Linear Programming and its Extensions

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Module No. # 01

Lecture No. # 19

Shortest Path Problem-Complexity Interpretation of Dual Variables post-Optimality analysis-Changes in the Cost Vector

So, let me just make you more comments about the shortest part problem, so remember if this is S P P and I showed you that, the restricted dual problem actually reduces to building up the path and we go on at each iteration you were adding 1 arc. In case, the theta value for obtained for only 1 arc then you are adding 1 arc, but you may be adding more than 1 arc at a time also, so the restricted dual problem was the path building problem, and there was no question of optimization at that stage, we were simply looking at feasibility.

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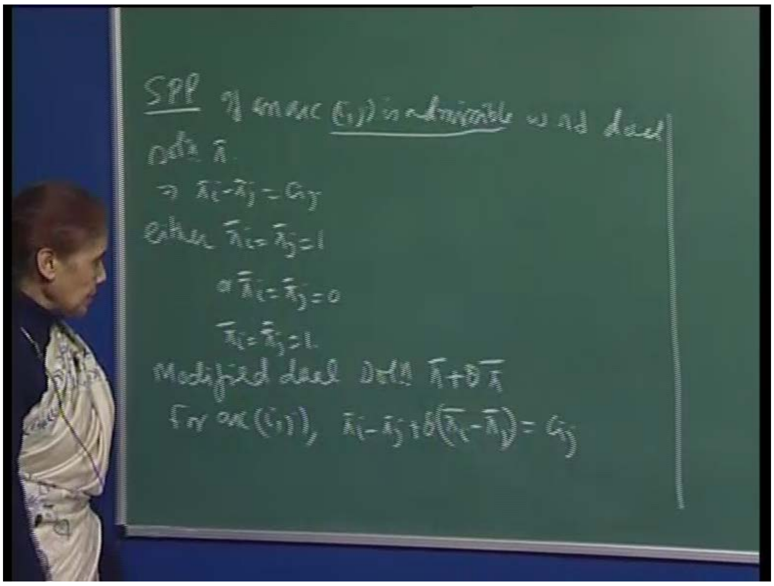

Now, let see that just a few comments about the complexity of the algorithm, it is how long it will take and what it all entails. So, see we said that if an arc i j is admissible with respect to dual solution, let say respect to dual solution pi; this means that your pi i minus pi j is equal to c i j this is what you mean by an arc been admissible, and then we saw that for the restricted dual, because the arc i j is admissible.

So, in case i is reachable from s then pi i had a value 1 and because i j is admissible, so this will also take a value 1. Similarly, if p is reachable from j then pi j bar value will be 0 and since this is admissible, so pi bar that means either pi bar i is equal to pi bar j is 1 or pi bar i is equal to pi bar j is equal to 0.

And in case i is not reachable from s or t is not reachable from j, in that case also we were putting this as pi bar j as 1; so in either case whenever an arc is admissible this difference and your modified dual solution is pi plus theta pi bar, and so for an arc i j it will be, for arc i j this reduces to pi i minus pi j plus theta pi i bar minus pi j bar, which is equal to again c i j.

Since, pi bar i and pi bar j for the admissible arc are equal, so this is 0; so this remains. Therefore, an admissible arc always remains admissible and when that happens this implies that.

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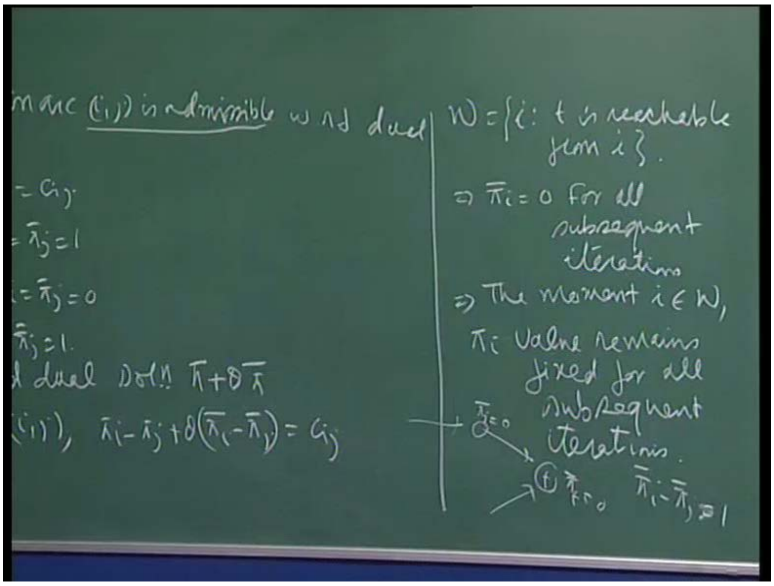

So, therefore for an admissible arc we can also see from here, that there will be no change, so this is it, now suppose I define a set w, as a collection of all node indices, where t is reachable from i.

So, the moment node is such that if t is reachable from that node that means that this implies that pi bar i to be 0 for all subsequent iterations. Because ones this is how we are building up, we always write down the optimal solution for the restricted dual and saying that if node is reachable, if t is reachable from that node then the value pi bar value is 0 and if it is reachable from s then the pi bar value is 1.

