Calculus Of Variations and Integral Equation

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Lecture No. #26

Welcome viewers to the six lectures of lecture series on integral differential equation under NPTEL course. Now, in today's discussions, we are going to discuss about Adomian decomposition method to solve the integral equation; that is a special type of integral equation that we are considering in last few lectures, that is Volterra integral equations of second kind. Of course, these Volterra integral equations of second kind is a non homogenous type.

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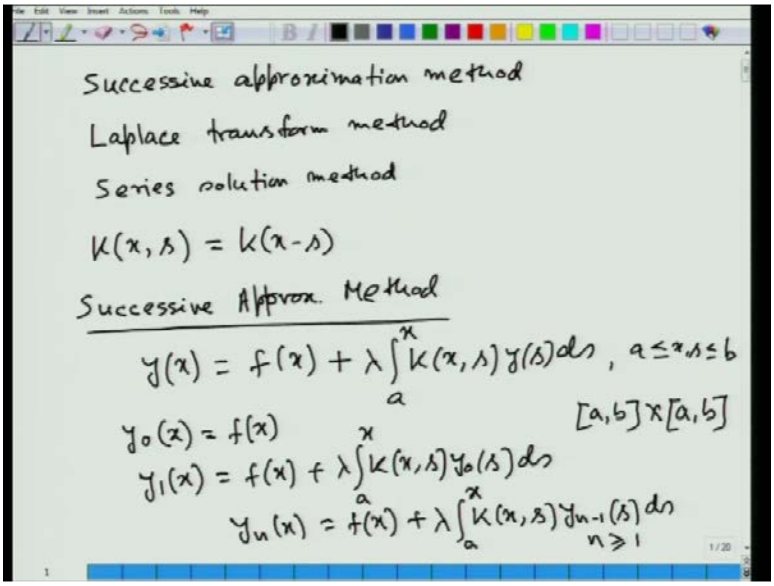

Now, you can recall in last 2 lectures, we have discussed about different methods for solving Volterra integral equations of second kind, which are non homogenous equation. Using the methods first of all we have use the method that is known as successive successive approximation method. Then we have considered the Laplace transform method, and lastly we have considered the series solution method. Among these three methods, the second method that is Laplace transform method deals with the problem, where cardinal of the integral equation satisfies a specifics form. That means, when the cardinal of the integral equation can be written into the form k(x minus s), that is the function of k(x,s) 2 variable functions can be expressed as a single variable function say t, where t is actually x minus s. That means, the entire cardinal can be written as a variable x minus s, taking x and s together under this factor that is x minus s.

So, these becomes a single variable function if x minus s is replaced by t, if cardinal satisfies these condition, then Laplace transform method is applicable. Just for a quick recapitulation, in case of successive approximation method what we have done - the successive approximation method, we have started with that y(0) is equal to f(x), where the given equation is y(x) equal to f(x) plus lambda integral a to x k(x,s) y(s) ds, where f(x) is continuous. In this case a less then equal to (x,s) less than equal to b, this f(x) is continuous over the interval (a,b), and k (x,s) is also continuous over the square of length sides of length b minus a; that means, over a region (a,b) cross (a,b), and partial derivative k(x,s) with respect to x is continuous over the same domain.

Then, first of all we started with y(0) equal to f(x), and you can recall that I have given some example, and also mention. Instead of considering y 0(x) equal to f(x), you may consider these equal to zero; that means, initial 0 approximation as y 0(x) is equal to 0 or you may considered y 0(x) equal to 1 or y 0(x) equal to x. But in general we can follow for the time being that y 0(x) equal to f(x) as the 0 approximation, then you can calculate y1(x) by substituting y 0(x) on to the right hand side of the this integral equation. So, that means, this is equal to f(x) plus lambda times integral a to x k of x comma s y 0(s) ds, and as a general step we can right y n(x) is equal to f(x) plus lambda times integral a to x k(x,s) y n minus 1(s) ds.

And these result is valid for n greater than equal to 1, and also we have discussed in detail the convergence of this method. So, that means, at every step we are getting a refinement of the function wise. So, first of all you are studied with y 0(x), then we have obtained y1(x), then we can calculate y2(x) and in general we can calculate y n(x).

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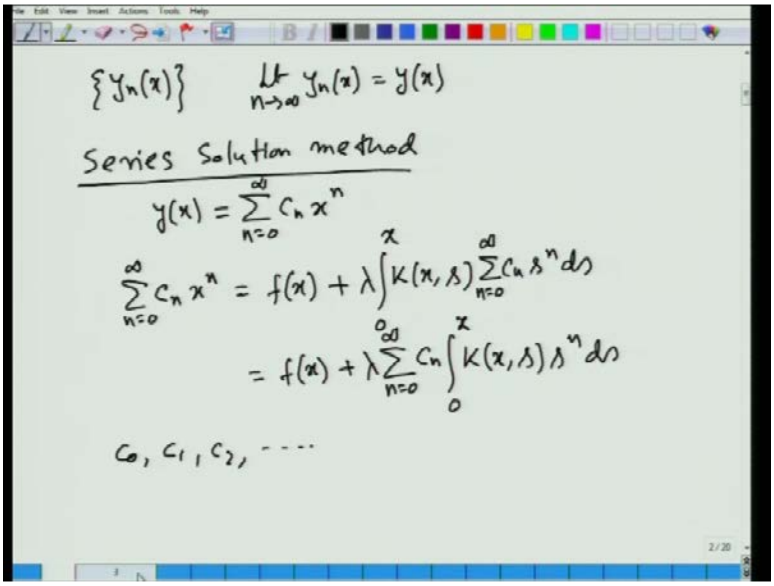

And of course, these sequence of function thus generated that is y n(x), these sequence converges uniformly as we have discussed; and if these sequences convergence uniformly, and if limit intense to infinity y n(x) this is equal to y(x), then this limit function is actually solution of the given Volterra integral equation. Now, in the second case that is in case of series solution method, in case of series solution method, we have assumed that solution of the integral equation can be obtained into the form, that is y(x) is equal to summation n running's from 0 to infinity C n x to the power n.

