**Linear Programming and its Extensions**

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**Lecture No. # 21**

**Parametric LPP-Right Hand Side Vector**

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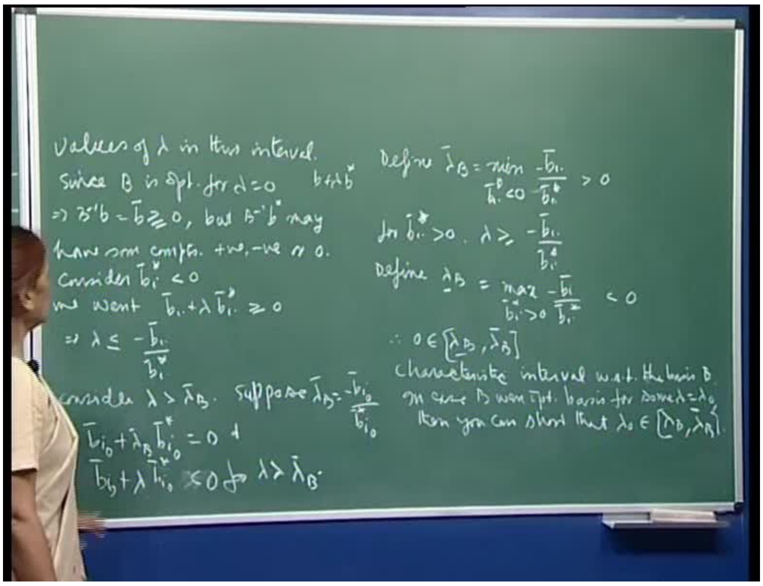

So, let me begin with the parametric right hand side problem; so, I should write it as, this is b plus lambda b star and x greater than or equal to 0. So, let me in the beginning assume that.., and then we will discuss when we have…; so, suppose for some value of lambda, and may be you can take lambda equal to 0, it makes life easy, just start with lambda equal to 0; suppose, for some value of lambda we have… and otherwise it will maybe some other value; in case you have an idea that was some value, you can immediately find an optimum solution, suppose some value of lambda we have an optimum basis.

So, it means that the basis B gives you a feasible an optimal solution, and you fix the value of lambda for the right hand side. So, you have some means of having that already, and we will in the course of the algorithm, when we discuss the algorithm, I will tell you how to obtain it, in case you do not have excess to an optimum basis right away.

So, now, since this is optimum you see, by changing the value of lambda the optimality criteria will not be disturbed right. So, that means we try to find out an interval for lambda, such that, B is optimal for all the values of lambda, lambda in this interval, right. So, basically we want to find out all values of lambda for which basis B gives you a feasible solution, because as I told you optimality criteria will not disturbed by changing the value of lambda, it is only the feasibility; so, what we are saying that we started with lambda equal to 0 and so B was an optimal basis when we put lambda equal to 0.

Remember, we the right hand side was lambda b star right, so we put lambda equal to 0, I had an optimal basis and therefore b inverse b equal to b bar, the optimal basic feasible solution is non-negative, but b inverse b star the signs may be anything, because we were optimizing then the lambda is 0, so some components here may be positive, negative, or 0; so, now, consider b i bar star less than 0, in that case we want to consider the values of lambda for which the current basis b will be remain optimal right, so I want this to be non-negative.

And therefore, this will give me that lambda is less than or equal to minus b i bar upon b i bar star, remember this is negative; so, the inequality gets reversed; and so, this is the value, therefore this implies that if I choose lambda for all b i bar star which are less than 0, I will choose the minimum of this ratios and that will come out to be positive right, because this is non-negative b i bar, and or it may be 0, but it will at least and this b bar star b bar r i, this is i right this is less than 0.

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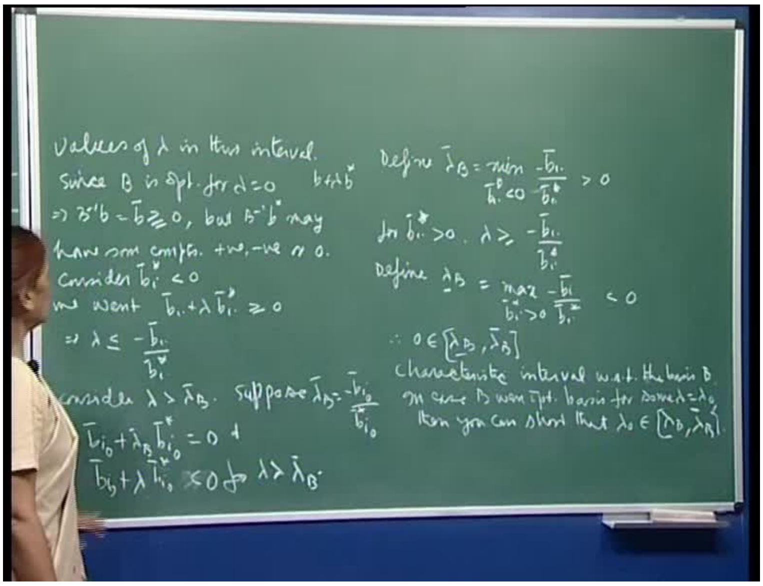

So, minus of this, this becomes positive, b i bar is positive or non-negative; so, the whole thing can be…, or I can put this, this can be greater than or equal to 0; if you have a degenerate basic feasible solution, that means, if some b i bar is 0, then of course this number will be 0 right.

And then similarly, if b i bar star is positive, because for b i bar star 0 I do not have to worry, this number will remain non-negative; so, when b i bar star is positive, then my lambda has to be greater than or equal to this ratio, and you can see that this is the negative number right, because this is for a non-negative, and this becomes negative, so the whole thing the ratio is negative.

And I will take the say that, that is the lower limit, that means, for lambda b lower bar I will define it as max this; then for all values of lambda greater than lambda lower bar B, this number will remain non-negative; and similarly, here for all lambda less than lambda upper bar B, this number will remain non-negative, that is the idea, so we therefore…, and so you see 0 will belong to this interval, and I will call this the characteristic interval with respect to the basis B for lambda. That means for all values of lambda in this interval the current basis B will remain optimal, and you can see that since you started to the value lambda equal to 0 this will always be in this interval.

Now, similarly, if you had an idea in you started with some other value of lambda, say lambda equal to lambda naught, and for which you could find an optimal basis, then a simple exercise you can show that whatever the characteristic interval that you find out I should have said this is lower bar, so that lambda naught then will belong to this characteristic interval right; and then the idea is that, you want to do this exercise for the whole of the real line, that means, for lambda varying from minus infinity to infinity, so here you will we which will continue with; so, consider a lambda greater than lambda B bar right and upper bar.

So, now, suppose lambda B upper bar was for corresponding to the ratio B bar i naught up on b bar star i naught, that means, for the i naught component this minima was attained; so, in that case you see that, for that particular component this number will be 0, and for all other values of lambda greater than lambda B upper bar this number will become negative. And so, you have a situation for dual simplex algorithm that means your optimality criteria satisfied, because that does not get disturbed by changing the right hand side and you have a negative variable in the basis, so you need to come up with the feasible basis, and therefore you will get optimal solution.

