Linear Programming and its Extensions
Prof Prabha Sharma
Department of Mathematics and Statistics
Indian Institute of Technology, Kanpur

Module No. # 01
Lecture No. # 14
Examples of Writing the Dual Complementary Slackness Theorem

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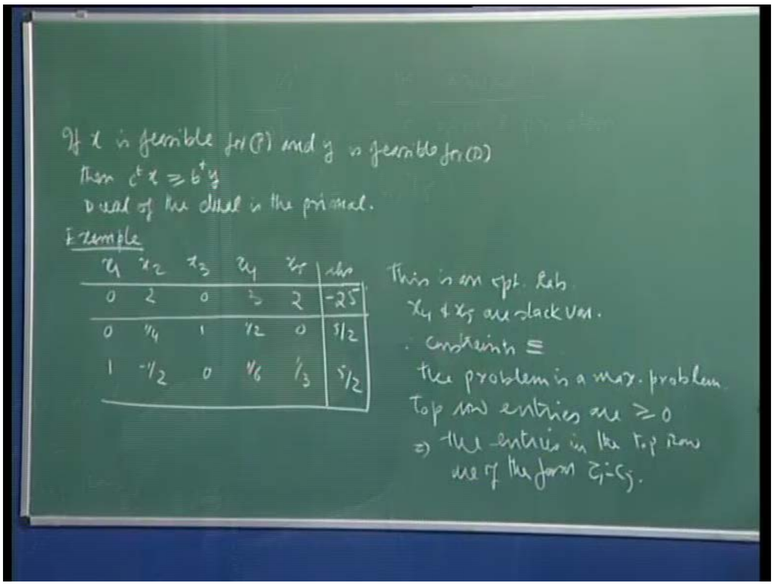

So, let me begin the recalling, the weak duality theorem. Remember the weak duality theorem said that, if x is feasible for P, and y is feasible for D, then C transpose x is greater than or equal to b transpose y, that is objective function value for the primal problem is greater than or equal to the objective function value for the dual problem. After this, I prove that dual of the dual is the primal.

So, you might say that, there is no difference; I can treat any problem as the primal problem and any other problem as a dual problem. So, then what happens to this inequality? So, that is where a word of portion is necessary, because this is the minimization objective function; this is the maximization objective function.

So, no matter what problem you call the primal or the dual, the inequality is between the minimizing objective function and the maximizing objective function. So, this is what you have to remember. So, I thought, I will just to revisit this and tell you about this, because after this result, one might say that there is no difference between, which problem you call the primal or which problem you call the dual.

So, here we are sort of calling a convention that our primal problem will always be the minimization problem and the dual problem will be the corresponding maximization problem. Now after this, because for writing the dual to a given LPP, there are many different forms in which an LPP can be written; so I thought, I will just go over the rules again and of course, it cannot ever be exhaustive. Depending on the situation, you will have to reformulate the problem, but the idea is that, you should always reduce your LPP to the standard form, and then, write the dual if you can, because it is easier to remember the rules for writing the dual for a standard form of the LPP. So, let me give you another example to show some more aspects of duality theory. Because one can just go on and on talk about a dual part of the linear programming problem and the relation between the two.

So, take up let us take this example, here I am giving you a tableau for the primal problem; let us see, x 1, x 2, x 3, x 4, x 5, and this is right hand side, and top row is 0, 2, 0, 3, 2; objective function value minus of the objective function value is 25, then the constraint part is 0, 1, 1 by 4, minus 1 by 2, 1, 0, 1 by 2, 1 by 6, 0 ,1 by 3 and the basic feasible solution is 5 by 2, 5 by 2.

And you are told that, this is this represents an optimal tableau for a linear programming problem and you have to infer lot of things. Immediately write now what can we infer and of course, so you are told that, this is an optimal tableau, then also that x4 and x5 are slack variables. So, once you know that x4 and x5 are slack variables, that means, your constraints are less than or equal to xi and if the constraints are less than or equal to xi, let us go by tradition. So, we conclude that the problem is a therefore, constraints are this xi and so the problem is a maximization problem. Now, if it is a maximization problem, top row what do you see? Top row the numbers are non-negative and if it an optimal table and that means, this has top row entries are greater than or equal to 0; this implies that the entries in the top row are of the form Z j minus C j, because if they were C j minus Z j, then this would not be optimal.

Because for a maximization problem, if the entries here for C j minus Z j, then you can improve the value of the objective function further. So, therefore, if we have assumed that the problem is a maximization problem, then the entries in the top row being non-negative implies that the entries are of the form Z j minus C j. So, there is some text books used this convention and therefore, I thought you should be familiar with this also and so this is all that we have been able to infer from the given tableau.

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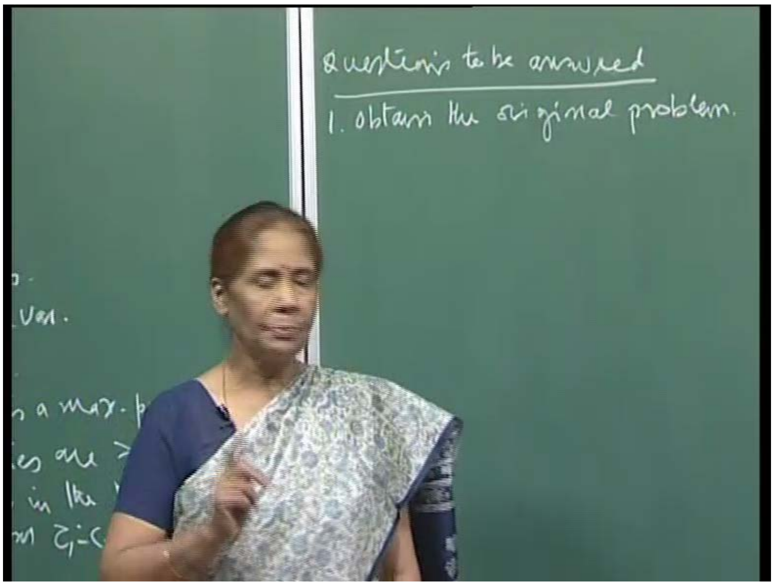

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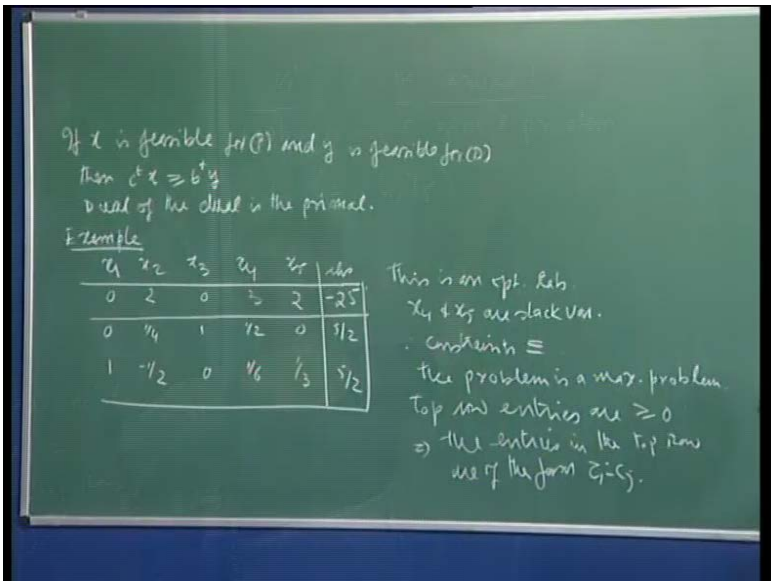

Now the question that you have to answer is, the first question is, so questions to be answered, first, obtain the original problem. So, that means, obtain the data; this is the data given to you for the final tableau and can we get from here, knowing that this is a maximization problem; x 4, x 5 are slack variables and top entries are cj Z j minus C j, can I get back the data for the original problem, that means, can I get the C j's, the matrix A and the right hand side B?

So, let us quickly do it. Now, first of all since x 4 and x 5 are slack variables, that means, when you started the simplex algorithm, these two columns were e1 and e2 and therefore, as I told you, because we use the product form the inverse, that at any iteration, the these columns will always give you the current basis inverse.

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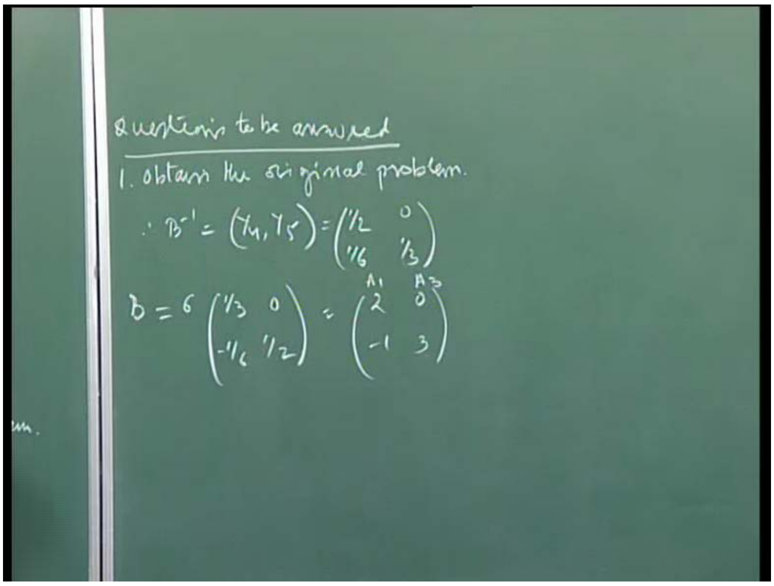

So, therefore, basis inverse is your columns right now Y 4 and Y 5, the current columns which are here, which is equal to half 1 by 6 0 1 by 3. So, this is your B inverse. So, we can quickly compute the B that means, the… So, what is your basis right now and this could be half 0, 1 by 6 1 by 3, so if the determinant is 1 by 6, so you divide that becomes 6; so 6 times and if you remember, this becomes 1 by 3; this becomes 1 by 2; this is 0; this is minus 1 by 6, which gives you 2 0, minus 1 3. And you can quickly verify that, this is the inverse, because you multiply by this you get 1; you multiply this with this, you get 0, then you multiply this with this column that this gives you 1 by 3 minus 1 by 3 0 and again, this is 1. So, this is the basis; you have the basis and now from here, because these two entries are 0s, that means, these must be the, these must be the basic variables, but again the column here is 1 0 therefore, this is the first basic variable; this is the second basic variable.