So, that means, it can be expressed as a power series of x, and up on substitution these series into the integral equation, we can get summation n running's from 0 to infinity C n x to the power n, this is expression after replacing y on the left hand side, on the right hand side f(x) remains annotator. And then we can write lambda integral 0 to x k of(x,s) summation n running's from 0 to infinity C n x to the power n ds, and from here assuming the convergence of the series we can inter change the summation, and integral sign. And we can write n equal to 0 infinity C n integral 0 to x k of(x,s) s n ds, and from here collecting the coefficient of equal powers of x, we can find out C 0, C 1, C 2 and so on.

And of course, equating some general term of the form either x to the power n or x to the power n plus 1 or x to the power n plus 2 as for your convenience from the both sides, we can calculate the redundancy consultation. After calculating these C 0, C 1, C 2, you can construct the series y(x), and if this series converges to a continuous function then that particular continuous functions is a solution to the integral equation. So, in this case sequence of function that is y n(x) converging to the solution, in these case we are calculating C 0, C 1, C 2 and so on, such that the summation in running's from 0 to infinity C n x to the power n these converges to the solution of the given problem.

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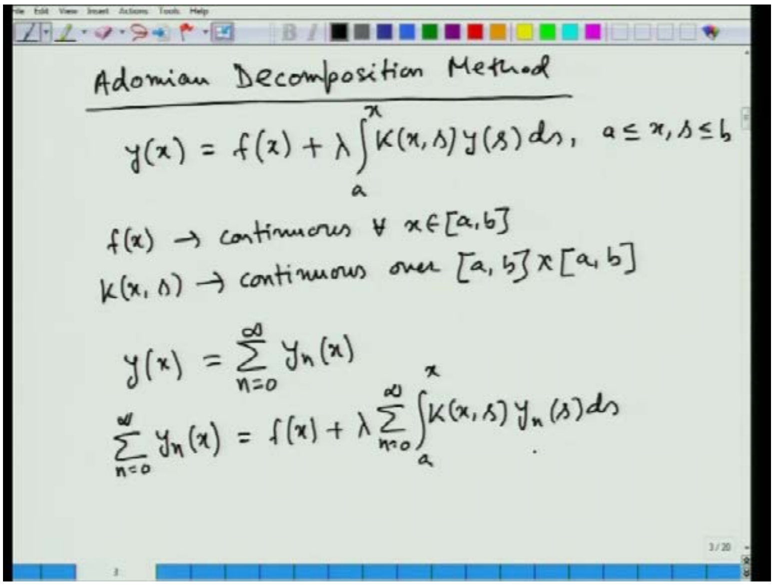

Now, we are going to considered the method that is called Adomian decomposition method. By theses Adomian decomposition method, we are going to solve this integral equations y(x) is equal to f(x) plus lambda integral a to x k of(x,s) y(s) ds. Again where a less than equal to (x,s) less than equal to b, where b is a finite number. For the applicability of this method, we need continuing of effects that is f(x) is continuous for all x belong to (a,b), and k(x,s); this is continuous over the square domain a, b cross a, b. If this condition is satisfied, then Adomian decomposition method is applicable.

Now, what is this Adomian decomposition method? This Adomian decomposition method is a first appeared in the area of solving ordinal differential equation, partial differential equation, integral equation, as well as some non-linear problems also, where the solution of y(x) is assumed to be into these summation format; that is n running's from 0 to infinity y n(x). And here we are going to construct one iterative method, such that after every step you will be having one terms resulting from the performance of the integration on the right hand side; that one terms of this particulars series. Such that this summation of the series convergences to the solution of the given problem.

Now, what is method first we will describe the method, and after that I will be coming to the proof of the convergence of this infinite series. Method says that we assumed this series convergence; that means, there exist solutions for this problem - and solution can be expressed as summation over n running's from 0 to infinity y n(x). We are not assuming any particular form of y n(x), that is most important point that you have to understand here. In case of series solution method, you have assume this will be of the form n running's from 0 to infinity C n x to the power n. So, that means, if power series of x. Now, at this point we are not at all assuming any particular form for y n(x), only thing is that every each of them will be continuous.

Secondly, in case of successive approximation method using the approximation of the last step, we can get a further modification or better solution for the given problem by approximation step by step. Now, here we are going to find, these functions y 0, y 1, y 2, y 3, and so on, in a systematic manner; such that up on summation of all these quantities will be having solution to this problem. And of course, in due course of time I will explain what is the advantage of this method, and also make a comparative study of this method with the other available methods.

So, if you assumed this y(x) equal to sigma n running's from 0 to infinity y n(x) is a solution of this problem. So, that means, this will satisfy this integral equation. So, if we substitute there, then we can find n running's from 0 to infinity y n(x), this is equal to f(x) plus lambda times summation in running's from 0 to infinity integral a to x k of(x,s) y n s ds.

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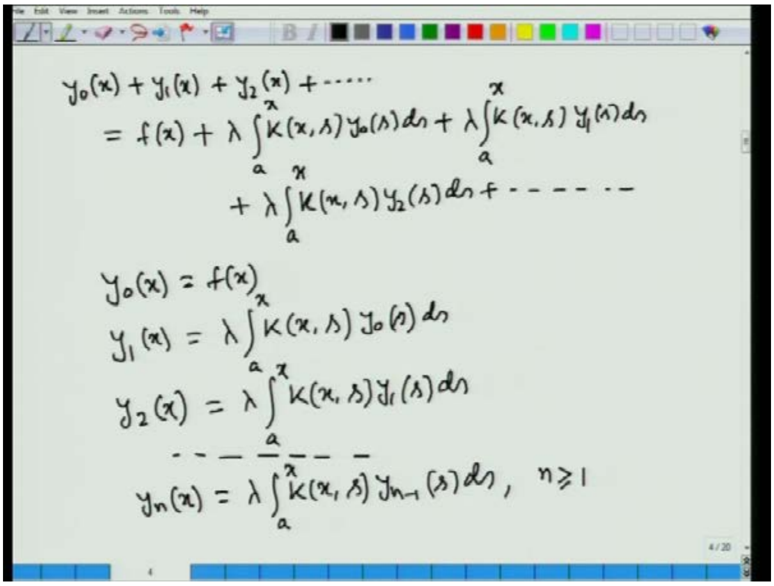

If we write the terms of these series explicitly, then we can find y 0(x) plus y1(x) plus y2(x) plus dot dot, this infinite series is equal to f(x) plus lambda times integral a to x k of(x,s) y 0(s) ds plus lambda times integral a to x k(x,s) y 1(s) ds plus lambda times integral a to x k(x,s) y 2(s) ds plus dot, dot. Now, our target is to construct a iterative method. Such that which some initial guess for y 0, we can calculate y1(x) once we have obtain y1(x), then we can calculate y2(x) and so on. So, here we are assuming y(x) equal to f(x) directly, this y(x) y 0 equal to f(x).