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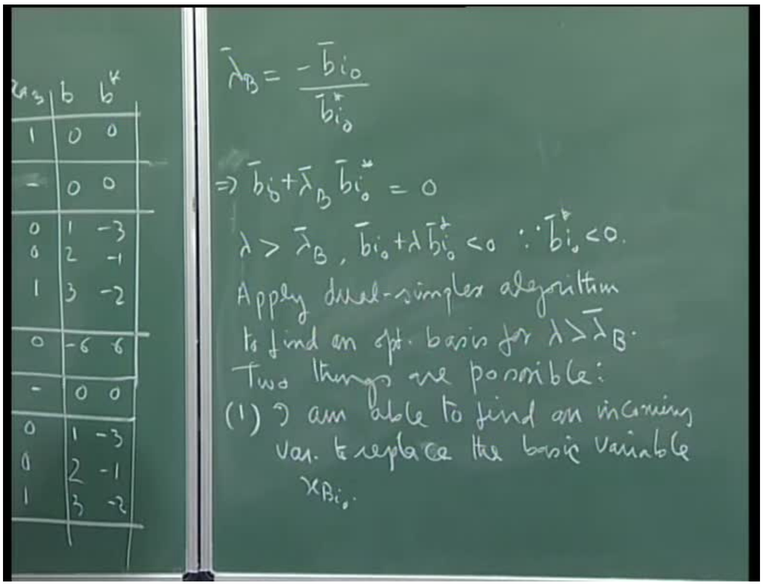


Now, what are the two possibilities, when you apply the dual simplex algorithm either you have…, so either or I should first say that, now apply dual simplex algorithm to find an optimal basis for lambda greater than lambda B upper bar.

So, applied dual simplex, because once you are variable becomes negative or basic variable, which we are saying is that the i naught basic variable will become negative for lambda greater than lambda B upper bar, so we need to apply the dual simplex algorithm, and I need to find an optimal basis for lambda greater than lambda B bar.

That means, we will work for feasibility since the optimality criteria is already satisfied, and a hopefully in one or two iterations we should get and basis which is in fact I should not say two iterations we should exactly in one iteration, once I remove this in feasibility, that is, basic variable which is non-negative I remove it, I get the adjacent basis, this it is important in one iteration; I will get a value, then I will get another interval for lambda for which the new basis will be feasible, and hence optimal because the optimality criteria will continue to be satisfied right.

So, now, suppose here what we are so the two things can happen; so, two things are possible, one would be that I am able to find an incoming variable to replace the basic variable of the x Bi naught, because this is become negative for lambda greater than lambda B upper bar, so I am able to find an incoming variable to replace the basic variable. That means, my new basis…, so the new basis, you can say B bar is B union A k slash Bi naught.

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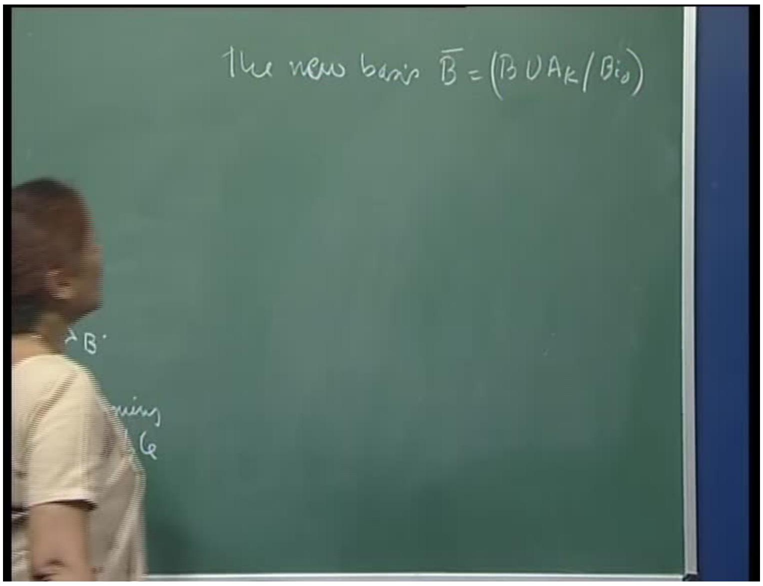

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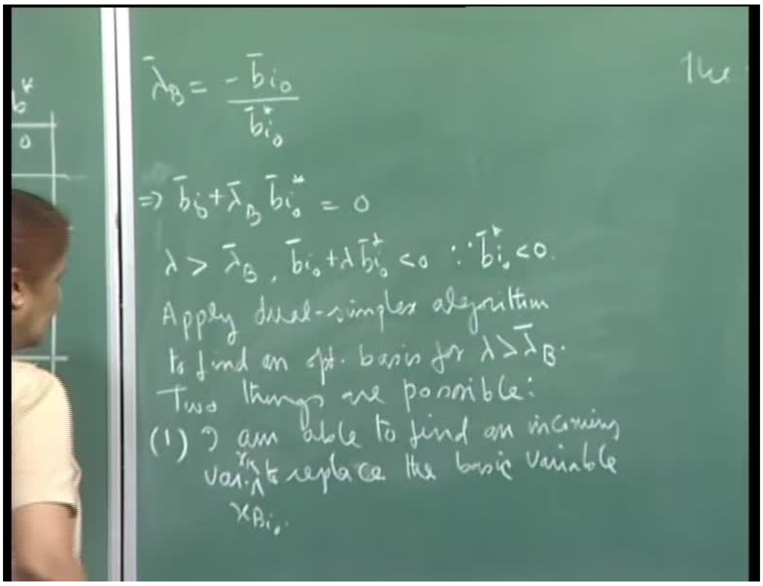


So, I have added the column A k, and I am able to find an incoming variable maybe I can say here insert here and say X k. So, I find an incoming variable X k, that means, in the i naught row I am able to find a negative entry, and therefore I can take the find the minimum ratio and decide on the incoming variable; and the new basis will be now B union A k Bi naught right.

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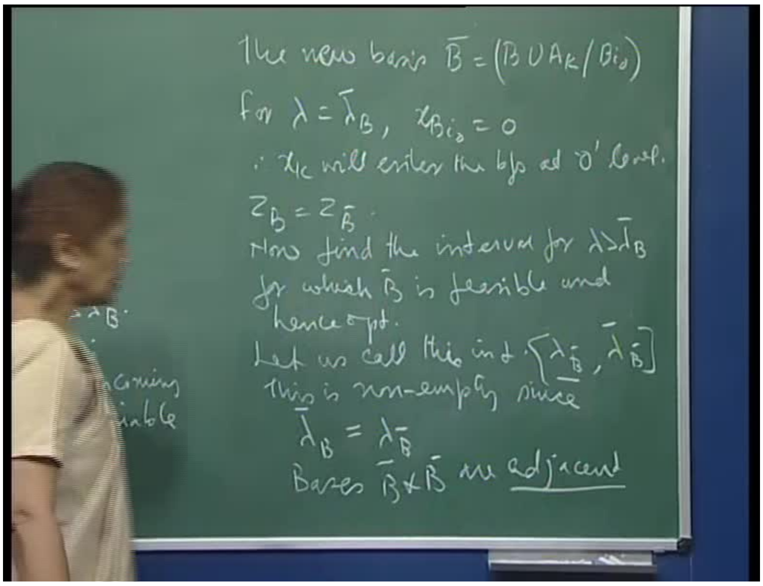


And you see that for lambda equal to exactly lambda B upper bar, your x Bi naught becomes 0, yes, because if this start becoming negative; once your lambda is greater than lambda B upper bar, because your Bi naught bar star is less than 0. So, at this point this is 0, and therefore your x k will also will enter the basic feasible solution at 0 level. So, you see that you have the case for alternate optimal solution, that means, your Z B is equal to Z B bar right, since the outgoing variable is at 0 level and the incoming variable is also at 0 level, so the two objective function values are the same, so you have an alternative basis with the same objective function value; and now, and so, therefore, and you will now find the interval for lambda greater than lambda B upper bar for which your B bar is feasible and hence optimal right; so, this is the thing.

