Linear Programming and its extensions

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Lecture No. # 20

Assignment 4, postoptimality analysis changes in B, adding a new constraint changes in {aij} parametric analysis

So, let me first discuss assignment 4 with you as I had promised, I will try to give you problems which depict all possible cases that can occur when you are solving the primal dual algorithm, and you are using the primal dual algorithm, and you may have to use an artificial constraint to obtain a starting feasible solutions starting feasible dual feasible solution.

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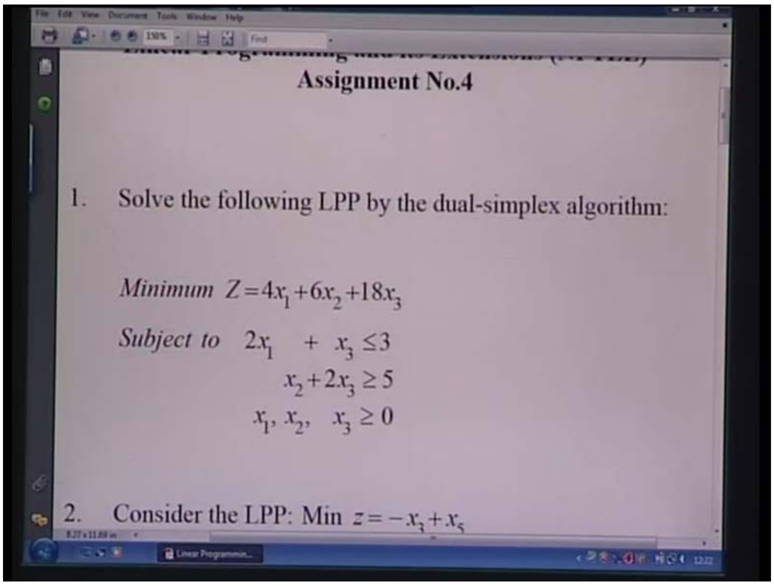

So, problem 1 is straight forward application of the dual simplex algorithm; so, you can see that the cost coefficients are all non-negative, but the starting dual solution made up of a slack and a surplus variable will be primal infeasible, and so you will have dual feasibility, and therefore you can proceed with the dual simplex algorithm.

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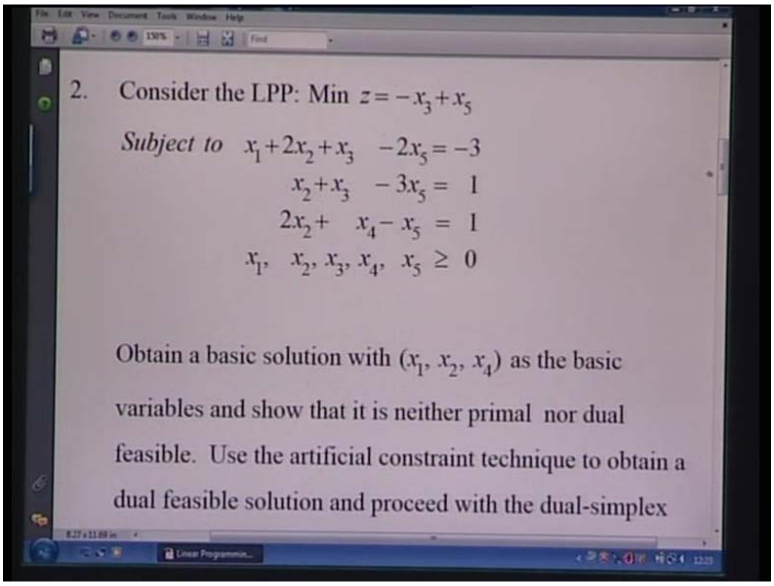

So, problem 2 I am saying that you obtain basic solution with x 1, x 2, x 4 as the basic variables; so, that means, from start your basis consisting of columns a 1, a 2, and a 4, and show that this is neither primal not dual feasible; so, by pivoting you will reduce this set of columns a 1, a 2, a 4 to identity column to the corporations all the elementary operations and then what you get at the right hand side will not be all non-negative, so it will not be prime feasible, and also you are C j minus Z j is the relative prices will also not be all non-negative.

So, the this basis is neither primal feasible not dual feasible; you have to use the artificial constraint technique to obtain a dual feasible solution, and proceed with a dual simplex algorithm; you should finally be able to show that the primal is infeasible, which we means that, you will be a we will be concluding that the dual is unbounded. So, when you proceeding with the dual simplex algorithm, remember what the condition is for detecting that the dual problem is unbounded, this will imply in turn that the primal is infeasible.

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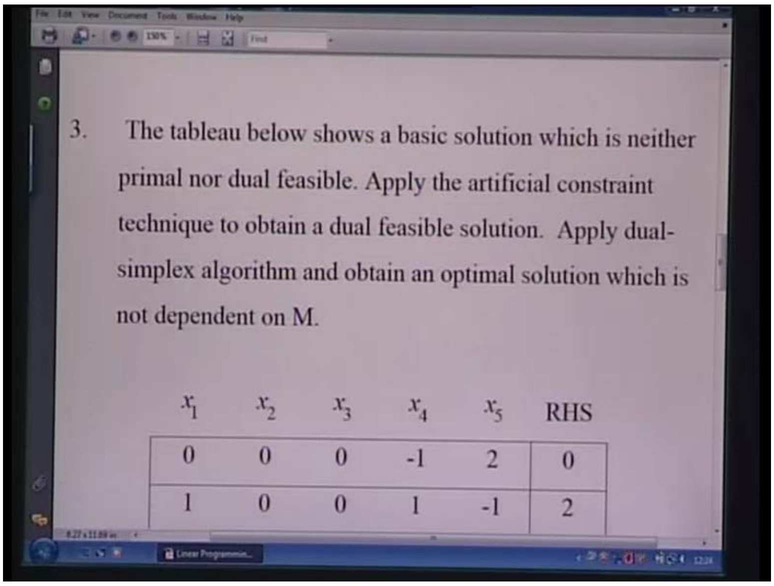

So, you should be able to show that… Now, let us go to problem 3, I am saying that the tableau below shows a basic solution which is neither primal not dual feasible again. And you are applying the artificial constraint technique to obtain a dual feasible solution. Apply dual-simplex algorithm and obtain an optimal solution which is no dependent on M.

So, I am trying to acquaint to with the different situations that are possible; and so, the solution is not dependent on M, which means that the artificial variable that you attach to the artificial constraint is not is present in the basis. Therefore, the artificial constraint is not tight, so you are objective function value will be not dependent on M, and so you can make the usual conclusions.

