Linear Programming and its Extensions
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Module No. # 01
Lecture No. # 15
Complementary Slackness Conditions Dual Simplex Algorithm Assignment 3

So, I will continue my discussion with the duality theory and the complementary slackness conditions. So, last time I had showed you that at optimality, this would be optimal value of the primal objective function, this is equal to C B B inverse b. And so this is nothing but your, because at optimality the primal objective function value and the dual objective function value are the same.

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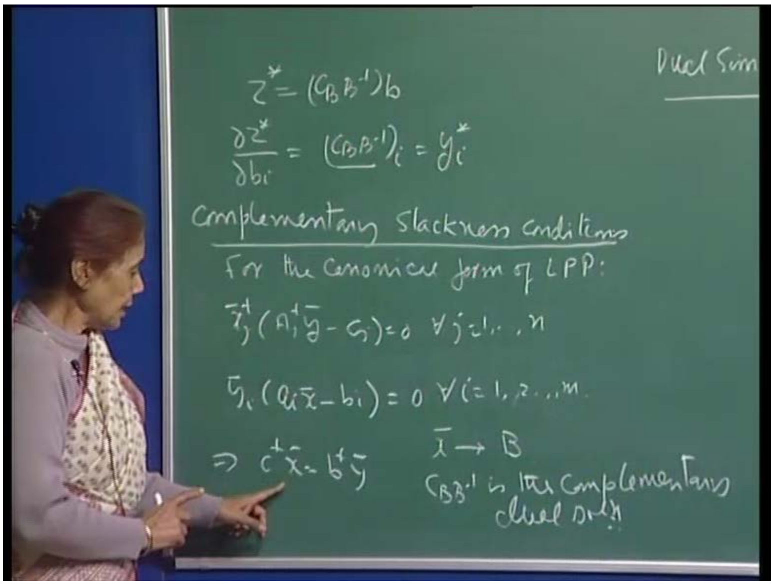

So, therefore, we can also look upon the dual variables, so when you take the partial derivative with respect to the b i, the i th component, this will give you the i th component of this which is your y i star. And so the dual variables can also be looked upon as a rate of change of the primal objective function with respect to the right hand side values given to you. And then, we will continue as I said you can as you go long, you will keep coming across many different interpretations of the dual variables.

Now again, I want to write down the complementary slackness conditions; because one can go on and interpreting them in many different ways complementary slackness conditions and I am stating them for the canonical form of LPP, yes, and this we said that this is x j transpose A j transpose, so this is y bar minus C j is 0 for all j varying from 1 to n, n for the dual variables, this will be y i bar a i x bar minus b i is equal to 0, for all i varying from 1, 2 to m. And we said that, if a particular primal constraint, dual constraint is satisfied as equality then, the corresponding primal variable can take any value, in fact it can take positive value, but the other way, if this has a positive value then this must be satisfied is equality, and if this is satisfied as inequality then the corresponding primal variable must take a 0 value, because the product has to be 0.

So, one can also interpret in terms of slack, see for example here, if you look at the nutrition problem, the right hand side specified minimum amount of i th nutrition that the adequate diet must contained. So, which means that here, if this is strict inequality that means, the diet that you have chosen has this particular nutrient more than the required minimum. So, this would be satisfied as strict inequality, and then for the y i bar will have to be 0; that means, the associated dual price that you attach to that nutrient would be 0 or if you want to look at it in terms of this or say you can interpret the value of y i bar.

So, this slack that means if there is slack here, then the corresponding dual variable must be 0, if there is a slack here then the corresponding primal value must be 0. And so you can go on interpreting this way, now another way to see, once you we when we prove this complementary slackness condition theorem, I showed you that for x bar y bar to be optimal for the respective problems, the complementary slackness conditions must be satisfied, and if the complementary slackness conditions are satisfied, then the pi by feasible pair x bar y bar, then the two solutions must be optimal for the respective.

So, it is if and only if condition, the theorem that we prove, so that means, if x bar and y bar satisfy the complementary slackness conditions, then this implies that C transpose x bar, I prove to you while proof this is this, the two objective function values are the same. Now, the idea behind developing alternate algorithm variant of the simplex algorithm is the dual simplex algorithm, so we will develop dual simplex algorithm.

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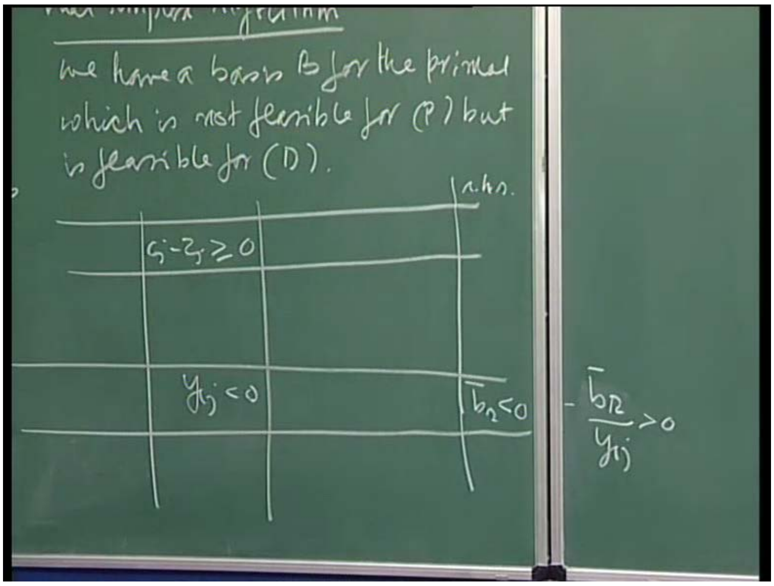

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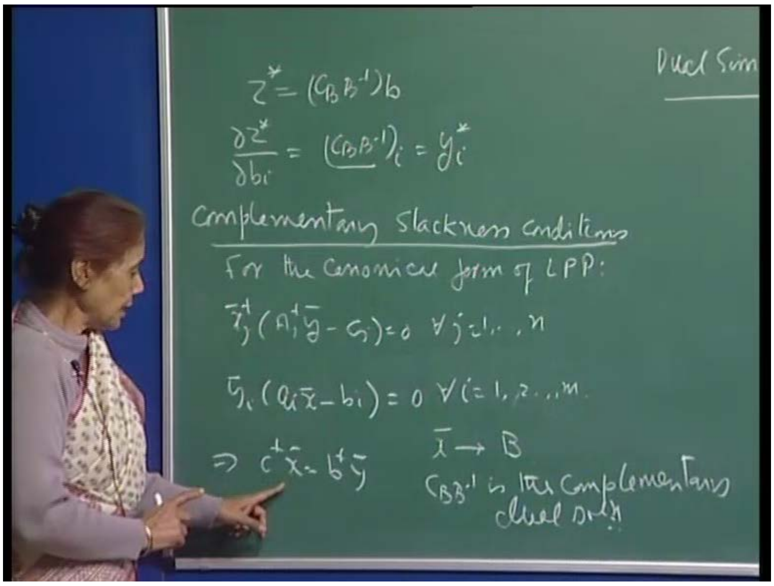

The idea is, suppose, for some reason it is possible for me to obtain a dual feasible solution very quickly, because remember obtaining a primal feasible solution you may have to go resort to a phase 1, phase 2 algorithm to come up to because, you do not know beforehand or right away by looking at the problem whether if the problem is feasible and how to get that starting feasible solution.

So, sometimes it may be possible to obtain y bar without much effort in that case, the idea would be that, and so when you can obtain a basis, so your y bar y bar is obtainable as C B B inverse, and by the way let me also point out here, that suppose x bar corresponds to the basis B; that means, x bar is a basic feasible solution corresponding to B. Then, we say that C B B inverse is the complementary - yes, this is important terminology - complementary dual solution.