So, for all nodes from which t is reachable then the pi r value will remains 0, this implies that the moment i belongs to w, pi i value remains fixed for that node; pi i value remains fixed for all subsequent iterations, because, I will not be adding any pi times when i add theta, pi bar times when i add theta for the corresponding with pi bar i value is 0. Therefore, the pi i value remains fixed

And here you can get a very nice interpretation for pi bar or pi i which I have put in as a assignment problem, assignment 4 I will be discussing next time, so there I have asked you to prove that pi i, the moment i become the member of w, then you can prove that pi i value is nothing, but the shortest path distance of node i to t, that means there is a path from i to t, whose length is pi i and this is 1 of the shortest path can be more than 1, but the value pi i value will represent the length of the shortest path from node i to node t. So, you have a very physical interpretation for the dual variable here.

Now, you see that at each iteration b at one node at least one node at every iteration to w, because remember you start with t, your t, your pi bar t value is 0; then we were looking at these arcs which are coming into t, and then we found out the 1 for which the length was a shortest.

So then this value, because from here t was a reachable, so this value became whatever you want to write here j, this became 0; so every time you only and since you add arcs for which your pi bar i minus pi bar j is greater than 0 which is equal to 1.

So, you will always add an arc for which pi j node, for which the pi bar value is 0 and this is 1; so you will always add an arc here, either here or here, such that the reachability to t is increasing, so from this node t will become reachable; so at each iteration we are adding at least one node to w, therefore what we are saying is that this algorithm will end will give you a shortest path at the end of at most, so let me write out here.

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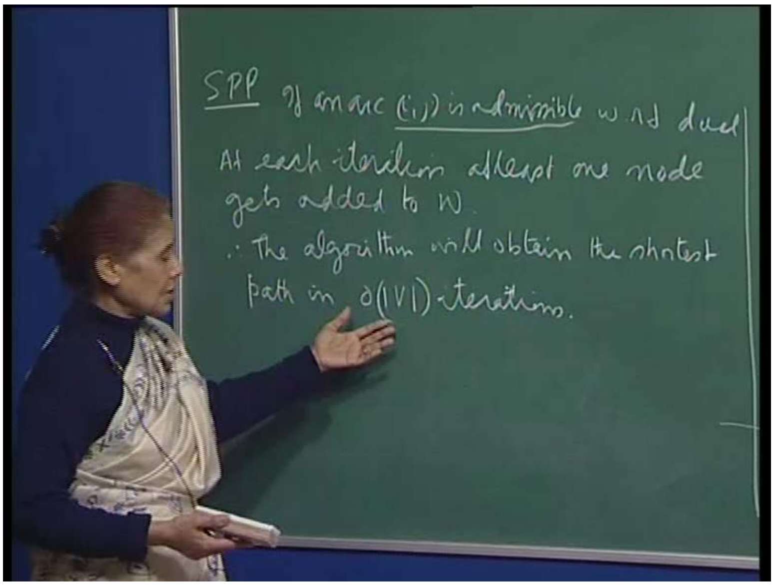

So, what we are saying is that w, the cardinality of w increases at each iteration, so at each iteration at least 1 node gets added to w, therefore the algorithm will obtain the shortest path in order v iterations.

So, at each iteration you are adding 1 node and total number of nodes is the cardinality of this set, a node set, so we will definitely reach that means we will be able to put the value of pi bar s equal to 0, because we have a path from s to t in at most this many iterations.

So, in the idea of wanting to know how much work you do for an algorithm is also very important and it is a development let say of the last I would say 30 years, (()) so that not only you want to make sure that the algorithm will give you the desired will obtain the desired solution for you, because an optimal solution and feasible solution whatever you want to put the restrictions on the simple problem.

So, algorithm should be valid, but you know you also want to have the consideration how much effort you are going to put into that getting that solution, so because one can always applied root four's and try to look at every feasible solution if possible, and then for example in the case of linear programming problem, you can say that let us just compute the value of the objective function at every extreme point, and so select the best one, but you can see that the number of basic feasible or basis can be very large and it may take you years to solve a reasonably size linear programming problem.

So, you do not want to do that kind of thing, if you want to develop an algorithm, you also want to make sure that it will perform or it will reach the objective optimal solution in a reasonable amount of time, reasonable effort is required. And here you see that this algorithm say that simply just many iterations you have to form, and at each iteration you saw that not much has to be done, because you simply have to update your dual solutions, so you have to find the value of theta, update your dual solution, then again find out the arcs for which are admissible arcs and continue building up the path.

So, nice an efficient algorithm and a good application of the primal-dual algorithm, this was the 1 reason I wanted to take up this example. And later on also we will have occasions to show you some more interesting applications of the primal-dual algorithm.

So, once these basic algorithms are in place, let us now look at another important topic is post-optimality analysis, and let me point out write in the beginning that for linear program simplex algorithm and its variants have this facility of being able to look at the post-optimality analysis, that means if some changes are made in the data, can you without much effort obtain a new optimal solution, because it possible that the current optimal solution may not remain optimal when you change the prices at the objective function or you change the right hand side of requirements or even the technology coefficients as we call them.

We see this a matrix A the things here may change or a new constraints get added or a new item is available for production, different scenarios will have different interpretation, so in any case if your problem data changes then you want to find out how much effort would be required to obtain a new optimal solution if the current optimal solution is not any longer optimal. Now, this is a feature of for the simplex algorithm and like if you look at the other linear programming solving algorithm Hashian and Karmarkars they will not have this facility, and if you go to interior programming problems of a non-linear programming problems, then of course you just cannot ever hope to, or may be one can do some kind of post-optimality analysis, but it is not straight forward.

So, let see and I hope to show you in couple of lectures from now, how you can look at the changes in the data and then try to modify your current solution accordingly. So, I will now go back to the bad problem, but not the housewife's diet problem, let me just address the problem, let me write it out here, I have given out the data here

So, this is you see in the U.S. dairy farming is a very big business, and so lot of care is taken to provide proper diet; so that good yield of the milk is available and the cattle help is also very important. And let me just take up a very simple example, so this is I will demonstrated through a diet problem for dairy farming.