And see what is happening is…, so you are finding an interval for lambda greater than lambda B bar for each B bar is feasible and hence optimal, and yes you want to also say that here that; so, let us call the interval, it is called this interval, it is called this interval lambda B bar lower this thing and lambda B bar upper bar, yes, and this is non-empty, since lambda B upper bar is equal to lambda B lower bar right.

We just saw that, because the basis B is in feasible for lambda greater than lambda B upper bar, we just saw that right. So, therefore, but at equal to lambda B upper bar it is an alternate optimal solution, so this interval is not empty; and therefore, you see that…; and so, therefore, the two values must be the same, because at lambda B upper bar B is also optimal solution, and beyond that it becomes infeasible, therefore the lower bar will be lambda B; for B bar the lower end of the interval would be equal to the upper end of the interval for B; so, this what and then and you see that the basis B and B bar are adjacent, this is very important.

So, only one column exchange, replace Bi naught by A k, and you got the new basis, and which also will give you an interval non-empty interval for which the new basis this feasible and hence optimal.

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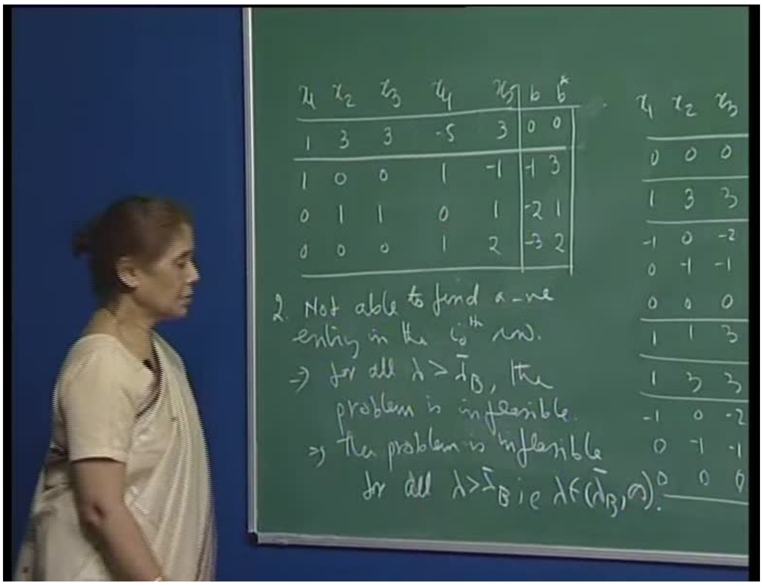


So, we do this; and therefore, and then we continued with this distinct will be…; and now, the other thing will be that, you are not able to find not able to find negative entry in the i naught with row right, that means, because the variable x B i naught is leaving the basis is not any longer a basic variable. So, to continue with the dual simplex algorithm I will need to find a negative entry in the i naught row, and if I cannot then I cannot proceed with the dual simplex algorithm.

So, this would imply this would imply that, for all lambda greater than lambda B upper bar the problem I mean the your prime problem is infeasible. See, the optimality criteria is not disturbed by lambda greater than lambda B upper bar, but as we saw that this number will become negative as long as lambda is greater than lambda B upper bar.

So, this will continue for all values of lambda greater than lambda B upper bar, and hence the problem will remain infeasible, because you will not be able to…, the dual will be unbounded, and hence the primal is infeasible. So, therefore, so, this implies that the problem is infeasible for all lambda for all lambda greater than lambda B upper bar, that is lambda belonging to lambda B upper bar infinity, so open interval on both ends.

So, therefore, I will either way be able to a…, either I will continue getting a new adjacent basis, and getting an interval, and these intervals will not be over lapping, because we saw that, just at the this end of the interval the first the lower end of the interval, the two basis B and B bar are alternate optimal solutions, and beyond that your basis B becomes infeasible.

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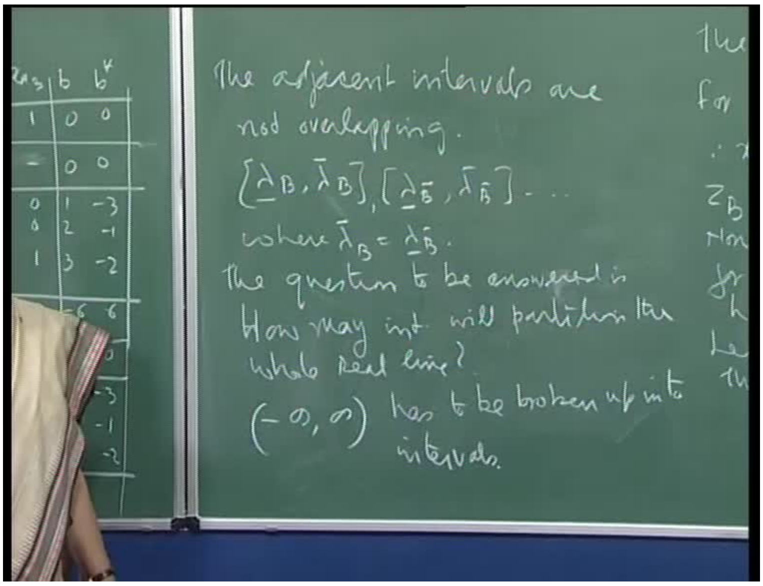


So, obviously, the interval that means here, the we had the…, so maybe I can set it here, so here that means, the adjacent intervals are not overlapping; so, for example, you had this lambda B lower bar lambda B upper bar, and then here you have lambda B bar lower bar among lambda B bar upper bar and so on, where lambda B upper bar is equal to lambda B bar upper a lower bar right and so on. So, the next consideration is…, therefore if this continues, either we will at some point encounter infeasibility, that means, unboundedness for the dual, and so infeasibility for the primal, and then beyond that for all values of lambda the problem will continue to be infeasible.

So, here the next question…; now, the question to be answered is how many intervals and will partition the whole real line right, that is, when you want to that means this would be so this interval minus infinity to infinity has to be broken up into intervals right; and the question is, how many such intervals; and now, since I have already given you the hint that only one basis is optimal for this interval, and the next will be in an adjacent basis, and since the intervals are not overlapping, you can immediately now answer this question that the number of intervals will be finite, because the number of basis is also finite.