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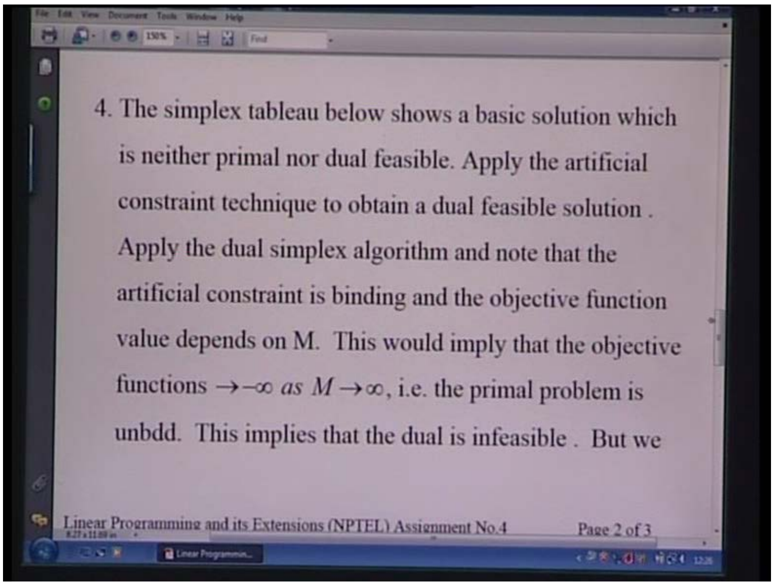

So, when you obtain the final tableau you will see that your then we go to problem 4, so the simplex tableau below shows a basic solution which is neither primal nor dual feasible apply the artificial constraint technique to obtain dual feasible solution. Then apply the dual simplex algorithm and note that the artificial constraint is binding that means your corresponding artificial variable is at 0 level in the optimal solution and the objective function value depends on M. This would imply that the objective function is going to minus infinity as M goes to infinity.

Yes, because remember your solution that you obtained is optimal; therefore, the coefficient of M in the objective function will be negative, and so as M goes to infinity the objective function value will go to minus infinity this implies that the primal problem is a unbounded; if the primal is unbounded you will conclude that the dual is infeasible. So, I am asking which dual, because you applied the artificial constraint technique to obtain a dual feasible solution; so, I want you to think about it as to what is happening here, because one way you are concluding the primal is unbounded, but that means, the primal which primal is unbounded at which dual is infeasible.

So, please just make sure that you know what is happening. I would like you to…, so this, this problem is an exercise, so that this is the table, and you are supposed to do it; well, I am saying that x 1, x 2, x 3 can be chosen as a starting basic variables, but the solution is not feasible - a primal feasible.

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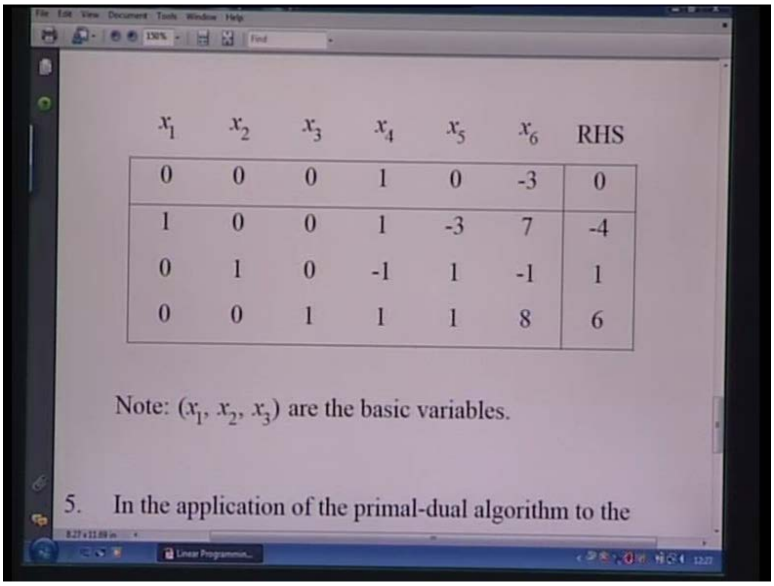

But it is well, it is neither…, yes, so you can start with the x 1, x 2, x 3 as the basic variables, you have a basis here, but you will have to add the artificial constraint to obtain a dual feasible solution.

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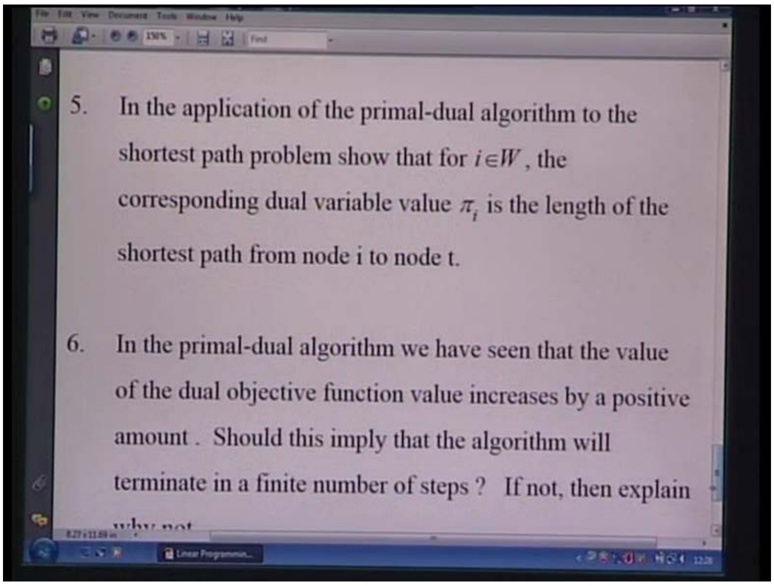

In the problem 5, I am saying that the application of the primal dual algorithm to the shortest path problem show that for I belonging to W. Remember I had defined the set W as the election of nodes which from which t is reachable, and the moment that happens you put node index in the set W.

So, the corresponding dual variable value pi i is the length of the shortest path from node i to node t, this you should be able to prove using the fact that y i got included in w; and therefore, using the fact that the corresponding dual constraints, some set of dual constraints are binding or are satisfied as equality, and from there you should be able to show that the corresponding dual variable pi i for i in w represents the length of the shortest path from that node 2 t.

Then problem 6 is again an exercise, the primal dual algorithm we have seen that the value of the dual of objective function value increases by a positive amount, yes, because you are when you continue with the primal dual algorithm the restricted primal value objective function value is positive, that is why you continue you modify the dual solution, because as long as the restricted primal objective function value is positive, you do not have a feasible solution for the original problem.

So, then you modify your dual, and that means, that your objective function value for the dual is increasing by a positive amount at each iteration; so, this should imply that the algorithm will terminate in a finite number of steps, but when should that happen, so if not then explain. So, I am asking you to sit down and just work it out; it should not be difficult you should be able to figure out why even though the objective function value for the dual is increasing at each iteration and dual is a maximization problem your, you cannot guarantee that the algorithm will terminate in a finite number of steps.

So, you either we making some conclusions, and then saying that you…, of course, you will terminate in a finite number of step, but why so I want you to explain that; so, this is the…, this is assignment 4, and I will continue with the my parametric and the and the sensitivity analysis in the lecture.

