Calculus Of Variations and Integral Equation

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Module No. # 01
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Welcome viewers, once again to the lecture series on integral equation under NPTEL course. Today, we are going to discuss about the some methods of solving non homogenous Fredholm integral equation of the second kind. Of course, there are some other theories named as Fredholm alternative that will be discussed in this lecture and afterwards I will be coming some other properties like Hilbert-Schmidt theory and Fredholm three theorems, for some special type of problems that will come in next lectures.

So, today we are mainly going to consider the Adomian decomposition method for solving non homogenous Fredholm integral equation, and also another method that is called successive approximation method or sometimes it is called iterative methods. For iterative methods, we will be considering the convergence criteria also. Now, before going to all these things, first of all we define a special type of kernel that is called separable kernel or degenerate kernel. And today’s entire lecture is confined within, those kind of, those special type of kernels that is separable or say degenerate kernels.

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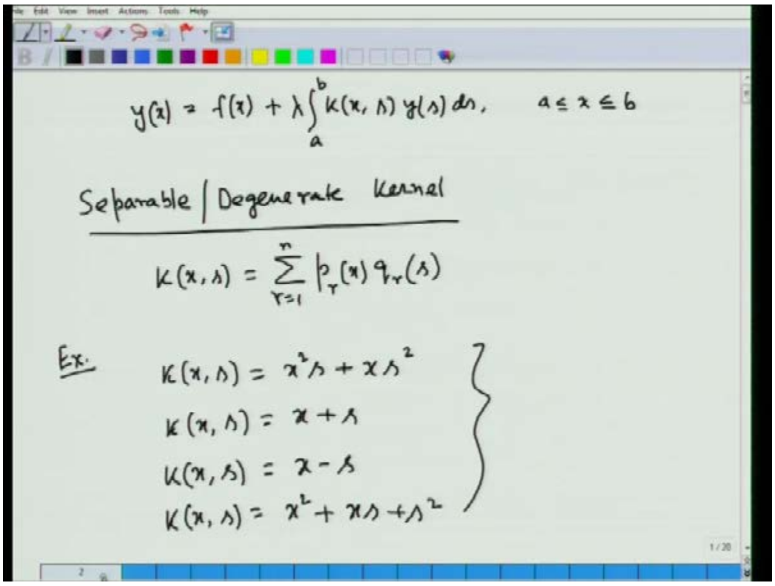

So, we will be considering these kinds of equations, that is y x is equal to f x plus lambda integral a to b K of x comma s y s d s, where a less than equal to x less than equal to b. And we will be considering the separable kernel, separable or sometimes it is called degenerate degenerate kernel, separable or degenerate kernels is defined by K x comma s, this is equal to summation r running's from 1 to n p r x q r s.

So, that means the kernel can be expressed as product of the functions of x and s and their finite sum. So, let us take some specific examples of such type of kernel for example, K x comma s this is equal to x square s plus x s square. Secondly, K x comma s that is equal to x plus s. Also, in case of Volterra integral equation where Laplace transform method was used, we have considered this type of kernel that is x minus s, this is also a separable kernel or degenerate kernel and other examples are K x comma s is for example, x square plus x s plus s square.

So, these are all, not all whether some examples of separable or degenerate kernels. Now, the point is that, sometimes for approximate solutions or in case of numerical solution of these type of Fredholm integral equation, that we are not going to cover within the this lecture series. Sometimes, it is possible that you have a kernel which is not a separable, but using Taylor expansion method, we can approximate the given kernel by a separable kernel and it is already proved in the theory, that those kind of approximation leads to very fastly converging numerical scheme, which are actually converging to the solution of the given problem, of course using some numerical techniques.

Now, at this moment, we can give certain sufficient criteria for the existence of unique solution of this Fredholm integral equation of the second kind, of course the equation is a non homogenous type.

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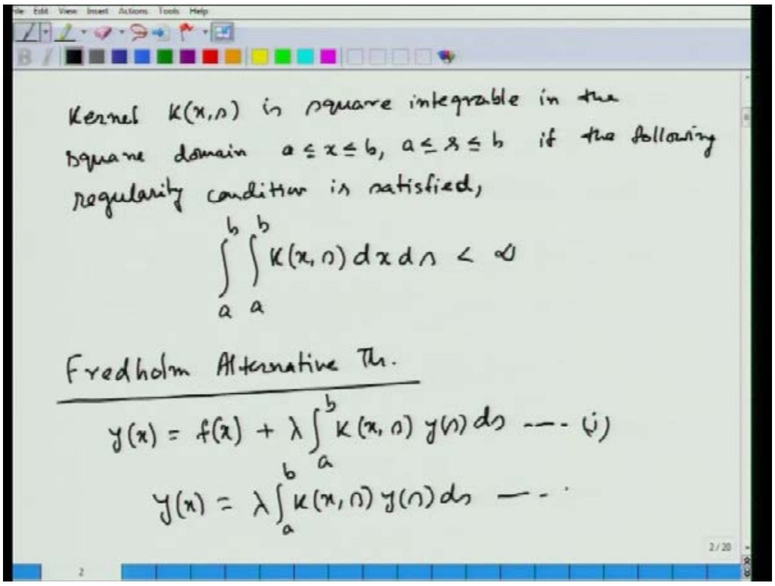

So, first of all for these criteria, we need the concept of square integrability of the kernel. The kernel K x comma s is square integrable is square integrable in the square domain which is defined by a less than equal to x less than equal to b, a less than equal to s less than equal to b if the following regularity condition is satisfied, and this regularity condition is given by integral a to b integral a to b K x comma s dx ds less than infinity. And of course, this regularity condition also involves the existence of this double integral. So, this double integral exists and this double integral is finite, then this kernel is actually called the square integrable, over the square.

And now, we state an important theorem, in due course of time we will be discussing about it in detail, that is called Fredholm alternative theorem. This Fredholm alternative theorem, actually relates between the solution of the non homogenous equation, that is y x equal to f x plus lambda times integral a to b K x comma s y s d s, and the corresponding homogeneous equation, y x is equal to lambda integral a to b K x comma s y s d s. And it states that, that if the corresponding homogenous equation, if the homogeneous equation, that is y x equal to lambda times integral a to b K x comma s y s d s, this particular problem admits only the trivial solution y equal to 0, if it admits only trivial solution that is y x identically equal to 0. Then the non homogeneous equations, that is y x equal to f x plus lambda integral a to b K x comma s y s d s this will be having one and only one solution.