So, given a basis for the primal problem, the C B B inverse will give me the complemented dual solution, and when you take y bar to be this, then you see the two objective function values will be satisfied, because forum for the basis B the value of x bar is d inverse b and so the corresponding coefficients when you multiply with, you get the value of c transpose x bar and if you take your complementary dual solution as y bar equal to C B B inverse, then you see b transpose y bar will give you the dual.

So, when you have complementary pair, there is another word, one can use when your complementary pair of primal and dual solutions; the objective function values are the same. So, we saw that, if there is a basis which gives you a feasible solution such that that basis also gives you a feasible solution for the dual, then the solution is optimal, because dual constraints are nothing but optimality criteria. So, if your y bar equal to C B B inverse is dual feasible, when the current basis will give you a primal optimal solution, and therefore, it will also give you a dual optimal solution, so this is a relationship between the two.

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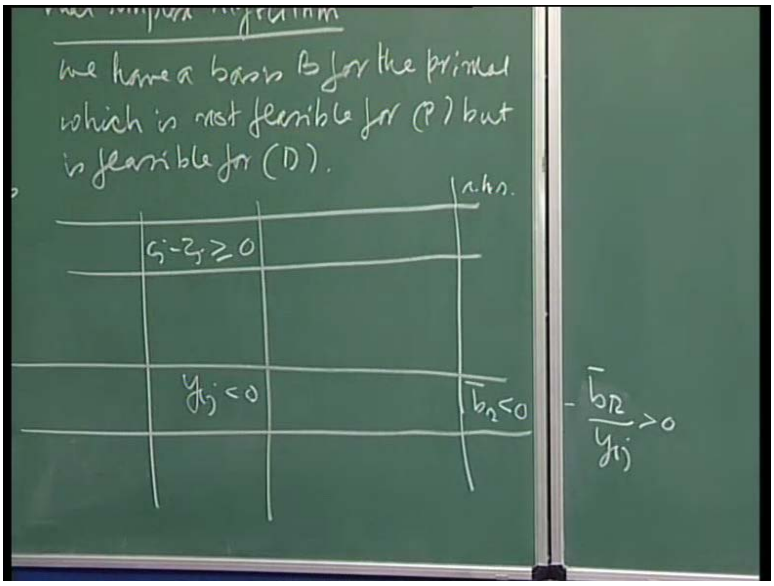

So, then the idea here is that if I have a basis B which is dual feasible; that means, the y bar satisfies the dual constraints or in other words, I have a basis for the primal problem, which satisfies the optimality conditions, but it is not feasible; that means, this solution here and what is it mean? So, suppose, we have a basis B for the primal which is not feasible for P, but is feasible for D - for the dual problem. It satisfies the duality constraints, then can be obtained a primal solution, this is the question. So, starting with a y bar which is equal to C B B inverse, can I work for primal feasibility, so this is the idea behind dual simplex algorithm.

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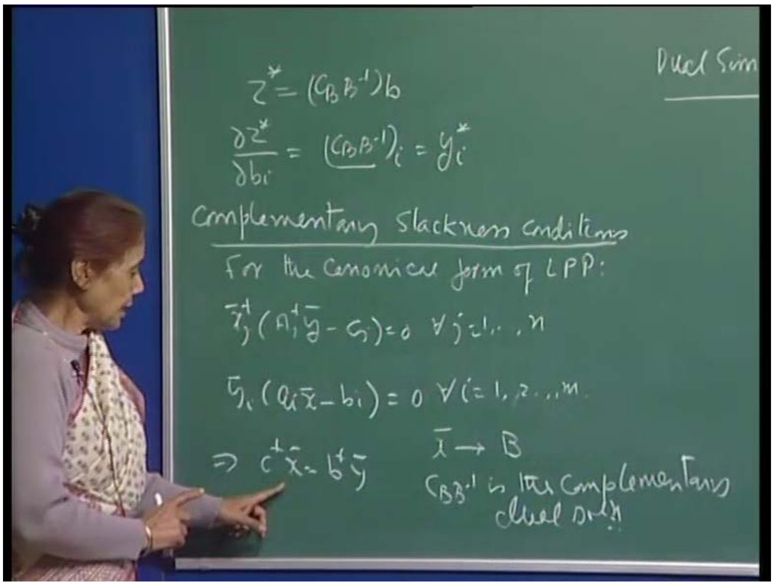

If you have a basis which is dual feasible, then we maintained dual feasibility and we work for primal feasibility. So, that means, the moment I have obtained a primal feasible solution I stop because then, because of the complementary slackness conditions I then have a dual feasible solution, and I have primal solution such that, the two objective function values are the same, and so they must be optimal by my complementary slackness conditions.

Let me repeat, so what we are saying is that if I maintained, in other words in the dual simplex algorithm, I will start with the basis which is dual feasible and if it is dual feasible and then, I work for primal feasibility, I all the time maintain this and therefore, at the end, when I end up with the primal feasible solution I have an optimal solution, so this is the idea.

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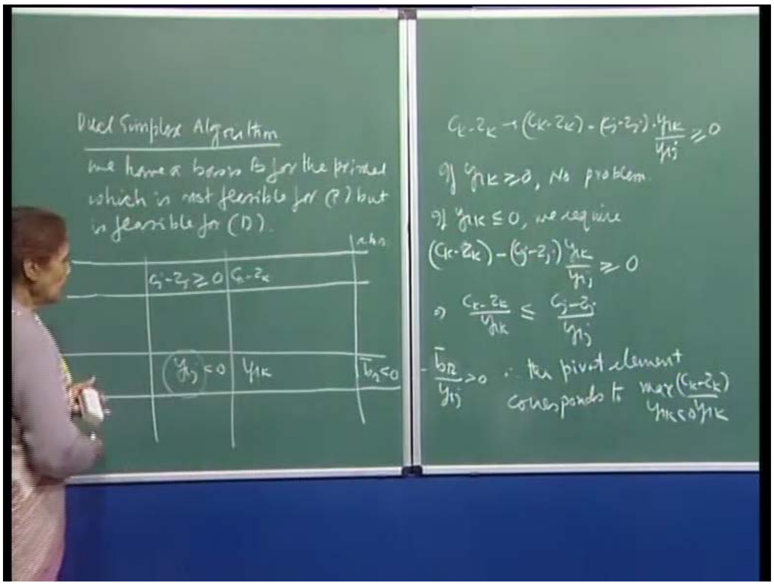

So, let me explain to you (( )) to a tabular and then we can work out in example, so remove this here, yeah, so the idea here is, see suppose, you have your C j minus Z j which are all greater than are equal to 0. This is your right hand side, and then I have the column y j, so let say, you have y r j and so (( )) we will need the row here, so that you look at this, and here we have b r bar which is less than 0. So, that means for the given basis for this basis B, you have a primal feasible solution, because this number is less than 0, if this was non-negative I would have a feasible solution. So, once you have this and C j minus Z j is all non-negative, so for the basic variables there will be 0, for others there will be non-negative.

So, optimality criteria is satisfied, how do I proceed so first of all my idea would be to make this non-negative, and so therefore, I need my pivot element to be less than 0. So, I will divide by y r j throughout, which will make this number b r bar - let me write it nicely - b r bar divided by y r j, so this becomes now positive b r bar is, because if it was 0 I would not bother, so only when it is less than 0, I bother about it, so b r bar upon j r j becomes positive I divide throughout by this. Then, so that means, I decide the exiting variable first, the exiting variable correspond to a negative value of b r bar, fine, and then among the corresponding row I choose the pivot element will have to be a negative element in this row. Now, I decide about the incoming variable or the column which will enter the basis.

