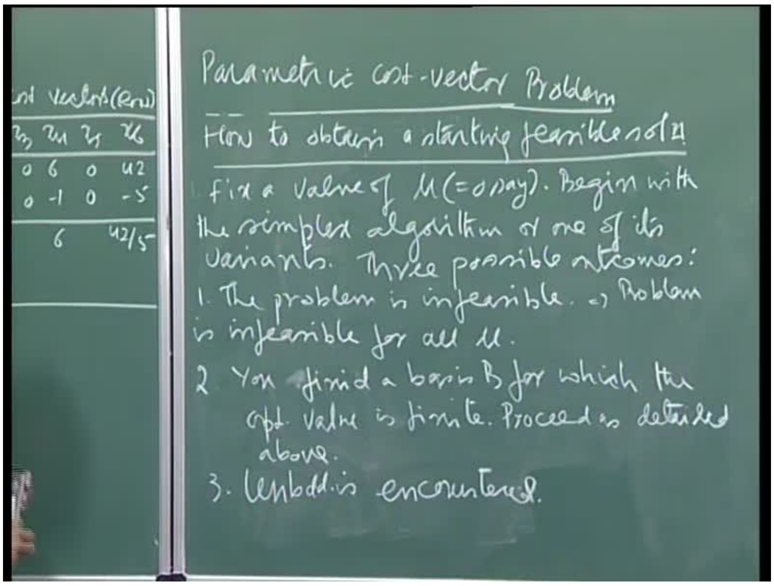

So, now, the question is how to obtain a starting feasible solution, so how to obtain a starting feasible solution is to fix a value of mu and normally you know how convenience sake we choose mu to be 0 unless you have reasons we choose some other value it defects. So, pick the value of mu; and you begin with the simplex algorithm or one of its variants or now you know enough for one of its variants right, it may be that the situations (( )) made for the dual simplex algorithm or you can we give the primal dual whichever is convenient or one of its variants.

So, variants, so, once mu is 0 it is a regular linear programming problem, you start with the use one of the methods to obtain a starting feasible solution and continue right; so, you may encounter what are the three things that…, so three possible outcomes are there right. So, one is the problem is infeasible; now, remember mu is I have fixed mu, and then I see that the problem is infeasible, so it is infeasible because value of mu has no role to play in the feasibility of the problem right.

So, we once I for some value of mu I encounter infeasibility, then the problem is infeasible for all of mu right; the problem is infeasible, this implies a problem is infeasible for all mu, because you are trying to find a feasible solution for A x equal to b x non-negative, so that is nothing to do with the value of mu. So, for any value of mu it turns out that the problem is in feasible, it is going to be infeasible for all mu right.

Second you find a basis B for which the optimal value is finite right; then are you already know I have we know spelled out in detail that once you have a optimal basis for some value of mu then you know how to proceed right, so is finite then proceed as detailed above as detailed I given you the details above. And the third thing is, of course, you encounter, so the third would be degeneracy is encountered; now, it written correctly I will rewrite is encountered; see, you come you come across the at degeneracy, the degeneracy unboundedness is encountered.

Now, here the value of mu is important, because for one particular value of mu, because unboundedness depends on your objective function value, and so for some mu it is unbounded then we have to discover for what other values of mu is it for what other the values of mu is it unbounded, so what will happen?

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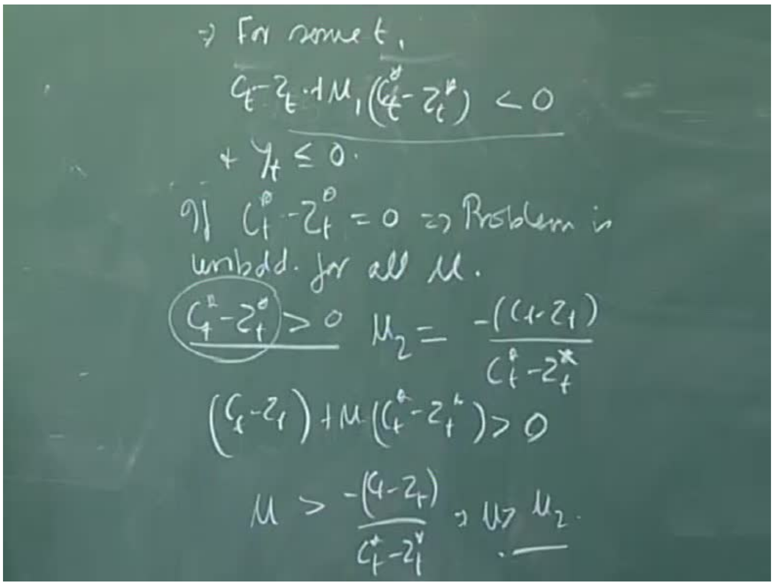

So, if here unboundedness is encountered, then so you want to look at the this things, that means, so which means this implies that, for some t your C j minus Z j bar plus mu naught, well, let me call it mu 1 whatever the starting value mu 1 C j I should say t here, this is t, this is t, so this is also t minus star minus Z t star, this is the thing but while I putting bars here; so, it is important, because once am I writing see either you write C t bar which implies C t minus Z t, so let me not write that bar here right.

So, this is less than 0, yes, this is less than 0, and your Y t is less than or equal to 0, that means, as just continuing with the simplex algorithm or with whatever algorithm you come across a column for which the current relative price is less than 0 and the column underneath is less than or equal to 0, so you cannot proceed with the simplex algorithm, and you say that the problem is unbounded right.

So, now, just see, if C t star minus Z t star is 0; if this is 0 what does it mean, it means that C t minus Z t is less than 0, and your Y t is less than or equal to 0; so, once C t star minus Z t star is 0, the relative price becomes independent of t; so, therefore, what would you conclude, this implies that problem is unbounded for all mu, because the moment C t star minus Z t star is 0, the role of mu is out, and this will continue to be satisfied the unboundedness criteria; then you can have C t star minus Z t star greater than 0, if C t star minus Z t star is greater than 0 then you can define mu t as minus C t minus Z t upon C t star minus Z t star, yes, and what do you see here C t star minus Z t star is positive.