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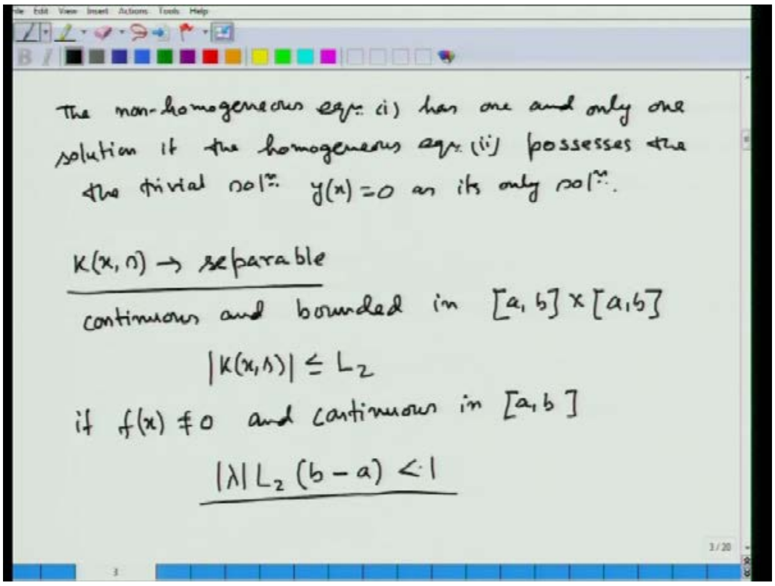

So, if we write this equation as number 1, this as number 2. So, then Fredholm alternative theorem states that, the non homogeneous equation the non homogeneous equation 1 has one and only one solution, if the homogeneous equation 2 possesses the trivial solution only, trivial solution y x equal to 0 as its only solution. So, this is actually statement of the Fredholm alternative theorem, and now we can state the sufficient condition for the existence of unique solution for non homogeneous Fredholm integral equation of the second kind.

So, that means we are going to state the sufficient condition for existence of unique solution for the Fredholm integral equation of the second kind. And first of all, we are assuming that K x comma x, this is separable, this K x comma s this is separable, it is continuous and bounded in the square a, b cross a, b. So, now, we can denote the bound of this K x comma s as less than equal to L 2. So, these are the property required for the kernel and if f x not equal to 0 and continuous, it is continuous in the interval a comma b, then the sufficient condition which will guarantee the existence of unique solution of the equation 1 is given by modulus lambda times L 2 times b minus a, this is less than 1.

You have to keep in mind that this condition is sufficient, but not necessary. So, that means, if this condition is valid, still problem may have unique solution, and you can consider one example, this is given in the book by (()).

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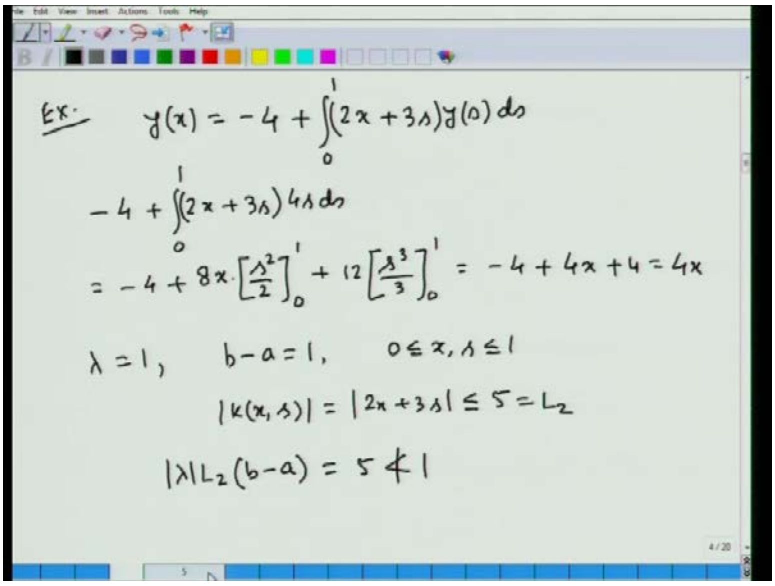

So, let us consider the equation y x equal to minus 4 plus integral 0 to 1 2 x plus 3 s y s d s, this is the given integral equation. Now, we just check whether y x equal to 4 x is the only solution or not. So, minus 4 plus integral 0 to 1 2 x plus 3 s, we are going to check whether y equal to 4x is a solution or not, so, y x will be 4 s d s.

So, after integration, this will be 4 plus 8 x, then integral of s, that is a square by 2, integral 0 to 1 plus 12 into s square. So, that means, s cube divided by 3 limit 0 to 1, and this will be equal to minus 4 plus 4 x plus 4, so this is equal to 4x only. Now, for this particular problem, for this particular problem you can check that lambda equal to 1 b minus a, this is equal to 1, and here 0 less then equal to x comma s less than equal to 1. So, that means, x and s they are confined within the square with what he says 0 0 1 0 1 1 0 1, then this kernel modulus K x comma s, that is equal to modulus 2 x plus 3 s, this is less then equal to 5.

And therefore, the quantity modulus lambda, so this is our L 2 modulus lambda, L 2 times b minus a, this is equal to 5, this is not less than equal to 1. So, that means, although this condition is not satisfied, still solution of the given problem exist. So, that means, the condition modulus lambda L 2 times b minus a, that is less than 1 is only the sufficient condition, but not the necessary condition.

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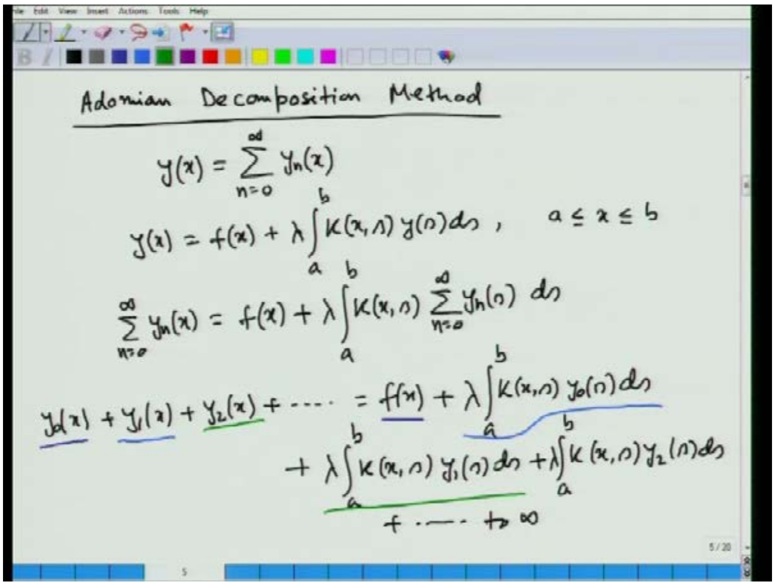

So, now, we consider the Adomian decomposition method adomian decomposition method for solving Fredholm integral equation of the second kind.