So, since we want to maintain dual feasibility what is the idea? See, how will this change, because if I make this pivot element, then I have to make a 0 here, which means that, this whole row gets divided by y r j and then, you multiplied by C j minus Z j and subtract. So, a typical this thing how will it look like, this will be C k minus Z k - some other entry if you take C k minus Z k - then the corresponding element here is y r k, you have divided by y r j and then you multiply by C j minus Z j.

So, this will go to C k minus Z k minus C j minus Z j into y r k upon y r j, and we want this to be non-negative. Now, I know that y r j is negative, so minus of that becomes positive C j minus Z j is non-negative. So, if y r k is greater than or equal to 0 no problem, the sign will continue to be non-negative, so I do not have to worry, it is only the negative y r k, but if y r k is less than or equal to 0, this sign can change here.

If then we require that C k minus Z k minus C j minus Z j into y r k upon y r j should be greater than or equal to 0 which implies that see if I will divide by y r k, which is a negative number then the inequality sign will change, which requires that C k minus Z k upon y r k should be less than or equal to C j minus Z j upon y r j. And therefore, when you choose the pivot element to be y r j, this ratio must be the largest among all ratios for which the corresponding entry here in this r th row is negative. So, in other words, the way you choose your, therefore, the pivot element corresponds to corresponds to max C k minus Z k upon y r k - y r k less than 0.

So, the maximum among all these; that means, first I decide on the existing variable which corresponds to a negative b r bar, and then in that row I look for all negative entries take the corresponding ratios and choose that as a pivot element which corresponds to the maximum ratio. And so, this will give me this and once I do this, then I make a 1 here and 0 is elsewhere as we do, and we do this for the whole tabular then what you get here? Will be new dual solution which gives you, so new top row you can say that dual solution will corresponds here, so the top row will satisfy the optimality criteria and we will look at this right hand side vector again, if there is any negative entry we will continue with the simplex algorithm.

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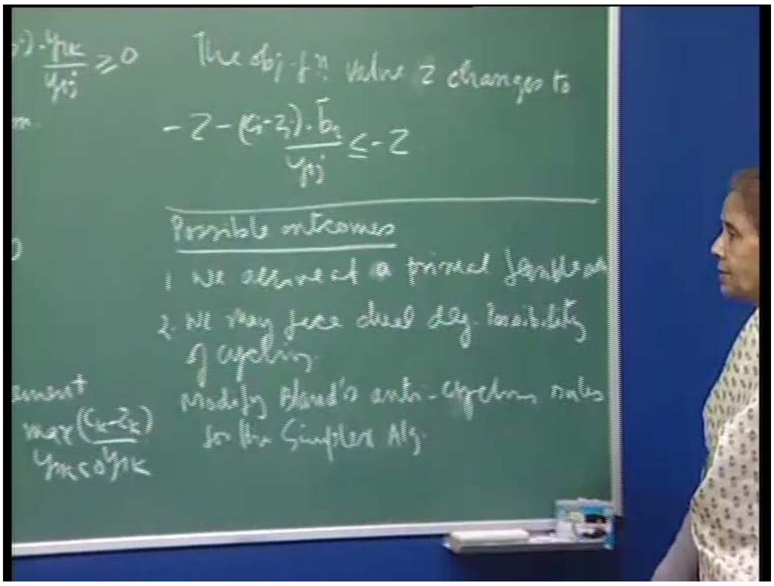

Now, another thing you can also see, how this number will change, so the current value if suppose, it is Z, the objective function value Z changes to S, so changes to Z and then what happens? Here, you have divided by y r j and then multiplied it by C k minus Z k to subtract. So, this becomes Z minus C j minus Z j into b r bar upon y r j and here, you see this is non-negative, this is negative, so this becomes positive and this is negative, so this is less than your Z.

Yes, so here what is possible is, your C j minus Z j may be 0, it is possible, so we will see less than or equal to z. So, the values definitely becomes lower; and remember, if you want me to write may be we will write minus Z because that is what our convention is, the value here is always minus of the objective function value. So, anyway this is becoming smaller then this, and when you take the minus of this whole thing, because that is the actually value it is actually going up, when you multiply by minus sign and this value will be bigger than Z - minus of this will be bigger than Z.

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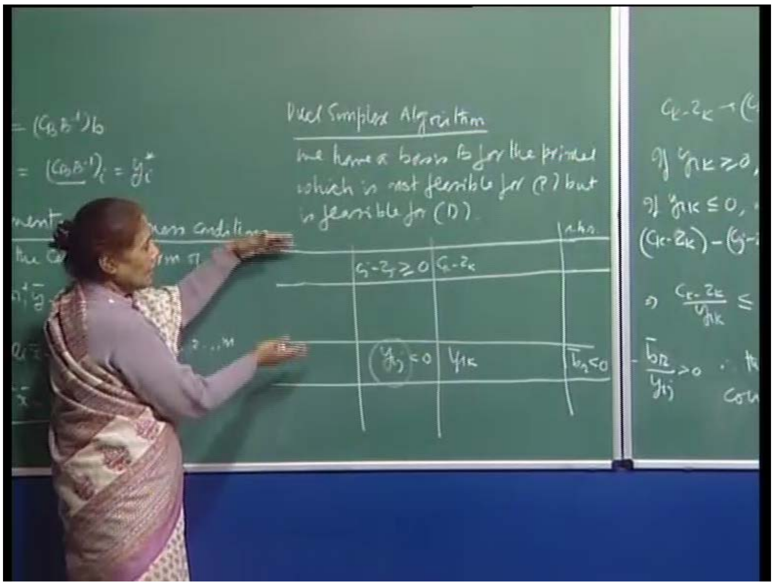

So, why is that happening remember, because we are working with infeasible solution for the primal, and now I want to if some of you have already become familiar with the simplex algorithm, you can see that we are actually solving the dual problem, but using the tabular for the primal problem. Because, primal feasibility implies dual, primal optimality implies dual feasibility, so I maintaining dual feasibility, and the moment and here a primal infeasibility would imply that the dual solution is still not the current solution is not optimal for the dual.

So, when we work towards primal feasibility trying to increase the primal reduce the primal infeasibility, then the value of the, we are actually solving the dual problem and that is why just see the pivoting this becomes your pivoting column as actually a row which is the column for the dual problem, fine because the matrix for the dual problem is transpose of the matrix for the primal problem, so these are the things which I am just pointing out, and you can sit down and look at them carefully, therefore, you are dual problem is a maximization problem. So, since I have maintaining dual feasibility, the value of the objective function will increase.