So, then if I choose my mu to be…, see remember you want to…, you want to see for what values of mu will this continue to be will not be satisfied, because I want to know well will I be able to get out of the unboundedness condition right, so you want to this to be.., so you look at here, this is the thing, and you want this to be…

So, this would be, that means, C t minus Z t I do it, and I bring it here plus mu t C t star minus Z t star, I want to this to be positive, so right not is 0, so that means, I take value of mu which is such that this is positive right, so which implies what, which implies that your mu should be greater than because this is positive should be greater than or equal to than equal since the minus of C t minus Z t upon C t star minus Z t star, because this is positive right.

So, that means, from mu so this implies that for mu greater than or equal to mu 2, the optimality criteria the degeneracy criteria will not be satisfied; for mu less than mu 2, for mu less than mu 2 the degeneracy criteria will be satisfied; so, that means, I can then proceed, so in case C t star minus Z p star is positive, then I can start with values of mu bigger than mu 2 mu 2 is this number right.

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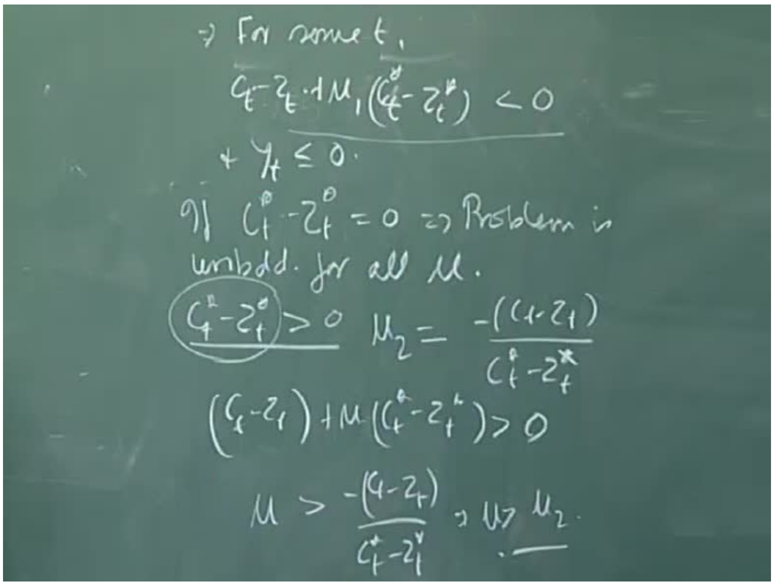

So, I will start with the value of mu bigger than mu 2, and in that case this particular relative price will become positive, and so therefore this will no longer be problem case, because this has become positive, and so Y t less than or equal to 0 is thus, is no problem right, and I will continue with the simplex algorithm, so this is it right; and of course, similarly, you can argue for C t star minus Z t star less than 0, in that case your mu 2 again will be this number, but then for mu less than mu to you can proceed with this simplex algorithm.

So, for mu greater than mu 2 the problem would then…, so depending on the sign of this number which would you would know when you are solving the problem; so, if this number is less than 0, then you will say that for all mu greater than mu 2, this number the problem unbounded; but less than that mu 2 number, the problem may not be unbounded, you do not know but you can at least proceed with the simplex algorithm, because then this number is…, this is no longer less than 0, I mean they are you are unboundedness criteria is gone right. Now, you again compute with the begin with the simplex algorithm, and depending on it is possible that you may again encounter unboundedness.

So, you will there will be another value of mu 2, I mean there will be another value of mu, we can call it mu 3, and so you will continue, but after a while you will get out of the unboundedness or the unboundedness may continue but anyway, so we know the method right. So, for some t, this is this, and so what we are saying is that if this is 0 problem is unbounded for all mu; and if this is positive then you have a mu 2; and for mu greater than mu 2, you will not have let me not write this right, because mu has to be strictly bigger than mu 2; and similarly, we can write it out for…

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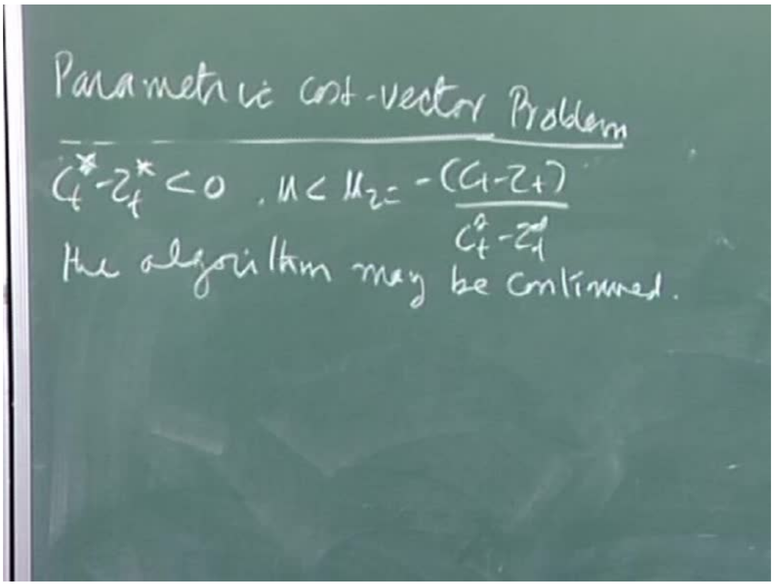

So, let me record it; so, that one wants to read at and look at it, and similarly for C t star minus Z t star less than 0, you want that for, yes, what did we say for mu less than your mu 2 which is minus of C t minus Z t on C t star minus Z t star the algorithm may be continued, so you continue with the algorithm, and so in any case all the stumbling blocks are over, you would know how to handle each situation right.