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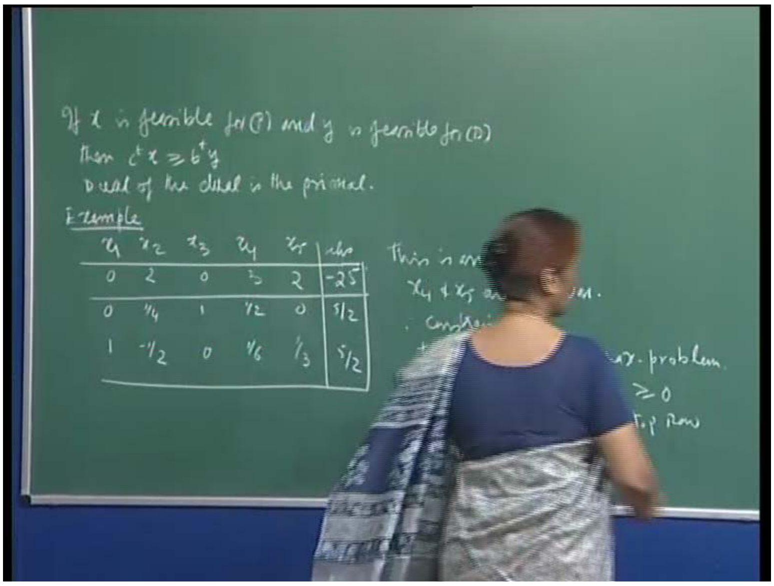

So, that means, now you have A 3; so this this gives you A 3 and this is your A 1 is it okay? Because this Z j minus C j are 0s, these must be the basic variables and since the column here, the basic column here is the Y 3 is 1 0, so this must be the first basic variable and this must be the second basic variable x1.

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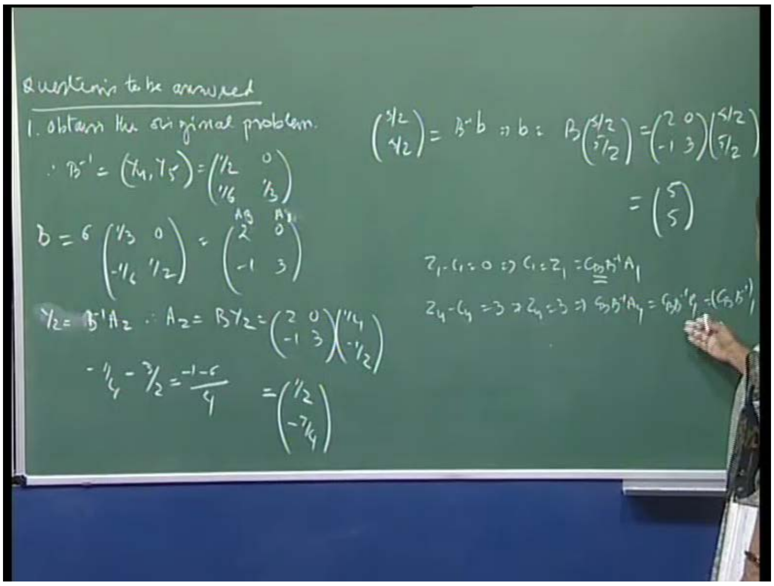

So, therefore, the columns here are A 1 and A 2; now you need to compute A 2. Because A4, A5 you know are e1 and e2, so I need to compute A 2. And A 2 is equal to… so to compute A 2, you know that, let me write it this way that your Y 2 is B inverse A 2 therefore, A 2 is BY 2. so whichever way you So, you have Y 2 and you have B; so B is, so this is 2 0, minus 1 3 and your Y 2 is 1 by 4 minus, so this is 1 by 4 minus 1 by 2.

So, what do you get from here? This is 1 by 2, and then, minus 1 by 4 minus 3 by 2, so minus 1 by 4 minus 3 by 2 is equal to 4 times this is 4 in the denominator minus 1 minus 6, so minus 7 by 4 minus 7 by 4; so this is you're a 2. Now, you have the matrix A; you have the matrix A here and we want to find out B. So, again we have that 5 by 2 5 by 2 is equal to what? This is your B inverse B, so whichever way you… therefore, this implies that your B is 5 by 2 5 by 2. So, the first question actually is familiarizing you with the, with the simplex tableau and what is where and so on, and then, we will come to the dual part.

So, this is equal to again your B is 2 0, minus 1 3 and this is 5 by 2 5 by 2 therefore, this gives you 5, and then, minus 5 plus 8 plus 15, so this is minus 5 plus 15; so 10, this is also 5. So, you get your B also, that means, now that lower part is with you and finally, we want to compute the cost coefficients. So, the cost coefficient you have Z 1 minus C 1 is 0, which implies that C 1 is Z 1, and Z 1 is C B B inverse A1.

Now, where do I get this A1? I have already, I have computed A1 here; have I done that this is okay, why have I put this? Sorry, this is A3 and this is A1, because the first basic column is A 3 and the second basic column is A1; so this is A3 and this is A1. So, C B B inverse A1, now remember because these are slack variables so and since we are writing Z j minus C j, so please figure out for yourself, that means, see, here what is happening, when you look at Z 4 minus C 4, this is given to you to be 3; now see this implies that Z 4 is 3, because C 4 is 0 and this implies.

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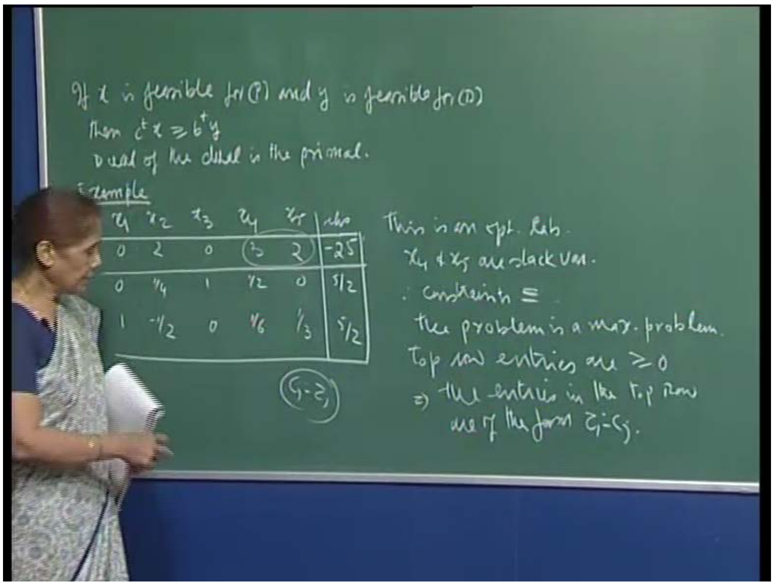

Now, what is Z 4? This is C B B inverse A 4, which is C B B inverse e1, which is C B B inverse first component; so that is equal to 3. And similarly, from Z 5 minus C 5 equal to 2, you will conclude that the second component of C B B inverse is 2. So, as I have been pointing out even earlier, that the dual solution would be available here… Now, we were most of the time the talking about the minimization problem and therefore, and since I was keeping C j minus Z j in the top row therefore, this was minus C B B inverse.

And But since here we are keeping Z j minus C j, so these entries will give you if you have slack variables here; remember surplus variables it changes again. So, for slack variables, this will give you the dual solution.

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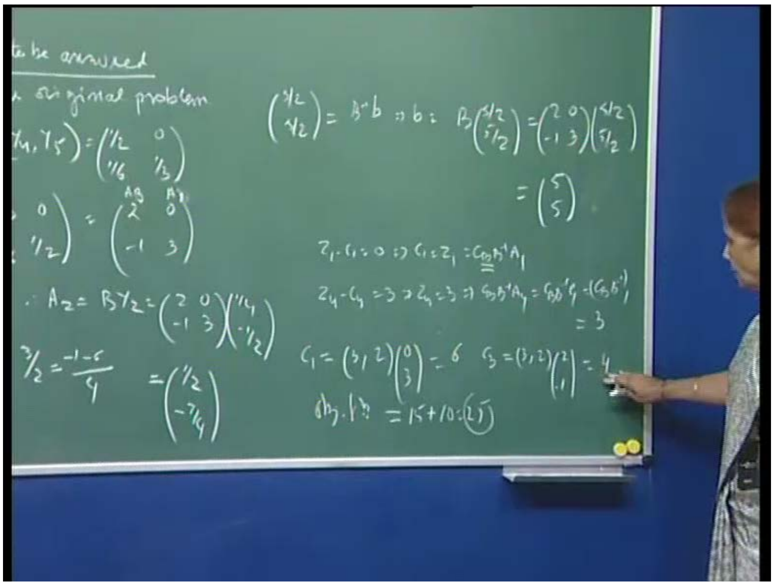

So, the dual solution is 3 2 or your C B B inverse is your dual solution. So, once you have this, therefore, from here, I will get C 1 as (3, 2) and your A 1 is 0 3; so which gives me this value as 6. And similarly, C 3, I can because I have C 3, you can compute which will be (3, 2) for same reason. Now, because Z 1 minus Z 3 minus C 3 is also 0, therefore, C 3 is Z 3, and Z 3 is C (3, 2) and the third column is 2 minus 1.