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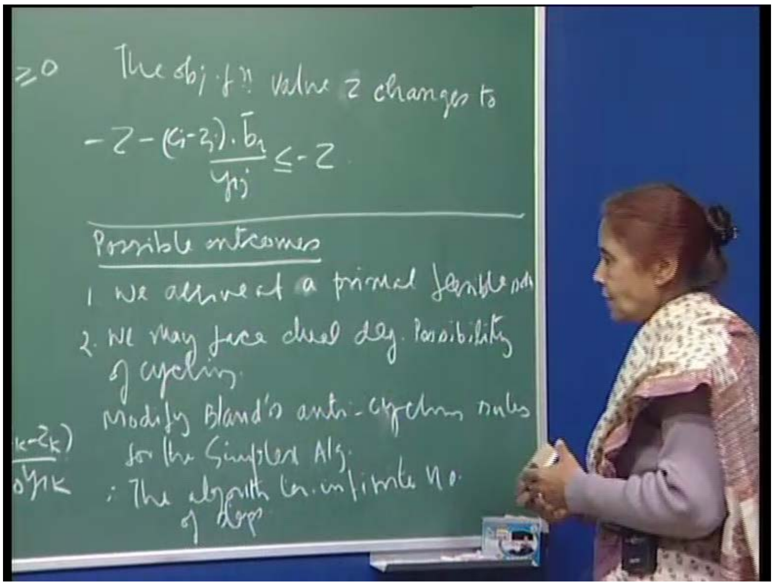

Now, quickly let us see, so what can happen is, what are the possible outcomes? One, we arrive at a primal feasible solution or if I arrive at a primal feasible solution, then I am done, I will see that I have an optimal solution for the primal as well as optimal solution for the dual. Second, and suppose, we may face dual degeneracy, that is, your C j minus Z j may be 0, but since you do not have primal feasibility, you will continue working for primal feasibility, so the entry here may be 0 for the pivoting column. So, we may face dual degeneracy and then possibility of cycling same thing, see, remember, you are working from with basic feasible solutions.

So, you go from one basis to another, but if you have degeneracy, it is possible that you may cycle. Now, here, may be right now, I can yes, so you have to revise or modify, so modify blanks anti-cycling rules for the simplex algorithm which should not be difficult. Remember, all blanks says that the first choice in each case has to be made be, in other words - so I will not write it down may be you can or maybe we need to do it fine, so I will quickly write down.

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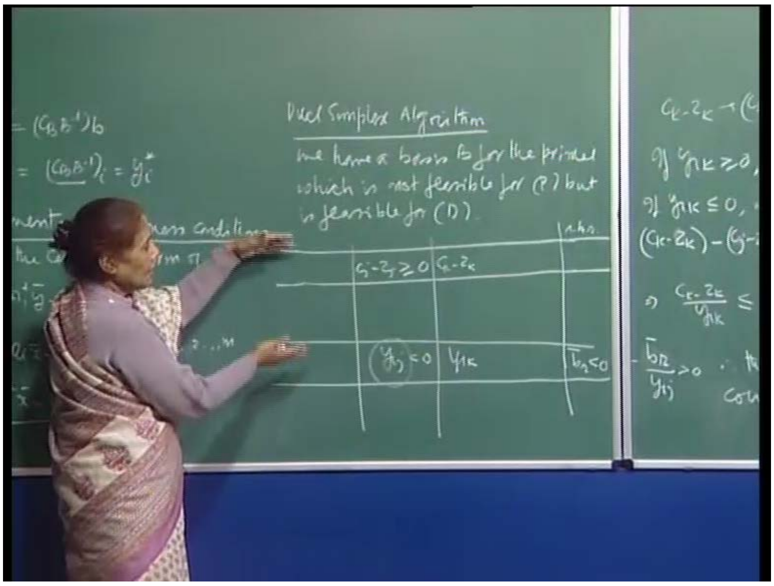

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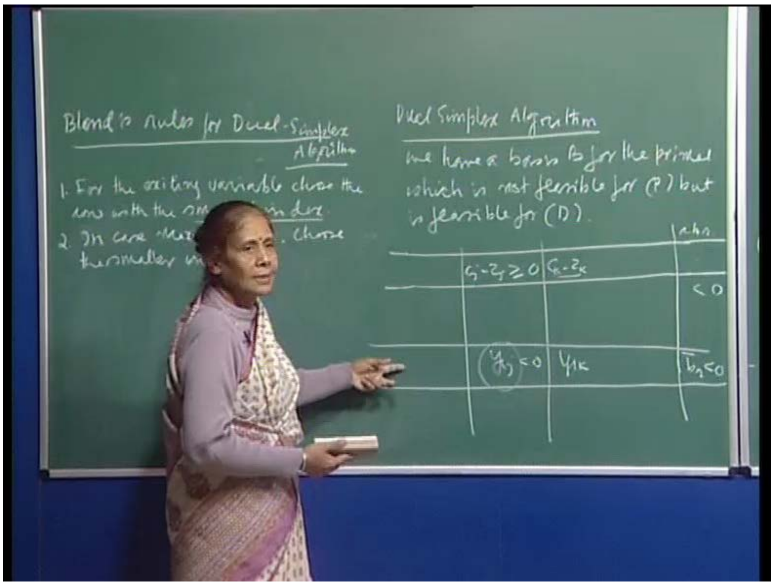

So, what will be the first, say for example if you have this less than 0 and you have another entry less than 0 here, it is possible now you have infeasibility when your primal infeasibility, you may have more than 1 right hand side number less than 0. So, what would be the first rule, so blank rules for dual simplex algorithm, so after be the first for the exiting variable choose the row with the smallest index, it has to be unique remember that is what bland said essentially.

So, unique choice is that out of these I will choose this one, because this row as a smallest index. So, you should correspond to the same thing again, maybe this is a little simplification, but what we understand by this is, that the outgoing variable has to be the smallest index basic variable this is this. And then, second one for in case of max ratio tie choose the smaller index column, so simple, should not be difficult for you to also sit down and modify it yourself.

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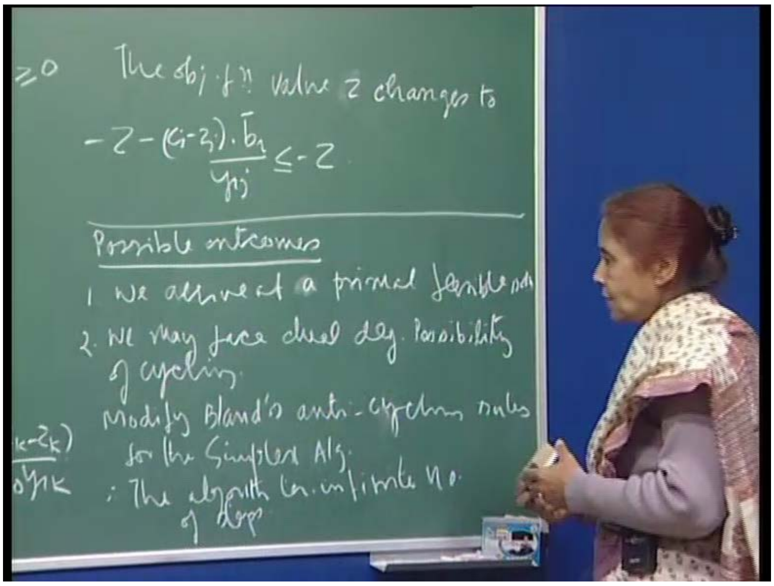

So, that means here again, if suppose I take a ratio here and I take a ratio here, both of them are equal and the other max ratios, in that case, I will choose this one to pivot on. So, these are, so once I do this, so then we may face dual degeneracy possibility of cycling modify Bland’s anti-cycling algorithm rules, and then for the simplest algorithm so then, you will prevent cycling, and hence, the algorithm, therefore, the algorithm will terminate in finite number of steps and why finite number of steps? Because, again I am working with the basis for the system a x less than or equal to b x greater than or equal to 0. So, in the number of basis is finite and since I am using Bland is anti-cycling rules, I will not use a basis more than once, and therefore, the algorithm will terminate.