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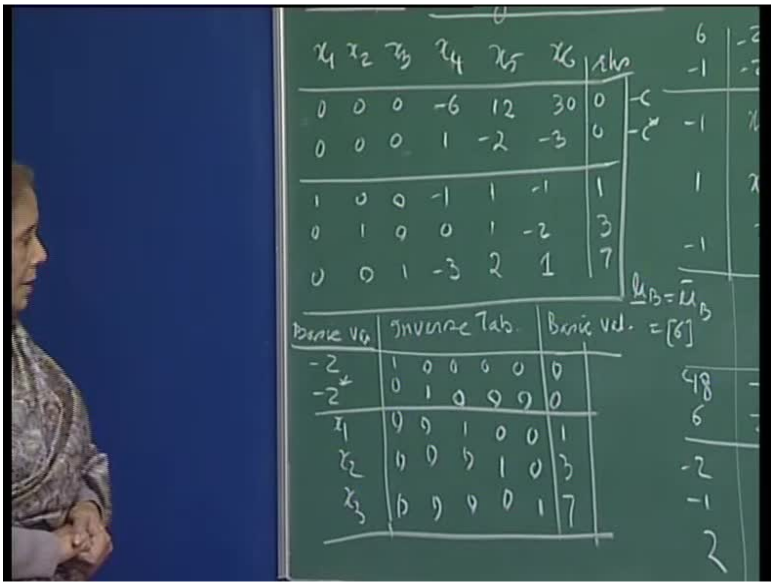

So, let us now look at the look at an example, and then if something is left out here hopefully we can cover it in that case; so, the problem is stated here, so this is your C, and this is your C star, and so that means, this corresponds to minus Z, and this corresponds to minus Z star; and then right now this is the tableau, so it shows you see immediately you see that x 1, x 2, x 3 is a feasible solution, because the right hand side….

So, in this case we do not have the problem of having to…, first find out starting feasible solution I had done it in the parametric right hand side case right; so, I thought in this case I will just begin with the simpler problem; so, here x 1, x 2, x 3 is a starting basis since the prices are all 0 here, C b and C b star, therefore, the C j minus Z j(s) are also the relative prices that we see C j minus Z j(s) because Z j Z j star are all 0s here right, so this is the thing right.

Now, lets me…, I will thought as this is a problem I will solve by using the inverse tableau technique; so, let us see this represents your starting basis, so the basic values are right now 1, 3, and 7, objective function values are 0s, and the your basis is x 1, x 2, x 3, and the inverse tableau is all identity, because this is identity, and right so this is a starting thing.

Then you can check here is optimality is satisfied, no, so we therefore we compute the…, so these are the actually these as…, I said these are the relative prices right, so from here you see that for the positive one you will compute, what, will you compute? I have written them, so that is for the other one, so that is the second tableau we will do it right here; see here the ratio would be 6 for positive one which will give you the lower value of mu, and here this is minus 12 upon 2 which is 6, so that means, in this case your mu B lower bar is mu B upper bar which is equal to 6.

So, this is a single terminals, I hope this is clear, because you see these are your relative prices respect to C and C star since your matrix is identity matrix by basis matrix, and these are 0s, so your C b b inverse is 0, so simply C j minus Z js are simply C js. So, therefore, for the positive one, there is one positive one, so here this is 6 upon 1 which is 6, so the lower value is 6 because only one positive; and for negative ones you have minus 12 upon minus 2 which will be 6 and minus 30 upon 3 would be 10, so the lower number remember, for the upper this thing you need the minima.

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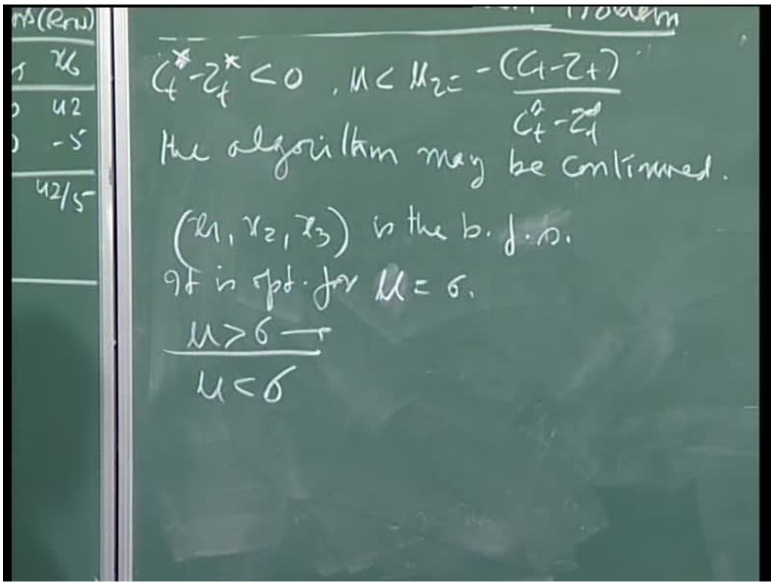

So, the minima would be same, therefore this is a single; so, that means, your basis, what we are saying here is that x 1, x 2, x 3 is the basic feasible solution, and it is optimal for lambda equal to 6; so, I did not have to do any extra effort to compute the starting basis, I caught it by inspection from here only; I could get an interval which the…, if this interval empty, that means, if this value was bigger than this value 6, then this interval would have been empty of aqueous interval, and then I would have concluded that B cannot be an optimal basis for any value of mu.