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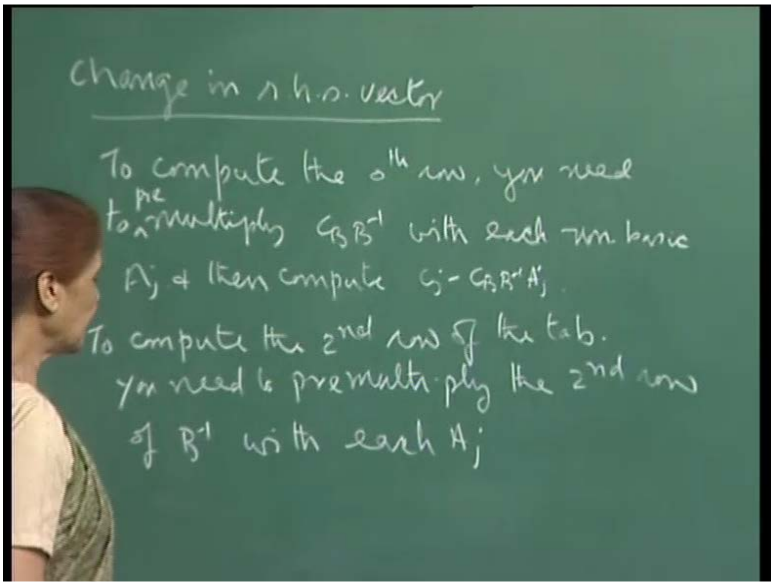

So, let me just get back to the calculations for the nineteenth lecture, and I had used the revised simplex algorithm. Let me just write out the…, we were doing the change in right hand side vector, so we were discussing this sensitivity analysis, and I had use the revised simplex version of the dual simplex algorithm; so, what was happening is that, we wanted to remove variable which became negative, because you increase the value of lambda. So, a particular variable basic variable became negative, so we had to remove it from the basis, and to do that I needed to say if I am computing, if I have using the dual simplex algorithms, then I need the top row, and the row corresponding to the outgoing variable.

Now, in the revised simplex you simply carry the extended basis inverse, so that means, you have to compute the top row and the second row, because here the second basic variable was going out of the basis, so you need to compute both the rows to take the ratios but determining the incoming variable and so this is what we did.

So, therefore, to compute to compute the 0th row you need to multiply C B B inverse, which is available to you in the extended inverse; you need to pre-multiply actually pre-multiply C B B inverse with each non-basic region, and that will give your…, so you will get the…, so C B inverse with…, so you will do with A j, and then you will compute, and then compute then compute C j minus C B B inverse A j once you get this quantity is the bracket from C j and that will give you the top row. To compute the second row of the tableau, you need to multiply, you need to pre-multiply the second row of b inverse which is again available to you; do you need to pre-multiply the second row with each A j corresponding to the non-basic variable with each A j.

So, once you have the second row and the top row then you can take the ratios and determine the incoming variable. And now let us continue, so we have consider the changes or the small change sensitivity analysis of the right hand side values of the cost prices, and now and then we also I think we worked out adding a new food if its available and so on.

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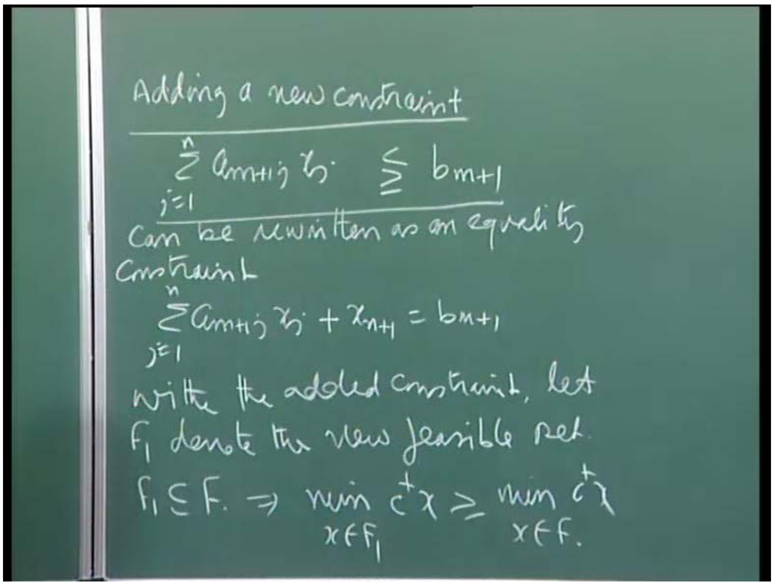

So, now, let us see if you add a constraint, the idea is that adding constraint, so adding a new constraint, so it is possible that the ministry or whoever the controlling authority is might decide that you need to have another nutrient, in that you must have a restriction on the another nutrient, and may be what the controllers may specify a minimum quantity or a maximum or equality whatever the situation.

So, say, you add a new constraint of the kind summation a m plus 1 j x j j varying from 1 to n, let us say less than greater than or equal to b m plus 1, so whichever the depending on, you know, the nutrient; if they do not want it in x s of whatever it is, but it is possible.

So, then you if you have a new constraint, so you can convert it to equality in case it is not equality, then you can be rewritten can be re written as an equality constraint, because you want equality constraint; so, saying that summation a m plus 1 j x j j varying from 1 to n, let may see it is a slack variable its less kind, so you saying x n plus 1 is equal to b M plus 1, yes, I am already points out something here before I do this.

At this point what you will do is that, you have an optimal solution; if its satisfy this constraint then we can conclude that the current solution is optimal, and why because you see if with the added constraint, let F 1 denote the new feasible set.

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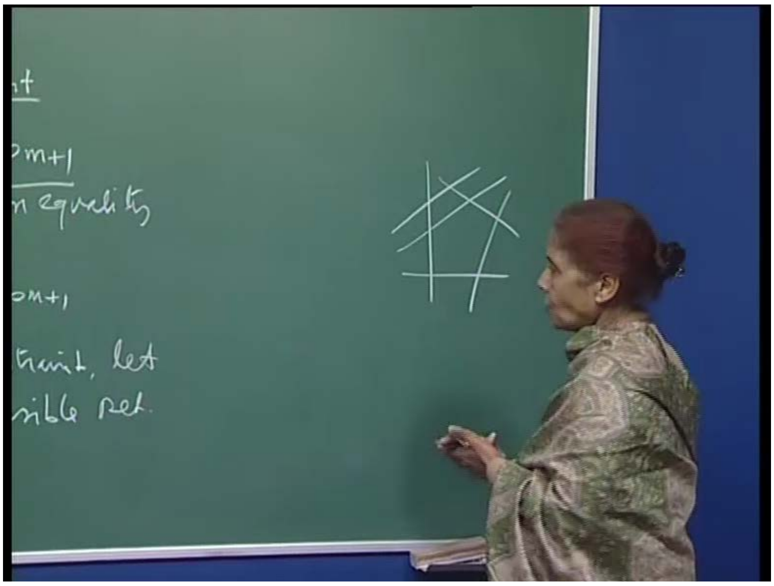

So, then F 1 is a subset of f, because you have added one more constraint initial feasible region; if it was this, now you have added another constraint or whichever way, so the regions become smaller, so definitely it may be either the same or it may it contain in F. So, this implies what, this implies that the minimum of c transpose x over x belonging to F 1 will be greater than or equal to minimum of C transpose x subject to x belonging to f; so, in case your current solution is feasible for the new problem it is in F 1, then it means that you cannot do better than what you already have, because here the minimum may be bigger, if the new if the original optimal solution did not belong to F 1.