So, this is 6 minus 2, which is 4 and let's quickly checks that my calculation is okay why how? Because now that I know C 1 is 6 and C 3 is 4 and the values are 5. So, how much will it be? So, this will be 6 5s is 30 and 5 4s is… 3 2s is 6 and minus 2 is this thing, so this becomes 4. So, if my this is my right hand side, your solution is 5 by 2 and 5 by 2 currently the optimal solution. So, therefore, what is the objective function value? Objective function value is equal to 6 times 5 by 2 which is 15 plus 4 times 5 by 2 which is 10 is equal to 25. So, my calculations are okay, because this number is here is minus 25 which is negative of the objective function value.

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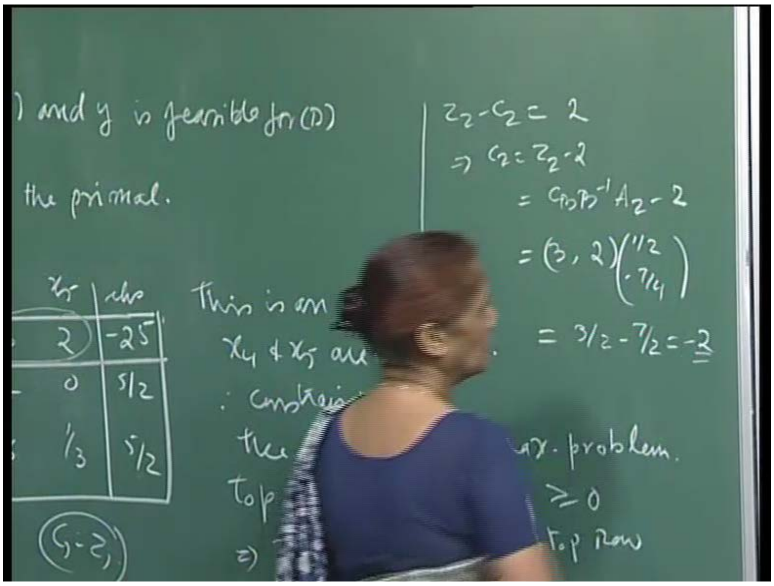

So, we compute… therefore, this is okay. Now, you need to compute C 2. So, let us now Z 2 minus C 2 is given to you to be 2. So, this implies that C 2 is equal to Z 2 minus 2, and Z 2 is C B B inverse A 2 minus 2. So, C B B inverse is (3, 2) and A 2 would be, A 2 we have computed here somewhere A 2 is 1 by 2 minus 7 by 4.

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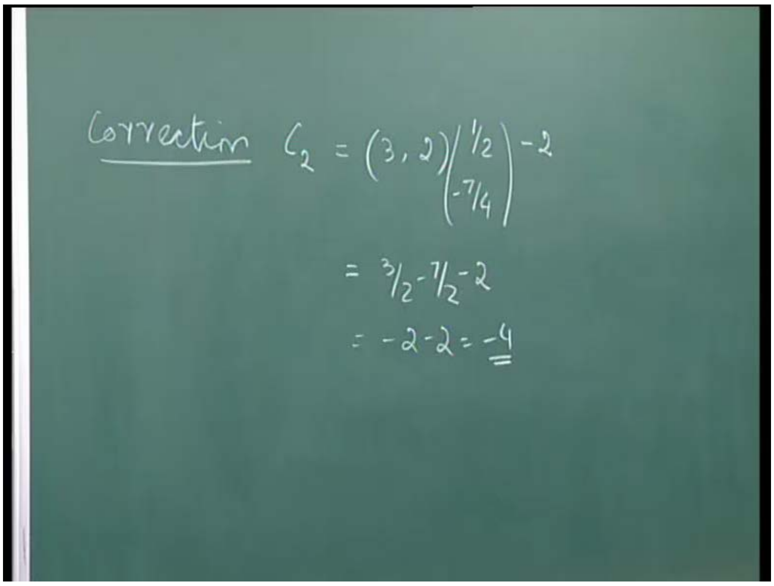

So, this is 1 by 2 minus 7 by 4; so that gives you 3 by 2 minus 7 by 2, which is 3 minus 7 so that is minus 4 by divide so minus 2. A small correction, while computing C 2, the minus 2 was left out; now please add that also, and then, the actual value of the correct value of C 2 will be minus 4.

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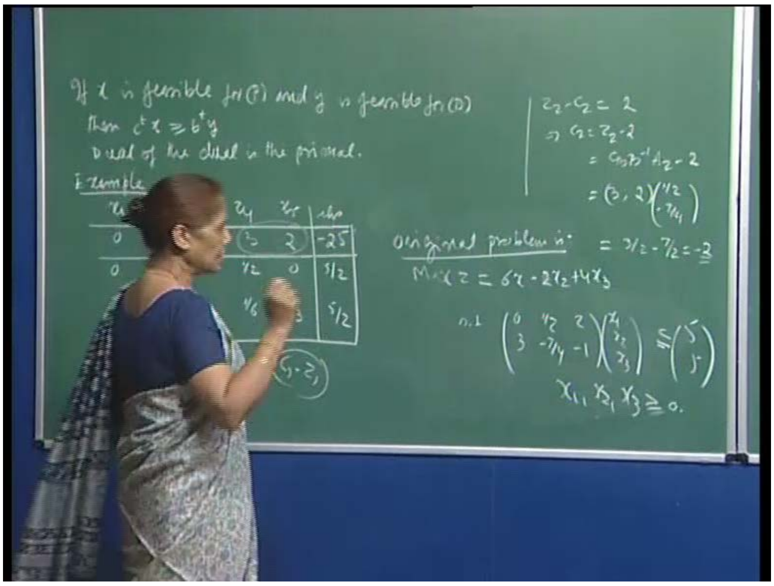

So, you can now write the let me write the original problem for you somewhere here, I need this data; this is the thing. So, the original problem is maximize Z equal to C1 quickly, C 1 is 6; so 6 x1 minus 2 x 2 plus 4 x 3, this is the thing x 4, x 5 are 0, subject to, so A1 your matrix A1 is 0 3; A 2 we found out is half minus 7 by 4, and A 3 is 2 minus 1; so this times x 1, x 2, x 3 and constraints are less kind; so this is this and this is 5 5, and x 1, x 2, x 3 is greater than or equal to 0; so this is the thing now you want to say that.

So, the second question is, write the dual to this problem and find out the optimal solution. So, since you have already found the optimal solution for the primal, then you know that the solution here would be the… because this is maximization problem and you are writing Z j minus C j. So, the dual solution is given by (3, 2) which is also the optimal solution; you already know from duality theory.

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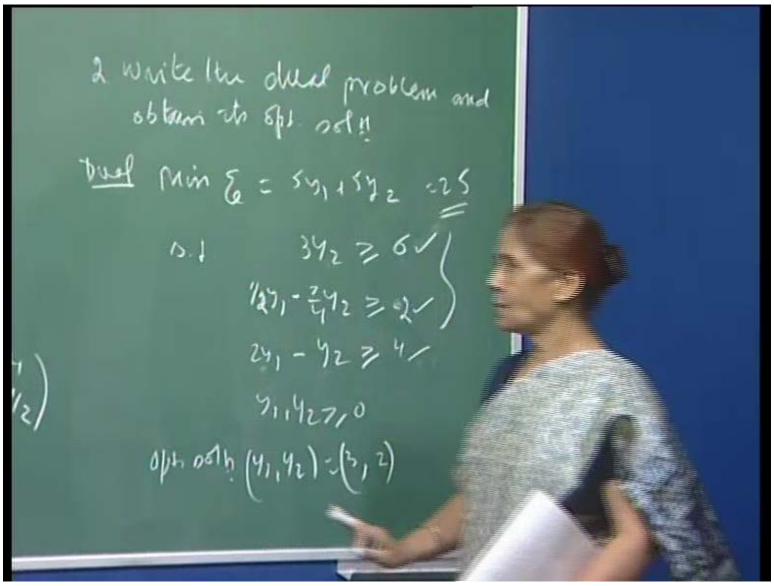

So, we can now answer the second question. So, the second question is, write the dual problem and obtain its optimal solution. And from our chart, we already know that since primal has a finite optimal solution therefore, the dual also has a finite optimal solution. So, the dual would be now in this case, it will be minimization problem, minimize psi equal to so these becomes to transpose, so this will be 5 y 1 plus 5 y 2 subject to y 2 greater than or equal to the objective function coefficient 6, then the other one is 1 by 2 y 1 minus 7 by 4 y 2, this is 2 which is greater than or equal to minus 2, and then, the third column is 2 by 1 minus y 2 is greater than or equal to 4; y 1 y 2 greater than or equal to 0.

So, I am saying that the optimal solution is y 1 y 2 equal to (3, 2) and just quickly verify that this is feasible. Here rewrite 0 3 therefore, 9 is greater than or equal to 6 this is satisfied, then here this will be 3 by 2 minus 7 by 2 which is satisfies the equality and but you have x1 also in the basis; so actually then 2 y 1 is what? 2 y 1 is 6 minus 2; so this is also satisfied as equality. So, what is happening here is that, actually this says that x 2 and x 3 should be this is this aspect needs to be probably there is some this thing in the because 3y 2 is greater than or equal to 6 this should have been all that satisfied as equality y 2 is 2.