But we were lucky we have got one value of mu for which again I am writing lambda, because this is mu is equal to 6, so this is the optimal solution; and therefore, when you we have to now a choice of considering mu greater than 6 or mu less than 6; So, suppose, we so suppose we continue with this one, if I do this then this number your c 5 minus z 5 respect to mu, this will become negative, because if mu is bigger than 6 this will become more than 12, so 12 minus something negative which is more than 12 there will be a negative, so that means your this x 5.

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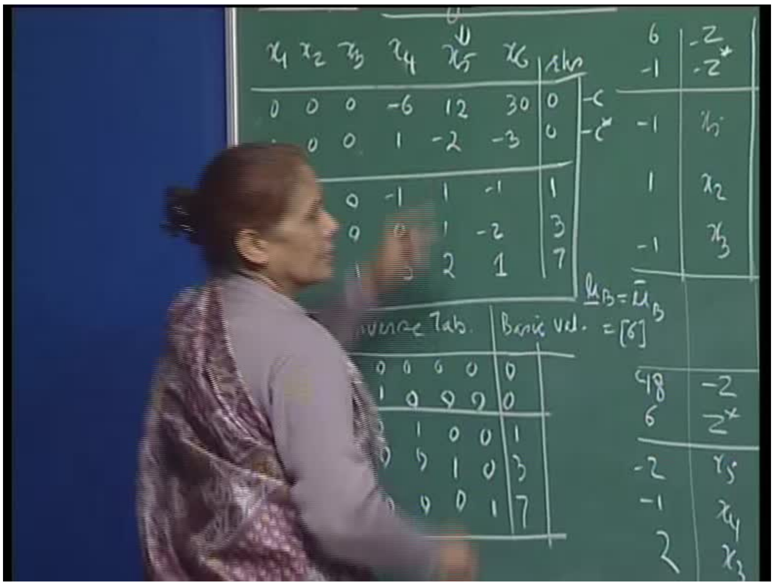

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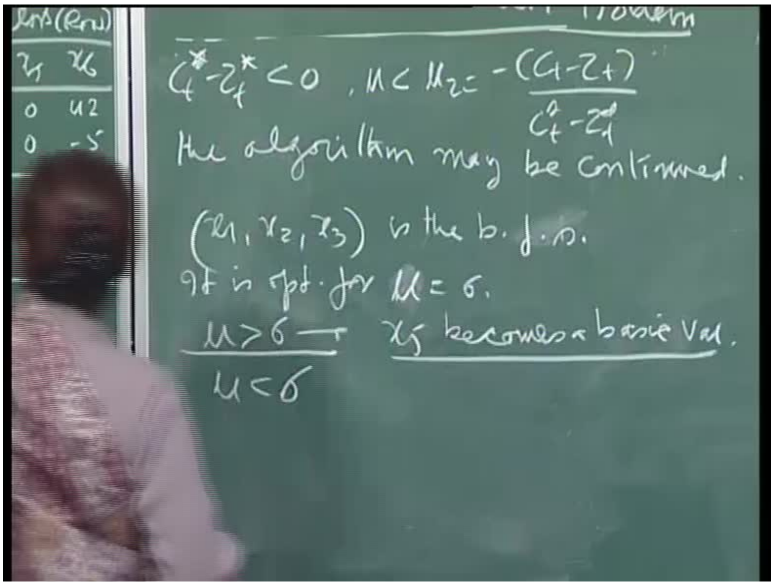

So, for mu greater than 6 x 5 becomes a basic variable, so x 5 becomes a variable basic variable, and we then this x they will show you how to…, so here x 5 becomes a basic variable, and then the idea is that you should be able to x 5…, if let me see if I am considering this thing for….

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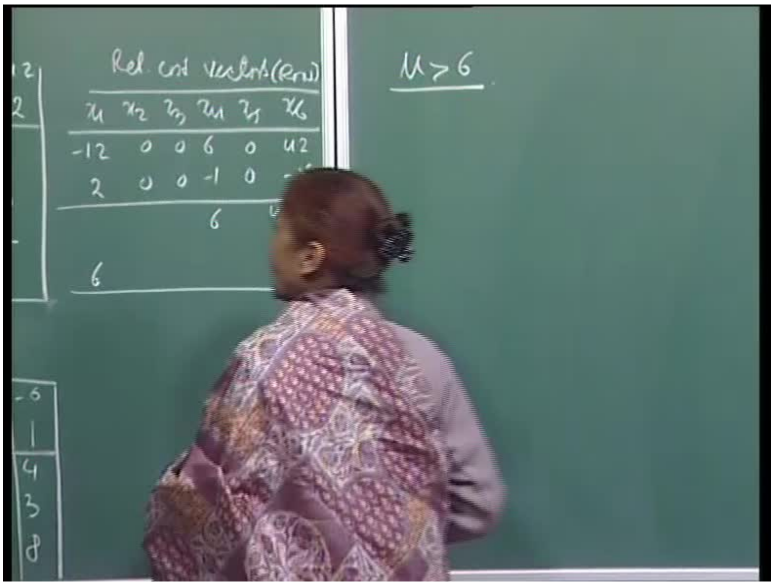

So, c here as I said that in this case this is the variable, if I am considering…, so I am considering mu greater than 6, this is the case we are looking at right now, and in that case this relative price will become for x 5 will become less than 0, it will be a candidate for coming into the basis. So, here for lack of…, I will doing the calculations on this side, but right now in this case I am writing this, because remember for the inverse tableau you write down the column pivoting column here, and then you do the pivoting so this column again, because my B inverse is identity this column comes here as it is, so this is 12, minus 2, 1, 1, and 2 right, and for the pivoting thing I have to take ratios 1 upon 1, 3 upon 1, and 7 upon 2.