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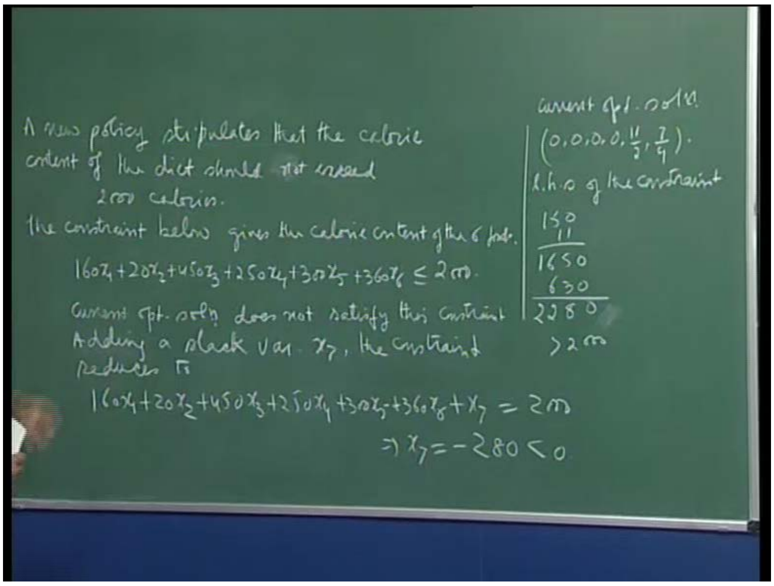

So, in case it does then you have the best; but if you do not have, then you immediately see that you will have a case for…, so that means, your constraint whichever kind will not be satisfied by the current optimal solution, but optimality criteria does not get disturbed by adding a constraint, so you have a situation made for eliminate made for the dual simplex algorithm.

So, then we can start with the dual simplex algorithm, and this is where I want to show you, because…; yes, so, you will add slack or a surplus variable I have added this, but the value here for x n plus 1 for the current optimal solution will not be a non-negative one, it will be negative because the current solution is not feasible. So, then you can start with your dual simplex algorithm.

So, let me work out an example and show you; the yes so, suppose, calorie content, so a new policy a new policy stipulates that the calorie content of the diet should not exceed 2000 calories, so that means, the constraints of the less kind; so, which means the end, the of course, you have to know what are the calorie contents of the 6 foods that you are uttering; so, yes, let me just have the data here; so, it says that, therefore, the this says that 160 x 1 plus 20 x 2 plus 450 x 3, so these are the numbers which are the calorie contents per unit of the different foods, 250 450 this plus 250 x 4 plus 300 x 5 plus 360 x 6 should be less than or equal to 2000.

Yes, and your current primal solution is given as 0, 0, 0, 0, 11 by 2, and 7 by 4; and you can just verify from here that, this would be 150 into 11 which would be 150 into 11 would be this, and then this would be 7 by 4, so 90 times 7 is 630. So, this is 0 8 this which is greater than 2000; so, the current optimal solution does not satisfy your new constraint, so we need to work start from very we are…, because so I can use the this thing and the optimality condition is satisfied by this basic feasible solution, so I am…, I can start my dual simplex algorithm.

Now, I can add a slack variable here to write it in the equality form, and therefore the constraint becomes 160 x 1 or maybe I should say that adding a slack variable adding a slack variable x 7, we the constraint reduces to 160 x 1 plus 20 x 2 plus 450 x 3 plus 250 x 4 plus 300 x 5 plus 360 x 6 plus x 7 is equal to 2000. And we see that since the right hand side after this point the value is 2 2 8 0, so therefore this implies that your x 7 is currently minus 28 0 which is less than 0.

Therefore, the current basis is no longer feasible when you added this constraint; and so, I need to find a different new basis, so that I get a feasible solution and hopefully optimal, otherwise I will have to work for it, so essentially, so the idea is that we will apply dual simplex algorithm.

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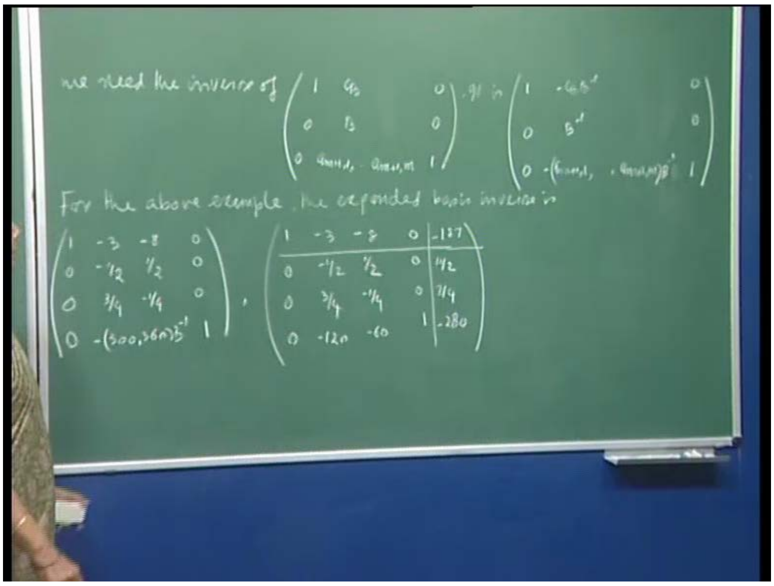

But since I have added a constraint my basis size is gone up, and so what I am saying is that, in general, if you add a constraint then these are the numbers corresponding again assuming that the first n variables are basic variables, so when you write out the a new basis the extended basis, this is the slack variable node added, and therefore these are the coefficients in the added constraint.

So this is the expanded basis, and I need the inverse of this; and again by a simple matrix methods you can see that the inverse of that expanded matrix would be…, so I can obtain it; once I have the basis inverse for this, I can obtain it by see this is already there, then here it will be become minus a m plus 1 plus 1, 1, so the same vector multiplied by b inverse and this; you can check for yourself that this is the inverse of this.

So, once you have the new basis inverse, let us we can then precede with the dual simplex algorithm, and of course, I have done in the last example I showed you how to take a do the revised simplex algorithm version of the dual simplex algorithm, so we will have to do the same thing. And now, if you go back to this example that, we have been discussing; so, here this was my basis inverse, this is your minus C B B inverse, so you have added since x 5 and x 6 for your basic variables, so 300 and 360, so this is what you add here, and this is b inverse, and so essentially this is your new basis inverse, so 3 by 3, and this is your right hand side.

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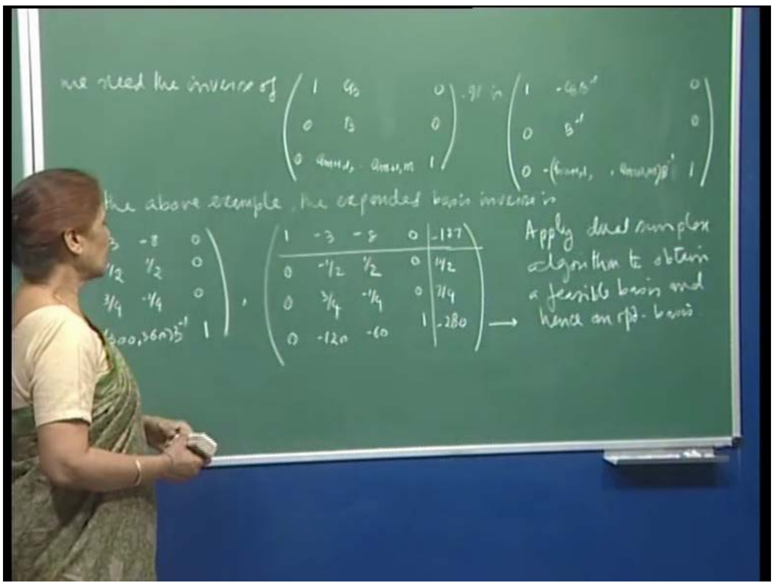

So, this is the final revised simplex inverse tableau for the current problem with the added constraint; and so, we will have to you x 7 from the basis, so that means, apply dual simplex algorithm apply dual simplex algorithm to obtain a feasible basis and hence an optimal basis, because the optimality criteria will not be disturb.