So, actually all the because the complementary slackness condition must satisfy in optimal solution and so you see that x1 and x3 are positive in the primal optimal solution. So, these constraints must be satisfied as equality. So, that is okay and you can quickly check that the value of the objective function is 15 plus 10 which is also equal to 25; so the calculations are okay.

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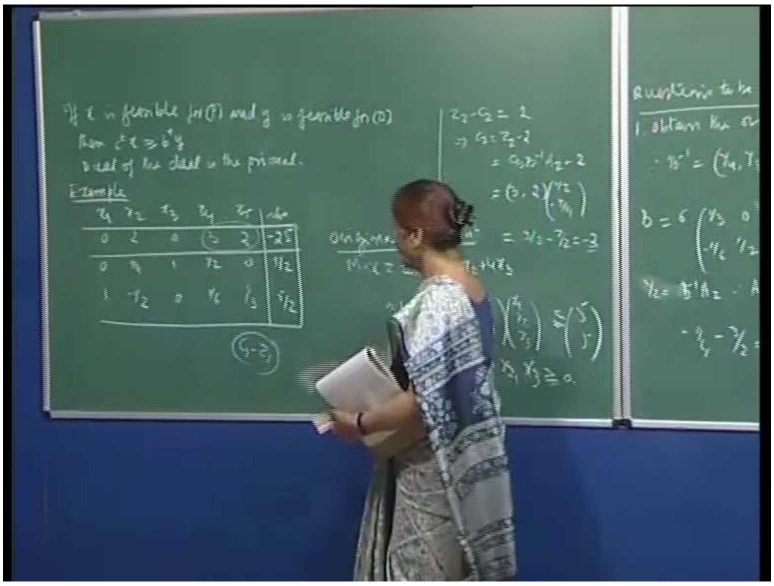

And you see the relationship between… therefore, just knowing that this is an optimal tableau for a linear programming problem, you could conclude lot of things and you could obtain your dual solution, dual optimal solution and so on.

So, the theory is very interesting and the more time you spend with it, the more you will learn to enjoy it and this is the idea I hope that end of the course, you really feel that you have picked up some tool that you can really enjoy and use to solve many day to day problems.

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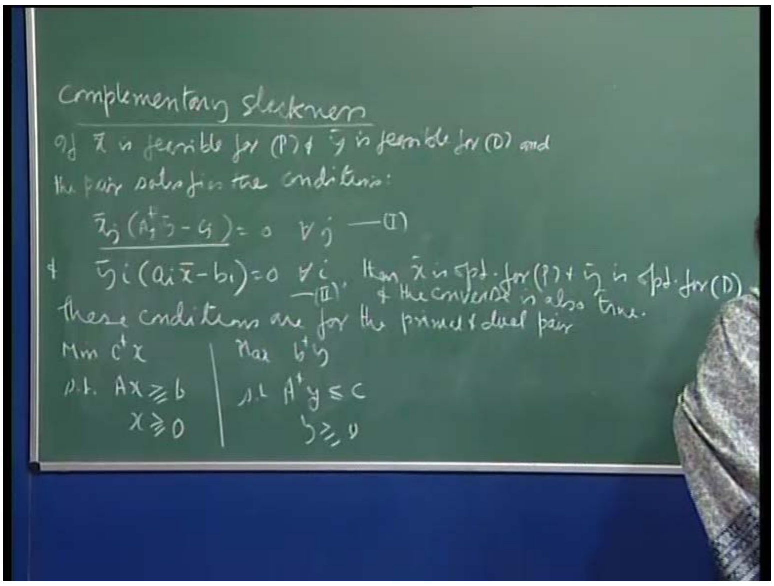

Now, let me go to another aspect of duality which is very interesting and that is complementary slackness. The idea here is that, look at the duality theory in a different way and that is, that we say that if x bar is feasible for P, and y bar is feasible for D and the pair satisfies the conditions: x j bar times this would be A j transpose y bar minus, C j is 0 for all j, and y bar i minus a i x bar minus b i is equal to 0 for all i; so this is j varying from 1 to n and this is.

Now, here I have stated these conditions for the canonical case. So, maybe I can say that these conditions are for the pair for the primal and dual pair minimize C transpose x subject to A x greater than or equal to b; x greater than or equal to 0 and the corresponding dual would be maximize b transpose y subject to A transpose y less than or equal to C y greater than or equal to 0. So, this is known as the canonical pair you may just inequality constraints have the greater kind; so, one has to keep this in track.

If I change the inequalities, then as you saw that your things change there. So, this is the pair and for this, I am writing. So, one it would be a good exercise if you sit down and write the complementary slackness conditions when you have the standard form of the linear programming problem.

Now, have I stated the theorem correctly? If x bar is feasible for P, and y bar is feasible for D and the pair satisfies the conditions this and this, then of course, this is just to note, then x bar is optimal for P, and y bar is optimal for D; the converse is also true. So, maybe I should stated as, if x bar and y bar feasible for P and D, satisfies these conditions, then of course, this is optimal and the converse is also true.

Now, actually you can try to understand what does that say. See, for example, this says just look at this, it say that if the dual constraint here, if the dual constraint is satisfied because the product is 0. So, at least one of the numbers has to be 0 either x bar j has to be 0 or this number has to be 0.

So, in case this particular constraint, the dual constraint jth the dual constraint is not satisfied as equality, then this is the negative number and therefore, x bar j must be 0, and similarly, here if the corresponding primal constraint is satisfied as strict inequality, then it says that the dual variable must be 0 and we look at the interpretations; I will come back to the economic interpretations of these conditions, but first let us just quickly prove the result.

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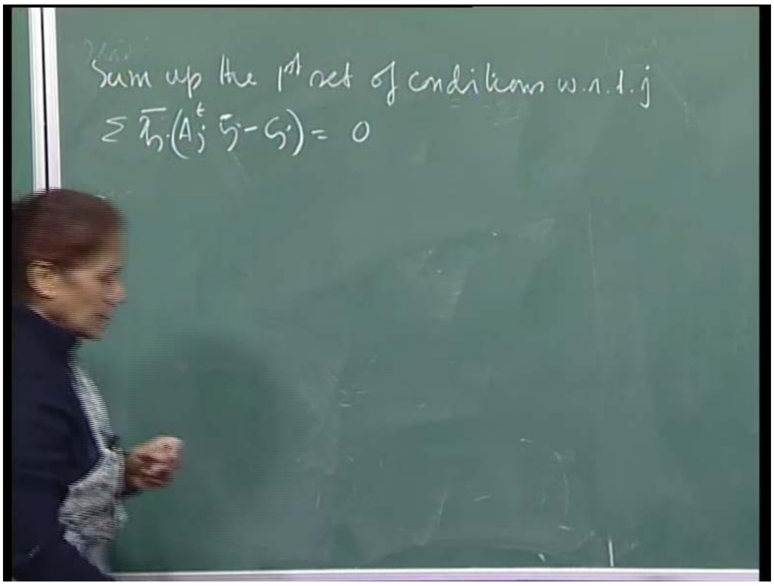

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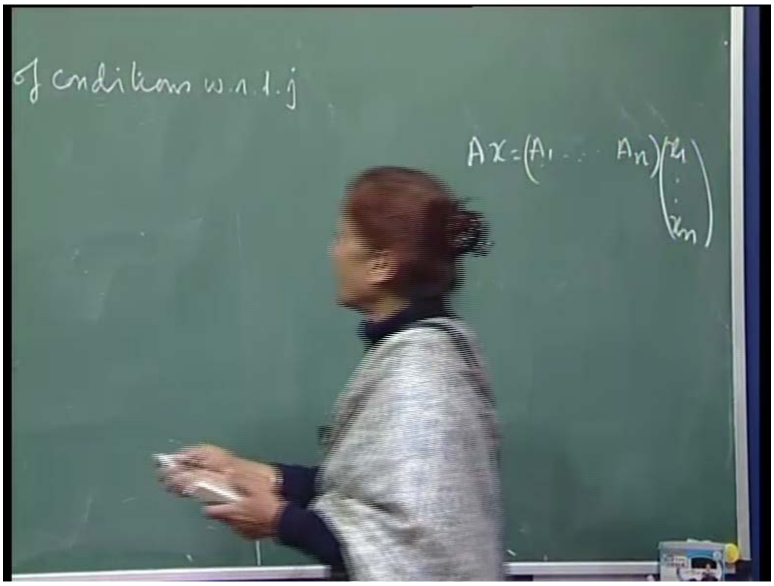

Let us first do it the first way, they satisfies these conditions. So, what does it mean? So, I will call this as the first set of conditions and this is the second set of conditions. So, sum up the first set of conditions with respect to j and you get what that summation x j bar A j transpose y bar minus C j is equal to 0 and this gives you what? See, here this is x j A j transpose y bar, so y bar is outside of this. So, actually this implies, that the summation if you do it here, I see this is the single number x j bar times A j transpose; if you adding up here x j A j… See, when you write this, when you write A x, you can write this as A 1 x 1 plus A 2 x 2; I can read this as, how will I read this? This is my matrix A x is A1 to A n, and then, you have x 1 x n. This is how you write it.