Now, if we equate this as term by term. So, first time on the left hand side is equal to first on the right hand side, then second term on the left hand side is equal to second term on the right hand side, proceeding in this way we can get these results that is y1(x) is equal to lambda times integral a to x k of(x,s) y 0(s) ds, then third term on the left hand side is y2(x) is equal to lambda times integral a to x k of(x,s) y 1(s) ds, proceeding in this way the general term y n(x), this is equal to lambda times integral a to x k of(x,s) y n minus 1 s ds.

And here n this is greater than equal to 1, because these recursive formula is valid for n equal to 1 and 1 was; that means, y1(x), y2(x) all these iterates can be obtained from the general formula by substituting n equal to 1, 2, 3, and so on, only there is separate definition for y 0(x). Now, before going to considered any particular example will be considering the convergence of this method. So, we have started with the assumption that y 0(x) is equal to f(x); you can calculate y1(x), because for the given problem cardinal x comma s is known. So, substituting y 0 is here, we can calculate y1(x), then y2(x) can be calculated up on substituting y 1(s) here. So, proceeding in this way you can get every iterates.

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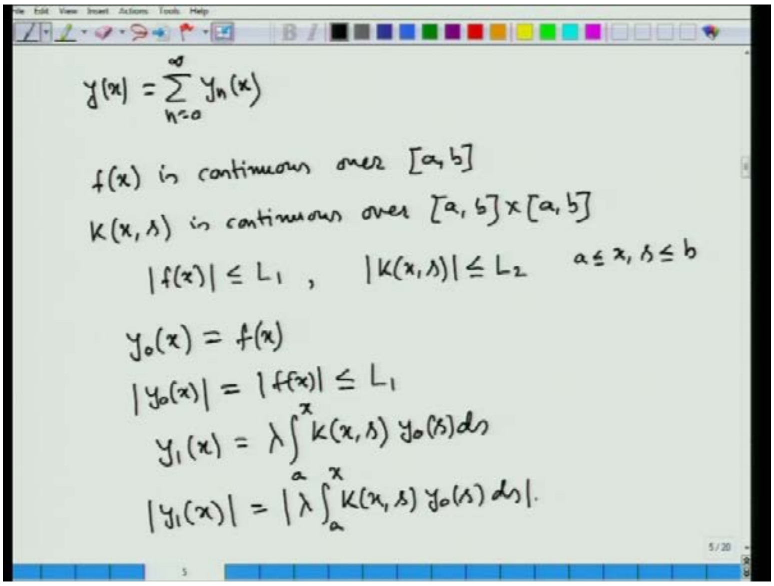

And hence the solution to the given problem is given by y(x) is equal to sigma n running's from 0 to infinity y n(x); this will be a closed form of function depending upon the problem, am if we are unable to identify any function in the closed form corresponding to this infinite sum, that is y 0(x) plus y1(x) plus y2(x) plus dot, dot up to infinity, then we have to leave the solution as is it is that appeared from the step by step approximation.

Now, we considered the convergence of this problem. You can recall that we have assume that f(x) is continuous we have assume that f(x) is continuous over the close interval (a,b), and k(x,s) this is continuous over the square domain a, b cross a, b; both of these are actually close sets. So, this continuous functions they are continuous about the close sets. So, they are exist to positive constants L 1, and L 2 such that f(x) less than equal to L 1, and modulus of k(x,s) less than equal to L 2. This result is valid for all x and s ranging in this particular range from a to b. Now with these assumption, you can recall f(x) is continuous. So, if f(x) is continuous, so first quantity that we are getting for the series of involved with Adomian decomposition method, this is a continuous function

- y 0(x) is continuous function. Further this y 0(x) satisfies this criteria that modulus of y 0(x) is equal to modulus of f(x) which is less than equal to L 1.

Now, definition for y1(x) this was lambda times integral a to x k of(x,s) multiplied by y 0(s) ds. So, taking modulus we can write modulus of y1(x) that is equal to modulus of lambda integral a to x k of(x,s) y 0(s) ds, this modulus.

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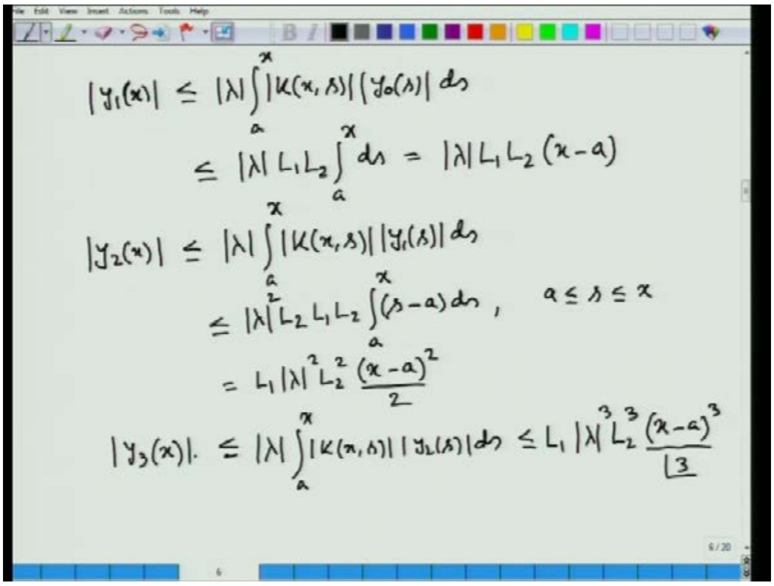

Then taking modulus inside the integral sign, we can write modulus y2(x) this is less than equal to modulus lambda integral a to x modulus of k(x,s) modulus of y 0(s) ds, and using the bounce for this y 0(x) is less than equal to L 1 this modulus, and modulus k(x,s) that is modulus of the cardinal is less than equal to L 2. So, using this results, we can find modulus lambda L 1 L 2 integral a to x ds, and this is equal to modulus lambda L 1 L 2 x minus a. Similarly, calculating sorry I am I have done mistake here, this will be y 1(x0, then y2(x) modulus as usual it will be less than modulus lambda integral a to x modulus k of(x,s), then modulus y 1(s) ds.