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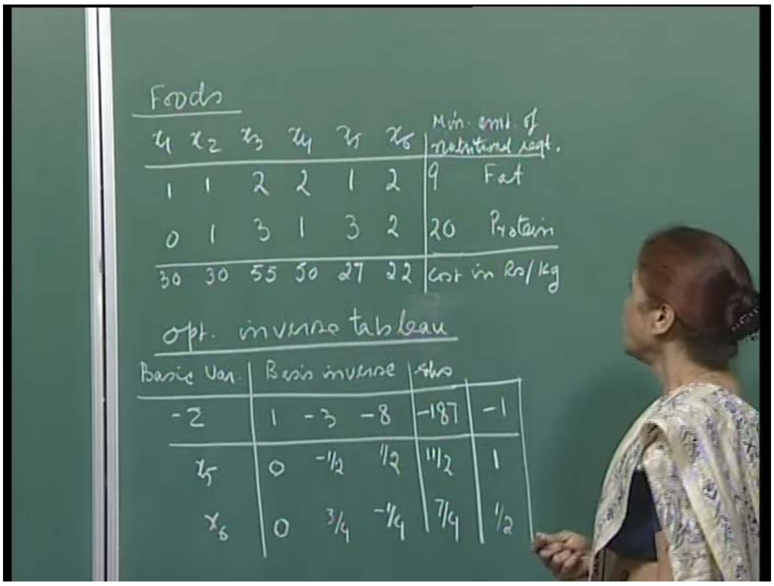

So, let us see, here I have written down a small problem, which says that we have 6 foods available, from which the diet for the cattle has to be made up, and 2 main requirements that have been specified.

So, minimum amount of nutritional requirement that the foods must have is a total diet must contain, is 9 units of fat and 20 units of protein, so whatever the diet that gets built up, that should satisfy the fat content of the diet should be at least 9 units, let us say 9 grams and then this it should be 20 grams of protein, and for different foods, this table shows you that; for example food 1 contains fat, so that means 1 kg of food 1 will contain 1 gram of fat and no protein.

Similarly, second food contains 1 gram of fat and 1 gram of protein, so these numbers will tell you the fat content and the protein content of the different foods, and so we said that through linear programming accent that the constraints will be there, then these are the cost 10 rupees per kg these different foods, so 30 rupees per kg for food 1, 30 rupees kg for food x 2 and so on.

So, given this we want to now look at the post-optimality analysis and this is your optimal inverse tabular, so here this is your basis inverse, these are your minus c b b inverse, the dual solution and this will be your optimal solution. I mean the minus of that, that means 187, this gives you the basic feasible solution; so that means what it says is that you should include 11 by 2 grams of a kilograms of food 5 and 7 by 4 kilograms of food 6 in your optimal diet.

So, now I will do the computations using the revised simplex method, because I thought you should also be familiar with the computations using the revised simplex method.

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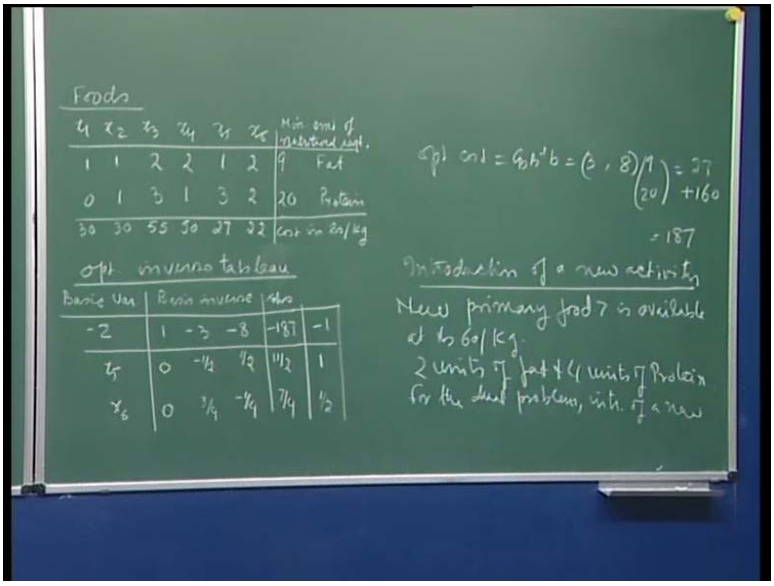

So, you can just make sure that this gives you the optimal cost, so immediately the optimal cost now is equal to c b b inverse b which is 3 8, and then your b is 9 and 20; so this makes it 27 plus 160, which is equal to 187; so 187 rupees is your current optimal cost.

Now, suppose we just want to look at various kinds of post-optimality analysis, I will now talk about introduction of a new activity, so this is the general term, but I will take up this example that means; now we have said there are 6 foods, and then you may want to include a new food. A new food has come into the market when you want to see, whether it will complete with the current diet, that means it may be a candidate for adding into your optimal diet and so on.

So, but when you say introduction of a new activity what we mean it that here, it may be if you have a production scenario, then it may be that a new product has to be reduced and then, therefore you it will form as a new activity, whatever it is. Basically, what we are saying is that a new variable is available and you want to know whether you can you should consider it in your optimal solution or not.