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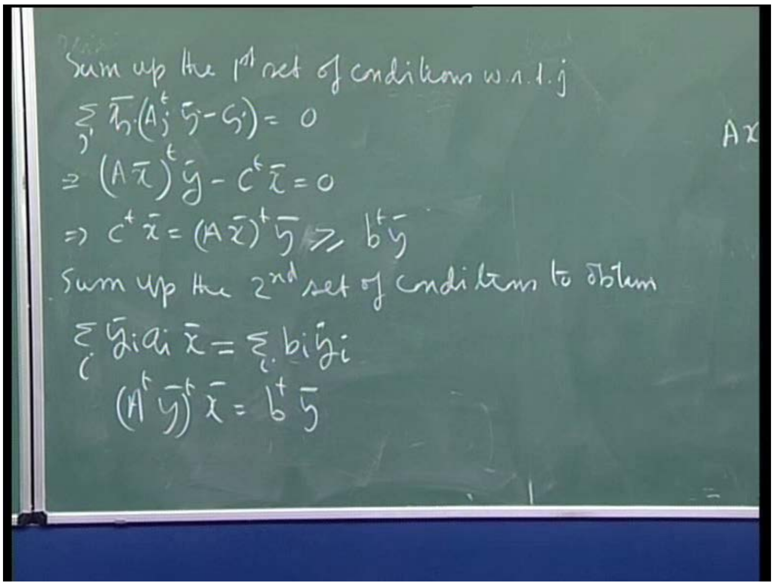

So, you write A 1 x 1 plus A 2 x 2 and so on, this is how you can rewrite this. So, same thing I am saying; so whether I write x j bar A j transpose or so it will be the same thing as because it is a single number, by the way this is if Aj bar A j transpose y bar j minus C j therefore, this implies that this is equal to A x bar; you can say, may be transpose y bar minus C transpose x bar is 0.

So, therefore, this implies that your C transpose x bar is A x bar transpose y bar which gives you and because x bar is feasible. So, therefore, you have A x bar greater than or equal to b, y is all non-negative therefore, what do you get from here? That Ax bar is… So, this is greater, because y bar all components are non-negative, this is greater than or equal to b, my inequality will be maintained, because components of y all are non-negative; so this is greater than or equal to b transpose y bar.

Then from here, from the second one, sum up second set of conditions to obtain conditions expect to i of course, we are summing of respect to i and here, you will get what? See, this is summation y i bar a i x bar summation over, I should have said summation over j here minus is equal to sigma does not matter what you write y i b i or this whatever it is.

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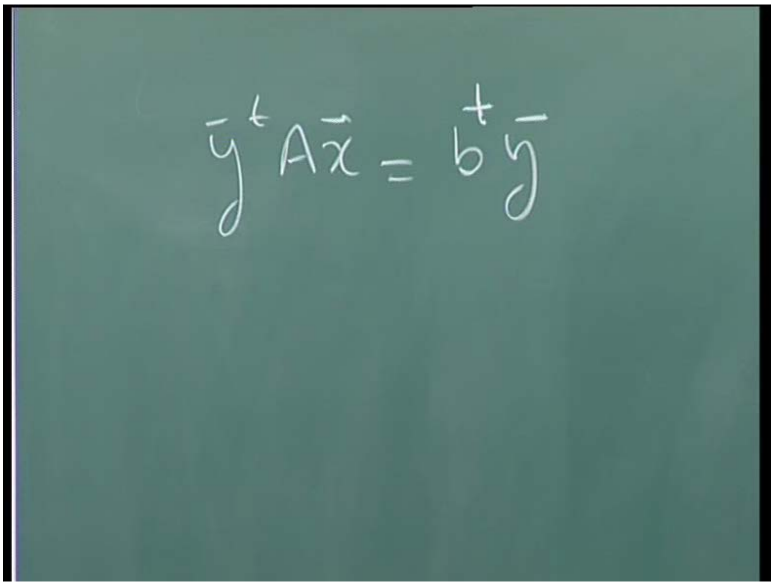

So, then this you can write as, see y i a i; so this is a row and so, I can write this as A y bar A transpose y bar times x bar, this is equal to b transpose y bar yes, because you are summing over i.

So, the correct equation is actually y bar transpose A x bar is equal to b transpose y bar. I had earlier written it as y bar transpose A transpose into x bar, which would not give you the right… See, on the right hand side, the term is 1 by so its a single number; so on the left hand side also, now it is a single number, because y bar is 1 by m; A is m by n; and x bar is n by 1; so the product is 1 by 1.

So, right now, the equation is valid. Now, that these two quantities, so here what you have, A transpose y bar; A transpose y bar is less than or equal to c and again all components of x are non-negative.

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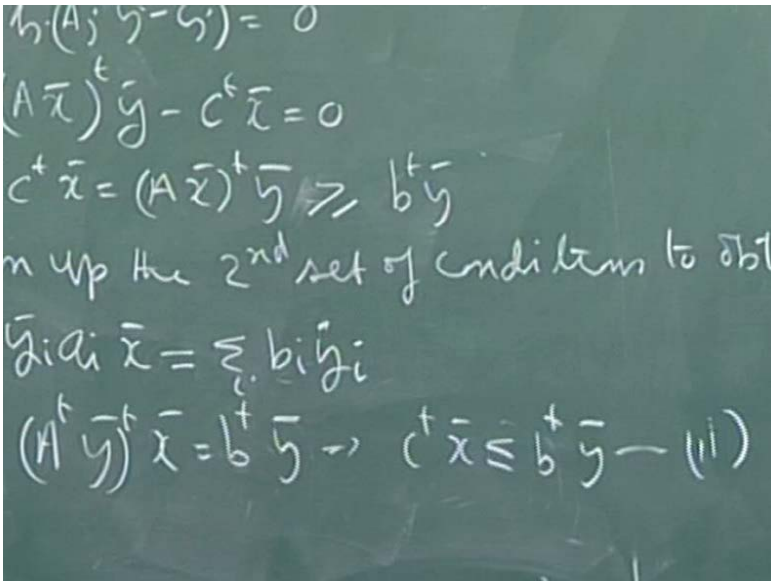

So, this implies that C transpose x bar is less than or equal to b transpose y bar from here, because y bar is feasible and all components of x bar are non-negative therefore, this is less than or equal to C transpose x bar is less than or equal to this and so this (( )).

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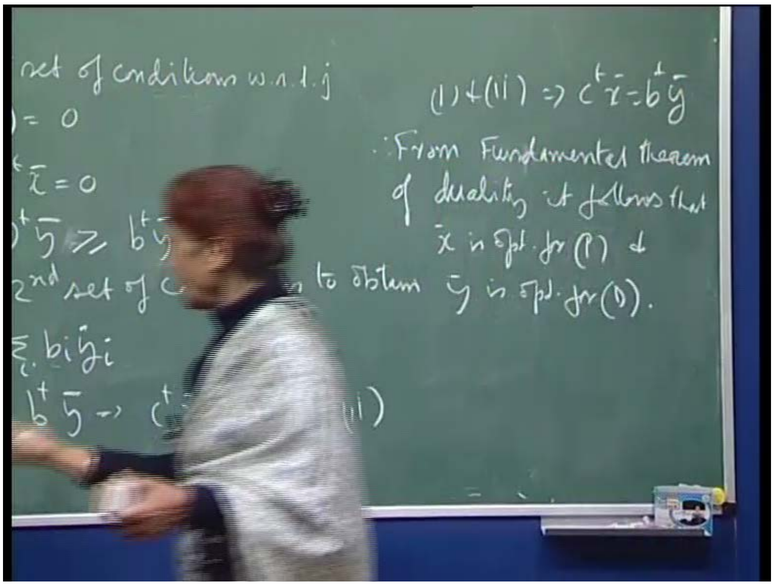

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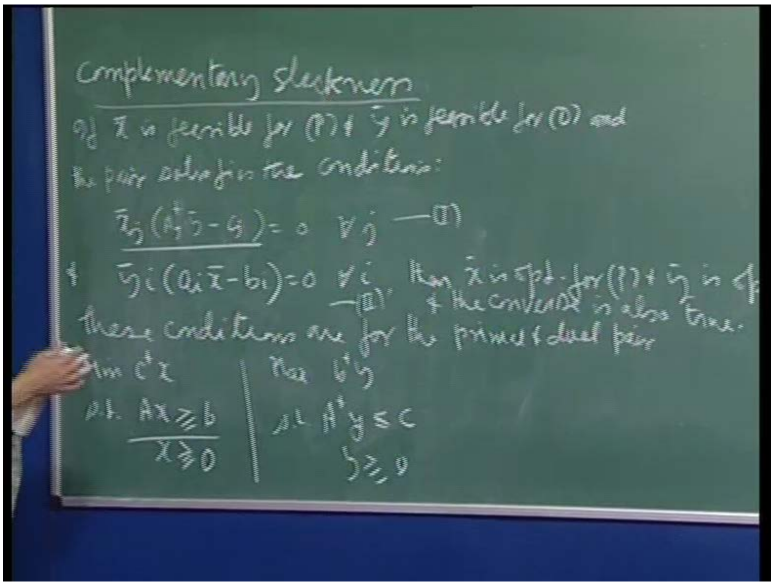

So, from here this is your 1 and 2. So, 1 and 2 imply that C transpose x bar is b transpose y bar and so what does it follow? And from therefore, from fundamental theorem of duality, it follows that x bar is optimal for P, and y bar is optimal for D. And now you can see the converse also is immediately obvious, I think see if so we said that x bar is feasible for P, and y bar is feasible for D, and x bar and y bar satisfy these complementary slackness conditions, then they are optimal. Now, what we want to say is that, if they are optimal, x bar is optimal for P, and y bar is optimal for D, then you just have to reverse the argument. Because in that case, by the fundamental theorem of duality, C transpose x bar would be equal to b transpose y.

