Mathematical Portfolio Theory

Prof. Siddhartha Pratim Chakrabarty
Department of Mathematics
Indian Institute of Technology Guwahati

Module 02: Basics of Financial Markets
Lecture 02: Bonds and stocks

Hello viewers, welcome to this lecture on the MOOC course on Mathematical Portfolio Theory. You would recall that in the last lecture we talked about Financial Markets with an emphasis on derivatives markets and we talked about the types of markets, the types of derivatives and finally, we concluded with the types of different traders. And then we did a broad classification of different kinds of securities that are available in the market and we had talked about Bonds and stocks and these are the two things that we will start discussing in a lot more detail today. One of the reasons why I am emphasizing an entire lecture on bonds and stocks is that a lot of times the portfolio that you have will involve a risk free and risky assets and the two simplest example of risk free and risk free assets are bonds and stocks respectively. And the question that we will try to answer while talking about modern portfolio theory is to look at how should we allocate our wealth between the risk free asset and the risky asset. So, we need to look at some of the basic terminologies and the way these two assets are actually designed how they function and what are the important terminologies that are associated with them.

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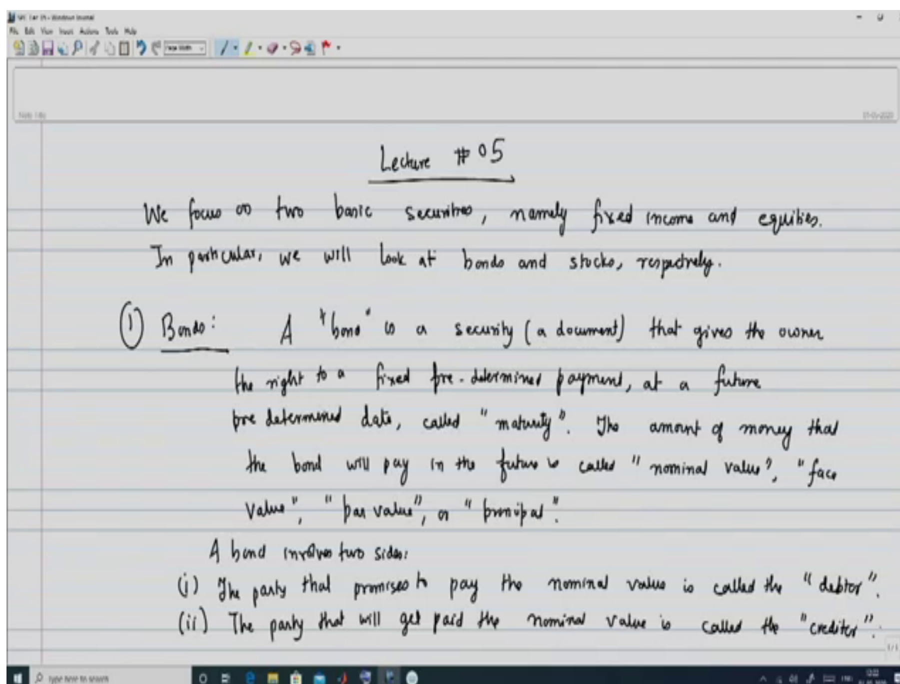

So, accordingly we begin this lecture now. So, as I said that in todays class we focus on two basic securities, namely, fixed income and equities and in particular we will look at bonds and stocks respectively.

So, let me first start off with a discussion on bonds. So, what is a bond? So, a bond is a security and by security I mean it is a document that gives the owner the right to a fixed predetermined payment, at a future predetermined date called maturity. So, the amount of money that the bond will pay in the future is called nominal value or face value or par value or simply the principal. So, just to recapitulate, what is the bond? So, bond is essentially a security or a document wherein the owner of the bond is entitled to asserted pre determine; that means, it is specified at the time of purchase of the bond for a predetermined payment at a future time which is also fixed. And this future time at which this payment is received by the owner of the bond that is what is known as the maturity of the bond and the payment which is predetermined that is received at maturity it has several nomenclature some of which are nominal value, face value, par value or simply the principal. Now, as this suggests that here you know we can clearly see that as was the case with other financial instruments that we looked at earlier there are going to be two parties to this particular arrangement. One which is going to receive this face value or par value or nominal or the principal at maturity and the other that is going to pay this. So, accordingly we now introduced terminologies pertaining to the two parties who are a party to this arrangement known as the bond. So, accordingly a bond involves two sides. So, the first one is the party that promises to pay the nominal value is called the debtor. And secondly, the party that will get paid the nominal value is called the creditor, ok. So, this means that there are two parties. So, one that will pay the amount this is known as a debtor and the one that will get the nominal value is called the creditor. So, essentially what the creditor does is that at the initial time they will extend a credit to the debtor that is they will pay some money and in lieu the debtor is going to issue them the bond. So; that means, that in practice this means that the creditor is extending credit to the debtor who are called as debtor because they now are in debt and then at the final time point that is the maturity the debtor now will pay the money namely the nominal value to the creditor. So, typically this will involve that you return the original amount of money along with something called the interest.