We are not going to discuss about the convergence of this scheme, but you can get this things in several books, the point is that we are assuming solutions of the given problem exist in the format y x equal to summation n runnings from 0 to infinity y n x, this is the targeted solution. Now, if we substitute these expression in the given equation, so that means, y x is equal to f x plus lambda integral a to b K x comma s y s d s. So, assuring the existence of solution for this problem, and solution can be obtained in the form y x equal to sigma n runnings from 0 to infinity y n x.

So, we can get summation n runnings from 0 to infinity y n x, that is equal to f x plus lambda integral a to b K of x comma s sigma n runnings from 0 to infinity y n s d s. Now, assuming the uniform convergence of this series, sigma n runnings from 0 to infinity y n x, we can and also the quantity K x comma s is square integrable, we assume that this summation and integration sign can be interchanged. So, after interchanging and expanding the series on the both sides, we can write y 0 x plus y 1 x plus y 2 x plus dot dot, this is equal to f x plus lambda integral a to b K of x comma s y 0 s d s, this is the first term. Then, plus lambda integral a to b K of x comma s y 1 s d s plus lambda integral a to b K x comma s y 2 s d s plus dot dot up to infinity.

Now, in order to define a recursive scheme, in order to find out this y 0 x y 1 x y 2 x and so on explicitly, such that their sum will be giving us a solution for the given problem. So, we have to equate each term on the left hand side with one terms of the other hand side, in order to get some recursive method. And for this purpose, we can equate in this way, first of all we can equate y 0 x with f x on the right hand side. So, that means, now y 0 x is known, with this known y 0 x, we can calculate y 1 x, if we equate it with this term, that is lambda integral a to b K x comma s y 0 s d s.

Then equating, now y 1 x is known because y 0 x equal to f x, y 1 x equal to lambda times a to b K x s y 0 is d s. So, y 1 x is known, and then you can calculate y 2 x equal to lambda integral a to b K x comma s y 1 s d s and so on.

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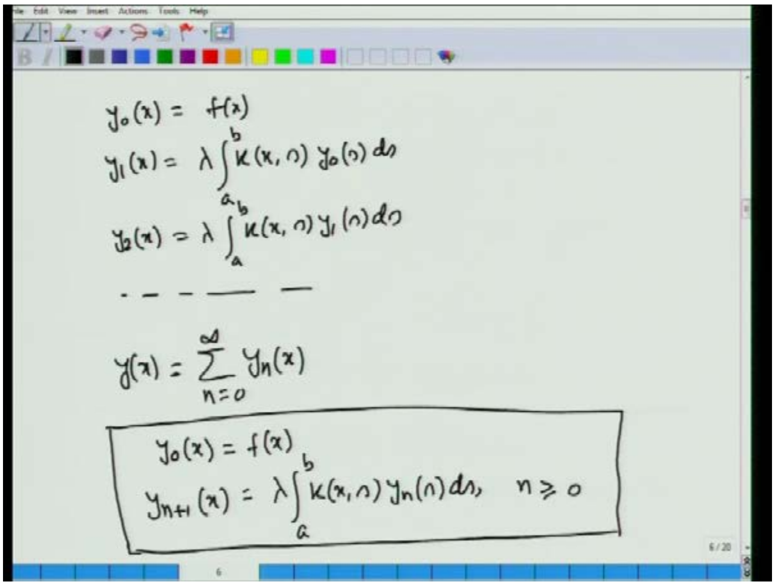

So, that means, recursively we can get the terms, that is y 0 x equal to f x, then y 1 x equal to lambda integral a to b K of x comma s y 0 s d s, then y 2 x that is equal to lambda integral a to b K x comma s y 1 s d s and so on. So, evaluating this y 0 x y 1 x y 2 x and so on, you can get the solution of the given problem, y x equal to sigma n running's from 0 to infinity y n x, and depending upon that problem you are going to solve by this method and iterates you are getting, you will be having the answer whether this solution will comes out to be a closed form or not, that completely depends upon the problem involved with.

And so, the concise recursive scheme is that, we can calculate y 0 x by equating it will f x, and then successive iterates that is y n plus 1 x equal to lambda integral a to b K of x comma s y n s d s, and this result is valid for n greater then equal to 0. So, this is the scheme, that is Adomian decomposition method to solve the Fredholm integral equation of the second kind of course, it is non homogeneous equation.

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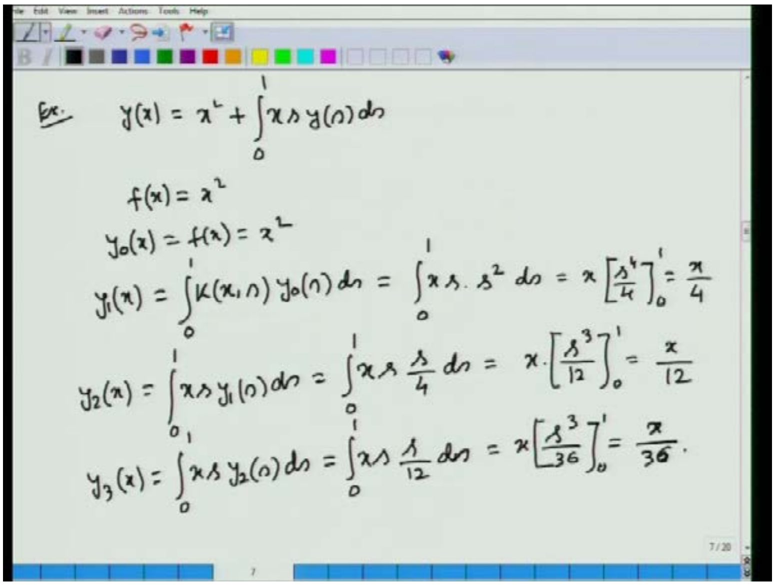

So, just for an example, we considered this example that y x, this is equal to x square plus integral 0 to 1 x s y s d s.

Now, here this x s is actually separable kernel, K x comma s is x s, so clearly it is a separable kernel, so as per the scheme, here f x equal to x square. So, therefore y 0 x equal to f x that is equal to x square. Next, we can calculate y 1 x is equal to integral 0 to 1, as per definition this is K x comma s y 0 s d s, now y 0 x is actually equal to x square. So, you will be having integral 0 to 1 x s, this is for kernel then s square this is actually y 0 s d s and after integration, you will be having x s to the power 4 by 4 integral 0 to 1. So, this is equal to x by 4, so this is the first iterate actually by substituting y 0 x there.