So, introduction of a new activity which means here, so I will just write out this, that a new primary food 7 is available at rupees 60 per kg, and this has 2 units of fat and 4 units of protein; so you want to know whether the solution change or would your current solution remain optimal. Now, the interesting thing here is that, you will be adding a column here, that means x 7 then you will write 2 and 4 here and 60 here, but if you look at the dual problem, then this new primary food that you have, the new food will now form constraint for the dual.

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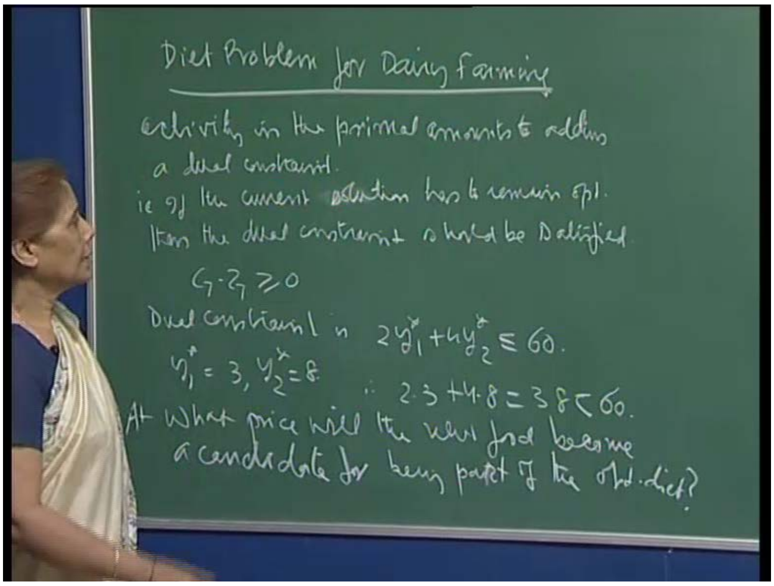

So, for the dual problem introduction of a new activity in the primal amounts to adding a dual constraint, so therefore for the dual problem the introduction of a new activity in the primal amounts to adding dual constraints, that is if the current solution has to remain optimal, then the dual constraint should also be satisfied. After all having a new activity for the primal means that here you may name this food as food 7, because you have food accepted to 6. So, new primary food 7 that we are saying, so essentially if the current basis has to remain optimal then you would want that your c 7 minus z 7 should be greater than or equal to 0.

If, it becomes less than 0 then you will say that, the current solution is not optimal because you have a variable non-basic variable which can be a candidate for entering into your basic feasible solution, therefore the optimality criteria continues to be satisfied, then we say that the current solution is optimal.

So, now here that means for the dual constraint would be (( )) is 2 units of this things are 2 y 1 star plus 4 y 2 star should be less than or equal to 60, but your y 1 star is equal to 3 and your y 2 star is equal to 8, this always gives you the dual solution minus of that, because this is minus c b b inverse, this row; and so minus of that is c b b inverse which is 3 and 8; so here you see therefore 2 into 3 plus 4 into 8 which is equal to 38 is less than 60.

So, the current solution is optimal and we do not have to make any changes, the new food which has been introduced is very expensive even though it has 4 units of protein and 2 units of fat, but the rice per kg is high and therefore it is not a candidate and it is not offering any competition to the current foods, which make up your optimal diet.

So, now the next question that is asked is, at what price will the new food become candidate for being part of the optimal diet, so this is the question and so I will address this question in a general way

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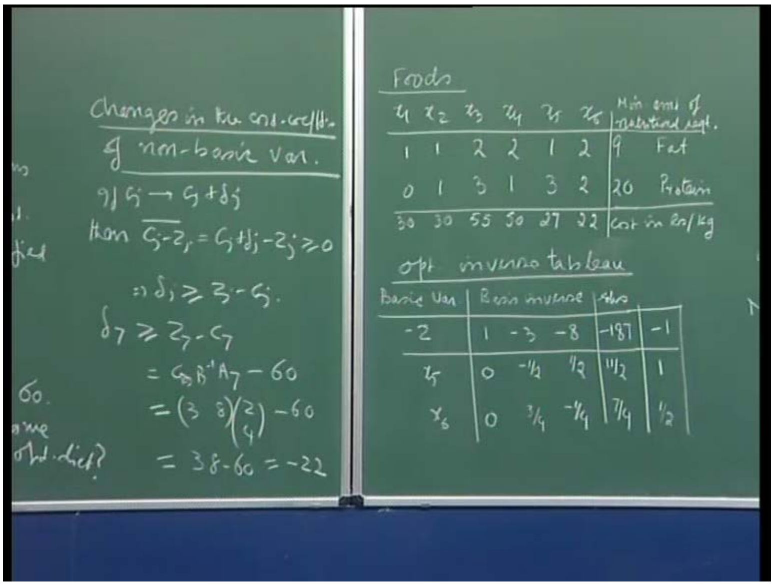

So, we want to look at changes in the cost coefficients of non-basic variables, so let us look at these non-basic variables.

So, if you want to look at changes in the cost coefficients are non-basic variables, then what happens is that your dual solution or your c b b inverse do not change, so that means for example, if c j goes to c j plus delta j, then your c j minus z j bar the new 1, because your c j has change will be simply c j plus delta j minus z j, because the z j has not change, z j is c b b inverse a j; so no changes have been made in c b or b inverse or a j; so this is this and this you want to be greater than or equal to 0.

So, you are looking for a value of delta j, so that the current solution is optimal, continuous to be optimal with this change, which implies that your delta j should be greater than or equal to z j minus c j.