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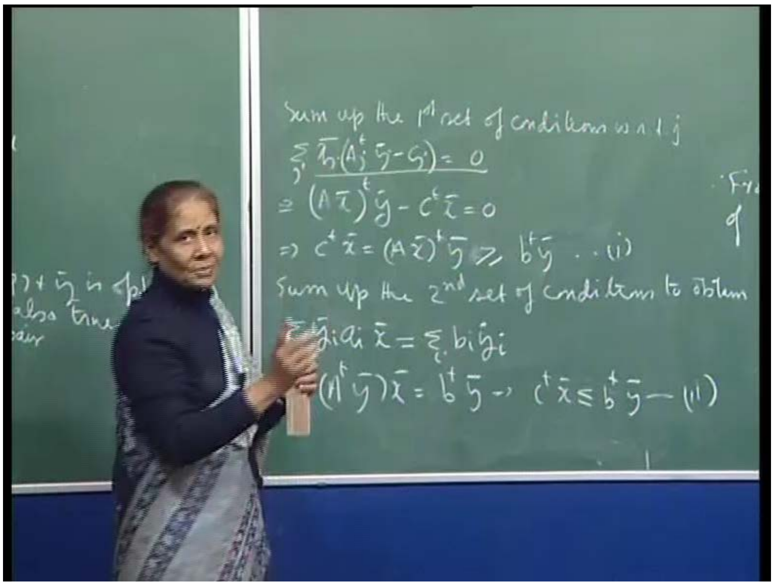

So, you will have equality and once you have equality, you see that these conditions will be satisfied, because when you sum up this with to respect to j and sum up this with respect to i, the two quantities are equal; therefore, the 2's… And you have already know that these two C transpose x is equal to b transpose y and so you can say that, the each of the conditions will be satisfied. Because once you have that this summation is satisfied as equality, then since x j bars are all non-negative so and these are all less than or equal to 0, so every term here is of negative sign.

So sum up of quantities of the same size equal to 0 implies each must be 0 and therefore, individually you can show that this is 0 and this is 0. So, I will leave the converse of the proving the converse, you can now write down the details; I have given you the idea, so that takes care of the complementary slackness theorem.

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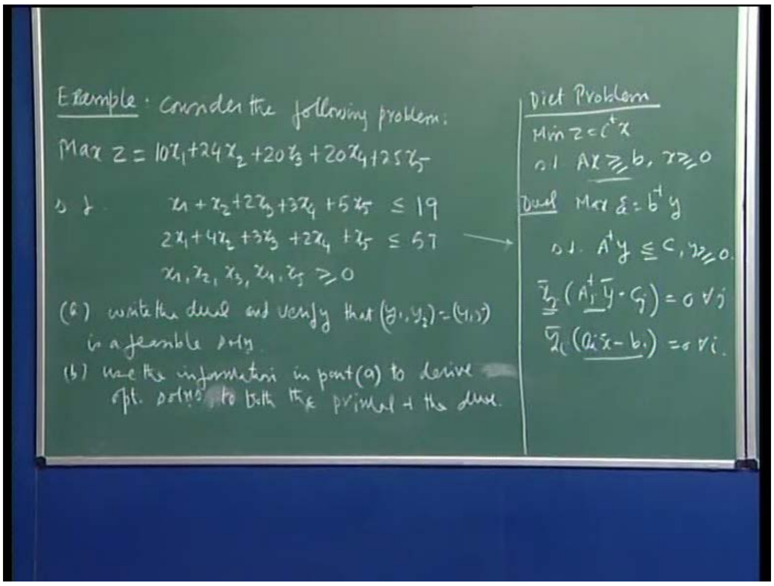

So, now, let me explain the complementary slackness conditions or try to give you some interpretations to this complementary slackness conditions through the diet problem that we had formulated last time. So, this was now the cost of a diet and this was adequate to divide the conditions for a diet to be adequate, that means, it needs the necessary requirements for the different nutrients. So, this was your problem and the dual would be of this kind.

So, when you say that x j bar times A j transpose y bar minus C j is equal to 0 for all j, and yi bar times a i x bar minus b i should be equal to 0 for all i; so these are our complementary slackness conditions. And we saw that if the x bar and y bar is a primal dual feasible pair and they satisfy the complementary slackness conditions and they are optimal; so at optimality what are we saying? See, for example, here remember, I gave you the interpretation of the dual variables as the value or the prices that a manufacturer attaches to the different nutrients.

So, here for example, this gave you the worth or the value of the jth food in terms of the nutrients it contains and cj is the market price of the food. So, what we are saying is that, in case the worth of the value of the jth food in terms of the nutrients is strictly less than the market price, then that corresponding x j bar should be 0 in optimal solution. That means, the house wife will not include that particular food in her diet, if the worth or the value of that food is less than the market price. Because in some sense, you will say that the food is not as good, and similarly, from here, you will say that and interpreted this way, this gives you the, you would say that a i x bar; so x bar is your diet, optimal diet, then this told you the total diet, what is the amount of the ith nutrient it contains.

So, this is the total amount of nutrient that your diet contains and if it is greater than b i, here, you have this; so, if it is greater than b i, then the corresponding y i bar, because then there is no value to the that nutrient, because your diet already contains that extra nutrient already more than what is required therefore, the value of the corresponding dual variable is 0.

And other way, you can say that, if you are including a food in if you are including jth food in your diet, then this must be 0. Because, that means, this is positive; so in that case, this constraint must be 0. That means, you will only include a food in your diet provided the value of the food in terms of the nutrients it contains is equal to the market price.

So, this is some sort of equilibrium condition, that whatever the optimal solution in the foods that get included in your diet, the corresponding value of the food with respect to its market price is equal; otherwise, you do not include it and so there is so many other… We will also talk of the other interpretation, because as we go along, we will very often keep visiting these complementary slackness conditions. So, I will come back to them half an hour.

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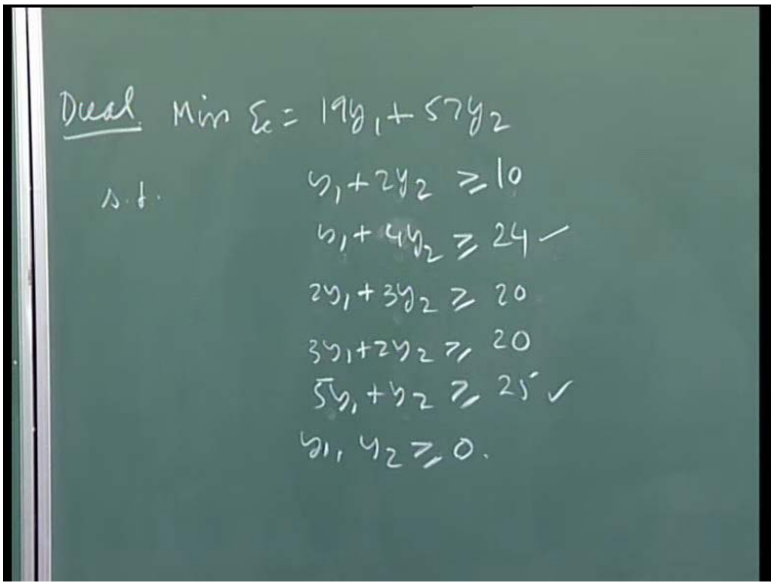

So, now, let us look at this problem and let me show you another way we can make use of the complementary slackness conditions. So, suppose you have been stated this problem, this is maximization problem; conditions are less than constraints are less kind, variables are non-negative. So, in fact, you can say that this is your dual problem; maximize the objective functions subject to less than kind constraints, non-negative variables. So, you want to so the what is your you are asked to write the dual of the problem. So, let us quickly write the dual; so dual of the given problem will be minimize, if you want, you can go through the exercise of converting this to a primal problem, then write the dual, but we have already done it. So, I will just straight away…

Because this is, this form, so the dual must be this form since the dual of the dual is the primal right; so I will use that concept. So, minimize psi equal to 19 y 1 plus 57 y 2 subject to quickly so this is y1 plus 2 y 2 and this is greater than or equal to 10, then the second column is y 1 plus 4 y 2 is greater than or equal to 24.

Then the third one is 2 y 1 plus 3 y 2 is greater than or equal to C 3, which is 20, then the fourth column is 3 y 1 plus 2 y 2 is greater than or equal to 20 and the fifth one is 5 y 1; so 5 y 1 plus y 2 is greater than or equal to 25 and your y 1, y 2 have both not.

So, you can quickly verify that, this is a dual feasible solution; 4 plus 10, 14 for greater than 10, then 4 plus what is it? 24, so I will mark this, because this is satisfied as equality and I will tell you why, then this is 8 plus 15 23, so this is satisfied; then 12 plus 10 this is also satisfied; then 20 plus 5 25, so this is also satisfied as equality. So, this solution that is the dual solution given to you is feasible for the problem.