Next, you can calculate y 2 x, that is equal to integral 0 to 1 x s y 1 s d s. So, this is equal to integral 0 to 1 x s times s by 4 d s. So, this gives you x s cube divided by 12 with limit 0 to 1, so this is equal to x by 12. And if we calculate one more term, then we can understand the trend about this iterates, that is y 3 x equal to 0 to 1 x s y 2 s d s, that is equal to integral 0 to 1 x s multiplied with s by 12 d s. So, this is equal x times s cube divided by 36 limit 0 to 1, so this is equal to x by 36.

So, therefore, second iterates y 2 x is x by 12, can be written as 1 by x by 4 into 3, then the third iterate y 3 x is x by 36. So, that can be written a x by 4 into three square.

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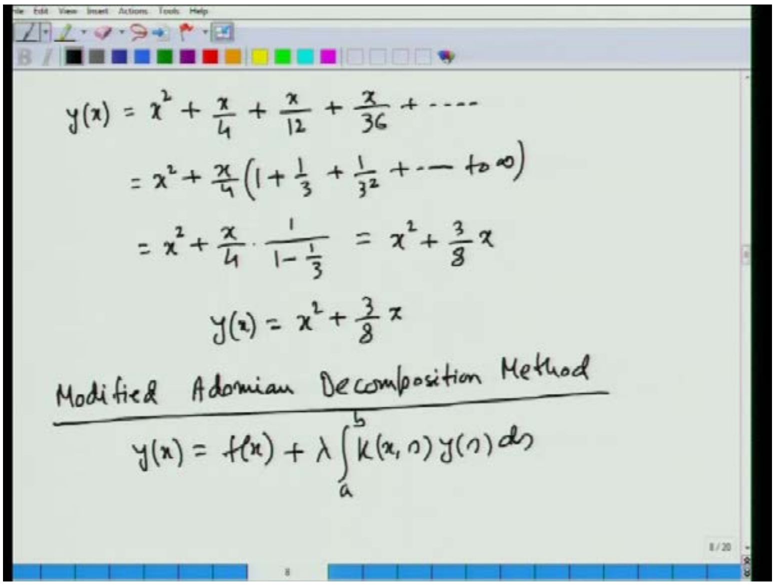

Now, if we consider the infinite sum, then we will be having y x, this is equal to x square, this is actually y 0 x plus y 1 x is x by 4 plus y 2 x x by 12 plus y 3 x x by 36 plus so on. So, therefore, you will be having x square plus x by 4 multiplied with 1 plus one third plus 1 by 3 square plus dot dot up to infinity. And this is an infinite geometric series, that is 1 plus one third plus 1 by 3 square plus dot dot up to infinity, with first term a as 1 and common ration is one third and therefore, its sum will be x square plus x by 4 multiplied by 1 by 1 minus one third.

So, this will results in two third and therefore, required solution is x square plus 3 by 8 x. So, the function x square plus 3 by 8 x, this is solution for the given problem that we have obtained using Adomian decomposition method. Now, you can recall, we have discussed this technique for Volterra integral equation, and there was another Adomian method, that was modified Adomian decomposition method. So, parallely what we have discussed in case of Volterra integral equation, similar method we can discuss, this is as follows, the modified Adomian decomposition method.

Similarly, what we have done in case of Volterra integral equation, this given equation is y x equal to f x plus lambda times integral a to b K x comma s y s d s. The main idea involved with that, we have to divide f x into two parts, f 1 x plus f 2 x, only for the reason that such choice of y 0 x equal to f 1 x, sometimes gives us opportunity to find out this integral that is a to b K x comma s y 0 s and so on very easily, which we will be discussing with help of suitable example.

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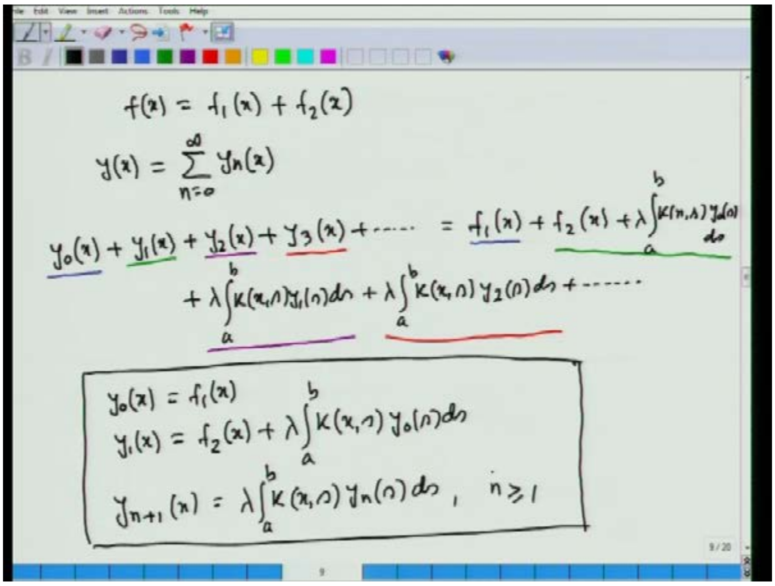

So, the concept is that, we can divide f x into two parts: f 1 x plus 2 x. So, similarly what we have done in earlier for the standard Adomian decomposition method, we can take y x that is equal to sigma n runnings from 0 to infinity y n x. And after substituting into the equation, and with the assumption that f x can be divided in two parts, f 1 x plus f 2 x , we will be having this result that is y 0 x plus y 1 x plus y 2 x plus y 3 x plus dot dot up to infinity. This is equal to f 1 x plus f 2 x plus lambda integral a to b K of x comma s y 0 s d s plus lambda integral a to b K of x comma s y 1 s d s plus lambda integral a to b K of x comma s y 2 s d s plus dot dot. And in this case, we will be equating first y 0 x with f 1 x. So, that means, y 0 x is known, next we equate y 1 x with f 2 x plus lambda integral a to b K x comma s y 0 s d s.

So, that means, we know y 0 x equal to f 1 x. So, substituting y 0 s, we can evaluate y 1 x, then y 2 x is equal to lambda integral a to b K x comma s y 1 s d s and so on. So, therefore, next we have to equate y 3 x with lambda integral a to b K x comma s y 2 s d s and so on. So, in this case, we will be having these results, that is y 0 x is equal to f 1 x then y 1 x, this is equal to f 2 x plus lambda integral a to b K x comma s y 0 s d s and then, rest of the iterates will be y n plus 1 x, that is equal to lambda integral a to b K x comma s y n s d s.

Now, you have to keep in mind that in these case, this iterates y n plus 1 x equal to lambda integral a to b K x comma s y n s d s, these follows from 2 and onwards where suffix of y is 2 and onwards. So, in this case n will be greater than equal to 1. So, this is actually the concise recursive scheme to obtain the solution of the Fredholm integral equation of second kind, which is non homogeneous by using modified Adomian decomposition method.