So, if as long as delta j is greater than or equal to this number your current solution will remain optimal, but if delta j becomes less than z j minus c j then you can see immediately that c j plus delta j minus z j will become less than 0, and therefore the new this will that means the basic the non-basic variable x j will become a candidate for being a part of the optimal basic feasible solution.

So, well at least it may not really be optimal, but at least it will become a candidate for entering and so you may have to do 2 or 3 iterations before you come to the optimal solution, so it is not a guarantee that it will become part of the optimal solution that is not necessary, but at least at the current basic feasible solution is no longer optimal, if this number delta j is there, so therefore now, if you want to answer this question for the new food that has been introduced.

So, then here I am considering delta 7 it should be greater than or equal to z 7 minus c 7 which is equal to what, this is c b b inverse a 7 minus 60; which is equal to c b b inverse is 3, 8 and a 7 is 2 and 4; so minus 60 which is equal to what, so this is 38 minus 60 which is equal to minus 22; 8 plus 2 is 10 and then 5 so minus 22, that means as long as delta 7 is greater than or equal to minus 22, the current solution remains optimal and you see that if delta 7 is equal to minus 22, then this number will be 0, c 7 plus delta 7 minus z 7 will be equal to 0; and so you will have k s for an alternate optimal solution, because in that case you can enter x 7, you can enter a 7 into the basis and then you will get an alternate optimal solution.

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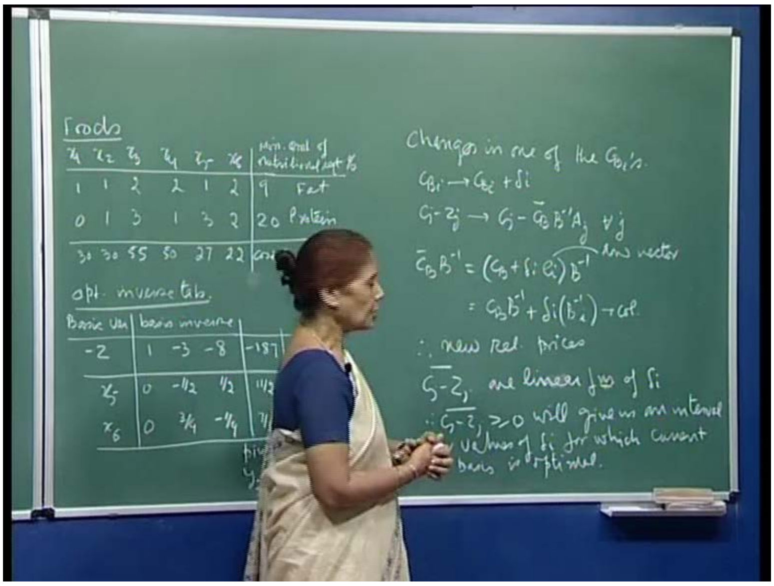

So, at delta 7 equal to minus 22 you have a alternate optimal solution, of a delta 7 less than minus 22, you would want to consider entering a 7 into the basis, so let us take the case, so suppose delta 7 is minus 23 that is it is less than minus 22, this implies that your c 7 plus delta 7 minus z 7 is equal to minus 1, see this was 38 and from here this computation we can immediately see that if my delta 7 is minus 23, then this number the c j plus z j minus z j it should be equal to minus 1.

So, I want to enter this and so obviously you need y 7, so y 7 is your b inverse a 7 and I made the computations here, so this is your c 7 minus z 7 and this is your y 7; so in the revise simplex, because we are not carrying the whole tableau, so we only make computations for the column, which is a candidate for entering the basis.

So, here you have y 7 just multiply b inverse with your 2 and 4 and you will get 1 and half, so then we take the ratios, because here this is 11 by 2 and this is 7 by 2; so this will be your pivot element. So, you pivot on this and then you will get the new solution and, so the optimality criteria will continue to be satisfied no problem, because this b inverse it is possible that you may not immediately satisfy the optimality criteria, but in any case you can now continue with the simplex algorithm, because you have the optimality condition is not satisfied. So, after you finish with this simplex algorithm, you will have a new optimal solution when your new primary food that you have added has entered the basics, so continue with this simplex algorithm.

So, now we want to look at the second kind of changes, this is changes in the cost coefficients of basic variables. Now, here the computations will have to be done more extensive way, because suppose the ith basic variable has change to this plus delta i.

So, then c j minus delta j would be c j minus c b bar b inverse a j for all j; and you see therefore for all j the things will change, because 1 of the basic variable cost coefficients has change. Now, c b bar b inverse I will write as, you see here; because c b, I am treating this as a row vector, remember I said that we will not bother to write transpose all the time; so this is a row vector.

So, now here the ith component of this vector has change, so this e i is also a row vector, so delta i e I, b inverse this is what your new c b bar b inverse will look like, and therefore when you open up the brackets c b b inverse plus delta i, and then this will be the, because you are multiplying this row unit row by the b inverse matrix. So, this will give you the ith column.

This will give you the ith column of the basis inverse matrix and this is your new this thing, and therefore there were new relative prices you see that, so here now this is the when you c b b inverse becomes a linear function of delta i, you substitute here then all the c j minus z j's would be the new c j minus z j was which I am denoting y bar will be linear functions of delta i.

So, that is clear and therefore, when I put the condition optimality condition for the current basis to be optimal, then all these should be non-negative and this will give me an interval for the values of delta i, because some components here may be negative, some components may be positive.

So, when I put the negative sign, so delta i will come out, there will be interval of values, so as long as delta i is in this interval, the current basis will be optimal, so once I get this then the moment delta i is outside the interval on either side, then I know that optimality criteria is not being satisfied, and I will know which one is a profitable basic variable to come into the basis and I can continue with the simplex algorithm.