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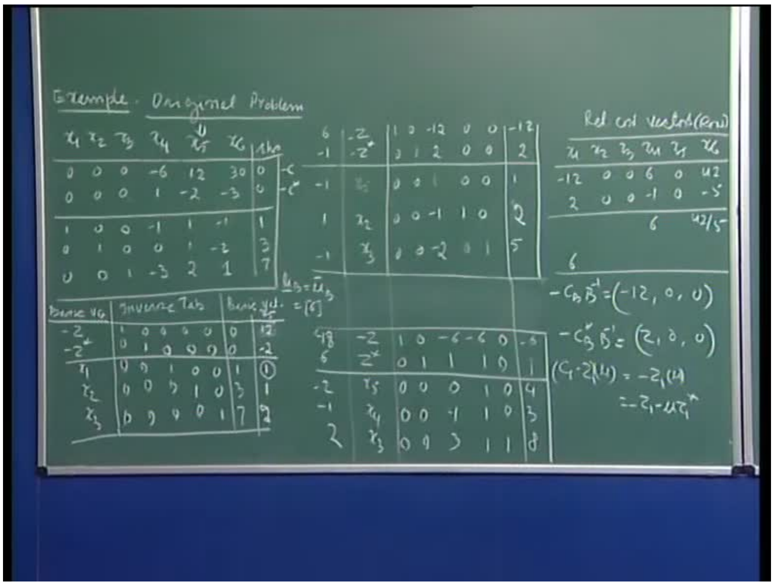

So, this is the smallest ratio. So, let us quickly pivot on this one, I have already done the calculations, which means that, nothing change is here, because this is one, then you subtract this row from here right which shown here, it should be 2 sorry, because you are subtracting this from here, so this will become 2, I am sorry, this is 2 fine, and then you are subtracting twice this from here right.

So, twice this from here, so that should be 5, this is for…, this one is 5, and then you want to multiply this by 12 and subtract, so this row you multiply by 12 and subtract, so what will it becomes, yes, it shows here right, because this will not change its only 12 here, so you minus 12 that becomes minus 12, and the objective function value will also this is one here, you are multiplying by 12 and subtracting, so your minus z is minus 12 right, and then you want to multiply this by 2 and add here; so, when you multiply by 2 this will becomes 2 right, and this will become 2, and this is 0.

so and then, off course, we have done the pivoting here, so this is your new tableau, this is your new tableau right the inverse tabular; and now, again we want to check look at the…; so, remember the only thing that in this case you have to do is, you have to compute these rows of the C bar and C bar star, because in inverse tableau so I need to know the ratios right. So, it that is not difficult, because see, you know, that this is your C b minus C b b inverse is minus 12, 0, 0 right, and your minus C b bar star B inverse is 2, 0, 0 right.

So, you need to compute say for example C 1 minus Z 1 mu I am saying here right, there was a notation, so this would be simply your C 1s, the original this thing I have with me that is 0, so that is 0, so I simply need minus Z 1 yes minus Z 1 whatever the value of mu, so minus Z 1 would be what, and so this is equal to minus Z 1 minus mu Z 1 star, this is what you have right. So, what is minus Z 1? Minus Z 1 is simply minus of C B star B inverse, so I have just this, and multiplied by the column for the a 1 there is only first entry here. So, the first entry is, so minus 12, so this is minus 12; similarly, the first entry of star thing is here right when 2 and 3 are basic variables.

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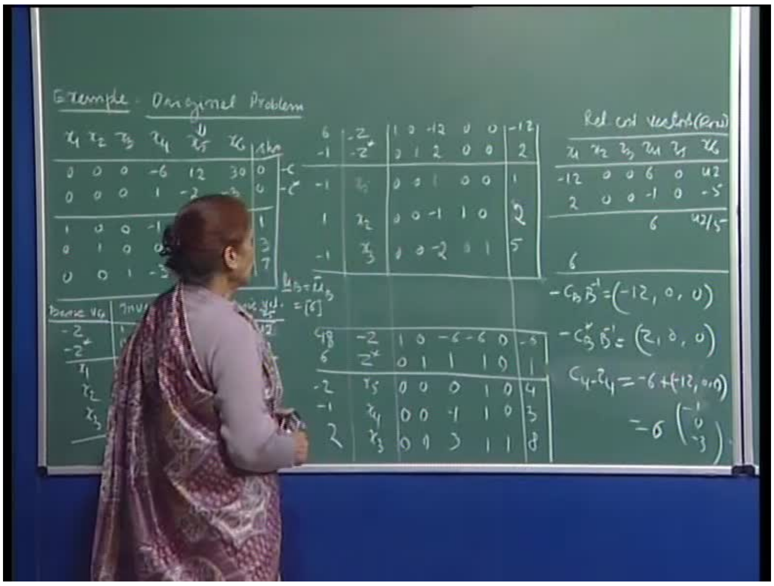

So, this is 0; and for x 4 what is your column, your x 4 the column is minus 1, 0, minus 3, yes, so you need to let me just remain show you the calculation; so, if you want to compute, so let me write C 4 minus Z 4 only, so C 4 minus Z 4 C 4 is minus 6, so this is minus 6, and then this is minus, so this which becomes this which is plus minus 12, 0, 0, times the column which is minus 1, 0, minus 3.

So, minus 1, 0, minus 3, so that becomes plus 12; so this becomes 6; so, plus 12 minus 6 that becomes 6 right; and similarly, you can compute minus 1, and then you can compute 42, and minus; so, now you want to compute the…, that means, for the new basis, the new basis consists of the basic variables x 5, x 2, and x 3 right for this the second basic variables. Now, you compute the lower and upper limits for the mu such that this basic solution basic feasible solution is optimal.

So, here for the positive 1 the ratio would be 6, just I am writing here, then for the negative 1s this is 6 and 42 by 5, so here again the new basis is optimal, because this is a minimum, and this is the maximum, this only one entry here, therefore what we have concluded is that, the new basis is again optimal for a single value of mu right.