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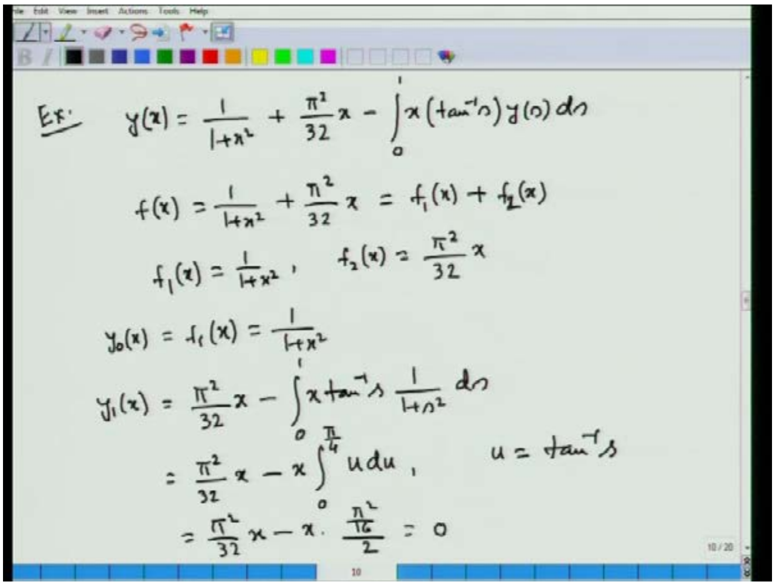

Now, for this problem, we consider again one example, where you can understand how to divide this two functions. This example is y x, this is equal to 1 by 1 plus x square then plus pi square divided by 32 x minus integral 0 to 1 x tan inverse s y s d s, this is the given problem. And now, you can see that x can be taken out of the integral sign, this tan inverse s y s, this can be easily integrated if we having initial iteration that is y 0 s is equal to 1 by 1 plus s square that is already present here.

So, that means, the given f x for this problem, f x equal to 1 by 1 plus x square plus phi square divided by 32 x can be taken as sum of two functions; f 1 x plus f 2 x where f 1 x, this is equal to 1 by 1 plus x square. And obviously, f 2 x will be equal to phi square divided by 32 multiplied with x. So, therefore, as per modified Adomian decomposition scheme, y 0 x, this is equal to f 1 x, so that means, this is equal to 1 by 1 plus x square.

And therefore, y 1 x, this is equal to pi square divided by 32 x minus integral 0 to 1 x tan inverses 1 by 1 plus s square d s. So, this will be equal to pi square divided by 32 x minus x can be taken out from the integral sign, and we can change the variable that is 0 to pi by 4 u d u where u equal to tan inverse s. And if you calculate this integral, then it will comes out to be pi square by 32 x minus x into pi square by 16 whole divided by 2, actually it will be u square by 2. So, pi square by 16 by 2 and this is equal to 0, now if y 1 x is equal to 0.

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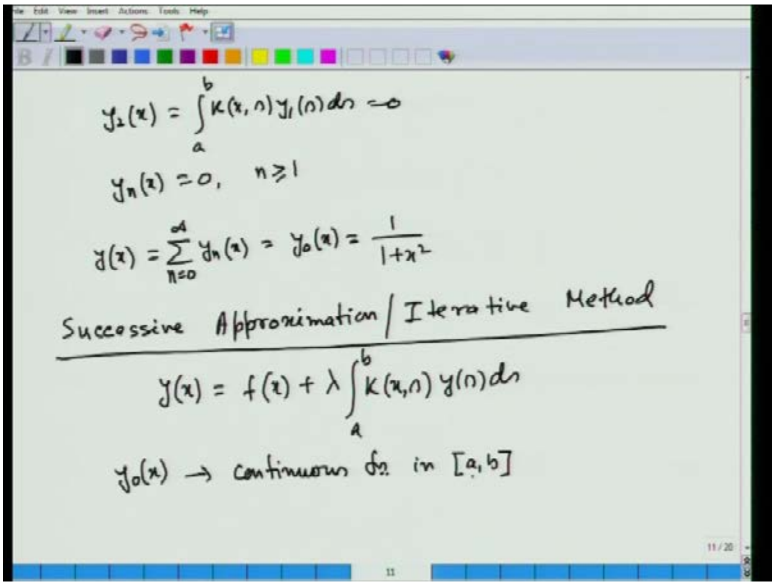

So, therefore, y 2 x that is equal to integral a to b K x comma s y 1 s d s, this is equal to 0. So, that means, we can write y n x, this is equal to 0 for n greater than equal to 2 and of course, this is valid for n equal to 1 also, that we have already derived. So, instead of greater than equal to 2, we can write this is greater than equal to 1. And therefore, this y x is equal to summation n runnings from 0 to infinity, y n x is nothing but simply y 0 x, and that is equal to 1 by 1 plus x square, this is the solution for the given problem by modified Adomian decomposition method.

So, next we are going to consider another method of solving this equation, that is successive approximation method, or in some of the books, you can find this as iterative method. This concept, we have also discussed in case of Volterra integral equations. So, the same technique we are going to apply here. So, the point is that we have to solve this equation, y x is equal to f x plus lambda integral a to b K of x comma s y s d s.

So, our target is first of all, we assume an initial guess for y and that will be denoted by y 0 x, you can recall in case of Volterra integral equation, we have described this may be equal to y 0 x equal to may be 0, may be 1, may be x. And in general, we can say any continuous function that is bounded within the closed interval a comma b will the serve the purpose, but of course, in order to solve any particular problem, you have to be careful for the choice of this y 0 x, because using y 0 x and substituting it into the integral equation on the right hand side; that means, under the integral a to b K x comma s y s d s, we can calculate first approximation y 1 x.

And using this first approximation y 1 x on the right hand side, we can get the second approximation y 2 x and so on. And ultimately our target is the solution will be y x, that is nothing but limit n tends to infinity y n x. So, the scheme is that we are assuming y x, that is any continuous function in closed interval a comma b that is 1.