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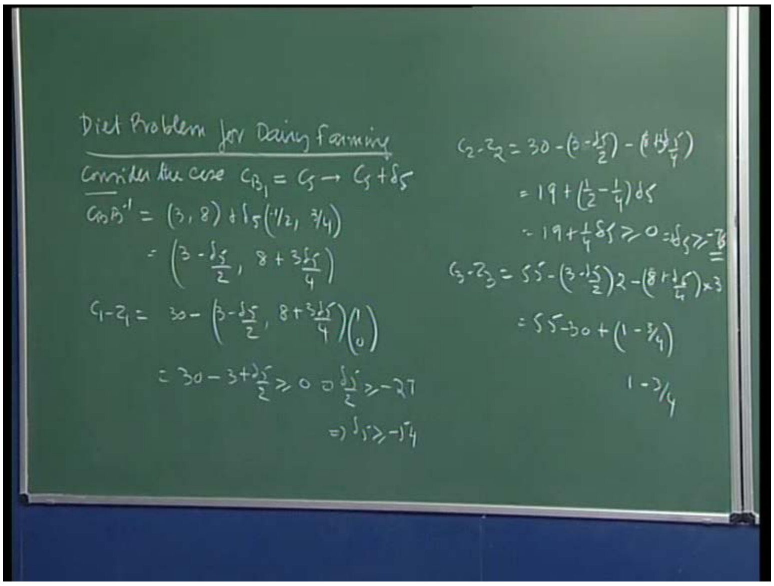

So, let us quickly do it first c 1 minus z 1, so c 1 minus z 1; so since I have c new c b b inverse c 1 is 30 minus 3 minus delta 5 by 2, 8 plus 3 delta 5 by 4 into your 1, 0. So, this is the equal to 30 minus 3 plus delta 5 by 2 should be greater than or equal to 0, which implies that your delta 5 by 2 is greater than or equal to 27; so minus 27 which implies delta 5 should be greater than or equal to minus 54; so this what you get here.

Then, if you do it for c 2 minus z 2, this will be again 30 minus 3 minus delta 5 by 2 into that is 1 and then minus 8 plus delta 5 by 4, because this is 1 one.

So, 8, 3, 11 which means that this is 19 plus delta plus 1 by 2 minus 1 by 4 delta 5, which is 1 by 4; so this is nineteen plus 1 by 4 delta 5 should be greater than or equal to 0, which implies the delta 5 should be greater than or equal to minus 76.

So, no need to worry about it, because I already have something which has to be, because if delta 5 is greater than this delta 5 is greater than minus 76, then c 3 minus z 3 would be 55 minus 3 minus delta 5 by 2 into 2 minus 8 plus delta 5 by 4 delta 5 by 4 into 3.

So, let us quickly do it that in fact I could use, so this is minus 6 and minus 24 minus 30 minus 6 minus 24, 30 and plus this would be 1,1 minus 3 by 4, you are all getting all this may be this is how it is going on this the 3 also z 3 also coming out to be the delta 5 by 4 are 3 times, now computing it for z 3 fine take a 2 and 3, so let me just make sure that the computation is ok.

So, this is hundred by 425 that is true, minus 5 by 4 that is make sure that plus this and this is minus, plus 1 minus 3 by 4. I am getting, have I made a mistake somewhere? Yes, see this is 3 delta 5 and I am doing it with, so this here it did not matter, but here it would have matter this is 3 3 delta 5.

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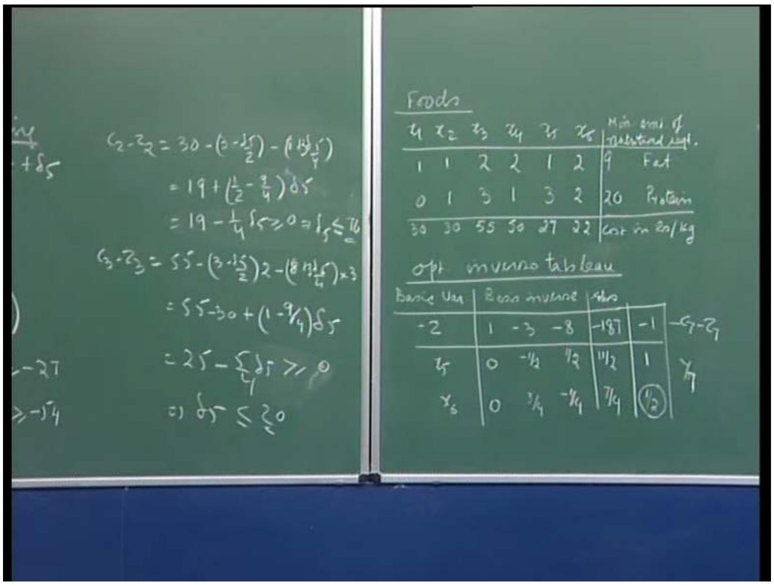

So, let us quickly make the correction, so this will be what then, so this is half and it will be minus 3 by 4; so this is equal to 19 and this will become 2; so minus 1 by 4 delta 5; so this is greater than or equal to 0 which implies the delta 5 is less than or equal

So now you get the other side which is 76, so here also this is 3 three delta 5, so quickly let us make it as 9 by 4 this is 9 by 4 delta 5; so this becomes 25 minus 5 by 4 delta 5 should be greater than or equal to 0, which implies the delta 5 is less than or equal to 20. So, here so far I have got the interval as minus 54 and 20, but I need to compute for z 4 also.