And I have already discuss with you, so what you will do is, in order to continue with the dual simplex algorithm we will need to compute the third row, because you now have the basis inverse, and you will need to compute the top row, you have the C B B inverse; and then once you have the third top row, and the third row you can take the ratios, and see that then continue with this side of the incoming variable and continue with the dual simplex algorithm.

So, what we have seen is that here definitely, because you do not have the whole tableau; when you want to proceed with the dual simplex algorithm or may be other version of this simplex algorithm, you need to generate the data, that means, you need to generate the top row, and you need to generate the row corresponding to the outgoing variable, but see the thing is that you still are not using as much space your storage requirement is very low, you are only making the necessary computations what you need to proceed with the algorithm.

And you are not carrying the whole tableau, because you do not need; if you have say 1500 variables, and you just need one coming into the basis, then why should you do calculations for the remaining 14 99, whatever the variable depend on the this thing also in our constraint. But anyway, so there is a definite saving in your computations when you use the revised simplex version and so I have try to show you this; but of course, for small problems you can when you want to do things by right hand, you may carry the whole tableau, and it will be much quicker of for you to see to decide on the incoming variable and so on.

So, as I said now all programming is done for the revised simplex version, because the whole tableau is never converted at every iteration only the necessary computations are made; so, I hope you have now obtained some feeling for the sensitivity analysis that we do after we have obtained an optimal solution.

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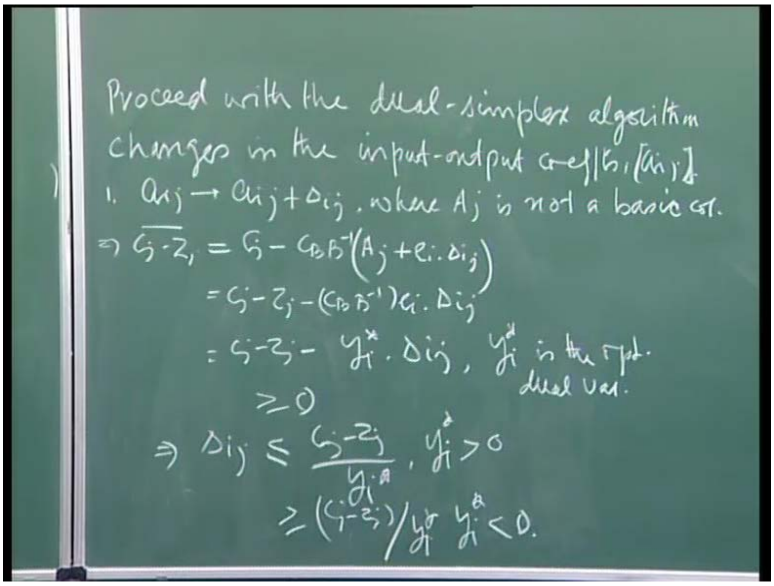

Now, changes in the other part is the changes in the input output, see then two names for input output coefficients, the coefficients outputs A i j; so, these are the these are the elements of the matrix, we call them input output coefficients, we can also call them technologic coefficients, because they reflect the technologies; see, for example, in the diet problem when you say that this is the amount of fat available in the first food, that means, that whatever methodology you have a extracting the how you prepare the diet; so, your content of the fat that you get from the first food will depend on the how you prepare the diet, sometimes fat may get burnt, and so on if you over whatever it is.

So, these coefficients are often called as the technology coefficients or the input output coefficients. So, we want to also look at how you can study the changes if you have if the input output, that means, you may have a better technology of preparing the diet, in that case these numbers may change.

So, can you make use of calculations that you have done so far in trying to take care of the changes in the input output coefficient. So, let us first let us look at the…, yes, so again two cases, one suppose A i j goes to A i j plus let say delta i j, where A j it is not a basic column. Therefore, what we will change, here in case A j is not a basic column, the only change that will occur would be in the relative price, that means, in the Z j. So, this implies that your C j minus Z j so bar will be equal to C j minus and this is C B B inverse I can write A j, because only this entry has change.

So, A j plus e i into delta i j, the only the ith entry ith entry in the jth column has change, so this is e i times delta i j; so, when you rewrite this will be C j minus C B B inverse is your Z j C B B inverse A j, and here it will be minus C B B inverse e i times delta i j, so that means this is the ith coefficient of element of C B B inverse which is nothing but the ith dual variable.

So, this will be C j minus Z j minus C B B, I should I have write that I say want to so say that this is equal to y i star into delta i j, y i star is the optimal dual variable; and so, you want this to be greater than or equal to 0; if you want to find out for what values of delta i j would your current solution remain optimal, so then this would imply that see I want to get this so this greater than or equal to 0; and of certainly, if so delta i j I am taking the situation depends delta y i star maybe negative or positive, so this implies that your delta i j is less than or equal to C j minus Z j divided by y i star if y i star is greater than 0 or it is greater than or equal to less than here in this case, because this comes to this side and you divide by y i star.

So, this will be this, otherwise it is greater than or equal to C j minus Z j divided by y i star, if y i star is less than 0, because if I have the standard formulation, then there is no restriction on the sign of the dual variables, otherwise for the canonical thing if that means if for minimization will have less, so the y s star would be the non-negative, so depending on what are the situations.

So, I immediately have this thing that, if delta i j has to be less, if delta i j becomes more than this, so delta i j becomes more than this, then the corresponding C j minus Z j will become negative, and so you will now have feasibility but would not have optimality. So, in either case, if your D i j exceeds the required limit, so as long as D i j in this case is less than or in this case is greater than then your current solution remains optimal you do not have to do anything, but if it exceeds these limits then you can proceed with the simplex algorithm.

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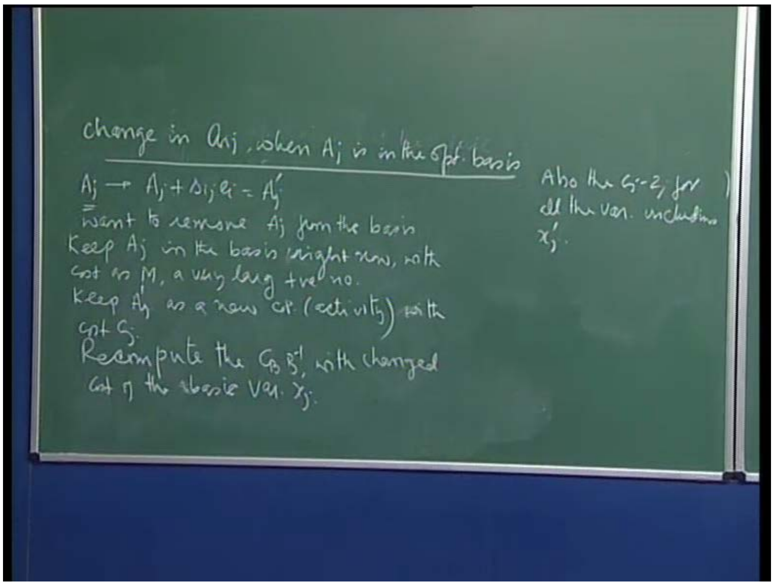

Now, in case the C i j changes for the basic variable, so A i j then you have to more work and let us see how we take care of it and then I show you through an example, so changes in A i j when A j is in the optimal basis, so we want to look at how to take care of this; see, the thing is that, that means in this case your A j will go to A j plus delta i j e i, and let me call this as A j prime.