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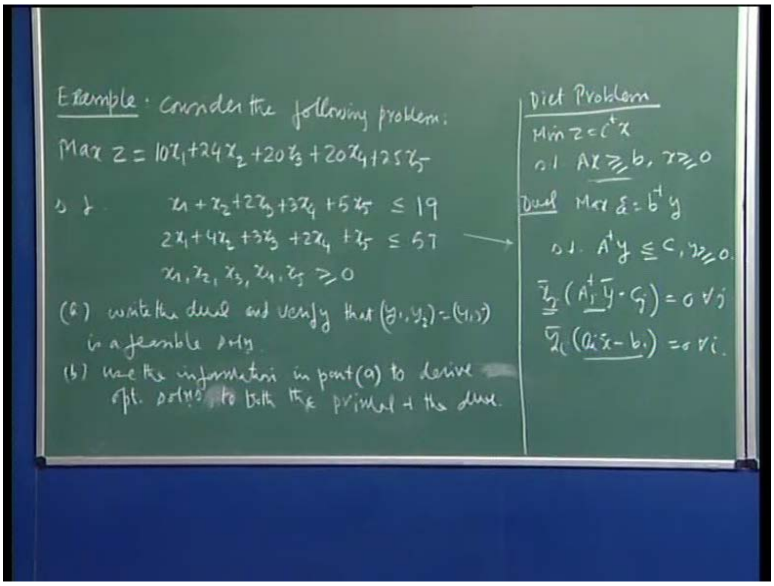

Now, let us use complementary slackness condition. What we said is, if we can find, so I have a dual feasible solution, if I can find a primal feasible solution such that the complementary slackness conditions are satisfied, then I can conclude that, because you are asked to find prime optimal solutions for both the primal and the dual.

So, let me take a gamble what I will say is, I already have a dual feasible solution; let me see if I can find a feasible solution for the primal, which together with this dual feasible solution satisfy the complementary slackness conditions. If I can find one feasible solution like this for the primal, then I know that both must be optimal. So, let and that is why I have marked this, because remember we said that here x j bar can be positive only if the corresponding dual constraint is satisfied as equality.

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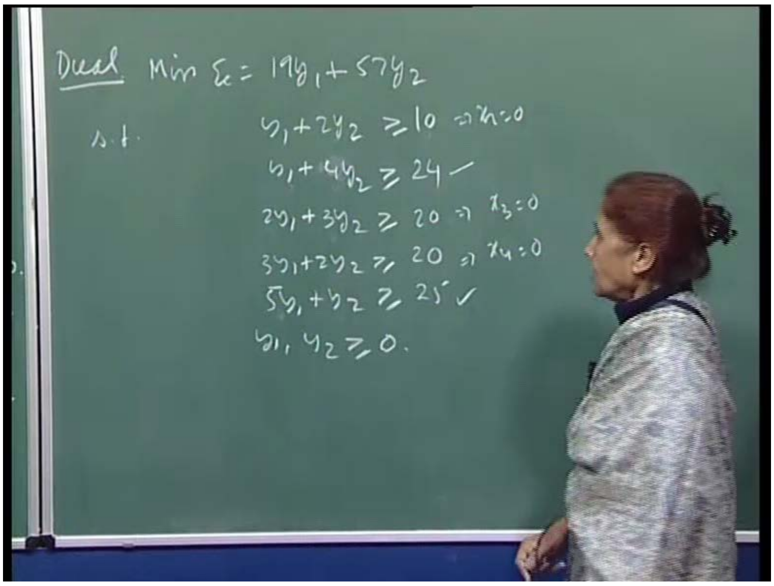

So, I have two such constraints, because this implies that your x 1 is 0, this implies that your x 3 is 0, and this implies that your x 4 is 0. If I have to find x bar which satisfies complementary slackness conditions along with this y dual feasible, then because of this, I must have whatever dual constraints are not satisfied as equality, the corresponding x j bar must be 0. So, that means, now I have to find primal feasible solution for which consists of the basic variables x 2 and x 5.

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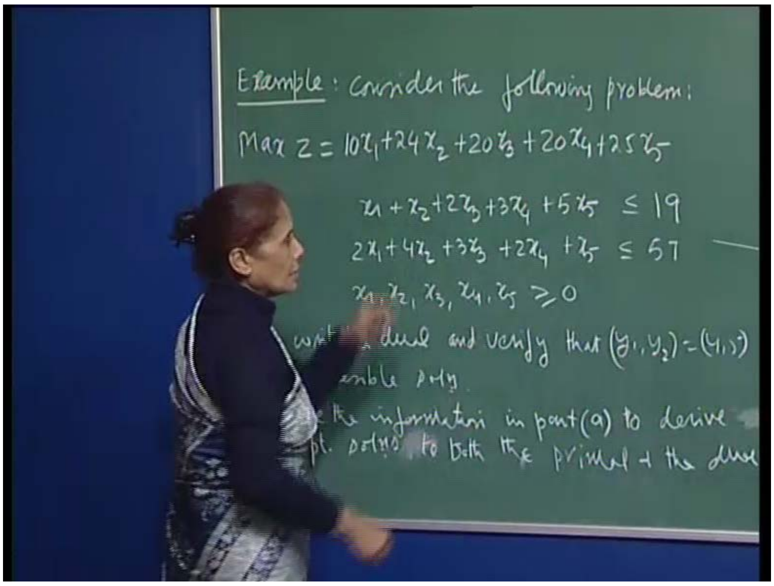

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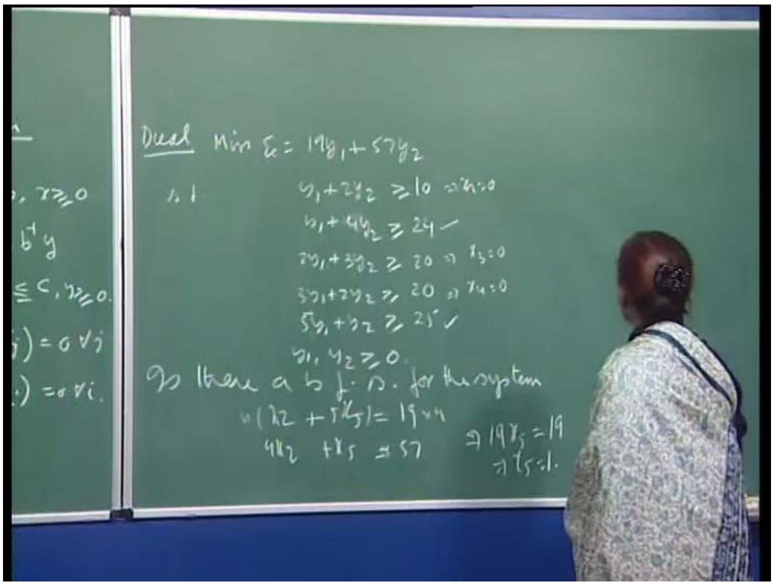

So, if x2 and x5… therefore, here you see, you make all these 0s, so what is your which implies so then when is there therefore, is there a basic feasible solution for the system what will it be? It is x2; so x2 would be x 2 4 x 2, and then, plus 5 x 5 and plus x 5; so this is 19 and this is 57. If I can find an x 2, x 5satisfying these constraints and they are non-negative, then I am done, because I have a basic feasible solution.

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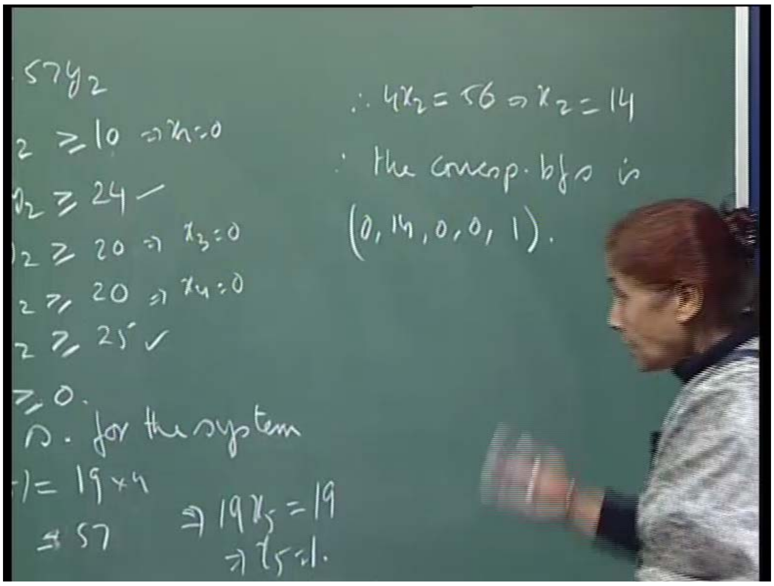

So, let us quickly multiply this by 4 into 4, and then, I subtract, so what will I get? 20 minus x 5 19, so this implies that 19 x 5 is this is 4 times and this is 3 times, so this is 19;

so this implies x 5 is 1; so, if x 5 is 1, then this becomes 56 therefore, 4 x 2 is equal to 56 which implies that x 2 is equal to 14.

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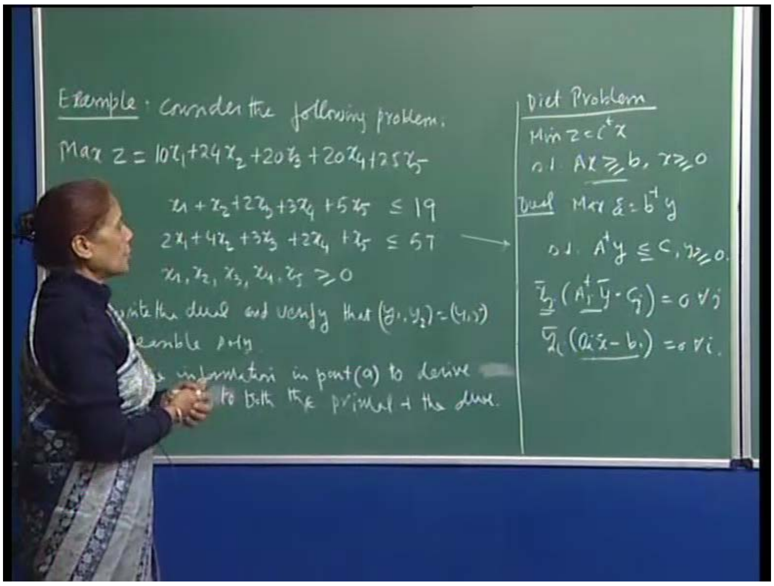

So, that means, I have a feasible solution. So, the basic feasible solution therefore, the corresponding basic feasible solution is 0, 14, 0, 0 and 1 and they should be optimal with y. Because the complementary slackness conditions are satisfied, yes you may say that I have not check these, but let us do it; we can verify that these are also satisfied actually this was enough.