And this is less than equal to modulus lambda, just be careful about here modulus k of(x,s) is less than equal to L 2, and modulus y 1(s) this is less than equal to this quantity. So, ultimately we get modulus lambda whole square L 1 L 2 integral a to x s minus a ds, because here x is greater than a, and within this integral you can recall that a less than equal to s less than equal to x. So, we can withdraw modulus sign from x minus a, when it is coming under this integral sign.

So, after performing the integration it is coming out to the L 1 modulus lambda whole square L 2 square x minus a whole square divided by 2. Similarly, for modulus y 3(x), we can calculate this is less than equal to modulus lambda, then integral a to x modulus k of(x,s) modulus y 2(s) ds. And after performing this integration, and substituting this limit we can find this will be less than equal to L 1 times modulus lambda whole cube L 2 cube x minus a whole cube whole divided by factorial 3, this will be the bound for modulus y 3(x).

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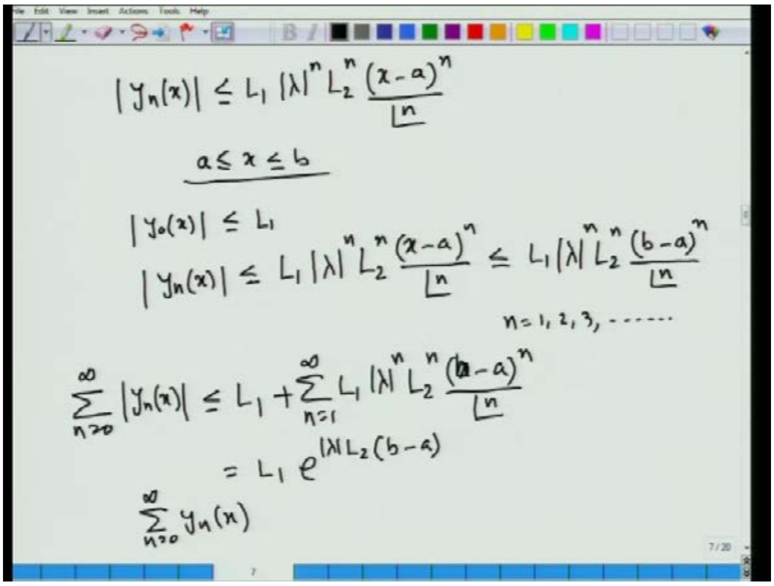

So, in general we will be able to prove that modulus y n(x), this is less than equal to L 1 times modulus lambda whole to the power n L 2 to the power n x minus a whole to the power n divided by factorial n, this is the modulus for y n(x). And now, we can recall that result we have assumed a less than equal to x less than equal to b. So, with this range for x, we can write modulus y 0(x) as usual less than equal to L 1, that we have obtained at the very faster from the assumption on effects, then from general term modulus y n(x) this is less than equal to L 1 times modulus lambda whole to the power n L 2 to the power n x minus a to the power n by factorial n. This is less than equal to L 1 times modulus lambda whole to the power n L 2 to the power n. Now, this x is always less than equal to b. So, therefore, b minus a whole to the power n divided by factorial, n these result is for n equal to 1, 2, 3, and so on.

So, for this entire range, it is valid. And therefore, summation n running's from 0 to infinity modulus of y n(x), we can write this is less than equal to L 1 plus summation n running's from 1 to infinity L 1 modulus lambda whole to the power n L 2 to the power n x minus a whole to the power n by factorial n. And taking L 1 common, we can write this is equal to L 1 times e to the power modulus lambda L 2 b minus a. So, that means, the summation of the modulus of that terms of y, j, x - these are uniformly bounded, and this bound is free from x term.

So, that means, the series summation n running's from 0 to infinity y n(x) converges uniformly. So, this is actually the proof of the uniform convergence of the series sigma n running's from 0 to infinity y n(x). So, based up on the continuity of the function f(x), and continuity of the cardinal x k(x,s), we can prove that these series of Adomian polynomials; these y n(x) sometimes called as Adomian polynomials. Is summation of these Adomian polynomials converges to the function y(x) uniformly, which is a continuous function, because from the every state you can easily verify that once y 0(x) equal to f(x) is continuous. Then if y1(x) will be continuous upon evaluating the integral, similarly substituting y1(x) into the formula you can get y2(x) is continuous. So, every y n(x) is continuous.

And also they are uniformly bounded, so this proves the uniform converges of the series to a continuous function, and if you call this particular continuous function as capital y(x), then y(x) is equal to capital y(x) is the solution of the targeted problem.

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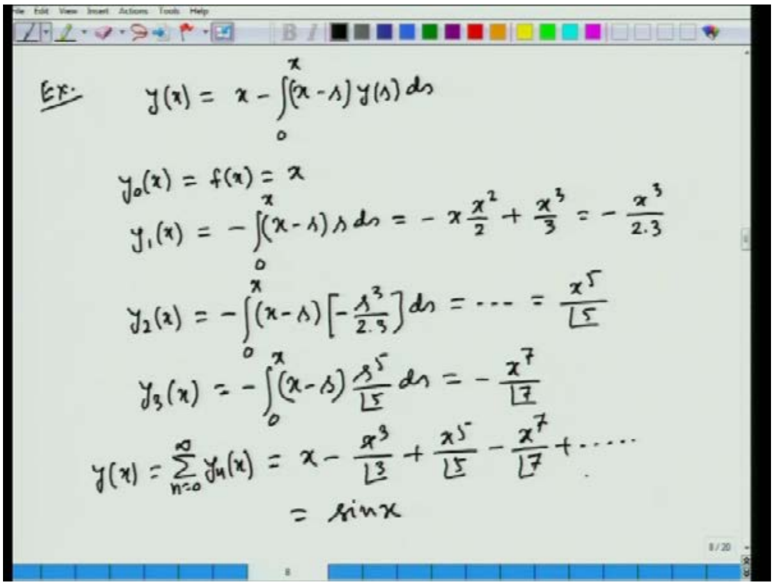

Now, we consider one example to understand this method. So, consider this example, this example already we have considered earlier also, using different methods to solve it. But again I am considering this problem in order to make a comparative study. So, this is x minus s y(s) ds. So, as per the method that we just discussed y 0(x) equal to f(x) equal to x, because x is the non homogeneous part, and then applying the formula we can find y1(x) this is equal to minus integral 0 to x x minus s s ds, and after integration it will be x into x square by 2 plus x cube by 3. So, that is equal to minus x cube divided by 2 into 3. And calculating the third term that is y2(x) this will be minus integral 0 to x x minus s, then y 1(s) is minus s cube by this. So, minus s cube divided by 2 into 3 ds, and this will results in after some calculation that x to the power 5 by factorial 5. And just one more iterates you can calculate, it will be minus integral 0 to x x minus s s to the power 5 by factorial 5 ds, it will results in minus x to the power 7 by factorial 7.