So, I picked up this example; so, here this the right hand side parametric problem, so this is your b, this is your b star right, and equality constraints also are given, non-negativity of the variables; so, right now I do not have a starting feasible solution, and that means, not I do not even have a starting optimal solution, so we use phase 1 right. For phase 1, I would use I would add artificial variables x a 1, x a 2, x a 3 to the three constraints and then now I want a starting feasible solution for phase 1.

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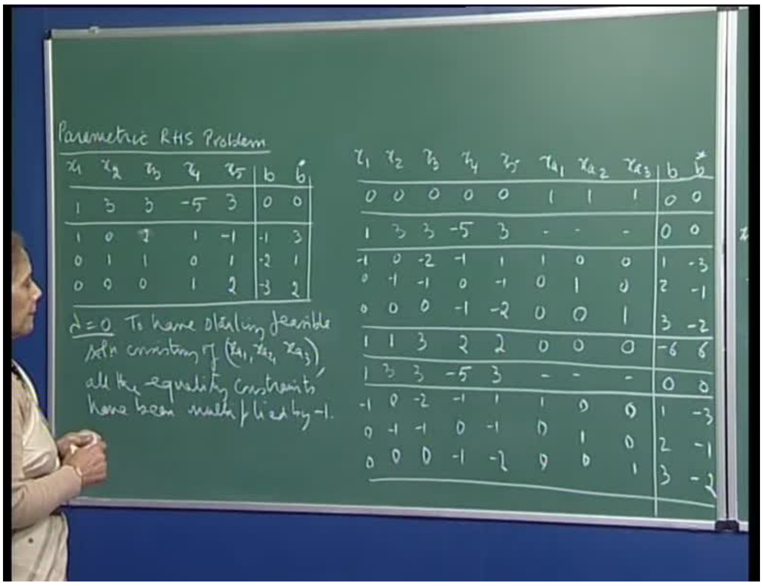


So, what I have done is, I have multiplied and so I am choosing, yes, I should say that, so choose lambda equal to 0, so that means, this side is not being considered right now, only this vector is being considered, and since I want to have a starting feasible solution, I have multiplied all the constraints by minus 1 right to make the right hand side non-negative right.

So, to have a starting feasible solution consisting of x a 1, x a 2, x a 3, all the equality constraints have been multiplied by minus 1 right; so, you have a starting feasible solution here, from here this is 1, this is 2, and this is 3, and you have to make these 0s. So, I add up the three rows and subtract, yes, so please just check that, this will not get disturbed, because so here the numbers then become 1 1 3, because minus 3 minus 2 2 then this is minus 2 2 so 2 right and this number is minus 6 and 6; now, what do you have, so here how will you read this tableau, remember this is minus z and this is minus z star.

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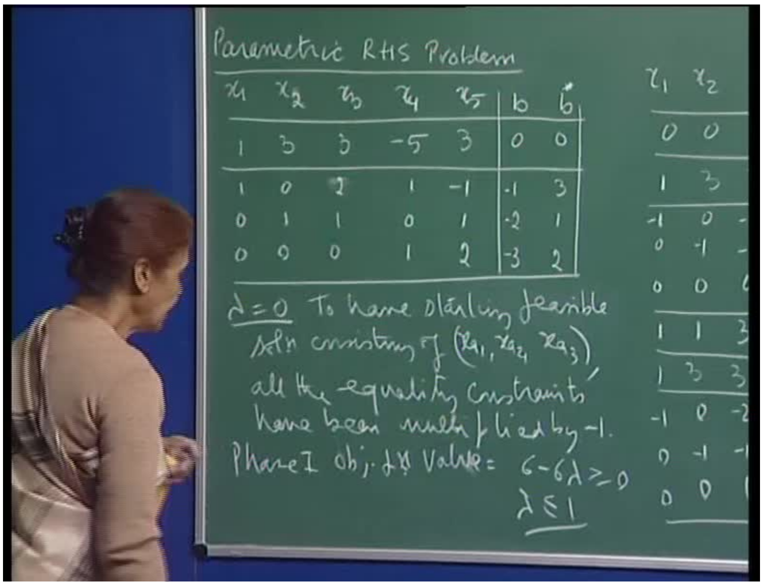


So, right now your objective function value, phase 1 objective function value value is equal to 6 minus 6 lambda right, so negative of this, so this number is 6, and this is minus 6, so this and this we want so this is greater than or equal to 0 for lambda less than or equal to 1, that means, in the interval…, so we will come…, so lambda less than or equal to 1, this will remain positive.

So, if this will remain positive, that means, I have not arrived an optimal solution for phase 1; the objective function value for phase 1 should be 0, which indicates that you have a feasible solution for the original problem, but what else before I talk of this thing I have also have to make sure that I have a feasible solution here, because see the optimal the relative cause here all non-negative. So, you have an optimal solution, the value of the objective function for phase 1 is 6 minus 6 lambda; but for what values of lambda is this solution feasible, because I can talk of always interpretations of optimality and so on only when I have a feasible solution.

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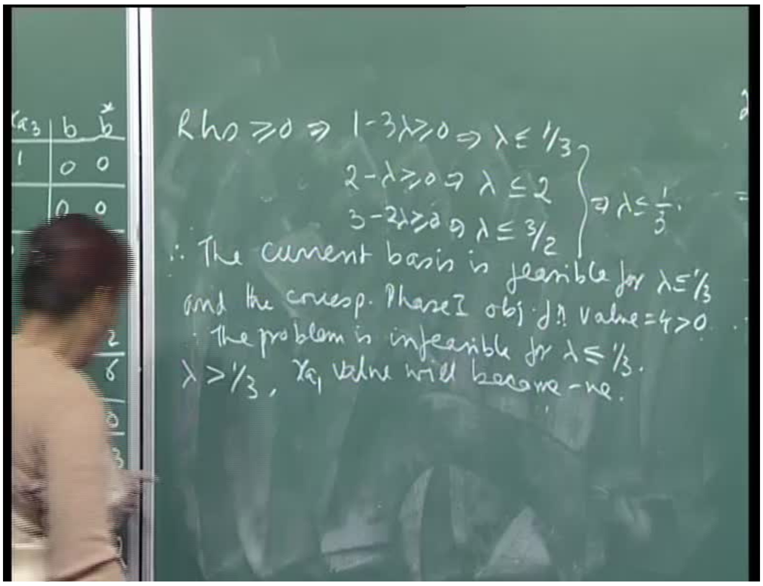


Right hand side greater than or equal to 0 implies 1 minus 3 lambda greater than or equal to 0, which says that your lambda should be less than or equal to 1 by 3 right. Then 2 minus lambda greater than or equal to 0 implies lambda less than or equal to 2; and 3 minus 2 lambda greater than or equal 0 implies lambda less than or equal to 3 by 2.

So, this all imply that lambda should be less than or equal to 1 by 3 for the solution to be feasible; this aspect I did not discuss, because when I was developing the algorithm for the parametric right hand side, I assume that I have a starting feasible optimal solution which is feasible. So, here I refer to the example; I will show you that, in case you are not able to find a starting optimal solution, then for some value of lambda how do you go about it.