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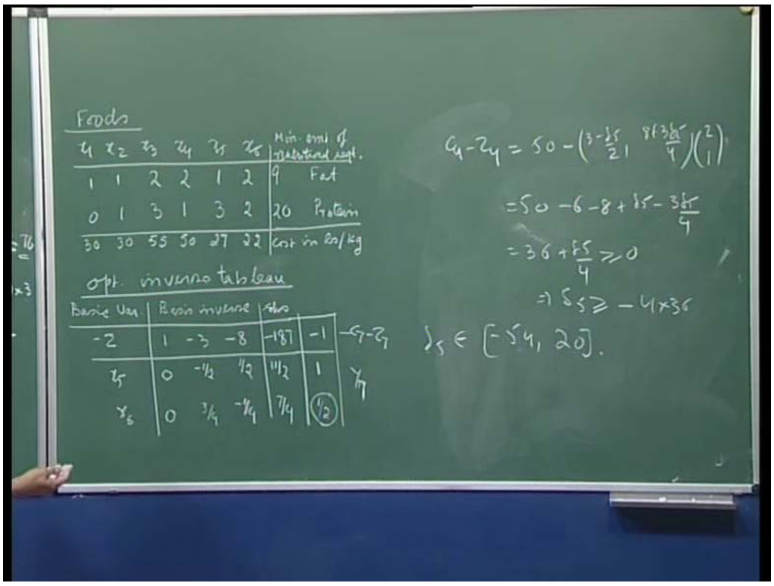

So, therefore c 4 minus z 4 will be equal to 50 minus 3 minus delta 5 by 2, 8 plus 3 delta 5 by 4 into 2, 1. See, I am doing the calculations in detail, so that no doubts left out, 50 minus this will be 6 and that'll be minus 8, then plus delta 5, because 2 into 2 and that is 1; so minus 3 delta 5 by 4, this should be fourteen; so that gives you 36 and so plus delta 5 by 4, this is minus 6 and then this is minus 8; so this is the number and plus delta 5 and plus minus 3 by 4, this should be greater than or equal to 0; which implies the delta 5 is greater than or equal to minus 4 into 36, that is a big number.

So, therefore the interval that you get for delta 5 is minus 54 and 20. So, as long as delta 5 is within this interval, so that means you can say that this does not really happen in the real life, that the price can go down by 54 that is what is happening here, but many case the numbers may be that kind, because it just an example to demonstrate how you can take care of changes in the cost or you know when new parameters come in, how will you see the changes in the optimal solution.

So, here what it this saying is the delta 5 can belong to this, and the current solution will remain optimal and you can see that when the moment delta 5 is equal to minus 54 or 20 you will have a an alternate optimal solution, because see minus 54 was happening for which one, this one; so that means in that case food 1 will may become a candidate for entering the basis and similarly, here for delta 5 equal to 20 you will have the food third food candidate for entering the basis, and then you will have alternate optimal solutions.

And beyond these numbers beyond this interval delta 5 the changes in the cost of food 5 take place beyond these interval, then of course you will not have a basic optimal solution and you will want to see the changes, because if delta 5 is around 21 then you will consider including food 3 in your diet and find out the new optimal solution and so on.

So, this is what we changes are in the cost coefficients, I have already taken care of adding an activity here. Now, let us see the changes in the right hand side that means suppose, there is a requirement that they fat content should go up or fat content should go down or the protein content should go up.

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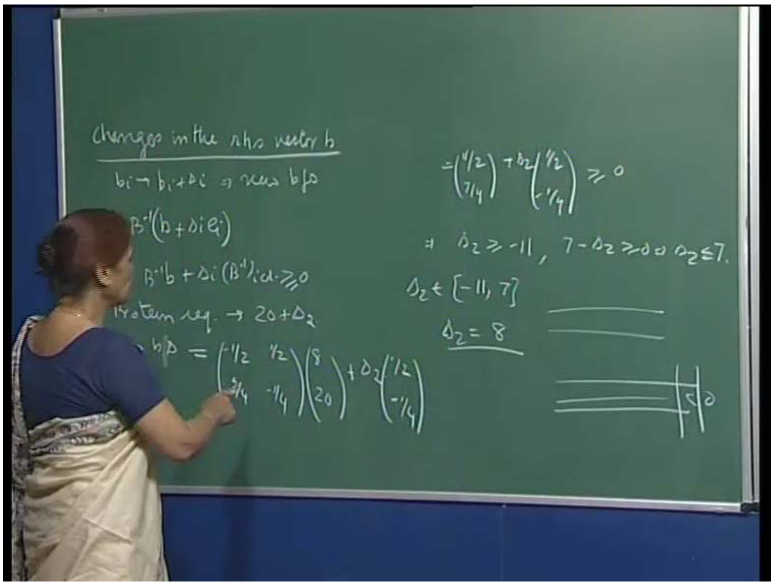

So, we will look at that kind of changes write away, changes in the right hand side vector b and suppose, what I want to say here is that if b i goes to b i plus delta i, then feasibility would be in question, because the optimality criteria will continue to be satisfied, since no changes made in c b or b inverse and so the b i goes to b i plus delta i would imply that your new basic feasible solution is equal to b inverse b plus; so here also I can write this as this, because e i e i, I am writing as a column vector, so you should always be able to see what the context it is, then accordingly read the thing, because it is too cumbersome to carry on the transpose all the time.