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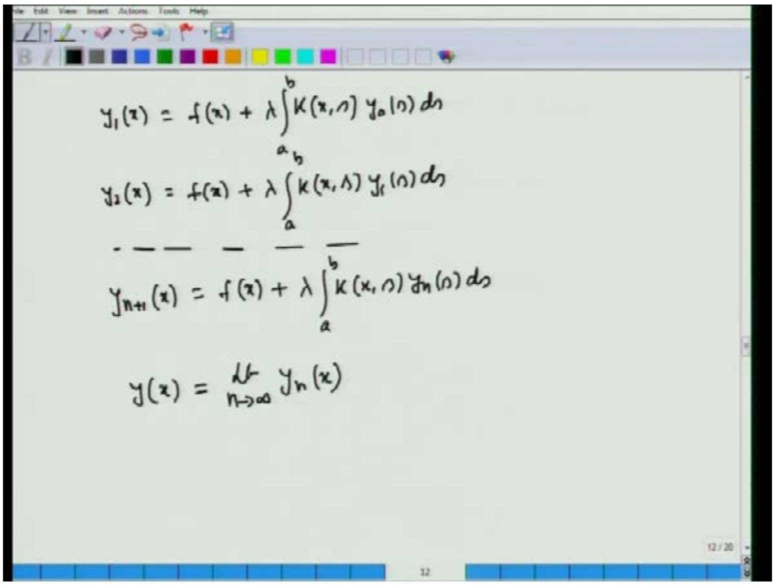

And then, we can calculate y 1 x, this is equal to f x plus lambda integral a to b K x comma s y 0 s d s, then y 2 x, this is equal to f x plus lambda integral a to b k x comma s y 1 s d s and so on. So, that means, in general, y n plus 1 x, that is equal to f x plus lambda integral a to b K x comma s y n s d s, and solution to the given problem y x is nothing but limit n tends to infinity y n x, this is going to be the solution of the given problem. And now, we are going to consider the convergence of this particular scheme.

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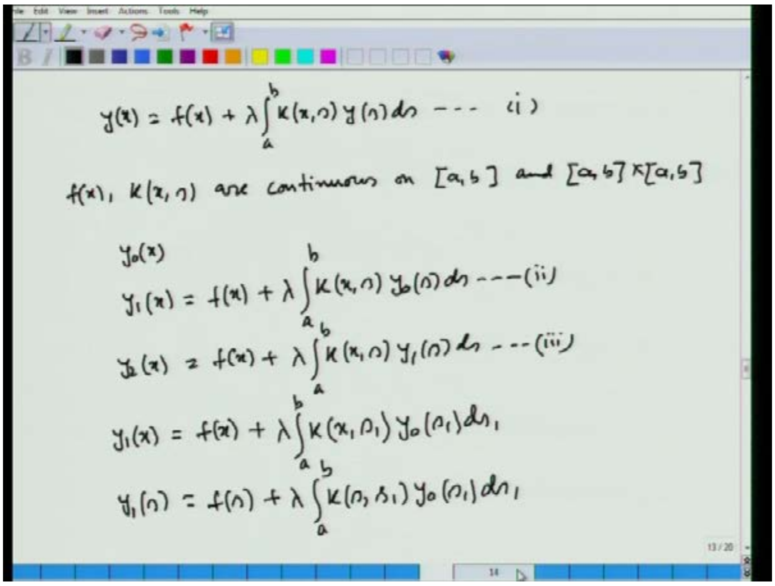

So, for convergence criteria, first of all we denote this equation y x equal to f x plus lambda integral a to b K of x comma s y s d s, this as number 1. Secondly, f x and K x comma s, they are continuous they are continuous on the interval a comma b, and a comma b cross a comma b respectively, and with this we are going to proceed for the proof of the convergence. Now, before going to prove the convergence, we need some notation and convention that we have to prepare in order to prove the result. So, first of all, you can recall that y 0 x is any arbitrary function from for which we can calculate y 1 x.

So, now y 1 x, this is equal to f x plus lambda times integral a to b K of x comma s y 0 s d s, call it 2. Then y 2 x, this is equal to f x plus lambda integral a to b K x comma s y 1 s d s. Now for the proof, we actually have to write this expression, that is y 2 x, by substituting the expression for y 1 x from equation 2. Now, in order to do this, what we can write from here without any loss of generality, y 1 x can be rewritten as f x plus lambda integral a to b K of x comma s 1 y 0 s 1 d s 1, because s is here the dummy variable.

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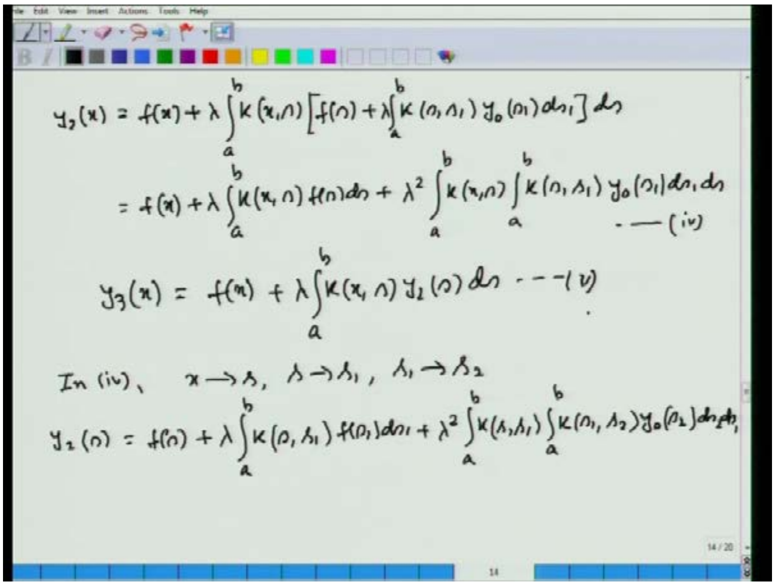

So, without any loss of generality we can change first s to s 1. And therefore, y 1 s; this will be equal to f s plus lambda integral a to b K of s comma s 1 y 0 s 1 d s 1. Substituting this expression for y 1 s into 3, we can write y 2 x, that is equal to f x plus lambda integral a to b K of x comma s, then f s plus integral a to b premultiplied by lambda k of s comma s 1 y 0 s 1 d s 1 d s. So, this is equal to f x plus lambda integral a to b K of x comma s f s d s plus lambda square integral a to b K of x comma s, then integral a to b K of s s 1 y 0 s 1 d s 1 d s.

Next, we are going to substitute these expression in the third iterate y 3 x equal to f x plus lambda integral a to b K of x comma s y 2 s d s. So, we need the expression for y 2 s, already we have the expression for y 2 x. Now, in order to write down the expression for y 2 s, we can use this change of variable in 4. If we call this expression as 4, that in 4, we are introducing this change of variables that, this x goes to s, s goes to s 1 and s 1 goes to s 2, then y 2 s, this is equal to f s plus lambda integral a to b k of s comma s 1 f of s 1 d s 1 plus lambda square integral a to b K of s comma s 1, integral a to b K of s 1 comma s 2 y 0 s 2 d s 2 d s 1, this is the expression.