So, y(x) is going to be these summation n running's from 0 to infinity y n(x), so this is some of that term x minus x cube by factorial 3 plus x to the power 5 by factorial 5 minus x to the power 7 by factorial 7 plus dot, dot up to infinity. And you can easily recall this is the series that is sin x. So, sin x is the solution of this equation that we just obtain using the Adomian decomposition method.

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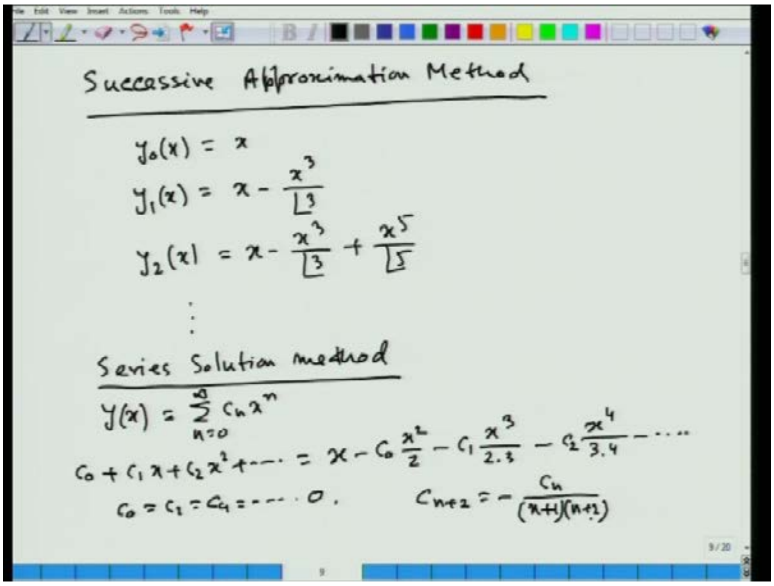

Now, just for a competitive study, you can recall that when you are going to solve this problem or you solve this problem by successive approximation method - for successive approximation method of the same problem we have obtained y 0(x) is equal to x, y1(x) this is equal to x minus x cube by factorial 3, y2(x) this was x minus x cube by factorial 3 plus x to the power 5 by factorial 5, and so on. So in this case, we have to integrate this entire function evaluated in terms of or written in terms of s, and substituting into the integrant pre multiplying it by x minus s, and ranging from 0 to x we have obtained the third approximation. But in case of Adomian decomposition method, the labor of integration is competitively less, as we have adopted in the case of successive approximation method.

So, instead of getting some extra terms at each iterates these y 0, y 1, y 2, up to y n, then sequence y n(x) convert this to the solution, but in case of Adomian decomposition method we are getting, each terms that is x minus x cube by factorial 3 plus x to the power 5 by factorial 5 at every steps of the iteration. And once we solve the same problem with the help of series method - series solution method, then little bit of difficulty was to construct the general nth term; general nth term to construct the recursive formula. And for that recursive formula, you can recall we have use this result y(x) is assume to be summation n running's from 0 to infinity C n x to the power n, and after substitution we have obtained C 0 plus C 1 x plus C 2 x square plus dot dot; this was equal to x minus C 0 x square by 2 minus C 1 x cube by 2 into 3 minus C 2 x to the power 4 by 3 into 4 minus dot dot.

And with some initial conditions, we have obtained C 0 equal to C 2 equal to C 4, all these quantities are exactly equal to 0, and then these results are obtained with help of these recursive relation that is C n plus 2 equal to minus C n by n plus 1 multiplied with n plus 2, but these recursive relation was not valid for some initial steps, that is from where we have calculated C 0 equal to 0, and C 1 equal to 1.

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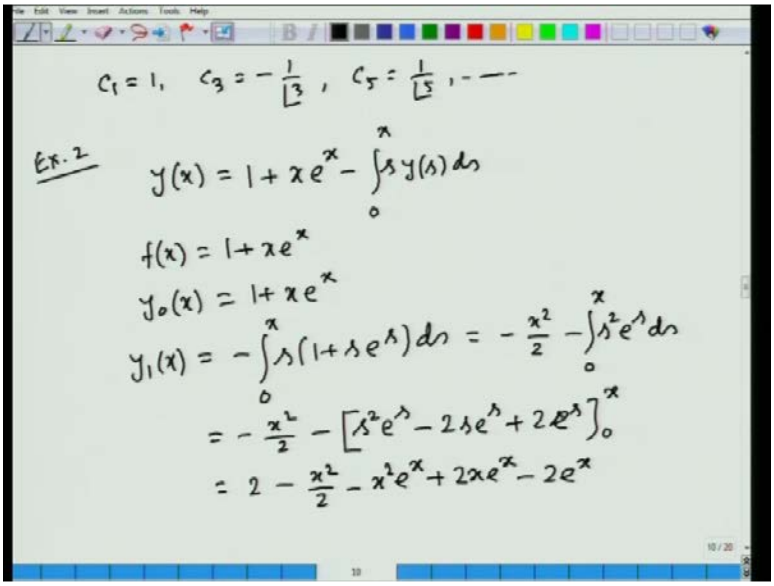

And with these iterates we have calculated that C 1 this is equal to 1, C 3 is equal to minus 1 by factorial 3, and C 5 is equal to 1 by factorial 5, and so on, and then substituting into the series, you have obtained the same solution. So, this is actually prominent difference between the 2 methods. Now, there are some limitations of these method; it is not true that every problem, it would be better to use Adomian decomposition method to obtain the solution. In order to understand the problem that may arise to apply this Adomian decomposition method, here you consider one more example. Suppose, we have to solve this problem y(x) - this is equal to 1 plus x e to the power x minus integral 0 to x is y(s) ds, this is the given problem.

So, clearly the non-homogenous part f(x) is equal to 1 plus x e to the power x, as per described method here y 0(x) is equal to 1 plus x e to the power x, and then if we going to calculate y1(x) minus integral of 0 to x s into 1 plus s e to the power s ds, this one.