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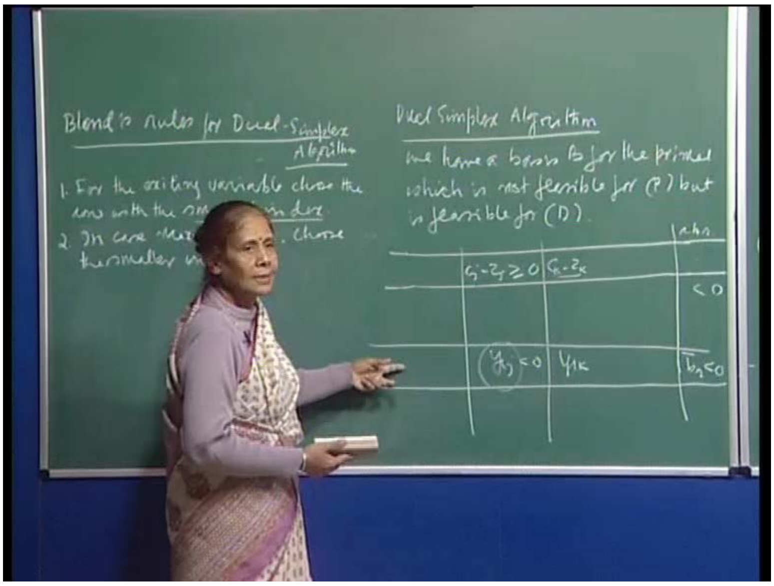

Now, what is the other possibility? Other possibility is that since, you see, remember in order to create this to remove this in feasibility, I needed negative element in this row to pivot on, so that when I divide by this the new variable that is coming in place has a positive value. So, but then it is possible that I may not have any negative entry here, all entries may be positive or 0, then I cannot proceed with my dual simplex algorithm. And what is that mean? That will mean that the dual problem is unbounded, and if the dual problem is unbounded the primal is infeasible, we have already looked at it.

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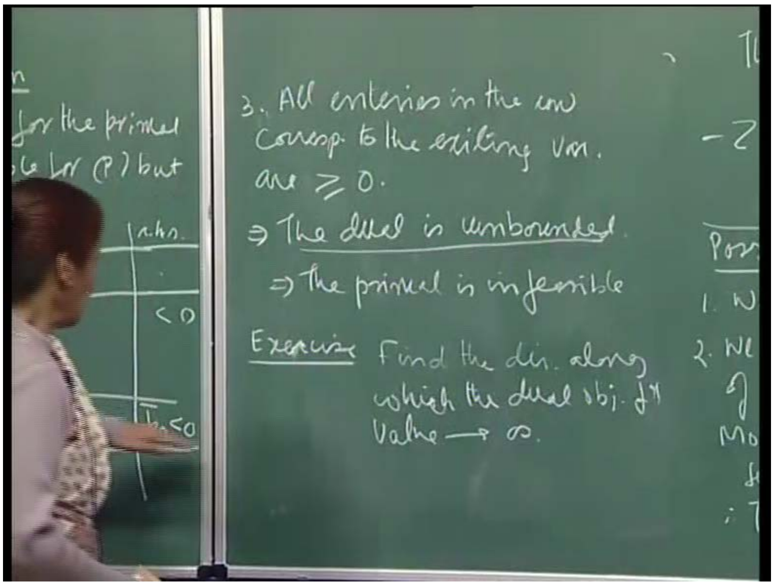

So, I said that possible outcome 1 2, and then 3 rd outcome will be all entries in the row corresponding to the exiting variable are greater than or equal to 0. This would implies as I told you can now proceed with the duals implies algorithm, this implies the dual is unbounded, which would imply the primal is infeasible. I think that this takes care of almost all the aspects, if there is something left out we will come back to it, but anyways. Now, this again I leave as an exercise, find the direction along which the dual objective function value goes to plus infinity.

Just like in the primal I showed you how exactly you write show that there is a direction, so here also, you should be able to show the direction along which the dual objective function value can be made as large we wish, and the any point on the direction remains feasible for the dual, so this is what your…

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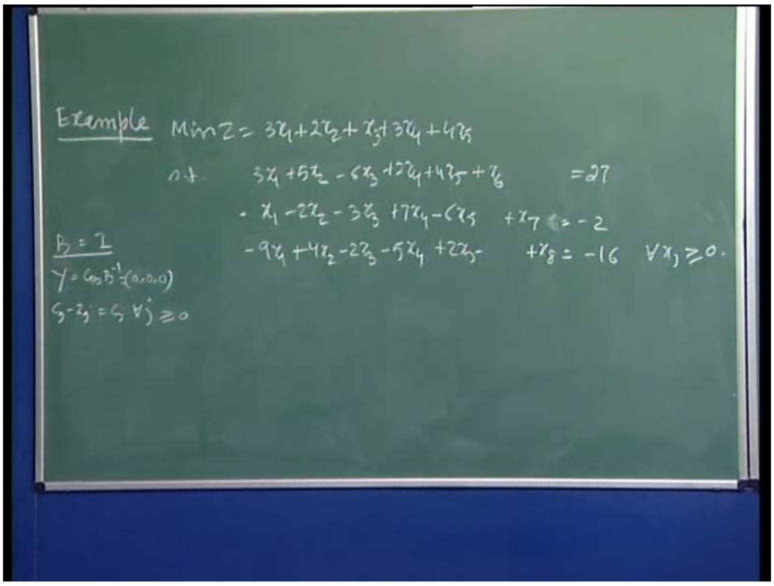

So, let us quickly take up an example, suppose you have this problem; minimize Z equal to 3 x 1 plus 2 x 2 plus x 3 plus 3 x 4 plus 4 x 5, subject to 3 x 1 plus 5 x 2 minus 6 x 3 plus 2 x 4 plus 4 x 5. Now, the in equalities less kind, so I will add a slack variable here, which will be x 6, this is equal to 27. So, x 6 is a slack variable and therefore, it does not appear in the objective function because the coefficient is 0, subject to this. See, the next inequality is greater kind, remember, I am trying to find a basis, so therefore, I will try to construct unit matrix among the constraint set.

So, I will multiply the constraint by, this is just to save time, now you can in fact rewrite minus 2 x 2 minus 3 x 3 plus 7 x 4 minus 6 x 5, so once I have made it less with the minus 2, then I will add a slack variable. I hope you understand what I am trying to say, fine, the constraint was greater kind, I multiplied it by the minus sign, so I made the constraint less kind and then, I add a slack variable, because I am looking for a basis - the starting basis - which will be convenient, I can immediately compute my dual solution. And the third constraint is also greater kind, so I will multiply by minus sign minus 9 x 1 plus 4 x 2 minus 2 x 3 minus 5 x 4 plus 2 x 5. And so, I will add a slack variable and this will be minus 16, yes, and all x j positive. Now, for all x j greater than 0, suppose, you have this problem, see you can see that here, because my starting basis is i – my b is i.

Therefore, your corresponding dual solution - complementary dual solution - C B B inverse, so this is C B B inverse this is of course all 0 0, because, your C B is 0, but what about your C j minus Z j, so since C B B inverse is all 0 therefore, this is all 0 0, so this is simply C j for all j, this is right yes, I am hurrying of a bit but does not matter, this is otherwise fine.

So, because your B is I, and your basis consists of slack variables whose cross coefficients are 0s, so all your C Bs are 0's, therefore, y you are dual solution is 0s and so Z j is all 0s, so C j minus Z j will be simply C js and these are in this case non-negative, you see all of them, so this is the basis which is dual feasible which satisfies the optimality conditions, but the corresponding primal basic solution is not feasible. So, this is till update the situation, where we can use the dual simplex algorithm to solve the problem. So, let us make the table and let us see how we proceed with the dual simplex, so steps are all clear and we will use in case of ties, we will use Bland's rule.