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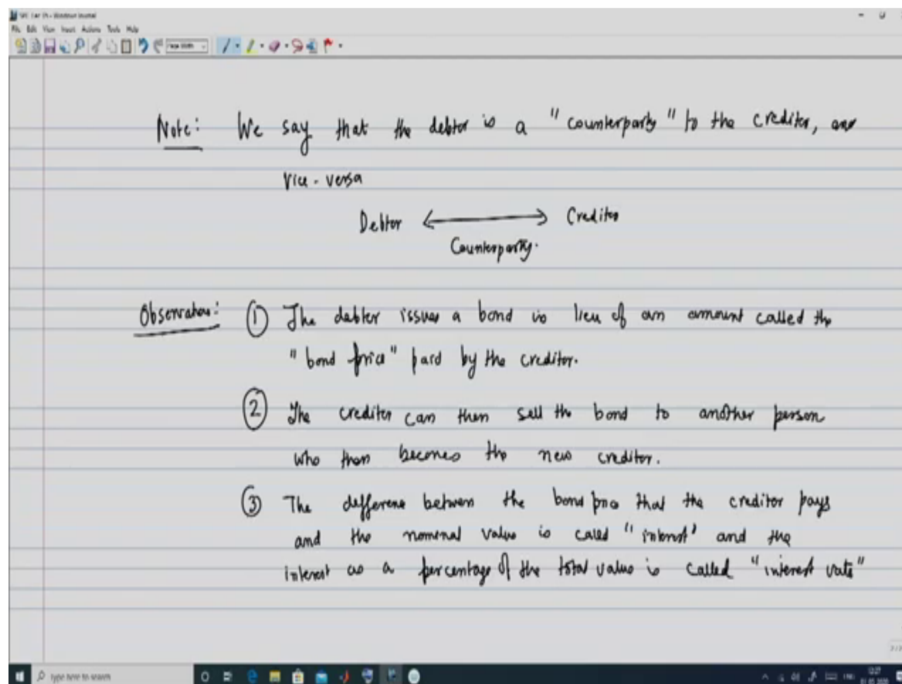

So, now we just want to let us make an observation here. So, this is a term that is used in general for any kind of financial instrument and I will define this in the context of a bond. So, we say that the debtor is a counterparty to the creditor and vice versa. So, this means that we have this debtor and we have the creditor and they are counterparty to each other. So, now we make some observations and also introduce a few more terminologies. So, the first thing that we talk about is that the debtor issues a bond in lieu of an amount called the bond price which is paid by the creditor. So, now, we have defined what is the bond price, that is

basically the amount of money that exchanges hand at the initial time. Now, the second observation is that the creditor can then sell the bond to another person who then becomes the new creditor. So, this means that suppose I have paid some money to a debtor say this is person A and I am the creditor and suppose that the maturity of the bond is 5 years. So, after 2 years I decide to sell the bond to some party B. So, what might happen is that I might the bond price might be 100 today and the maturity is 5 years then after 2 years, I might sell the bond to 100 for 120 to some party B. So that means, that the party B they will pay me 120 and then I basically give them this document for the bond to party B and inform party A that they do not need to pay me the nominal value or the face value, but instead they need to pay this to party B. So, in that case instead of me under this arrangement the party B becomes now the creditor or the counterparty to the debtor. The next point that I want to make is that the difference between the bond price that the creditor pays; remember that initially it is the creditor who pays to the debtor and the nominal value is called interest and the interest as a percentage of the total value is called interest rate.