So, obviously, you use phase 1 to first obtain a feasible solution; so, this say that lambda less than or equal to 1 by 3 allows you to gives you the feasible solution and the corresponding objective function value for phase 1 would still be positive right; for lambda equal to 1 by 3, this would be 6 minus 2, which is 4 which is still positive. So, no feasible right; therefore, this implies…, so therefore the basis the current basis may be I should since imply that the current basis it is feasible for lambda less than 1 by 3 and the corresponding phase 1 objective function value equal to 4 is also positive; therefore, the problem is infeasible problem is the original problem, this infeasible for lambda less than or equal to 1 by 3 fine.

So, we continue with the dual simplex algorithm, because for therefore for lambda greater than 1 by 3; see this one your x a 1 value will become negative as long as soon as lambda is greater than 1 by 3, this value will become negative.

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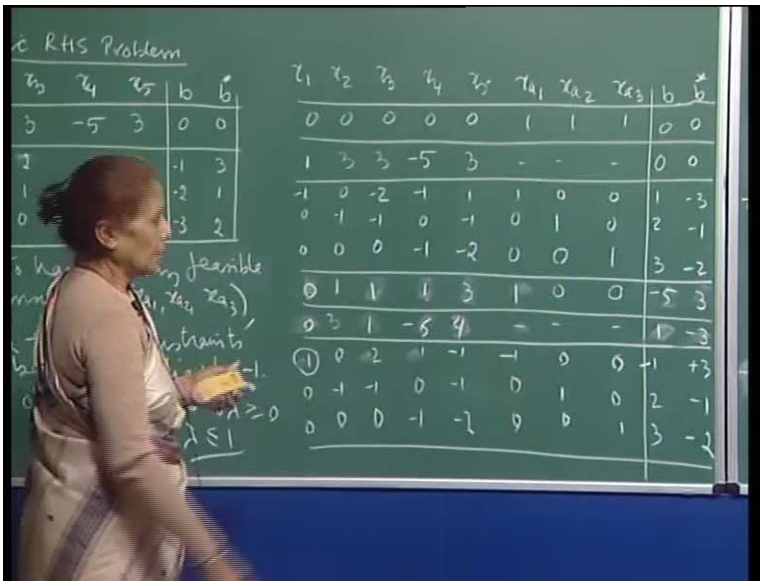

So, you have a case for…, so I should do it here may be because that so this would be yeah, so from here we come here, and we say see that this one here has become negative, so we apply dual simplex algorithm for phase 1 right, and so you look at the negative entries here and take the ok sorry you have to do it here only.

Because this is the right table fine; so, when you do it here what happens? This is the ratio minus, this is minus 1, this is minus 3 by 2, this is minus 2, so the maximum one in this case is this; so, to maintain dual feasibility you we will pivot on this element right. So, this is not difficult, because all are these are 0s, so this it should be straight forward let us quickly do it; so, that means, I make a 0 here, and of course I make 0 here, also you just add add the row so this becomes 0, and this remains 1, this becomes 1; see, the test is that, for we have that to test that you are making the right calculations, this should remain non-negative right, so this is 1, this becomes 3, this is 1 right, and then this becomes minus 5 and this is 3.

And similarly, you add this row to this, so this will be 0, 1, minus 6, 4, this I do not need to calculate this, because in phase 2 these variables will not be appearing, so we minimize our minimize our calculations, and this becomes 1, and minus 3, because I just added the rows yes, and then multiply the whole thing by minus sign. So, this is this, this is gone, this is gone minus 1, minus 1, this becomes minus, and this becomes plus, say you see what we wanted to check here that the intervals will be overlapping, see you see the optimality criteria his…, say satisfied for the new basis.

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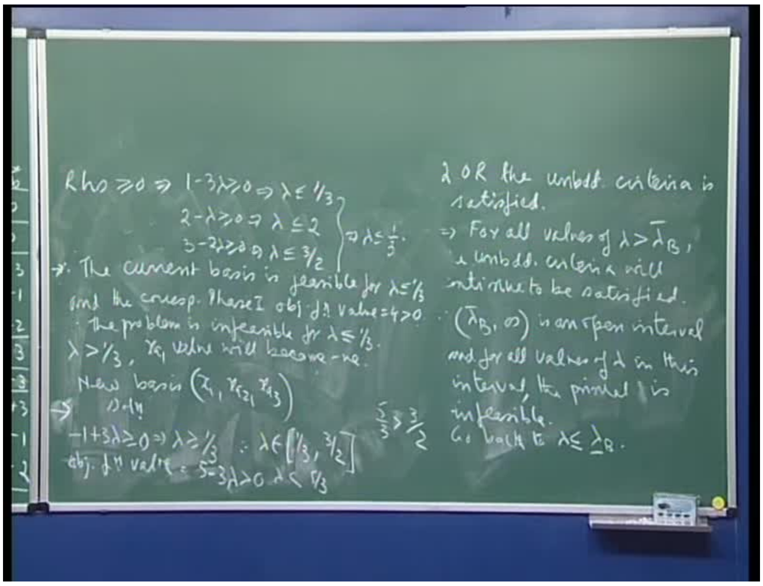


So, the new basis consists of…, what is your new basis? New basis is now x 1, x a 2, x a, I mean new basic variables new basic the solution basic solution consist of this right, so and so the your new basis is a 1, a 2, and a a 3, so these are the three columns which form your new basis. And you see for feasibility for this now you will require that minus 1 plus 3 lambda greater than or equal to 0, which we implies that lambda is greater than or equal to 1 by 3 right; and here is that at lambda should be less than or equal to 1 by 3, and these two will give you lambda less than 2 and lambda less than 3 by 2, these 2.

So, immediately, you see that the interval is…; therefore, the characteristic interval in this case is 1, well it is not the characters; the characteristic interval always refers to an optimal solution; so, we right now have 1 by 3 to…, what is the smaller value there 3 by 2 let me write it bigger. So, therefore, for lambda belonging to 1 by 3 to 3 by 2, because a moment it is let us keep it close, you see that for and for up to 3 by 2, so this is for feasibility; but for the phase 1 objective function to be non-negative, so objective function value is equal to 5 minus 3 lambda, and this would be greater than 0 for lambda less than 5 by 3 right.

So, now, which is smaller 5 by 3 or 3 by 2, I think 5 by 3 is less than, no, it is the other way, greater than 3 by 2, so that means, the problem is still infeasible, because this must become 0; and this will become 0 only when lambda is equal to 5 by 3, but my current basis is feasible only for lambda in the interval 1 by 3 to 3 by 2.