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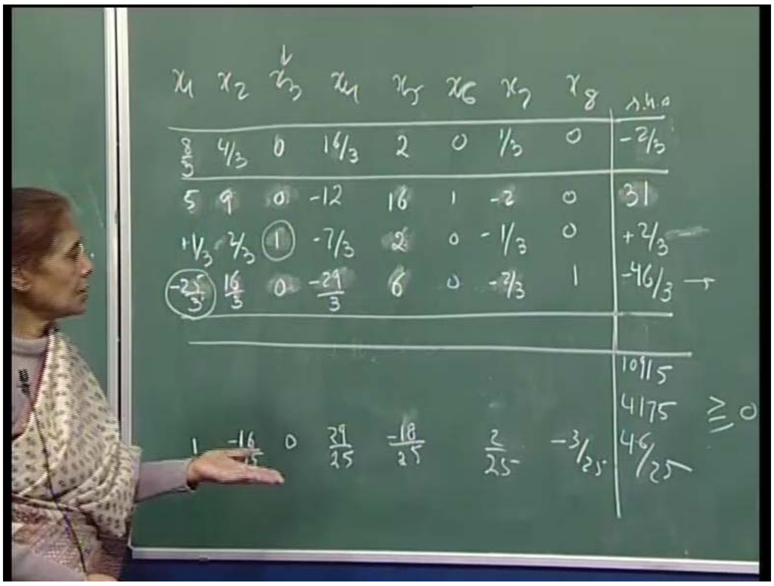

So, this would be let us write down the table this could be x 1, x 2, x 3, x 4, x 5, x 6, x 7, x 8, all so this is 3, 2, 1, 3, 4, 0, 0, 0 this is your right hand side the current value is 0. The objective function then you have 3, minus 1, minus 9, 5, minus 2, 4, minus 6, minus 3, minus 2, then 2, 7 and 5, plus 5 finally 4, minus 6 and 2, 1, 0, 0, 0, 1, 0, 0, 0 ,1 and your basic solution is, 27, minus 2, minus 16. Now, by Bland's rule as I said, this is the second basic variable, this is the third basic variable, so we will choose the exiting variable to be the one which corresponds to the smallest index. And now here you look at the negative entries and take the corresponding ratios, this gives you minus 3, minus 1, minus 1 by 3 and minus 2 by 3.

So, this is the largest, so the negative numbers the smallest one will correspond to the maximum ratio. Here, there is no tie and therefore, this is your, so you decide on you say that x 3 will become basic and x 7 will now become non-basic. So, let us quickly do the pivoting, so this becomes minus 7 by 3, 2, minus 1 by 3, and you see this will become 2 by 3 then you just simply subtract this from here.

So, when you subtract this what will happen here? This will be 2 by 3, so 3 minus 1 by 3 would be 9 8 by 3 then, 2 minus 2 by 3, 6 minus 2 4 by 3 this becomes 0. The check is that, now no entry here should become negative, if it does that means you have made a mistake somewhere, you have not chosen the max ratio. And this will be plus, so 9 plus 7 16 by 3 then this is 2 and this becomes 1 by 3 and this is minus 2 by 3.

So, from 0 the value of the actual objective function has gone from 0 to 2 by 3 as I said it should increase, yeah, so then we make 0's here, and this will that means, you have to multiply this by 2. And so let us quickly just do this, I think what I will do is, I will use the calculations done in this thing, so that we say time yes, so this becomes 5, 9, 0 as we have to do is minus 12, 16, 1 and this is minus 2 and this entry becomes 31.

Then here, it will be minus 25 by 3, 16 by 3, 0, minus 29 by 3, 6, 0 minus 2 by 3, 1 and this becomes minus 46 by 3. And so here again, now you have only one negative entry, so this will be the exiting variable take the ratios, this is 8 by 25, 16 by 29 and this is minus half, just check this comes out to be the pivot element, so I will continue with this. The numbers have become a little cumbersome that is why I have not completed that tabular here, but at any case when you pivot on this what is happening is, that I have shown the last row, the top row will remain non-negative, the numbers will become little cumbersome.

But here you see the right hand side is all greater than 0. And so, this will be the end of the dual simplex algorithm, then this basic feasible solution will be the optimal solution for the primal and whatever dual solution I have here, remember this is your C B B inverse. I have shown it to you last time, that when you have a starting initial basis as I then, whatever you have here will give you the minus of this C B B inverse, so this is minus C B B inverse for minus of that will be your dual solution.

So, what you read here would be your current dual solution or among these columns what you have here, would be your B inverse you can compute the dual solution again but many case, because you started with the slack variables, this will be minus of this the entries here would be your dual solution, this is your primal solution and you have optimality because the two objective function values will be the same. So, I will show some more the interesting situations here and then later on also show you why dual simplex algorithm is really very, very, useful because when you do post optimality analysis and so on, we use the dual simplex algorithm comes very handy we can use it.

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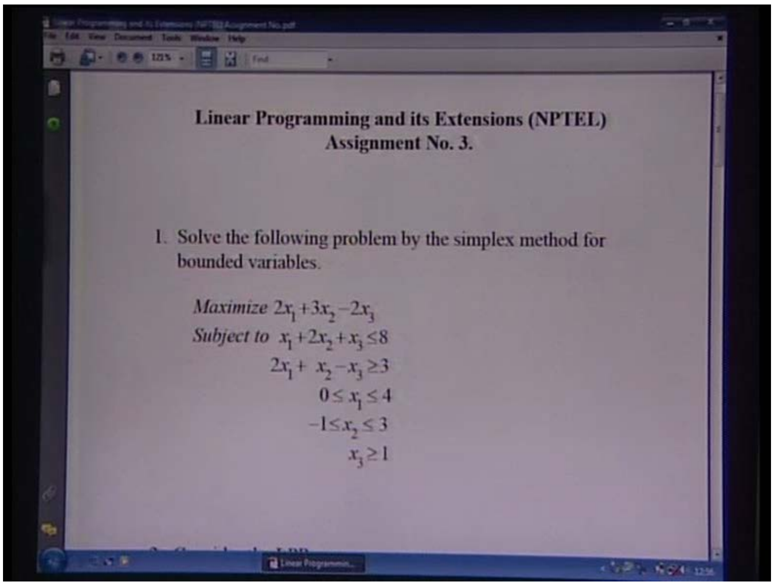

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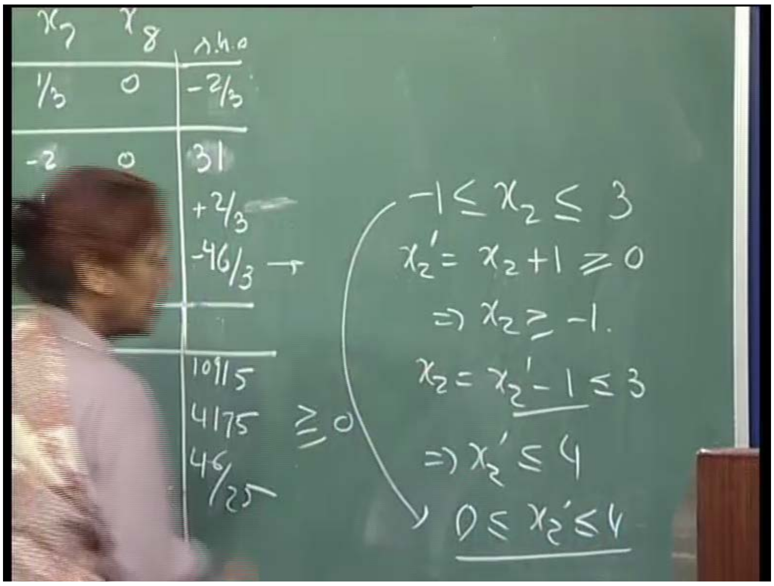

Let me now discuss the assignment sheet 3 with you, and the problems I have chosen such a way that they are little different from what we have discussed in the class, in the lectures, see for example question 1, it is an upper bounded variable problem, but we have lower bounded constraints also. For example, you have x 2, you have minus 1 less than or equal to x 2 less than or equal to 3.