See, b inverse b plus delta i e i which becomes your b inverse b plus delta i and then this is the ith column, so this is the ith column and therefore again you would want this to be greater than or equal to 0. If, you want your current solution to remain optimal, then you want to know what are the kind of changes in the ith right hand side number which can be observed by the current solution, that means it continues to remain optimal, and so here again you see you will have any you get it interval for the values of delta i and within that interval if your delta i is stage, then your current bases will remain optimal otherwise you will have to then in this case, because feasibility would be lost; because the current basis will not be any longer feasible, so we will have to do the dual simplex version of the algorithm.

So, now here in our case, suppose the protein requirement has been change to 20, so currently the protein requirement was 20. So, suppose 20 plus delta 2 protein requirement was 20. So, suppose we change it to 20 plus delta 2. In that case your new basic solution would be b inverse, so I will just take it from here, you already have it in your earlier slides, so this would be minus 1 by 2, 1 by 2, 3 by 4 and minus 1 by 4 this times this stage as 8 and this is 20 plus delta 2 times.

So, this I already know what the numbers here are so anyway delta 2 and remember it is the second column, so second column will be half and minus 1 by 4. So, in other words your new basic solution would be 11 by 2, 7 by 4 minus plus delta 2, 1 by 2 and minus 1 by 4, so let us quickly do it, and this should be greater than or equal to 0.

So, this implies from the first 1 that your delta 2 should be greater than or equal to minus 11 from here, 11 plus delta 2 is greater than or equal to 0; so this and from here you get that 7 minus delta 2 should be greater than or equal to 0 which implies the delta 2 should be less than or equal to 7; so that means as long as your changes in the in the protein requirement are within this interval, the current basis will remain optimal that means if you cut down the protein requirement or you increase the protein requirement up to 27 grams per diet then the current solution will remain optimal.

But, if you want to make delta 2 as 8, that means your protein requirement has gone up to 28 grams for the diet, then you see the current basis will become invisible, because here when you put delta 2 equal to 8; so then this would be 7 minus 8 is not greater than or equal to 0. So, the current basis is no longer feasible, and that means will have do the revise simplex version here, because the second basic variable, the currents basic variable has become negative, so I need to remove it from the basis.

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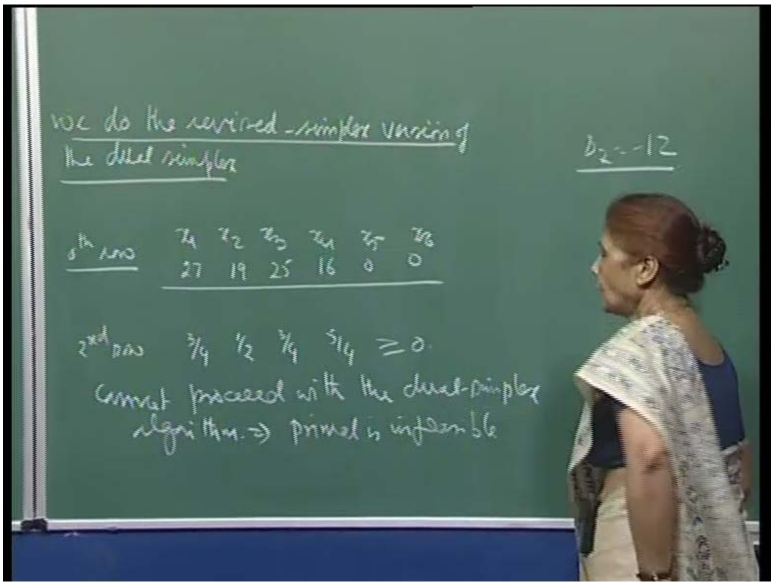

So, I thought again through this example, I will try to do the revise simplex virgin of the dual simplex algorithm, so you get a chance to look at. Therefore, now what you need to do, because remember when in the dual simplex algorithm, if you have a negative entry here less than 0, then you need the entries here and you need the entries in the top row, because you have to take the ratios of the negative entries here with this and then decide the incoming variable.

So, now for the revise simplex algorithm again you do not have the whole tableau, so we have to make this computation; so I quickly did the zeroeth row that means I already have c b b inverse which has 3 comma 8, so I compute the c j minus z j for all the non-basic variables and then I need compute the second row because my second basic variable has become now negative. So, this is my second row here, but what do you see here see everything is greater than or equal to 0.

So, therefore cannot proceed with the dual simplex algorithm, and what does it imply, this implies that primal is infeasible, because the dual is unbounded. If you have this situation what the dual simplex algorithm, that means the outgoing variable all the entries are greater than or equal to 0, then this implies that the dual is unbounded and therefore primal is infeasible.

So, that means I cannot find my problem becomes infeasible if my protein requirement is 28 unit, 28 grams for the diet, because then with the given data I cannot find a feasible solution, so the problem is infeasible. So, therefore that means I cannot go beyond this.

So, now I would like you to work it out here, that if in case delta 2 is minus 12, so again you will have a case for infeasibility for using the dual simplex algorithm, because now the first food; see here if you put delta 2 as minus 12, then this is 11 minus 12 which is minus half. So, which will be negative and therefore you have to use the dual simplex algorithm and see that if you can get lowering your protein requirement, can you get an optimal solution, a new optimal solution; because the current basis is no longer optimal.

So, you see that how interestingly we can, because this is the versatility of the simplex algorithm, that you are able to do all kinds of post-optimality analysis, see the changes in the parameters, and see the effect on the current solution or you know without doing too many new calculations, you can use the current calculations; you get a new basic feasible solution, so we will continue with some more post-optimality analysis in the next few lectures.