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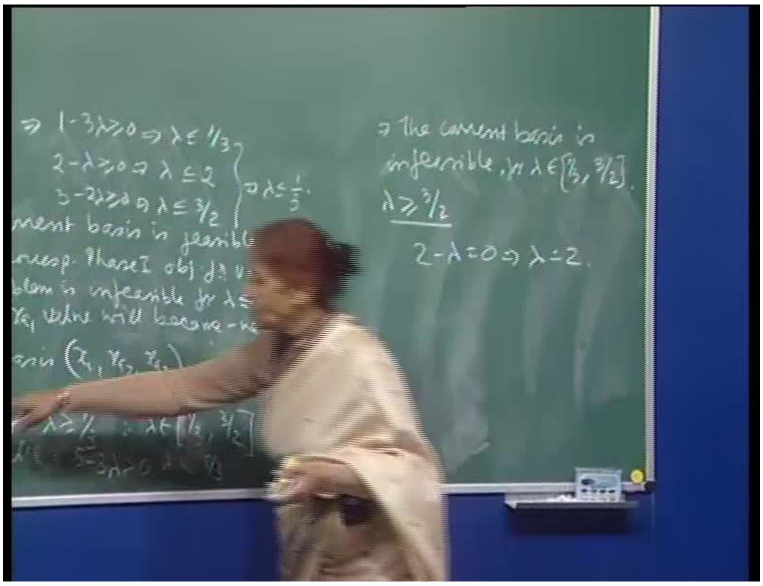

So, I must continue further, because I still have not found a feasible solution right; so, lambda, let me write it here, so this implies that the current basis is infeasible, so 5 by 3 is bigger is infeasible is infeasible right so up to 3 by 2 and 1 by 3. So, this is any way positive, so here this is the next variable, so this is the next variable which will leave the basis; because now let means I must consider infeasible for lambda belonging to 1 by 3 to 3 by 2, so I am considering lambda greater than 3 by 2 or equal to sign you can write, but so this is the next step, and we can quickly do it here.

So, in this thing you have minus 1 minus 2, so the ratio…, so here again you going to pivot on this right, and you can quickly do it; so, this just add this to this, these remain undisturbed, this is 0, this is 1, and nothing happens here, this becomes no sorry this remains 0, this is the thing, so this becomes 1; and then when you add this here, this is minus 2, and then this is 1, and I simply have to add this here.

So, this again not much of calculation, this is 3, I am making it 1 here not necessary, then you add this here this becomes 2, and this is 1; see you see the two things match; so, this is 2.

And now, you see that, for this would be 0, so when would this be 0, 2 minus lambda equal to 0 implies lambda is 2; and here you see the interval you would also do this plus this minus minus plus, so you can immediately see that lambda greater than 2, because this will remain feasible. So, you see that here this becomes lambda equal to 2, this becomes 0, and your solution is feasible for lambda equal to 2; so, here this was the thing we got up to here, then I have to make a 0 here also.

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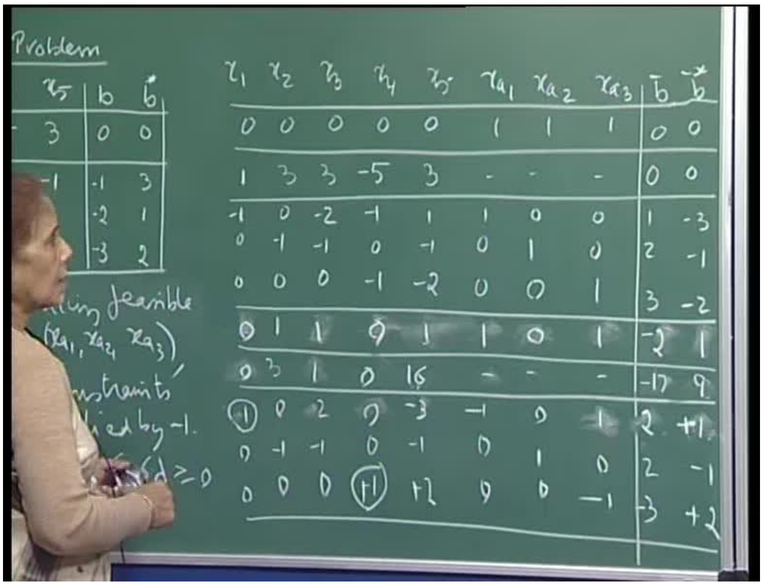


So, now, 6 times you have to add here, so these numbers do not change, this number will change, this has to be 0, then 6 2s 12 12 plus 4 16, so this becomes 0, this becomes 16, and this I do not need to calculate, but the objective function value has to be calculate 6 times I am multiplying and adding, so minus 18 will be minus 17 right, and 6 times you are multiplying so 12 minus 3 would be 9.

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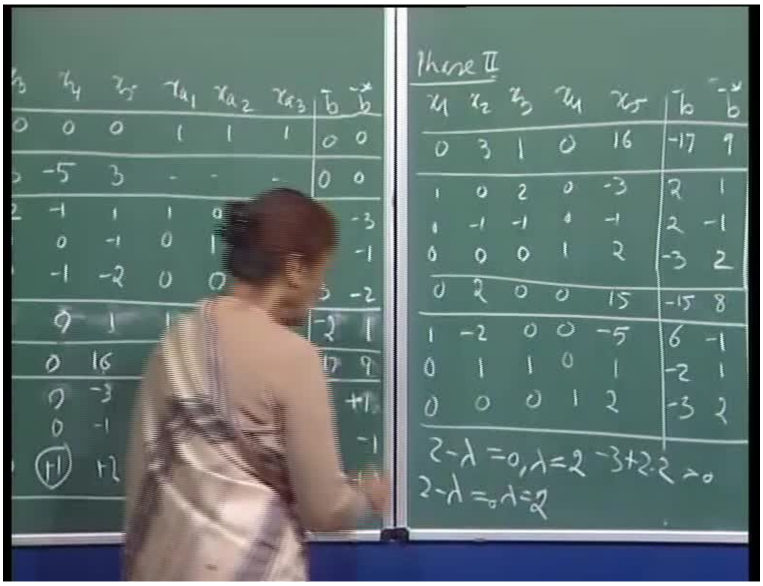


So, that is shown in the table here; and you will see that, yes, so the calculations was that this number becomes 2 minus lambda becomes 0 for lambda equal to 2, and the current solution this is all positive, because 2 plus lambda, lambda is positive is this one is 2 minus lambda. So, this is also 0 for lambda equal to 2; for lambda less than 2 for lambda the anyway so lambda 2 that is its feasible, but the objective function value will be 0 only at lambda equal to 2; for lambda less than 2 this will still be positive, 2 minus lambda; for lambda less than 2, this number will still be positive.

So, you attain feasibility only when lambda is equal to 1, and at lambda equal to 2 this is are also feasible, because this will be minus 3 plus 2 into 2, which is positive, so you have feasible solution for lambda equal to 2.