So, we actually want to see at this point when once you have already obtain an optimal solution, and then you realize that know your A j should have been A j prime, then it will require a few more iterations, because I cannot trace back my calculations; so, the idea here is, but we will use the current solution, and which would mean that see what I do not want A j column A j anymore.

So, that means, what we will say is that, let us treat the corresponding variable x j as an artificial variable now; it is in the basis I want to drive it out, and one way we learnt the technique may be from the…, I did not discuss, I did mention it is the bigger method, but we will talk about it now.

So, the idea here is want to remove A j from the basis; it should not figure in any optimal solution, because A j is no longer a feasible column for me, therefore we will use the big M method; so, what we will do is, keep A j in the basis now with cost as M a very large positive number large positive number.

Because if M is a very large positive numbers since we are minimizing the cost M A j, if this possible to obtain a feasible solution without A j, then this will help us, because M is very large, so an optimal solution x j should not figure provided I have a feasible solution from among the other variables, so a very large positive number, and this is what we call is a big M method, and keep A j prime as a new column.

So, let me say you added a new activity as a new column or activity as we call it in this case in our case it is a new food, as a new column with cost price with cost C j, so this is our legitimate column now, and this is not legitimate we want to drive it out of the basis; so, therefore, the one more column we get added; and now the idea that, and so you will proceed with the simplex algorithm, but before that so because A j is currently in the basis and you have changes objective function the cost, you have change the cost C j to M; and remember in the when we were discussing the price ranging for a basic variable you have to re-computed.

So, here re-compute the C B B inverse with changed cost of the basic variable basic variable x j, because the very cost x j, so I will compute this, and therefore we compute this and also the C j minus Z j for all the variables including x j prime x j prime corresponds to your A j prime.

So, you will do this, and then you will start with the simplex algorithm, because you want to drive out, so obviously once because you take M as a very large number, there will be at least one legitimate variable whose relative price would be negative, and so it will be candidate for entering the basis, and we continue with the simplex algorithm. Now, I did not discuss this, this is known as the big M method, when you do not want to particular variable in the… but you need it as a device, because in this case I have a ready mate tableau, I have a basis inverse everything corresponding to A j being in the basis.

So, therefore, it helps to start the computations from there, therefore currently A j is in the basis, but I want to drive it out; so, then the idea is that, you can give it a very big price, so that it is not profitable to keep it in the optimal basis; so, as the process of the simplex algorithm it will go out of the basis, if you have a regular this thing from there.

So, let me work out a problem and then I will discuss what can be the possible implications of, you know, adding a giving a very big price to a variable when you do not want it in the big S.

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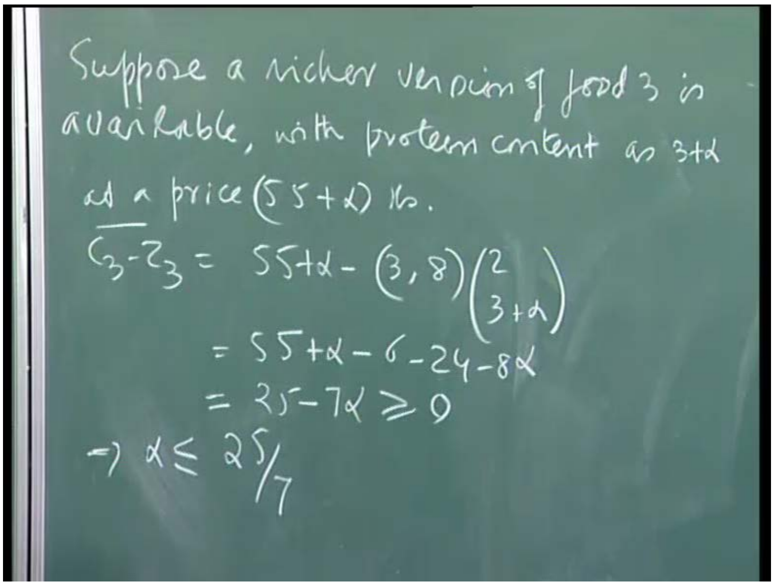

So, let me give you the data here, this is the suppose a richer version of food 3 is available; so, what I am saying is that, suppose a richer version of food 3 is available, which means, what that in the market with protein content, so with protein content as 3 plus alpha the originally it was 3 a 3 plus alpha at a price 50 plus alpha rupees.

So, now, you want to know food 3 is not part of your basic feasible solution optimal basic feasible solution, so you want to know when would this food become competitive, when would it become a candidate for being included in your optimal diet; so, obvious question would be to find out what the C j minus Z j would be, so therefore let us find out.

So, this c 3 minus z 3, the new this thing would be 55 plus alpha minus, remember our dual solution was 3 comma 8, and the this thing would be…, your protein content is the second one; so, the first yes I have to go back to the this thing, so yes, we were computing this whichever n is there, so the new this thing would look would be as 55 plus alpha minus.

This would be food 3 with fat content 3 plus alpha, so this would be there is a plus alpha here, yes, this is 3 plus alpha, so this will come out to be 6 minus 24 minus 8 alpha, so the whole thing comes out to be this is 30, so 2 5 minus 7 alpha, we want this to be greater than 0 which implies that your alpha is less than or equal to 2 5 by 7.

And this remember, the formula I gave you that had the y i star here, but here it is not y i star, it would have been y i star 8, but since I had the price also changed, and I did not take that in my calculations; when I was discussing the change in the A i j corresponding to a non-basic column, I did not take the change in the price, I kept the price as it is.

So, with the change in the price the number becomes 7 here, and this, so this alpha is less than or equal to 25 by 7, then it says that the food would not be competitive enough to be included in the optimal diet current solution is optimal even with this new data with the change data; but in case alpha is greater than 25 by 7, then C 3 minus Z 3 will become negative and you can proceed with your simplex algorithm to enter a 3 into the basis and to stop when you get the optimal solution. So, this is the calculation for change in A i j corresponding to a non-basic column. Now, suppose, change in A i j when A j is in the optimal basis, so x food 6 is included in your optimal diet, and let me give you the…., so that can to the fat content.

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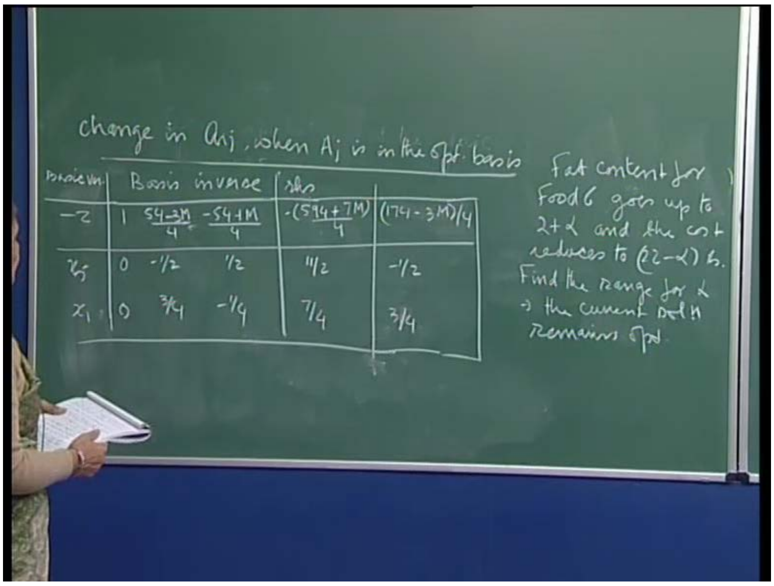

So, suppose fat content for food 6 goes up to 2 plus alpha and the cost reduces to 22 minus 7 rupees; so, the fat content has gone up, and therefore the food the price is come down. Now, you want to again find the range for alpha, such that, the current solution remains optimal current solution remains optimal; so, we will need to find out because the price also has changed, the price has change for a basic food.