And you can clearly understand, one part can be evaluated very easily, that is the first part x square by 2, but for the second part we need the repeated application of the bipher's integration, that is s square e to the power s ds. And if we evaluate this integral completely, then you can find this will be x square by 2 minus with bipher's integration, it will be s square e to the power s, because first step we have to keep s square unaltered that this is u, this is v.

So, by integrating e to the power s s square e to the power s minus 2, it will be integral of s e to the power s, and after integration it will be s e to the power s, and then plus 2 e to the power s this limit 0 to x. And after substituting the limit we can find this will be 2 minus x square by 2 minus x square e to the power x plus 2 x e to the power x minus 2 e to the power x. Now, from these expression, now you can clearly understand it will be very difficult to calculate y2(x) with these as the integrant. So, we have to be very much careful for the choice of the particular method with which we are going to solve this equation. So, in this case the standard Adomian decomposition method is not giving us very easy way to obtain the solution of this problem.

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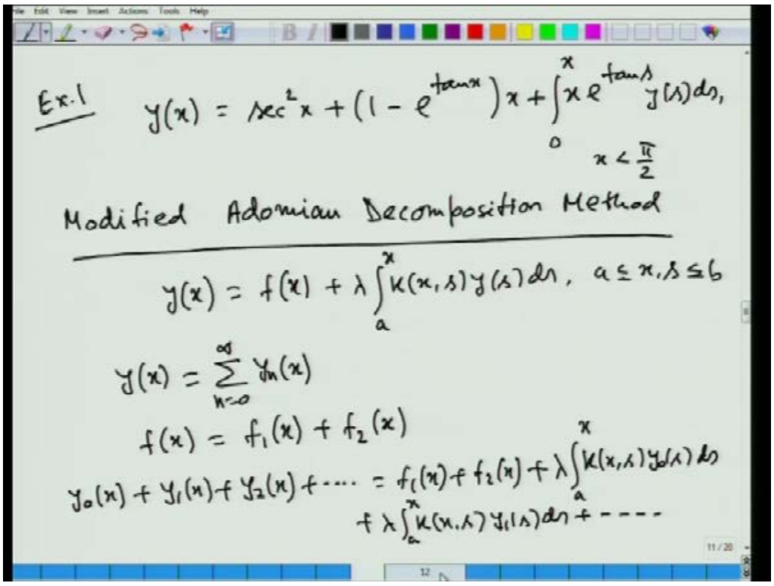

Now, this Adomian decomposition method is modified further that is called modified Adomian decomposition method, and these particular method is applicable for the problems, like suppose we have to solve this problem y(x) is equal to sec square x plus 1 minus e to the power tan x multiplied with x plus integral 0 to x x e to the power tan s y(s) ds, where x is less than pi by 2. Now, in this problem if we choose this entire quantity that is sec square x plus 1 minus e to power tan x into x as f(x), then again it will be very difficult to apply the Adomian decomposition method. So, in these case we have to use the modified Adomian decomposition method; this modified Adomian decomposition method says, the target equation is y(x) equal to f(x) plus lambda integral a to x k of(x,s) y(s) ds, a less than equal to (x,s) less than equal to b.

We are intended to get solution into these form n running's from 0 to infinity y n(x), modification is that we have to divide f(x) into 2 parts: f 1(x) plus f 2(x), this decomposition means that sometimes we have to take f 1(x) consist of 1 or 2 terms, such that the integration in the successive steps will be very much easier. And just for a quick difference you can have a look at this expression, this integral equation under this integral sign contains x e to the power tan s times y(s) t s. So, variable of integration is s. So, we can take x outside, so in these case, if we having this y(s) as sec square x which is already present here in terms x.

So, then it will be very easy to integrate - this integrate evaluate the integral. So, for these kind of problem, here f(x) can be divided into 2 parts, such that other iterates can be obtained very easily. So, we have to decompose f(x) into 2 parts as f 1(x) plus f 2(x), and then we have to substitute into the expression that you have described earlier. And if we substitute there, so we will be having terms like y 0(x) pus y1(x) plus y2(x) plus dot, dot; this is equal to f 1(x) plus f 2(x) plus lambda times integral a to x k(x,s) y 0(s) ds plus lambda times integral a to x k(x,s) y 1(s) ds plus dot dot. And here, we have to equate y 0 with f 1(x), then y1(x) is equal to f 2(x) plus the first integral in a appeared on the right hand side, and then y2(x) equal to as we have adopted earlier that is lambda times integral a to x k(x,s) y 1(s) ds, and so on.

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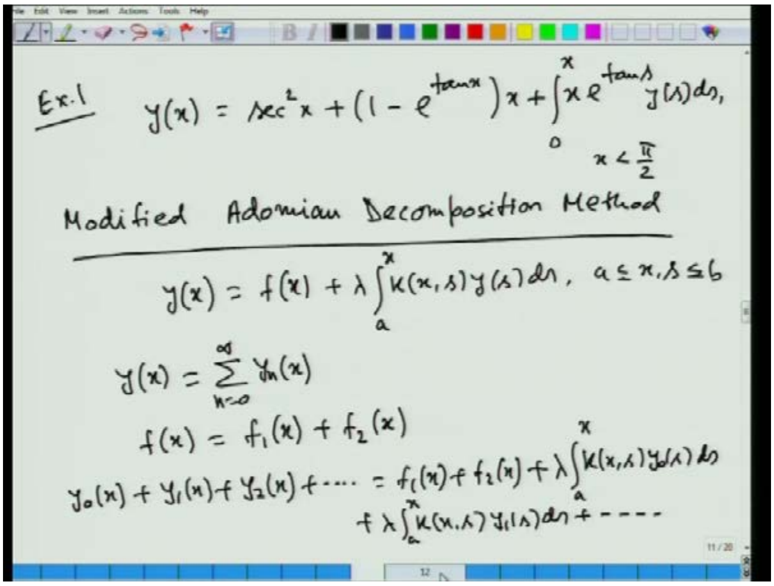

So, that means, in the modified Adomian decomposition method, the iterates will be y 0(x) is equal to f 1(x), y1(x) this will be equal to f 2(x) plus lambda times integral a to x k of (x,s) y 0(s) ds, and then y2(x) is equal to lambda times integral a to x k(x,s) y 1(s), and then other (( )) iterates will be similar. So, that means, it will be a to x k(x,s) y 2(s) ds, in this way. So, that means, general iterates will be y n(x) is equal to lambda integral a to x k(x,s) y in minus 1 s ds.