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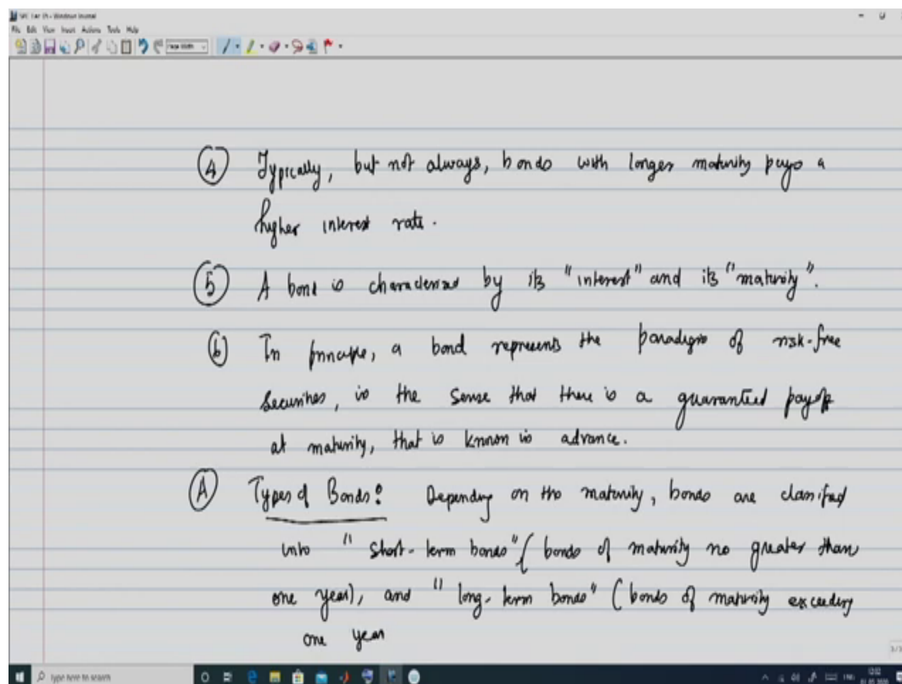

Next I will make a observation that is typical, but not always necessarily true. So, typically, but not in all cases bonds with longer maturity pays are higher interest rate. So, for example, if we have a 10 year bond and a 5 year bond, it is more likely that the interest rate that is being paid on the 10 year bond is going to be more than that of a 5 year bond. Next observation: a bond is completely identified or characterized by two things, namely, its interest and its maturity. So, one typical example of a bond is a fixed deposit. So, when you are looking at a fixed deposit, typically what you are going to look at is that what is the maturity or what are the different maturities that are being offered and once you have identified the maturity for which you would like to invest you then start looking at the interest rate or vice versa. So, this means that the two key things that are of interest when purchasing a bond is basically when you are going to get your money back that is what is the maturity and basically what is the rate of return that you are going to get that is the interest and these two things completely characterize a bond. Now, I am going to make an important observation that in principle, a bond represents the paradigm of risk free securities, in the sense that there is a guaranteed payoff at maturity that is known in advance. So, let me illustrate this with a very specific example. So, suppose you have purchased a fixed deposit which is bond and then you have invested an amount of 100 and the interest rate is 10% per annum and then you have the maturity is 5 years and the interest is calculated as a simple interest convention. So, this means that at the end of 5 years on your investment of 100 you are going to get 10% each year; that means, 10 each year amounting to a total of 50

at the end of 5 years. So, in that sense a bond is said to be risk free because setting at time $t = 0$ when we invest 100 in that bond, you know for certainty that at time $t = 5$ years which is the maturity you will receive a nominal value of 150 and nothing less or nothing more. So, in that sense and in that perspective a bond is considered to be a risk free asset. However, bonds can do suffer from some kinds of riskiness that I will elaborate later in the lecture. So, now what we do next is we just talk about a couple of things specific to bond. So, first of all I will talk about types of bonds. So, depending on the maturity, bonds are classified into short term bonds which are basically bonds of maturity no greater than one year and long term bonds these are bonds of maturity exceeding one year. So, this was one classification that if the maturity of the bond is less is one year or less then we call it a short term bond and if it is if it exceeds one year then you call them the long term bond. Another classification of bonds that you look at is basically whether we are receiving one final nominal value or whether you are receiving periodic payments.