So, let us see what are the both the dual constraints are satisfied as primal constraints. Because here you have x 2 is 14 and this 1; so this is 19, then 4 4s are 4 14s are 56 plus 1 57, so both the constraints are satisfies equality, because y 1 bar and y 2 bar are both positive.

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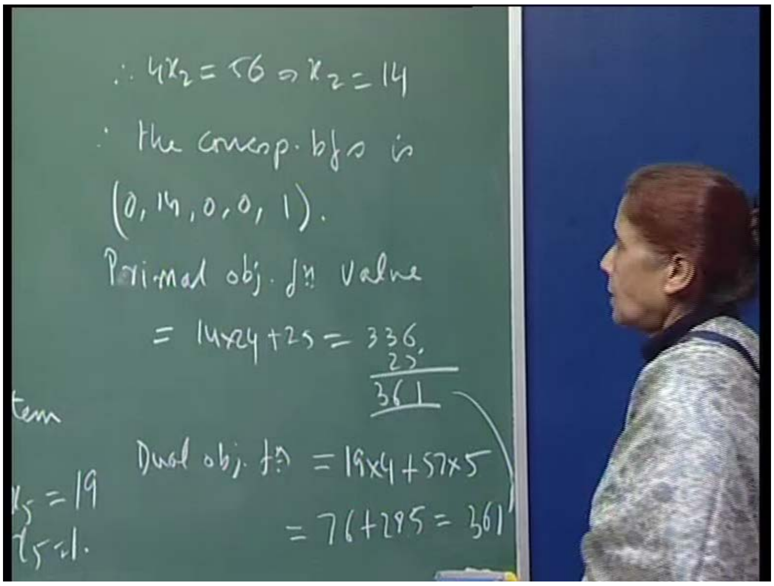

y1 bar is 4; y 2 bar is 5, so you I have this. And now, we have some more checks; we can make sure that the two solutions are optimal and how do we do that? We will compute the objective function values. So, the primal objective function value is equal to x 2 is 14 so 14 and 2 24 14 into 24 plus 25, let us quickly do the arithmetic, 4 4s are 56, 5 4 2s are 28 and 3, 33.

So, this is 1 5 and 6- 361 and dual objective function is equal to 4 and 5 y1; so this is 19 times 4 plus 57 times 5. So, let us quickly do it, 76 plus 5 7s are 35 5 3 5 5s 25 2 8 1 and 7 8 15 6 so the two values are equal.

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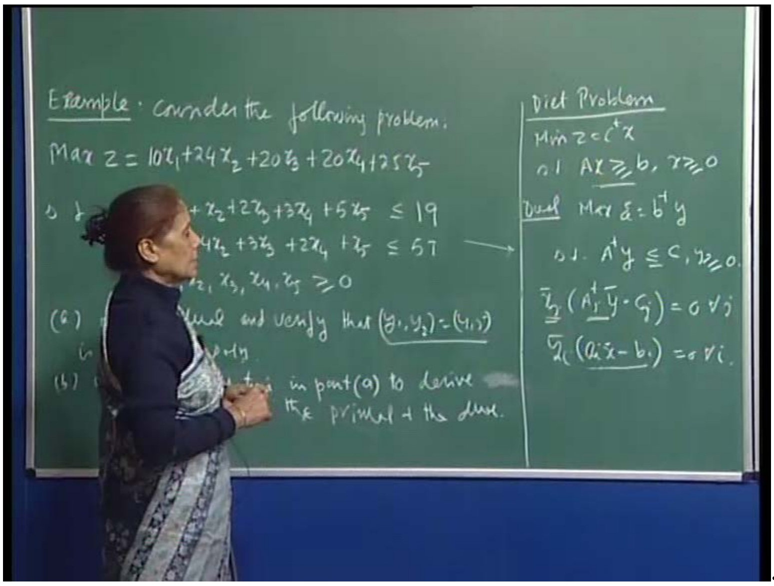

So, either way you check the solution is optimal. So, you see this is this is also a big tool, because I had feasible solution for the dual of course, the question will arise how did I obtain it? Suppose I will show you situations, which are very naturally arise, where you might have a dual feasible solution or it may be very convenient to lay your hands on a dual feasible solution, and then, you want to find out, whether you can you we have a corresponding primal feasible solution for which together with this dual feasible solution satisfy the complementary slackness conditions and therefore, you can easily obtain optimal solutions for the two problems.

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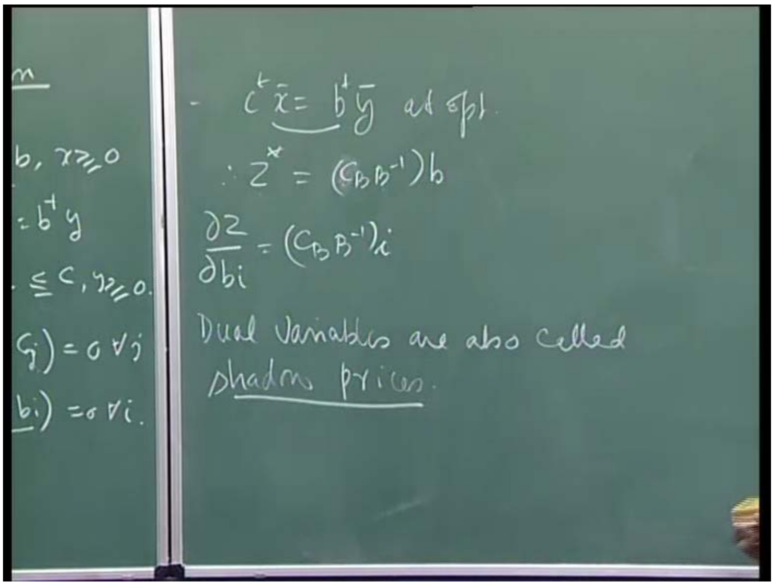

Now, as I said that, the story about the duality theory is not over, because one can go and on. Of course, I will show you, I will through using these complementary slackness conditions, in the next few lectures we are going to develop variance of the simplex algorithm. But one more thing I wanted to show you here and that is you know, you can another interpretation for the dual variables that we can have is, see at optimality we have said that your C transpose x bar is b transpose y bar at optimality.

Therefore, your Z star may be you can say is actually you can write this as C B B inverse is times b; remember I am reading this always as a row vector. So, if b is your corresponding optimal basis, then this is what you have. Because at optimality, the two objective function values must be the same; so Z star is my optimal value and this is which is equal to this.

So, therefore, you can also say that delta Z by delta b i, if you want to look at the rate of increase of Z star with respect to increase in the value of b i, then this tells you, this is C B B inverse of i. So, you see a nice interpretation, dual variables are and of course, I should also give you the name which is very common, dual variables are also called shadow prices. So, the shadow prices you have the name, relative prices, shadow prices and as you read the literature you will find out any way.

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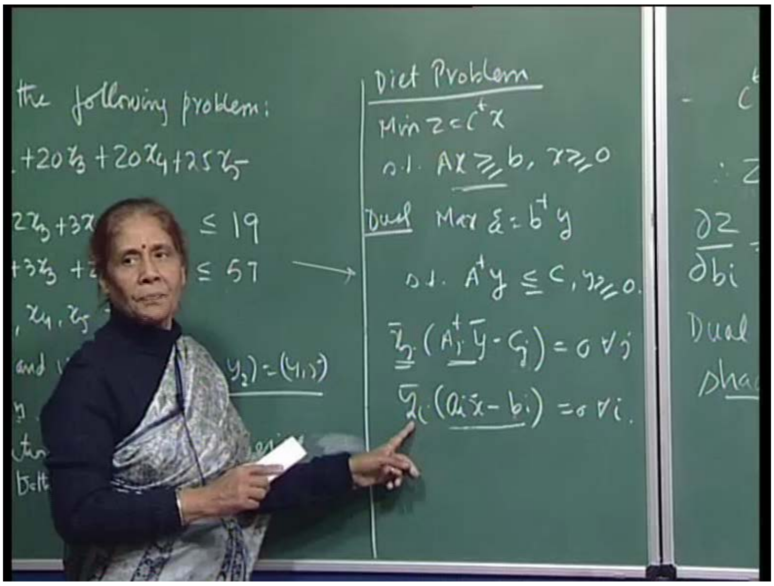

So, this that means the dual variables also tells you the rate increase of the objective function respect to the right hand side vector. So, in other words, the y i bar for the diet problem tells you what? That is what I am saying, that in case your this constraint is not satisfied as equality, that means, the nutrients present in your diet that you have chosen are the ith nutrient present in your diet is greater than the required 1, then the worth of that nutrient is not there, that means, if I increase that particular nutrient in my diet, it is not going to help me, because I am already meeting the minimum requirement.

And therefore, y i bar is 0, that means, at optimality if you already have enough of that nutrient, why would you want to increase the content of that particular nutrient and so y i bar would be 0, that means, the rate of in case you do not want to increase the content of this particular nutrient in your diet, because you are already meeting the requirement.

And many other ways of looking at it of the at the dual variables. As we go on, we will talk about it more and more. And so, next time, I will try to yes, there will be an assignment sheet that I will discuss, you know, consisting of problems based on what we have discussed so far, and then, I would like to developed variants of the simplex algorithm using these complementary slackness condition.