So, in the lectures, I am not handled lower boundary constraint, but it is not the difficult because, since we have the algorithms built for non-negativity of the variables. So, here, I can simply say that write x 2 prime as x 2 plus 1, so that when you say that x 2 prime has to be non-negative this could imply that x 2 has to be greater than or equal to minus 1. So, this lower boundary constraint can be taken care of by making this transformation and then, insisting that x 2 prime remains non-negative which our algorithm manages without any problem, so then this will happen.

So, therefore, we will make the transformation x 2 is equal to x 2 prime minus 1, everywhere in your problem, you do this transformation and that will make, so what will be the new upper boundary constraint on when you say that x 2 has to be this is this, and this has to be less than or equal to 3, this will imply that x 2 prime has to be less than 4. So, your new constraint this one gets transform to 0 less than or equal to x 2 prime less than or equal to 4, so this how you will take care of, so you will convert the lower bound constraints to the upper bound constraints by the following transformation. Similarly, you will make a transformation called x 3 greater than or equal to 1.

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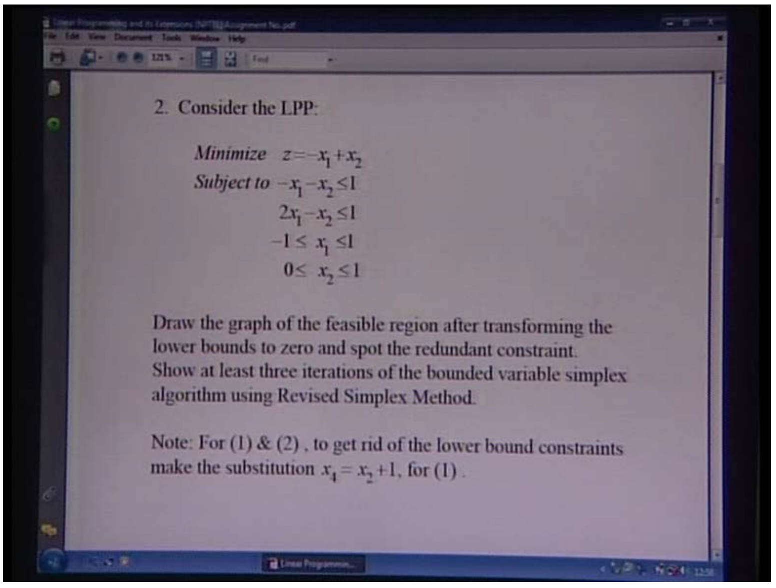

So, similarly, if we look at problem 2, problem 2 also has lower bound constraints and I am asking you to draw the graph of the feasible region after transforming the problems, it is 2 variable problem, so you can transform the problem, and then draw the graph and so your lower bounds will be 0. Now, you should be able to spot the redundant constraint here, once you draw the graph you will see that 1 constraint if you remove it does not change your feasible region, so it become redundant. And then I wanted to show three iterations of the bounded variables simplex algorithm using revised simplex. So, just try to see, if you can sit down with the revised simplex algorithm and for this bounded variable problem try to see how you will proceed. So, it will be interesting if you can, it will help you to understand, and so this note that I have just explained tells you how to transform the problem from lower bounds to 0, as the lower bounds.

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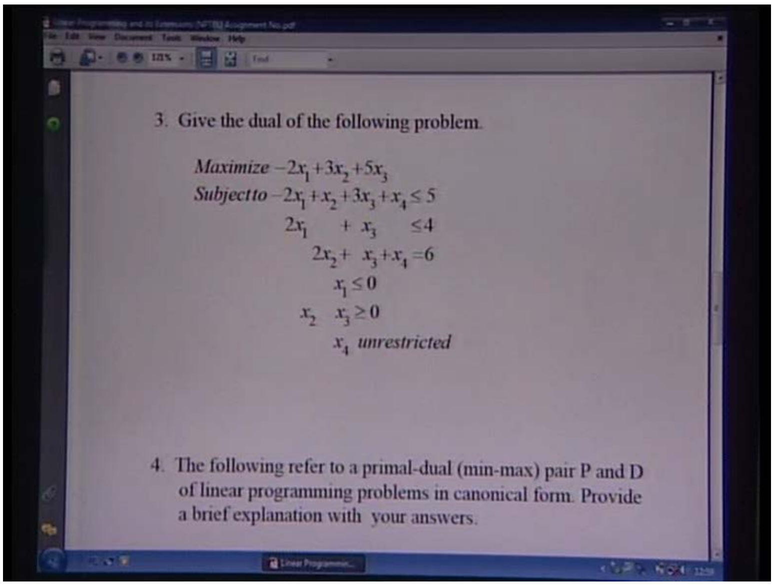

Problem 3, yeah, let us just read problem 3, so I have asking you to give the dual of the following problem, now here I have x 1 less than or equal to 0, and then x 4 I am restricted, so unrestricted you can take care of by writing x 4, as x 4 equal to x 4 plus minus x 4 minus for x 1 less than or equal to 0, you have to replace it by x 1 prime which is equal to minus x 1.

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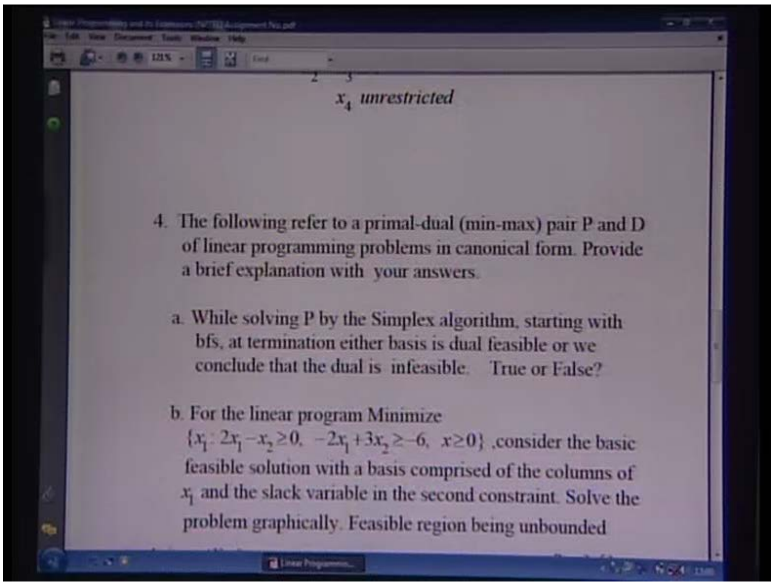

So, everywhere in the problem we will make the transformation and then, you have the in the regular form and you can now write the dual of the corresponding problem. Then, the following refer to a primal dual min max pair P and D, so my primal is P a primal is minimization problem that dual is maximization problem, and there in canonical form which I have told you, so provide a brief explanation with your answers.

So, read the statements carefully, I will just go through one or two of them with you and then you can. So, while solving P by the simplex algorithm starting with basic feasible solution at termination either basis is dual feasible or we conclude that the dual is infeasible, true or false. So, now I have change the wording a little, either you will determination either basis is dual feasible which means that it is optimal dual feasibility means, optimality of the primal, or that dual is infeasible what does infeasibility of the dual mean, yes.