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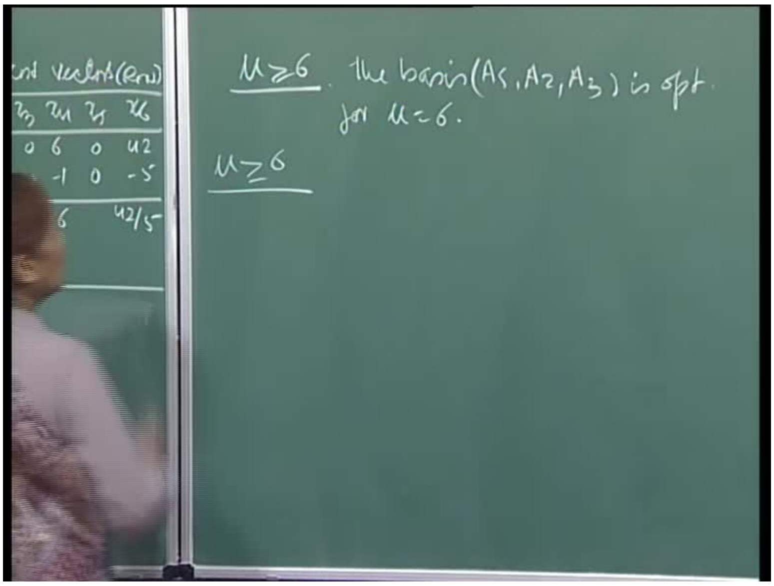

So, that is the conclusion; so, for mu greater than 6 so 12, so, well, we have that the basis or the basis actual I say A 5, A 2, A 3 is optimal for…, I should have said here this for mu equal to 6 only, so that means, I still have to then proceed further. So, in a way you can say that, may be I would not…, so that is it fine, so then but what will you do now, because you have a new basis, so you have to again check, so for now again you want to look at mu greater than or equal to 6.

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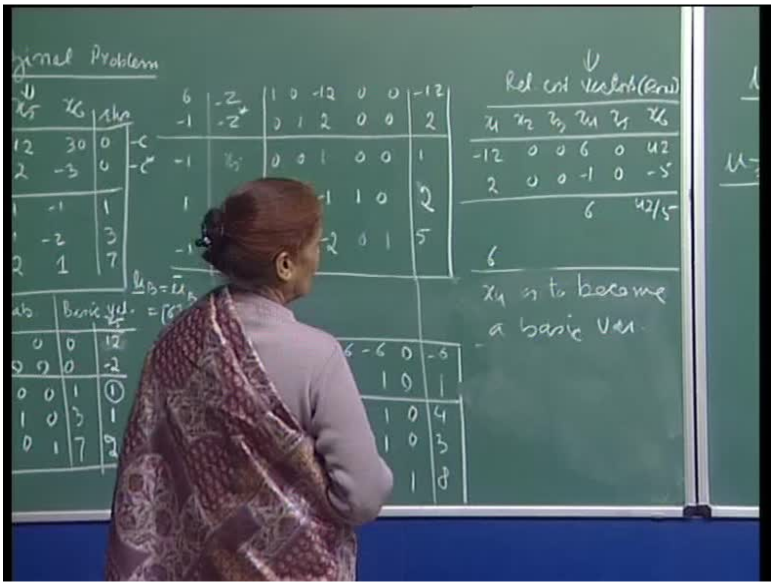

So, in that case this will be a candidate for entering the base right x 4, so x 4 is to become a basic variable, x 4 is to become a basic variable, and I already have this numbers here, so what I am showing is, well, why am I choosing 6 minus 1 and not minus 12, 2, what is happening for x 4, because you see here I am choosing fine here, this is ok.

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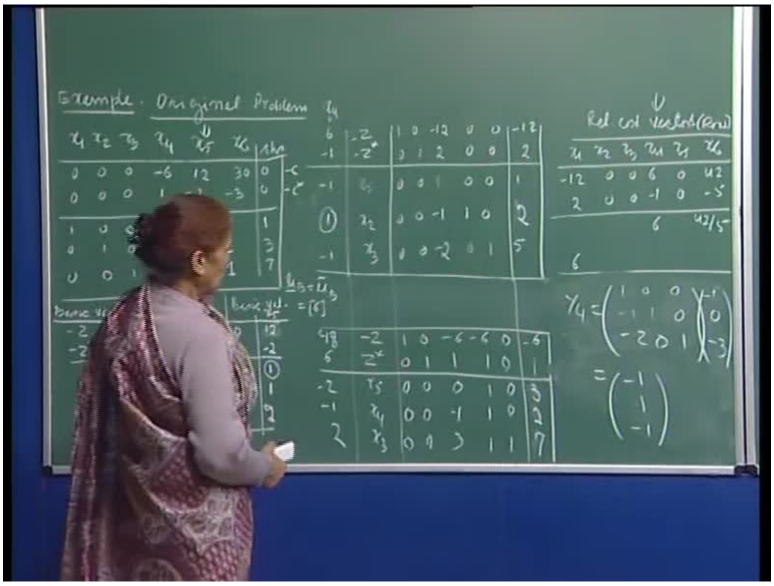

This is your x 4, this is the corresponding to x 4, this is a thing, so just to minimize space being and doing it this is 6 minus 1 right and the corresponding column is for x in the Y 4, the y 4 column you have to write down which is minus 1, 1, and minus 1.