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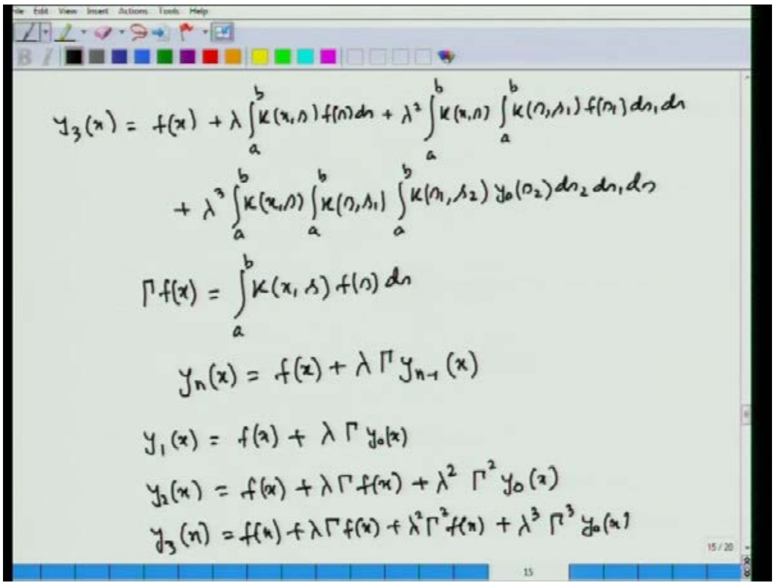

Now, with this expression y 2 s, if we substitute in 5, in these expression if we substitute in 5, then we will be having that y 3 x, this is equal to after substitution and rearrangement, it will be f x plus lambda integral a to b K of x comma s f s d s plus lambda square integral a to b K of x comma s integral a to b K of s comma s 1 f of s 1 d s 1 d s plus lambda cube integral a to b K of x comma s integral a to b K of s comma s 1 integral a to b K of s 1 comma s 2 y 0 s 2 d s 2 d s 1 d s.

And now, if we introduce this notation that gamma f x, that is the integral operator gamma, that is defined by integral a to b K of x comma s f s d s. So, therefore, the general iterative scheme can be written as y n x is equal to f x plus lambda gamma operated up on y n minus 1 x, this is the general iterated scheme, and if we use this notation from expression what we have obtained for y 1 x y 2 x and y 3 x, then you can find in terms of this integral operator we can write y 1 x, this is nothing but f x plus lambda gamma y 0 x, this is the first one.

Then y 2 x, this is equal to f x plus lambda gamma operated up on f x plus lambda square gamma 2, this will be operated up on y 0 x. Similarly y 3 x, this is equal to f x plus lambda gamma f x plus lambda square gamma 2 f x plus lambda cube gamma 3 y 0 x and so on.

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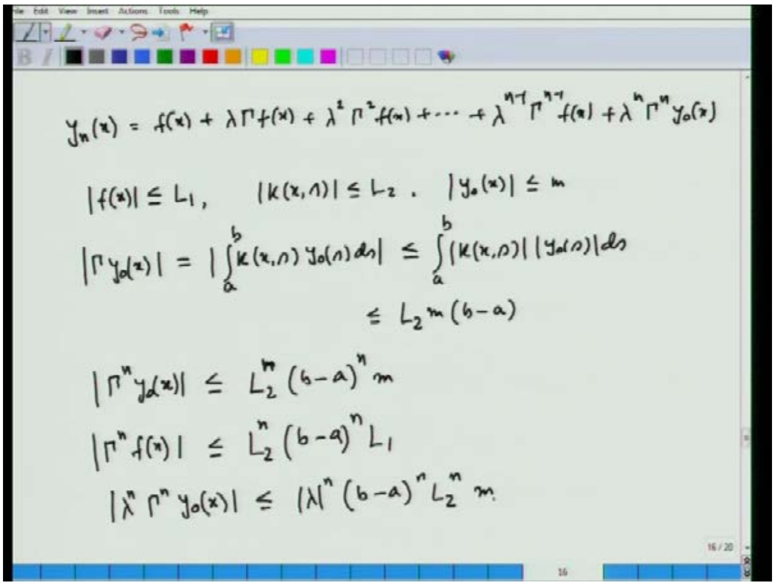

So, at the nth iterate we can find y n x, this is equal to f x plus lambda gamma f x plus lambda square gamma 2 f x plus dot dot up to lambda to the power n minus 1 gamma n minus 1 f x plus lambda n gamma n y 0 x.

And now, our target is to prove that limit n tends to infinity lambda to the power n gamma n y n x, this goes to 0 under certain condition and therefore, y n x will be f x plus lambda gamma f x plus lambda square gamma 2 f x plus dot dot up to infinity. And for this purpose, we assume that modulus f x less than equal to L 1 because f x is bounded, already we have defined that modulus K comma s less than equal to L 2 and modulus y 0 x, this is less than equal to m.

So, therefore, gamma y 0 x, this will be equal to modulus integral a to b K of x comma s y 0 s d s, this is less than equal to integral a to b modulus K x comma s modulus of y 0 s d s, and this will be less than equal to L 2, then m times b minus a this one. And in general, gamma n y 0 x repeated application of this argument will leads us to this will be less than equal to L 2 to the power m b minus a whole to the power n, this will be actually n, L 2 to the power n b to the power n b minus a whole to the power n multiplied by m. And similarly, modulus of gamma n f x, this will be less than equal to L 2 to the power n b minus a whole to the power n, this multiplied with L 1.

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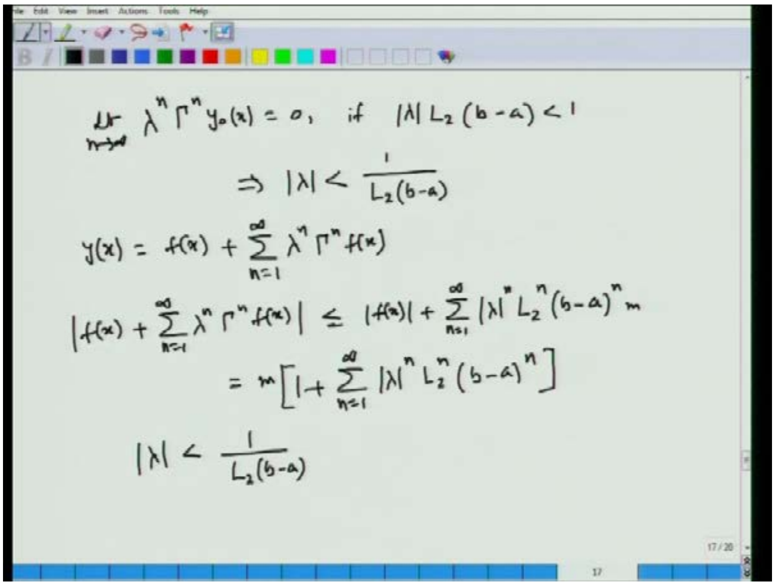

So, therefore, modulus lambda to the power n gamma n y 0 x, this will be less than equal to modulus lambda whole to the power n b minus a whole to the power n, this multiplied by L 2 to the power n into m. And therefore, this limit converges to 0, that is limit n tends to infinity lambda to the power n gamma n y 0 x, this will be equal to 0 whenever modulus lambda times L 2 times b minus a less than 1 and these actually implies a restriction on lambda that is modulus lambda less than 1 by L 2 times b minus a, this is the convergence criteria.