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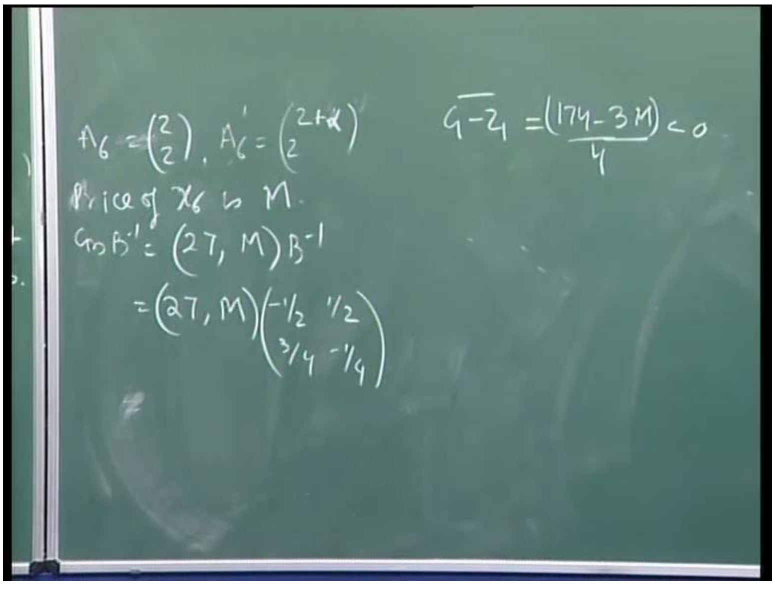

So, what we will saying is that, we will…, so you are A 6 will currently, so that means, this has gone up to 2 plus alpha, and the cost reduces to this, so what are we doing now, we are adding your column A 6, therefore has gone to originally your column A 6 was 2 2, so A 6 was 2 2; now, your A 6 prime has become 2 and 2 2 plus alpha and this, because the fat contents refers to be first, so this is 2 plus alpha, and A 6 is in the basic now, A 6 prime is not.

So, we will keep A 6 prime in our tableau some wherever we have stored the original data, so that to be there A 6 is 2 2; but now, we say that change C the price of price of a x 6 is M, now price of x 6 is M, so which means that your C B B inverse has become because x 6 is the second one, and what is your price for this thing for the first one, yes, x 5, so this is 27, and this will be M, and this will be b inverse, so this will be your new C B B inverse which means that this will be 27 M and yes so B inverse is minus 1 by 2 1 by 2 3 by 4 minus 1 by 4.

And so, that turns out to be which I have shown you the computations here, so these are the computations, so your new C B B inverse will be this, and therefore the new objective function cost could be this without the minus, and then this is your current basic feasible solution with A 6 as the activity column, so this is it, then it turns. So, we will start with the simplex algorithm, because I want to make use of the calculations that I have, so therefore yes with this new C B B inverse I need not write it here, it is all given shown on the table that is where. So, now, you start with your C 1 minus Z 1 new this thing, and it turns out that the C 1 minus Z 1 is equal to 174 minus 3 M upon 4 which is less than 0, because M we are taking to be very large number.

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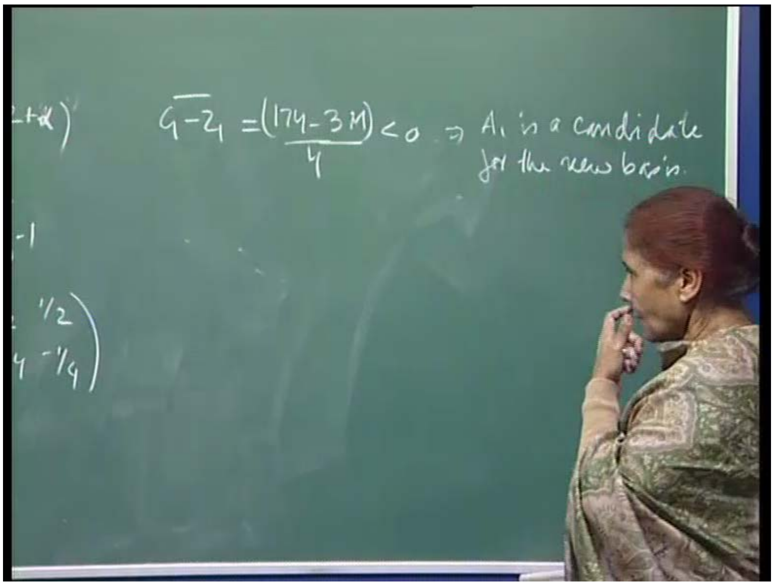

So, the moment M has a negative sign, no matter what number here is, I have what we say that M is sufficient enlarge, so that that means that this is less than 0; so, the this implies that a 1 is a candidate for the new basis, and I have already done the calculations here.

So, this would be the your y 1, this column is y 1, and this is your C 1 minus Z 1, so we will pivot on this just want to show you the calculations a little bit; so, pivoting would mean what, see here yes you have to decide on the pivoting elements, so since this is negative, this is your pivot element, and so you want to make 0 and 1, 0, 0, and 1 here, and which means that the second that means now your x 6 will go out of the column.

Because currently A 6 was the second column in your current optimal basis, so pivoting on this one means that A 6 will go out; therefore, you would have gone and what would, that mean, that if A 6 is going out that means it is no longer in the basis to check whether your calculations are right or not, these M should disappear from your calculations.

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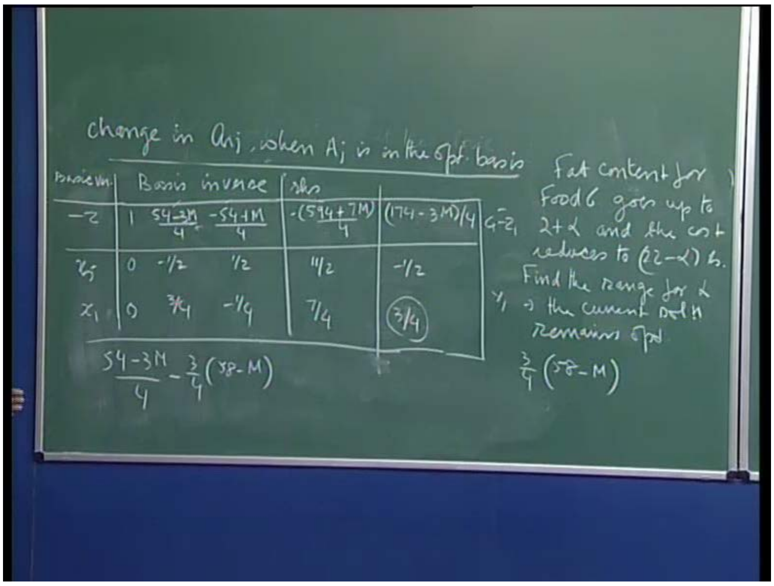

Because they would not be…, because your C B will not be a function of M, therefore C B B inverse would not have any component of M, and so your new calculation new top row should be independent of M; and I will quickly show you how, because you see this number you can write this 3 by 4 times this is 58 minus M, because we this is 3 by 4 already. So, that means, this row I multiply by 58 minus M and subtract; fine, if I multiplied by 58 minus M and subtract I will get this number, so just do that; so, here this is 54 minus 3 M by 4 minus you have 3 by 4 58 minus M, so you see minus 3 M plus 3 M it will cancel out.