Now, a 4 you have to write down, because your basis the inverse has changed, this is your new inverse; if you want we can do the calculation quickly, so what is Y 4? Y 4 this is see 1 0 0, minus 1 1 0, minus 2 0 1, and the original column is minus 1 0 minus 1 0 minus 3, so this comes out to be minus 1, and then this could be plus 2 minus 3, so this one is 0, because now this is 1, this is 1, and then this will be 2, and minus 3, so minus 1; so this is your Y 4, so minus 1 1 minus 1, and these are your relative prices respect to C and C star, so then you want to pivot here, and the pivoting element of course will be just this, and calculations are not difficult, because what you have do is simply add this row to this and add this row to this and that should be immediately…

So, here this becomes 0 0, this becomes 1 and 0, this becomes 3, so this calculation you have to just incorrect, so this is 3, and this would be 2, so this will also change because you again adding, so this becomes 7, this is 7, so this becomes 0 1 0, then you add to this, so then this will be 0, 1, and 3, why is it sorry you have to add this to this, you have add to this, so this this becomes this is the 0 1 1, and 1 0, this is what you have right, and 6 times you subtract 6 time this row, you subtract from here, and you can verify that this is these are the calculations; the only thing is that, yes, you are multiplying by 6, so this is 2, and you are multiplying by 6 and subtracting.

So, 12, how come I have minus 6 here, this is not correct, because you have 6 times, you have to subtract, so minus 24, so become minus 24 right. And then you just adding, so this will become 4, see the number thus have not change just that, so this is 4, so I am glad the calculation correction have been made, so this is it. And now, again in the same way as I showed you we have to compute the we have to compute the relative prices here.

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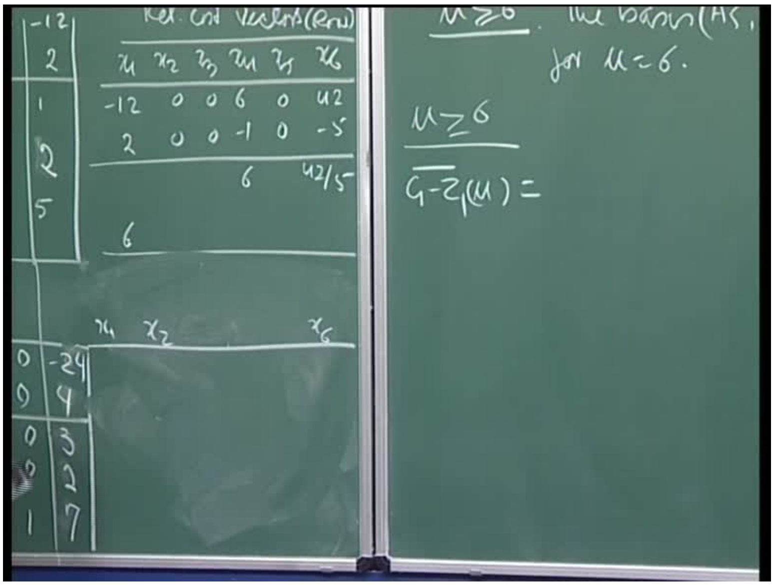

And either we can do it or we can make use of the calculations I have already done, you need to compute again x 1, so you have basis consist of x 5, x 2, and x 3, so actually I need to compute them for x 1, x 4 should become positive, and x 6, these are the things right, because I know that my basis consists of x 5, 4, and 3; this basis I entered x 4, so x 4 replace x, so this is, that means that I have to consider x 1, x 2, and that is it, these 3 I should need to compute x 1, x 2, x 6, quickly let us do it, and so this is your thing; so, what is your C 1 minus Z 1 bar, this function of mu, so what is it, so this could be minus 6, what is your C 1 C 1 is 0.

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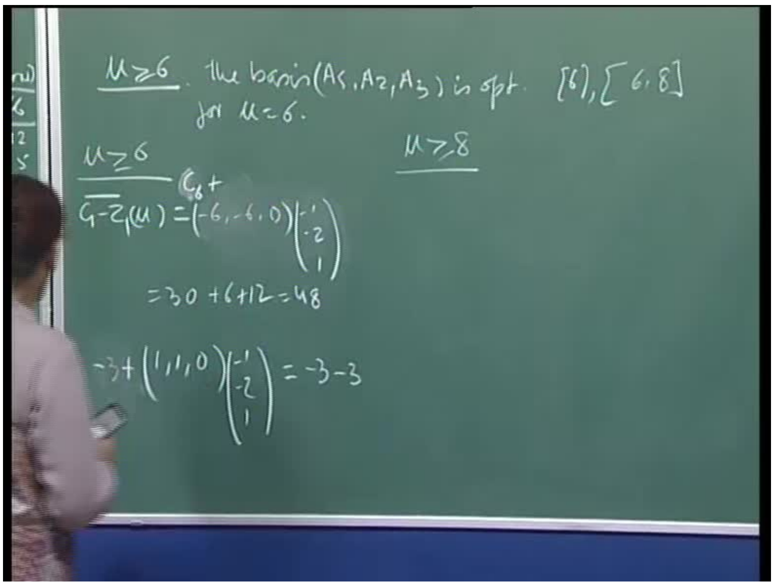

So, this would be simply minus 6 minus 6 0 times a 1 which is 1, so this is minus 6, this is minus 6; and what is your other vector, the other rector is from here 1, 1, 0, 1, 1, 0, remember this is in the form minus C b b inverse. So, I do not do to anything here, so 1, 1, 0, in and a way 1 1 1 0 0, so that is 1, so this will be 1 fine; then for x 2, x 2 you are writing minus 6, minus 6, 0, and then x 2 column is 0 1 0, so it becomes minus 6, this becomes minus 6, and the other one is 1 1 0, that 1 is e 2 the, so this will be simply 1, so this is again 1 right.

Now, for x 6, x 6 it is minus 6, minus 6, 0, times, your this is this minus 1, minus 2, and 1 right; so, but and C 6, so this becomes plus, because C 6 minus this thing, and you are writing minus this right, so this is it, so C 6 is how much 30, so this should be 30 plus 6 plus 12, yes, plus 6 plus 12, and this is 0, so this is 48, and your 1 1 0 the star thing this would be…, yes, so that will be 30 plus sorry not 30 what is the price here that is minus 3.