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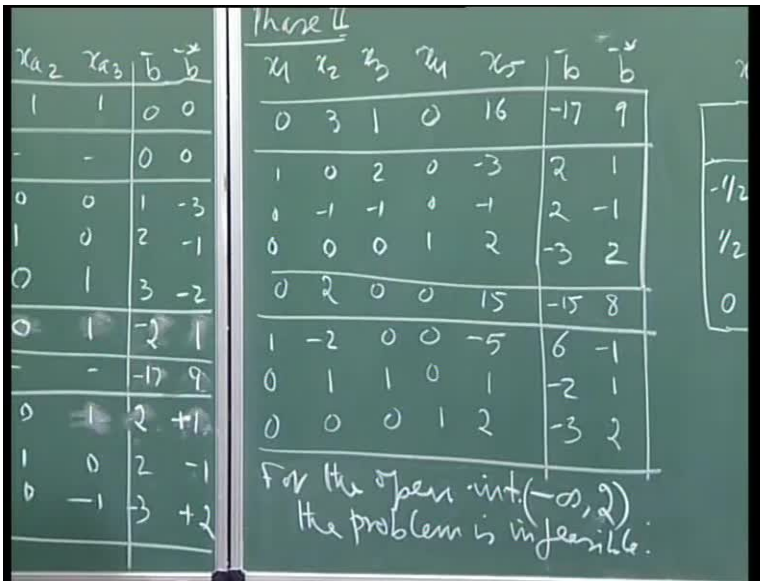


So, therefore, at lambda equal to 2 phase 1 ends, and so here is that means we have obtained that the interval or the open interval that means your problem we have finally shown that the problem is infeasible. So, for the open interval minus infinity to 2, the problem is infeasible, so fine this problem is infeasible; now, I begin my phase 2; so, phase 2 for lambda greater than 2, this number will become negative right 2 minus lambda, so when lambda is bigger than 2 this will become negative. So, this will be a candidate from going out, now top row does not have to be recalculated, because that was carrying it along with me all the time as we do in phase 2.

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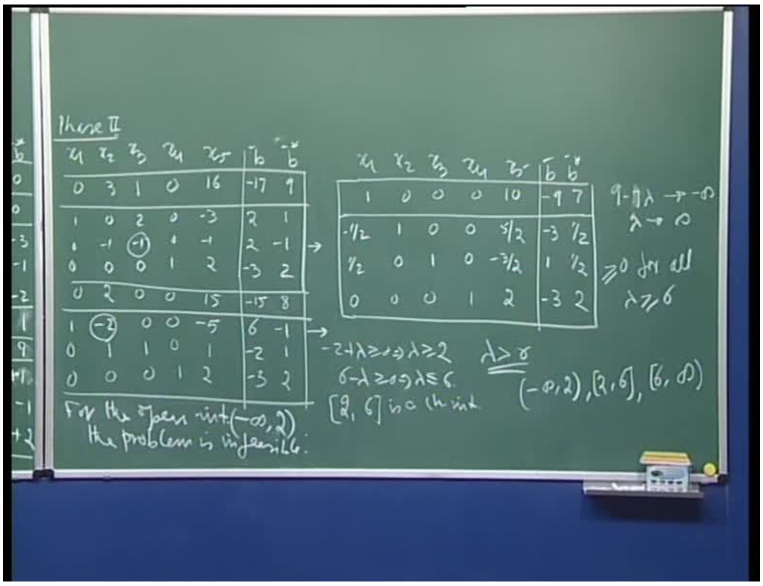

So, here let us take the ratio, so, this will be minus 3, minus 1, minus 16, so this is maximum, so this is the variable which comes into the basis, I mean, the pivoting so the x 3 will come into basis, and I have calculated it here; so, you multiply this by minus sign, so the sign changes here, then you simply have to added here, and this is the number right; you can immediately see that 2, 0, 15, and this becomes minus 17, minus 15, and this becomes…, why is that because if you just add then this is minus 1, and now this is the thing right.

So, phase 2 and nor we simply have to worry about, because we already have a feasible solution and an optimal solution, but the interval; so, from here it tells you that minus 2 plus lambda greater than or equal to 0 implies lambda greater than or equal to 2 and 6 minus lambda greater than or equal to 0 implies that lambda is less than or equal to 0. So, now you have a closed interval which is the characteristic interval; and for 2, 6 is a characteristic interval, and the basic feasible solution is x 1, x 3, and x 4 right, x 1, x 3, and x 4.

So, for this basic feasible solution, this is a characteristic interval; and you see that beyond fine so for lambda greater than 6 is what you have to now look for, so lambda greater than 6 this will become infeasible, become negative, so you have to again applied dual simplex algorithm, and this will be the pivot interval sorry the pivot element, this is the pivot element, so you have to just add to make 0s to make a 0 here, this is 1, 0, and

10, so 1, 0, 0, and 10, these are the new values of the minus z and z star, and you have done this right.

Now, you see the these things are positive, and you can see that because lambda is already greater than 6, so this is greater than 0 for all lambda greater than or equal to 6, because this will be here; in this case it will be half into 6 will be 3 so 3 minus 3 0 and beyond that these are non-exultant.

But here what is the objective function value 9 minus 7 lambda right; so, the solution the current basis is feasible for all values of lambda greater than or equal to 6, but this goes to minus infinity as lambda goes to plus infinity right, because the coefficient here is negative respect to lambda, so 9 minus 7 lambda will go to minus infinity, therefore the problem is unbounded.

So, see the whole span, you have seen everything, you are seen that up to 2 the problem less than or equal to…, less than 2 the problem is infeasible, the interval that you found the optimal interval, the characteristic interval that you found is from 2 to 6, and beyond 6 the problem remains feasible, but the objective function is going to minus infinity. So, it is unbounded right, because I can choose my lambda to be as larger is I wish; and therefore, your character so here that means, you have now broken up your real line, you have partitioned the real line; so, anyway let me write the conclusion; so, I have shown you it is going to be minus infinity as lambda goes to plus infinity.

So, in other words, for any finite value of lambda you have an optimal solution; as long as there is a lambda value of lambda which is bigger than 6, this basis since there was an optimal solution, and the corresponding objective function value will be there right; so, you have partitioned is…, you have actually now three basis; well, here you could show for this is a basis for which the problem is infeasible, then you had a basis which gives you optimal solutions for all values of lambda here; and then so therefore, the in partition of the real line is then 2, 6, and 6, infinity, this closed but this is ok.

So, that in a way shows you the whole thing; now, let us quickly discuss some aspects; so, so what the question that I did ask in the beginning was that, since we have we have partitioning the whole real line minus infinity to infinity will it be how many iterations we will have to go through. So, now, the first question that you can immediately you can clean that, see once you leave a value of lambda you will not return to it, and that is why I took place to show you that the intervals are over are not overlapping and so on.