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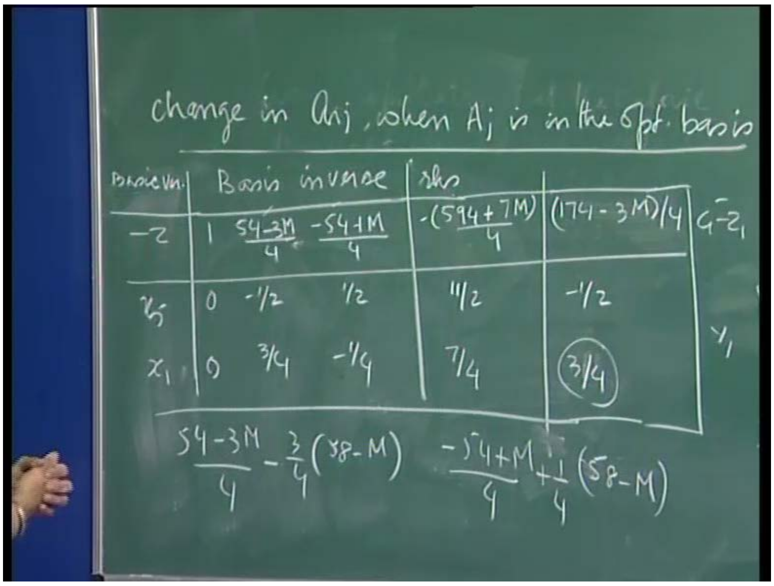

So, without wanting to do we just want to show you that here plus minus 3 M plus 3 M, so this number M will go out from here; and similarly, if you look at this 1 you are multiplying this row by 58 minus M, so then minus 54 plus M upon 4, and this will be plus 1, because your subtracting 1 by 4 times 58 minus M.

So, here again M by 4 and minus M by 4, this will go up; so, once the C B B inverse, that means, your calculations are ok, because the new C B B inverse is independent of M, and then here also the new objective function will be there; when you make a 0 here and then 1 can continue with the calculations here. So, your new basis is x 5, x 1, and depending on the sign of then we will again start computing your new C 2 minus Z 2 and C 3 minus Z 3, and if I satisfy the optimality criteria, then this is my optimal solution, otherwise I will continue with the simplex algorithm till I satisfy the optimality criteria.

So, this sort of takes care of most of the changes here, you might say that we have not a really discussed this technique of adding of making the cost coefficient of a variable very large, so what are the possibilities, because since we do not want that particular variable; in this case, you see if I do not come to a situation where this vector is going out of the basis, that means, if it continues to be in the basis then I do not have a feasible solution with the new column, because this continues to be in the basis.

So, what I will do it that may be after I discuss the parametric part also we will come back and visit the big M method and I will try to discuss the theoretical aspects of that, so we will do that later on. So, the next topic in the post optimality analysis would be parametric analysis, and let me just introduce you to the topic and then we can work out the detail in the next lecture.

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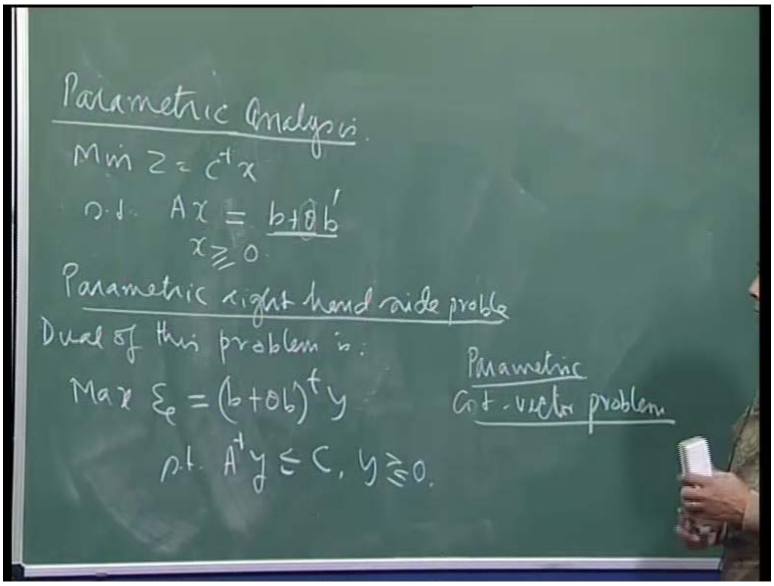

So, the other part is…, and the reasons are very obvious why we want to include this also parametric analysis; see, it consider a situation, see we have a linear programming problem minimize Z equal to C transpose x subject to…, let me consider the canonical case and x greater than or equal to 0.

So, in the diet problem or may be some other situation, where is a manufacture and his reducing some items, and these are the raw materials available consider equality that means say the product mix problem. So, you have the raw materials available to you or given here and then you want to produce something, and (( )) minimizing or maximizing does not matter, but this is a situation; what happens is that, some of these items, not all may be some are under governments control, that means, they will not be freely available

So, in that case there may be there may be a policy that in batches the raw materials, some of the raw materials may be released by the government, and this may happen at the last moment.

So, therefore, this becomes parametric in the sense that, this controls the amount which is available to you in the market; depending on again the situation, how things are available, how the production is, there must be some places where these items are being produced; so, the government will have a control, and will decide to release certain items in the market from time to time; but as a manufacturer you have to be sure that you are prepared, because this may happen at the last moment, but you have to set up your production levels much before and you have to plan.

So, therefore, you may want to know what would be your production levels depending on the value of theta, and again this simplex algorithm allows you to really do this kind of parametric analysis. So, we will call it this is known as the parametric right hand side problem; so, this makes lot of sense; and then if you look at its dual, dual of this problem is maximize, this equal psi equal to b plus theta b prime transpose y subject to a transpose y less than or equal to C y unrestricted.

So, because we also the primal and the dual also well interconnected, it make sense to also look at the parametric version of the cost function; so, therefore, if this is parametric of the right hand side the dual will become parametric, so this would be known as parametric cost vector.

And I did the sensitivity part first, because sensitivity part allowed you to only change 1 C i j or 1 A i j or 1 b i at a time; now, here what is happening is that, more than 1 b i's will be changing according to the b prime and so theta is the parameter; so, b prime is given to us say theta, so again there is only one parameter, but the thing is that your right hand side vector may change by more than one component, so this is the idea.

So, we will now try to look at this, and again through examples I will try to or going to with the methodology and so on, and we would not make it very rigorous, but I will try to just work out examples and the first this thing you can understand what is happening.