Now, here this iterated formula is valid for in greater than equal to 2. So, that means, if we combine these results for modified Adomian decomposition method. So, first iterate y 0(x) is equal to f 1(x), y1(x) is equal to f 2(x) plus lambda times integral a to x k of(x,s) y 0(s) ds. And then general iterate y n(x), that is equal to lambda times integral a to x k of(x,s) y n minus 1 s ds, where n greater than equal to 2. So, this is the actually compact form of the Adomian decomposition method that is of modified type.

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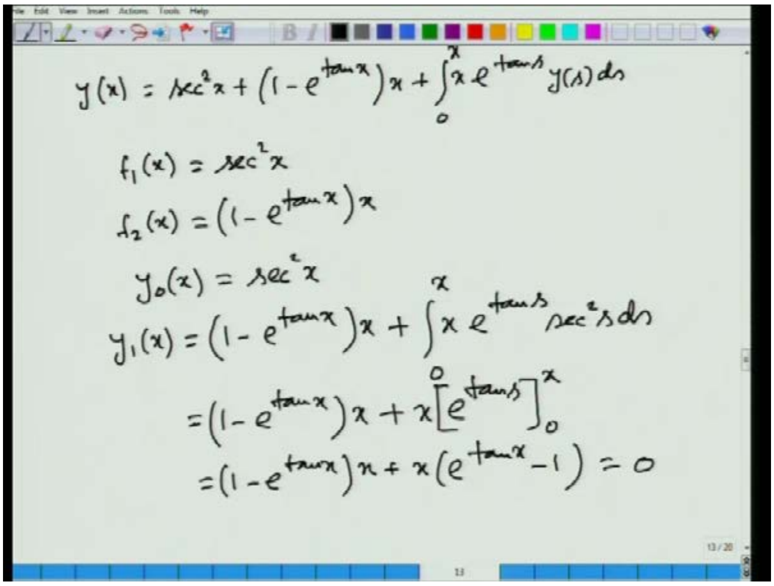

Now, with these methods we solve this problem with which we have started these discussion on modified decomposition method. So, given problem was y(x) is equal to sec square x plus 1 minus e to the power tan x multiplied with x plus integral 0 to x x e to the power tan s y(s) ds.

So, we are assuming that f 1(x) this is equal to sec square x, and f 2(x) this is equal 1 minus e to the power tan x these multiplied with x. So, with these f 1 and f 2. So, our first component y 0(x) is equal to sec square x, and interestingly if we calculate y 1(x). So, this will be 1 minus e to the power tan x, these multiplied with x plus integral 0 to x x e to the power tan s multiplied with sec square s ds. Now, here the variable of integration is s. So, we can take this s outside the integral sign, and then assuming this tan s equal to u, this integral will be reduced to integral e to the power u du. So, then after evaluating the integral, we can find 1 minus e to the power tan x multiplied with x plus x e to the power tan s this limit will be from 0 to x. So, after evaluating you can find this tan x multiplied x plus x e to the power tan x minus 1.

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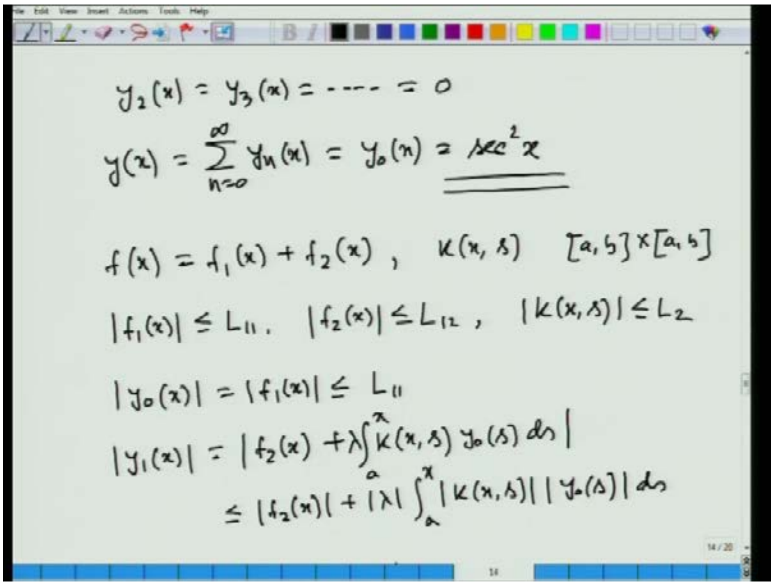

So, this is exactly equal to 1. So, once this y1(x) is exactly equal to zero. So, immediately you can understand that y2(x), y 3(x), and so on, all these components are exactly equal to 0. And therefore, y(x) that is assumed to be n running's from 0 to infinity y n(x), this is nothing but simply y 0(x) is equal to sec square x, this is actually the solution of the given problem.

So, that means, if we going to apply any other method, it would be very difficult to find solution of this particular integral equation. You can verify with yourself by the method of successive approximation, that will be little bit problematic, because it is entire expression. If we assume this as y 0(x), then it will be difficult to calculate y1(x), and so on. Series solution you can clearly understand that is difficult, because this integrant involve e to the power tan s. So, there is no possibility to obtain this integral in a closed form, apart from this applying some iterative formula or some reduction formula.

So, based possible way to solve this integral equation is the modified Adomian decomposition method. So, you have to keep in mind that based upon the given problem, you have to choose the method appropriately. Before coming to the end of today's lecture, we can look at the convergence of these method; in these case we have assumed that f(x) can be expressed as summation of 2 functions f 1(x), and f 2(x). So, again these two functions are actually continuous functions k(x,s) this is continuous over this square a, b cross a, b. So, we are assuming that modulus of f 1(x), this is less than equal to L 11, modulus of f 2(x) this is less than equal to L 12, and modulus of k(x,s) this is less than equal to L 2.

So, what will going to happen in the successive steps: First step modulus of y 0(x) that is less than equal to sorry, it will be exactly equal to modulus of f 1(x), and this is less than equal to L 11, then modulus of y1(x) this is equal to modulus of f 2(x) plus integral a to x with lambda k(x,s) y 0(x) ds; this is less than equal to modulus of f 2(x) plus modulus of lambda integral a to x modulus of k(x,s) modulus of y 0(s) this ds.