And whenever this condition is satisfied, then we can say y x is equal to f x plus sigma n runnings from 1 to infinity lambda to the power n gamma n f x. And we can prove that this series f x plus sigma n runnings from 1 to infinity lambda to the power n gamma n f x, this series converges absolutely and uniformly because this is less than equal to modulus f x plus sigma n runnings from 1 to infinity modulus lambda whole to the power n L 2 to the power n b minus a whole to the power n, this is into m. So, this is equal to m times 1 plus sigma n equal to 1 to infinity modulus lambda whole to the power n L 2 to the power n b minus a whole to the power n.

So, this is again a geometric series and under the same condition that is modulus lambda less than 1 by L 2 times b minus a, this has a limit that is convergent and it has a limit that can be obtained as equal to 1 and r is nothing but modulus lambda L 2 times b minus a. So, you can derive the condition and therefore, this is a infinite geometric series which is convergent whenever modulus lambda less than 1 by L 2 times b minus a and hence the scheme, that is f x plus sigma n runnings from 1 to infinity lambda to the power n gamma n f x, this converges uniformly and absolutely.

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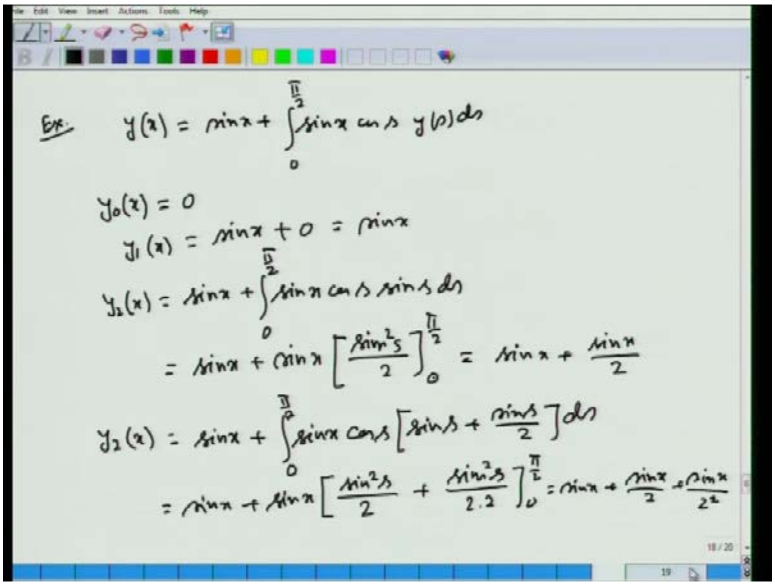

And before ending, we just consider one example, that how to apply these scheme. Let us take y x equal to sin x plus integral 0 to pi by 2 sin x cos s y s d s. And here, you can take y 0 x this is equal to for example 0, if you take y 0 x equal to 0, then y 1 x will be sin x plus 0 that is equal to sin x because contribution from this integral will be 0 because y 0 s is 0 then y 2 x, this will be equal to sin x plus integral 0 to pi by 2 sin x cos s, this is the kernel and y 1 x is sin x, so this will be sin s d s.

So, this is equal to sin x plus sin x sin square s, this divided by 2 integral limit from 0 to pi by 2. So, this will be equal to sin x plus sin x divided by 2. I am not writing here this is equal to 3 by 2 sin x, because if you calculate the next iterate y 2 x, this will be sin x plus integral 0 to pi by 2 sin x cos s. Now, here you will be having sin s plus sin s by 2 d s. So, this is equal to sin x plus sin x will come out, then sin square s divided by 2 and here you will having this sin square s divided by 2 into 2 limit 0 to pi by 2. So, this expression will be equal to sin x plus sin x by 2 plus sin x by 2 square.

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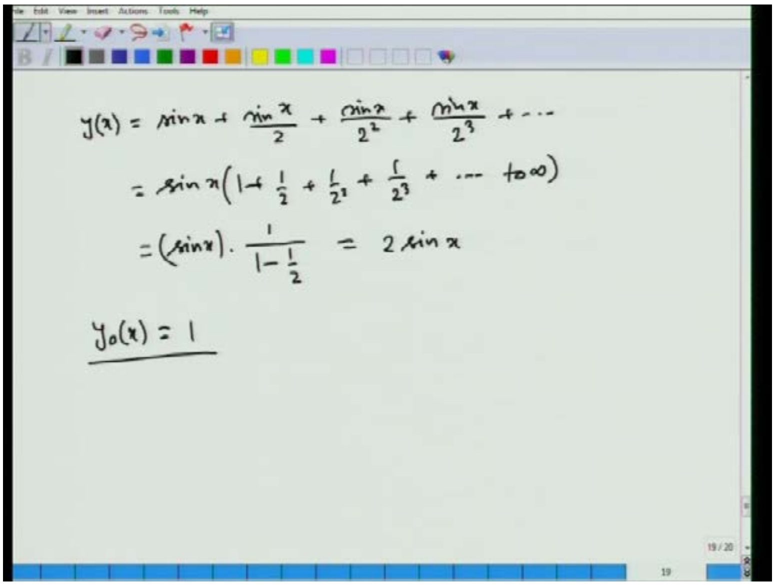

So, therefore, the sum will be y x, this is equal to sin x plus sin x by 2 plus sin x by 2 square plus sin x by 2 cube plus dot dot. So, this is sin x multiplied with 1 plus half plus half square plus half cube plus dot dot up to infinity. This is again an infinite geometric series with first term a and common ratio that is half. So, result will be sin x this multiplied with 1 by 1 minus half. So, this is equal to 2 sin s

And just for as example, you can verify the same result you can obtain by assuming y 0 x equal to 1. In that case, y 1 x will not come out to be 0, it will be sin, x y 2 x will be sin x plus sin x by 2 and so on. And ultimately the sequence y n x, this will be convergent to y x as n tends to infinity and the answer will be same what we have obtained here, that is 2 sin x.

So, in today's lecture, we have discussed about the Adomian decomposition method and modified Adomian decomposition method for solving non homogeneous Fredholm integral equation. And then we have proved the iterative scheme, that is convergent absolutely and uniformly convergent for Fredholm integral equation and we have discussed one example, by which you can understand how this solution actually obtain for the prevent Fredholm integral equation. Thank you for your attention.