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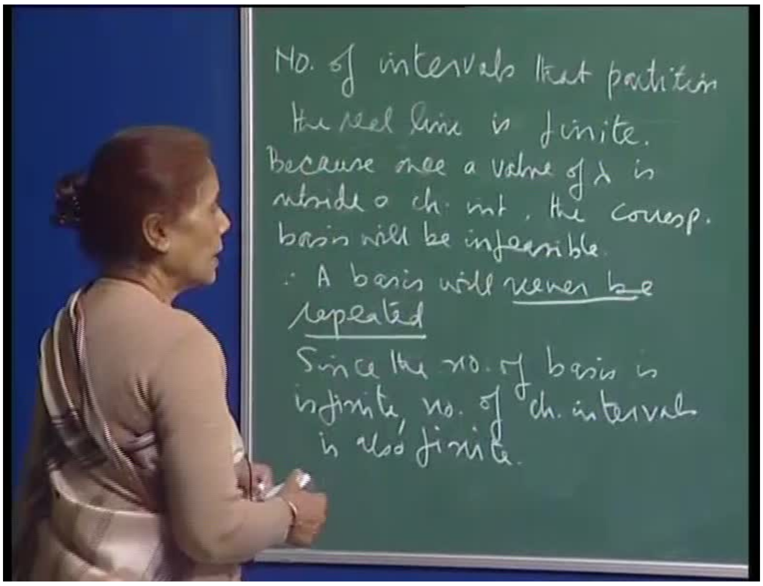


So, what we want to say is that the number of intervals that partition the real line is finite in number is finite right, why because once you go beyond a value of lambda you will not return to it; can you whatever we have calculated so far you see that, value of lambda here a particular basis is optimal, but when I go beyond this then this basis does not remain optimal; in fact, it does not remain feasible here in this case is finite, why because once a value of lambda is outside maybe a is outside a characteristic interval, the same basis or may be the corresponding basis I should say the corresponding basis will be infeasible for it right, in this case the current basis would be in corresponding basis would be infeasible right.

So, therefore, a basis will never be repeated never be repeated; this is important thing; so, we will never be repeated, a basis will never be repeated, since the number of basis is finite, number of characteristic intervals is also finite, so that shows you…; and in this case, because the partition the whole real line got partition into three intervals only right; and the second thing is that, yes, you observed that the right hand side and the objective function were linear functions of lambda all the time right.

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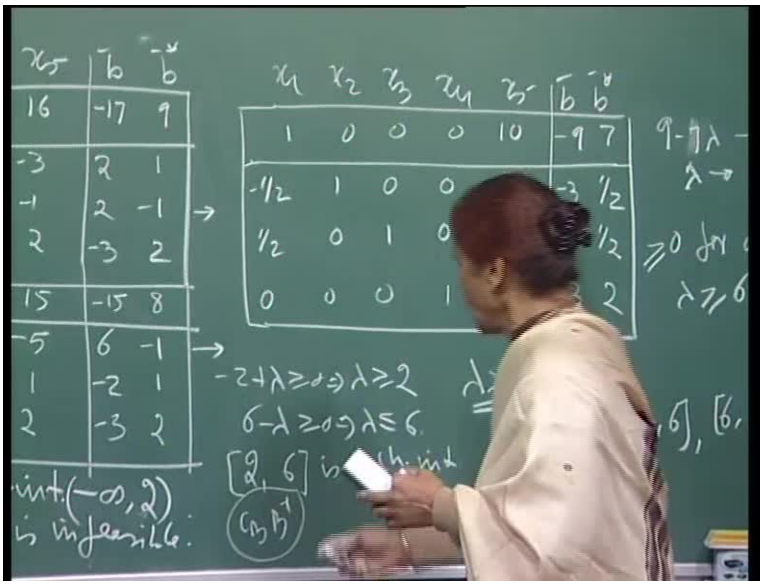

But for a particular characteristic interval, for example, here if B was the basis, then your this continued as the dual solution, a remained fixed; so, for a characteristic interval the dual solution remains fixed, but the right hand side and the objective function values are linear functions of lambda.

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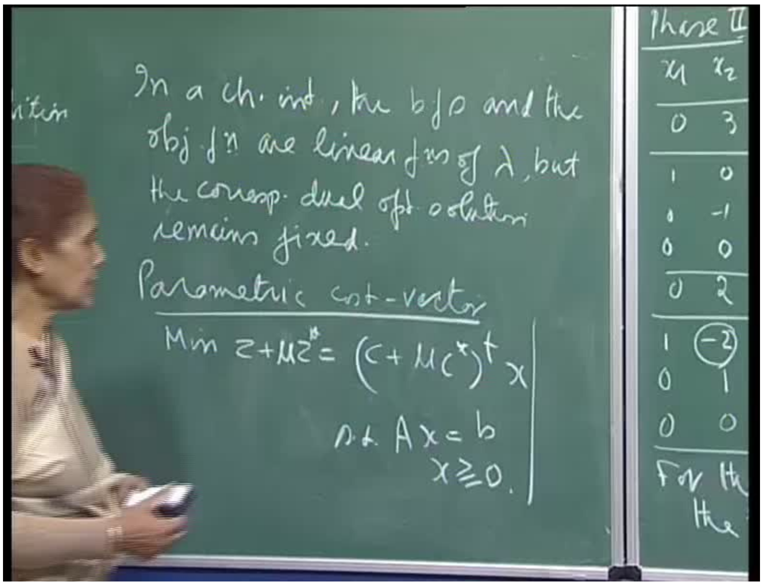


So, we can just make that comment also here in a characteristic interval the basic feasible solution and the objective function are linear functions of lambda, but the corresponding dual optimal solution dual optimal solution remains fixed right; if this not and that is why our optimality criteria is not disturbed remains fixed.

So, I think, if you simply the only point I did not really develop on or left out for you to answer is that the basis are not overlapping, and I hope you can once you go through this, then you can yourself sit down and show it very easily that the basis the adjacent basis are not overlapping.

So, you see once you have this kind of calculation now with I told you in the beginning,, if the government announces the value of lambda right at the last moment the manufacturer have already knows what are is optimal solutions for the different values of lambda, and so one he can immediately plan his production levels accordingly, because he knows what the optimal solution is and how the things will behave.

So, I keep repeating this that in many more than one way in fact in so many ways we have already seen how versatile you simplex algorithm is fine, and any other method the alternate method that have we been developed have other strong points, but they do not have this feature, so well inherent in then that you can do post optimality analysis. So, the next step would be now to…, yes, so I will try to…, because they are some points of departure when you handle the parametric cost vector; so, the next attempt would be to look at, so that means, we would be looking at the problem, parametric cost vector, and this will also have it is applications.

So, we want to now look at minimize Z plus lambda Z star which would be equal to C plus lambda…, may be let us use another parameter here, let us say it is mu, and this will be mu c star transpose x subject to A x equal to b x greater than or equal to 0. So, the idea is, here how do we though one can say that once you have a the algorithm for the right hand side parametric, right hand side we can write the dual here and then look at it.

But that is fine you can do it, you can treat it as a take the dual of this, and the apply the parametric right hand side algorithm, but let us see how what would be the ramifications, and it will be interesting to see if you handle this directly, because now in this case the feasibility will not be disturbed, it is only the optimality which would be a matter of concerned, and we can in this case keep applying the simplex algorithm right.

Because once you have a starting feasible solution and the same concerned that they are in some sense that how do you get the starting feasible solution, and then once you have a starting feasible solution, then suppose you can find out the interval for which continuous to be optimal and so on.

So, here the concerns here would be unboundedness right, and of course, because once you know that the problem is feasible, then it is the this objective function which plays a role; so, if the parameter mu will have a role to play in your…, now here what is that was this thing minus 2, and this this goes to minus sign, so that is fine; we can here the conclusion is because as my lambda goes on increasing, this value becomes smaller and smaller, so as lambda goes to infinity, this goes to….

So, for any finite value of lambda in this interval the I have the same basis continuous to be optimal; so, this would be the material for the next lecture where we will try to look at what are the issues if I handle it as it is instead of looking at its dual.