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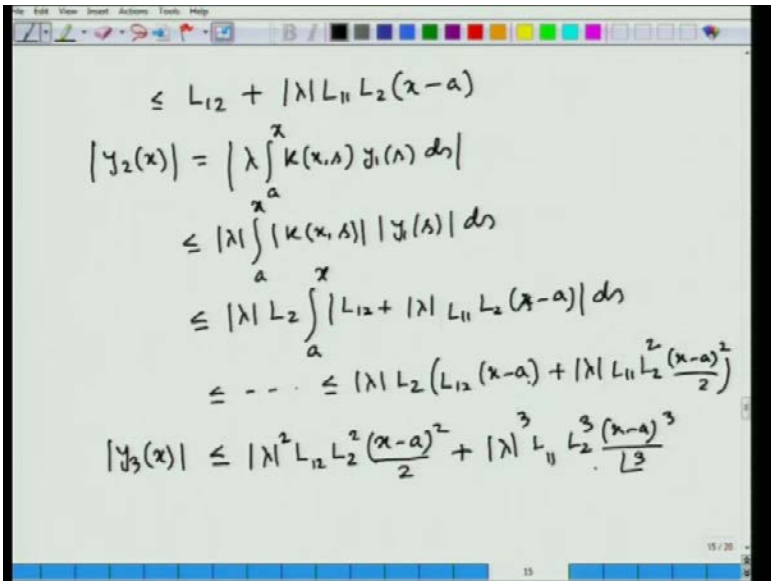

And this will be less than equal to L 12 plus modulus of lambda L 11, L 2 times x minus a, just have a look at this expression. Here modulus k of(x,s), this is less than equal to L 2, this modulus of y 0(s), this will be less than equal to L 11. So, ultimately it will results in modulus lambda L 11 into L 2 multiplied by x minus a. Similarly, modulus of y2(x) this is equal to modulus of lambda integral a to x k of(x,s) multiplied by y 1(s) ds, which is less than equal to modulus lambda integral a to x modulus of k(x,s) modulus of y 1(s) ds.

This is less than equal to modulus lambda L 2 integral a to x modulus of L 1 2 plus modulus lambda L 11, L 2 times s minus a this modulus ds, and after applying this modulus that is mode of this is less than equal to modulus of L 11 plus this quantity, and L 11 is positive. So, ultimately you will be having that modulus of L lambda into L 2 times L 1 2 multiplied by x minus a plus modulus lambda L 11 L 2 x minus a whole square divided by 2. So, we are having one particular terms of the form that is L 11 L 2 modulus lambda x minus a, this is coming from here. And here will be having this modulus lambda L 1 2, it will be L 2 square modulus x minus a whole square by 2.

So, in this way, if you calculate modulus of y 3(x), it will be less than equal to modulus lambda whole square L 1 2 multiplied with L 2 square, then x minus a this whole square divided by 2 plus modulus lambda whole cube L 11. L 2 cube x minus a whole cube divided by factorial 3, this will be L 1 1.

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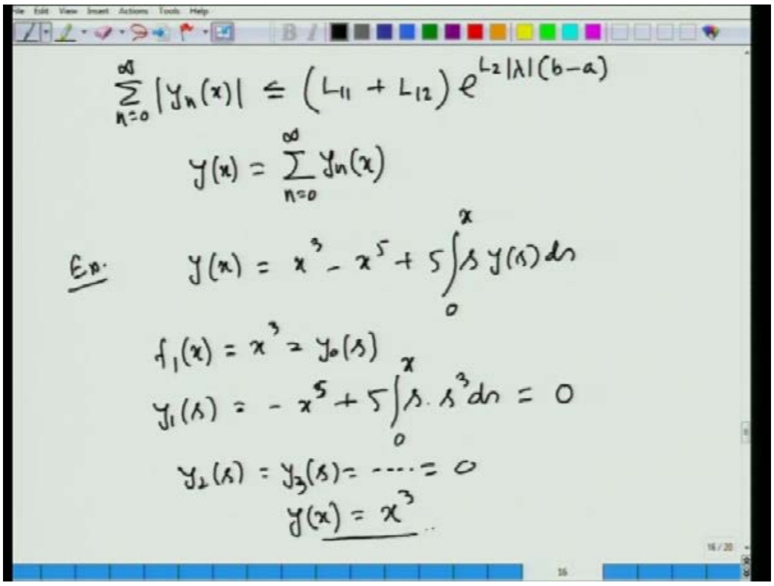

So, in this way you will be having modulus of y n(x) is summation of these two, and afterwards you will be able to prove that summation n running's from 0 to infinity modulus of y1(x). This will be actually less than equal to L 11 plus L 1 2 e to the power L 2 modulus lambda times b minus a, this is the result. And therefore, this sequence some of the series is uniformly convergent, and this solution is given by y(x) is equal to sigma n running's from 0 to infinity y n(x). So, this is the final solution of the given problem.

So, we have started with the Adomian decomposition method, where we have assume the solution into these form y(x) equal to summation n running's from 0 to n y n(x), and with the definition of the first iterates y 0 equal to f(x), and then y n(x) equal to lambda integral a to x k(x,s) y n minus 1(s) ds, n greater than equal to 1. We have obtained this is method is called the Adomian decomposition method, and then we have considered one example, where we have seen that it is difficult to carry out the calculations for y 0, y 1, and etcetera, if we use the standard Adomian decomposition method. So, we can adopt the modified Adomian decomposition method to solve this kind of problems. And before coming to the end, just for a quick example you can consider this one that y(x) is equal to x cube minus x to the power 5 plus 5 integral 0 to x s y(s) ds.

If you just have a clever look at this problem. So, if you are able to substitute here y(s) equal to s cube. So, after integration it will produce an x to the power 5. So, this case again the modified Adomian decomposition is very useful. So, f 1 equal to x cube, if you use this is equal to y 0(s). So, y 1(s) will be equal to minus x to the power 5 plus 5 integral 0 to x s into s cube ds; this is equal to 0, because this will results in s to the power 5 by 5, so minus x to the power 5 plus s to the power 5 equal to zero. So, y 2(s) and other expression y 3(s) all of them will be identically equal to zero. So, easily you can find y(x) equal to x cube. This is a solution for this problem. So, today I can stop at this point.