So, minus 3 plus this, and again you write minus 1, minus 2, 1, so how much is this minus 3, then minus 1, and minus 2, minus 3, so minus 6, so this becomes 48 minus 6, so this the…, so the ratios here would be 6, 6, and this would be 8, or may see the idea is that you write the upper thing here 8; see, immediately, you see that the new basis that, you have which consist of the basic variables x 5, x 4, and x 3, so for this set of basic variables, you are interval now is, so that means, characteristic interval is 6 and 8.

So, this is the characteristic interval before that you had a single turn this, and below 6 we have yet to consider; so, these are the two characteristic intervals that you have obtained right, and yes, that part I did not complete with you; so, right therefore, now, if you want to consider, so for mu greater than or equal to 8, immediately, because this only the value right these are the lower things; so, here x 6 becomes a candidate for coming into the basis right, and I have shown the thing here; so, we need to compute this for what I can do is can simply show you the…, so this is this is your basic feasible solution.

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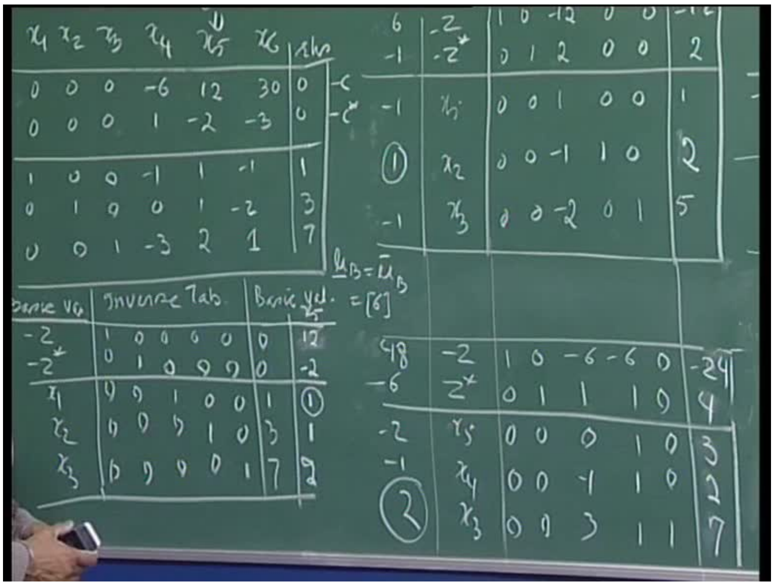

So, what we need to do is, so I have written out 48 and 6 for the… I wrote out the numbers, and that is we should have been minus 6 sorry this should be minus 6 right, because that is the upper limit so 48 minus 6 and this is your 6. So, this is the thing now, and lets quickly the pivot element would be simply this, because this is only positive one, and so we pivot on this, we make 0s here, we make 0s here; so, the new tableau if I want to write down 48, minus 6, minus 2, minus 1, and this is 2; so, let me quickly show you that the inverse tableau would be…, so here.

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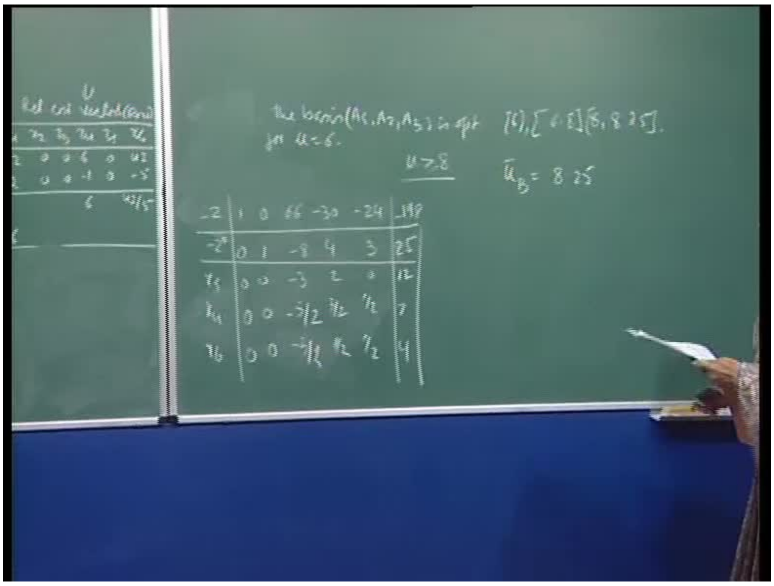

So, this is your minus z minus z star to this, and so this I am showing you after pivoting, so this table is after the pivoting, and the numbers are 1, 0, 66, minus 30, minus 24, 0, 1, minus 8, 4, 3, this number becomes 198, we will I will check, and then we will correct it in the next lecture possible.

Because since I made the correction here, so the calculations have to be carried forward, and your new this thing x 5, x 4, and x 6, because x 2 gets replaced x 3 gets replaced by x 6, and the basis inverse this calculation are 0, 0, 0, 0, 0, 0, this is minus 3, minus 5 by 2, and minus 3 by 2, this, this is 2, 3 by 2, 1 by 2, 0, half, half, this side also will have to be checked, because I have been carrying it 3, it is actually 2, so we will make this correction right; now, this is I am writing it as this without this thing, so this is the new tableau.

And you see that here the upper limit is, see this is giving you 8 which is your upper limit here, so the lower limit because for the positive ones, you get the lower limit that gives you 8, sorry I mean maxima the maxima would be the maxima would be this one, because this would be less than 8 right 4, 7, 7.2, and this is 8, so this is the upper limit, the lower limit for the new basis; and the upper limit now is mu b upper bar is 8.8, 8, 8 are 64, so 0.25.

So, that means, your new characteristic interval is 8.25, and so we continue with this, because this correction also has taken to be care off and the objective function values may also be…, they well they may not change, but this is we are pivoting on this element and not on this, so I will continue with that.