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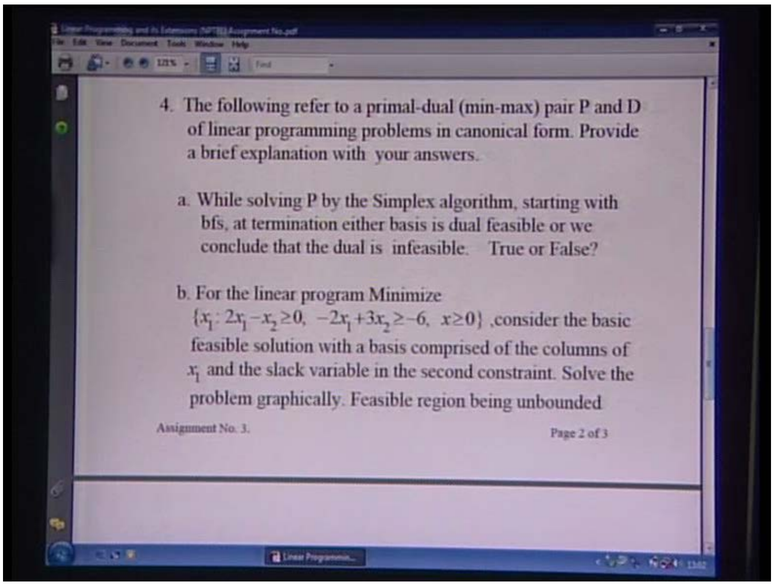

So, figure it out that is what I want you to. Then, for example look at, yeah, so I have look at problem 2 at that B part, for the linear program minimize, so I have given you the linear programming problem where the objective function is to minimize x 1, you have certain constraints x is the vector with the x 1, x 2 as the components. So, B is there now, fine consider the basic feasible solution with the basis comprised of the columns of x 1 and this slack variable in the second constraint.

So, that means, if you add these slack variables x 3 for the first constraint, x 4 will be for the second constraint. So, consider the basis, consisting of the first column corresponding to the x 1 and the fourth column corresponding to x 4. Solve the problem graphically and the feasible region being unbounded, see when you draw the graph you will see that the feasible region is unbounded, but then it turns out that the objective function value is finite.

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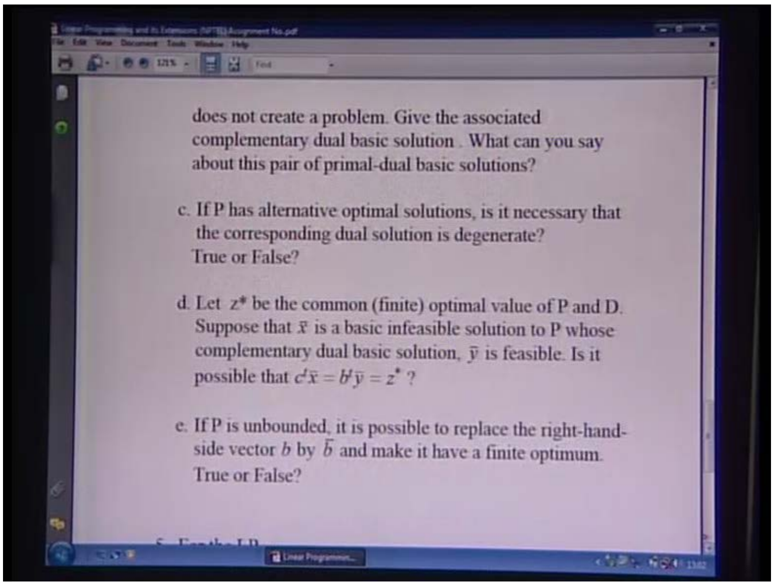

So, I just thought I will give you can, yourself see that it is not necessary I am asking you give the associated complement dual basic solution, you do all that then what can you say about this pair of primal dual basic solutions. So, what can you say about it fine if P has alternative optimal solutions is it necessary that the corresponding dual solution is degenerate. See, if P has more than 1 optimal solution, I am asking you, is it necessary that the corresponding dual solution is degenerate? If you try to answer it from the definition of the dual, or maybe constructed example where you can show either way - I mean - which I if you saying it is true then you should be able to construct an example, or if its fault then you should be able to construct an example. D part Z star be the common optimal value of P and D, suppose, we know (( )) it is finite.

Suppose, that x bar is a basic infeasible solution to P, whose complimentary dual basic solution y bar is feasible. So, we know it C B B inverse if for x bar the basis is B, so it is infeasible, but the dual is feasible. Is it possible that C transpose x bar equal to b transpose y bar is z star? Ok, so you understand that I am asking you that the x bar when, by looking at the dual simplex algorithm, you already have seen that you start with the basis, which is primal in feasible and it is dual feasible, but it is not equal to the optimal value the objective function, because you work for it, I had to do two iterations to arrive at the optimal solution for the corresponding problem.

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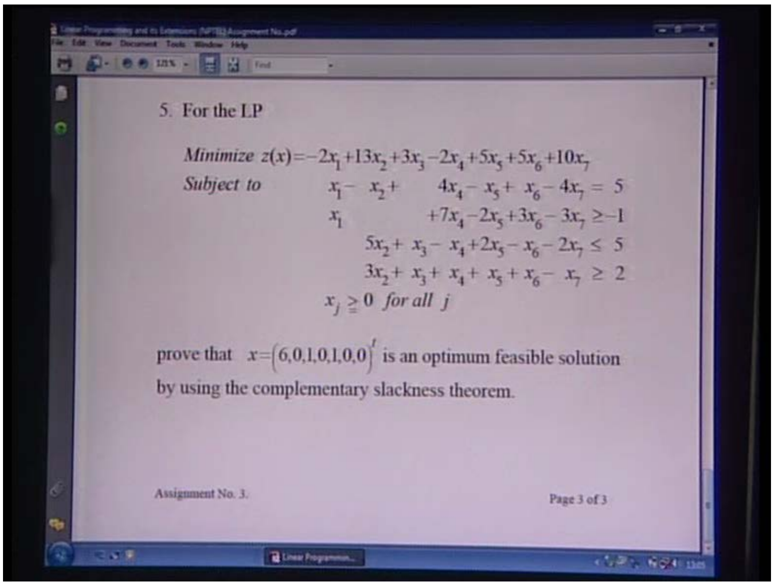

So, obviously the answer to these false, that it will not be equal. Then, the E part is, if P is unbounded it is possible to replace the right hand side vector b by bar and make it have a finite optimum. See, P is unbounded means, that the dual is infeasible; if P is unbounded that dual is infeasible. Now, what we are saying is that if you replace the vector b - right hand side vector - by the vector b bar can you make the dual feasible, this is the question and just think does b have a role to play in the dual feasibility.

If you can answer that, then you can answer this part. The final problem is, yeah, this is an exercise in writing the dual and then for verifying the complementary slackness conditions. So, here is a dual, carefully here is a linear programming problem, you have to write its dual very carefully, because I have all kinds of constraints equality less greater, so you have to be careful in writing the dual and once you do that then, I have given you prove that this is an optimum feasible solution by using the complimentary slackness theorem.

So, I have given you a primal feasible solution, and I am asking you whether you can, by using complimentary slackness conditions, confirm or verify that it is also optimal. So, by see remember, when you have this primal solution, you will now which constraints are satisfied as equality, so the once which are not satisfied as equality the corresponding dual variables will be 0s.

So, you will have a smaller; that means, you can now find corresponding dual feasible solution, if you can find a dual feasible solution given this basis, then you can show that this is also optimal for the primal, and the dual solution that you find will be optimal for the dual problem. So, I have tried to make an assortment of problem, so that while you work them out you will have a better understanding of the theory that we have discussed so far.

So, this is up to complementary slackness conditions and then, we continue with the assignment 4 will be with respect to the dual simplex algorithm and other variance of the simplex algorithms.