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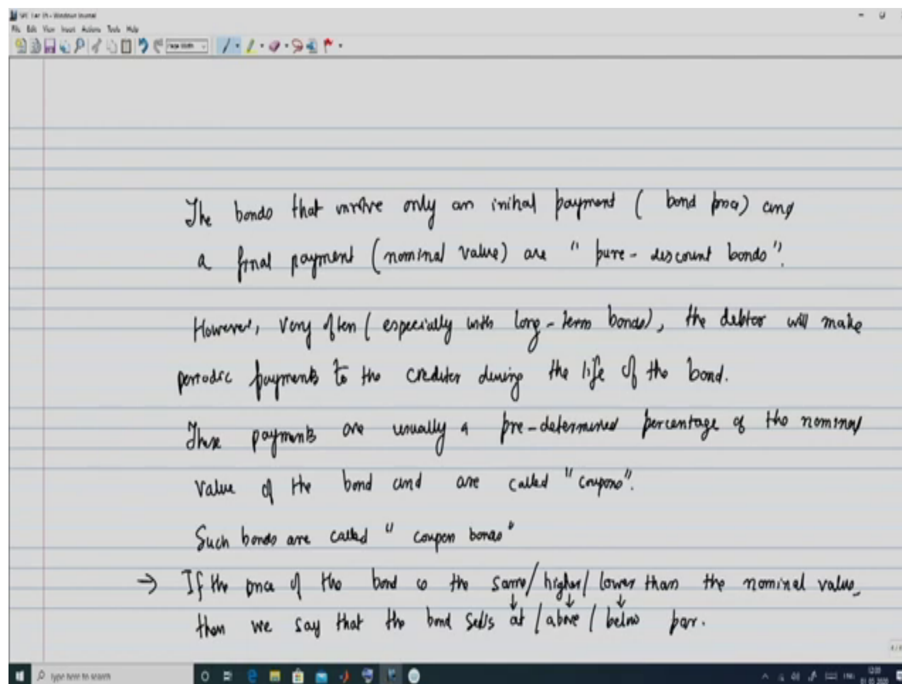

And accordingly, the bonds that involve only an initial payment and remember that this is the bond price and a final payment and recall that this was the nominal value are what are known as a pure discount bonds. So, the reason they are called pure discount bond is because that you make an initial payment. So, you make a payment of 100 for a period of 5 years and then at the end of 5 years you receive 150. So, if you take 150 and divide by $(1 + 0.1) \times 5$, then you get 100. So; that means, the price of the bond can be obtained by just a single discounting of the nominal value which is why they are called the pure discount bond because it involves just one discounting resulting from just one final payment. However, the thing becomes slightly more involved if instead of just one nominal payment at the maturity there is a series of payments that are happening between the time which you are purchasing the bond and the maturity. So, to accommodate that we introduced another terminology which are basically called the coupon bonds and let me elaborate on that. So; however, very often and this is especially true with long term bonds, the debtor; that means, the party which is issued the bond will make periodic payments to the creditor during the life of the bond. Now, these payments how are these payments calculated. So, these payments are usually a predetermined percentage of the nominal value of the bond and are called coupons. So, how does this arrangement of several payments at intermediate time points differ from the pure discount bond? So, suppose what you are doing is that you get into a 5 year coupon bond which essentially pays coupon on a yearly basis. So, suppose you invested an amount of 100 which is the bond price with 10% per annum being paid. So, if this 10% interest that is being paid annually is paid actually at the end of each year instead of total amount of

150 at the end, so, what you do is that you basically get one 10, 10, 10, 10. So, four payments of 10 at the end of first second third and fourth year and in the final year that is at maturity, you will receive the last interest of 10 plus the nominal value of 100. So, here the coupon payments are 10% of the nominal value of 100. So, coupon bond involves coupon payments in the intermediate time points and at maturity there are two things that are paid the final coupon and the nominal value. So, naturally such bonds are called coupon bonds ok. So, just to wind up one more terminology that I want to introduce is the following. That if the price of the bond is the same or higher or lower then the nominal value then we say that the bond sells at above or below par. So, this means that if the price of the bond is the same as the nominal value then we say that it sells at par. If it is higher, then the nominal value then we say it is trading above par and if it is less than the nominal value we say that the bond is trading below par ok. So, far what we have discussed is we looked at some of the characteristics of the bond and primarily interest and maturity and we talked about long and short term bonds and you looked at pure discount bond and then extended it to a coupon bond which essentially is collection of several pure discount bonds. And so, this sort of sums up the classification of types of bonds. The next thing that we want to look at is why should we invest in the bonds in the first place.

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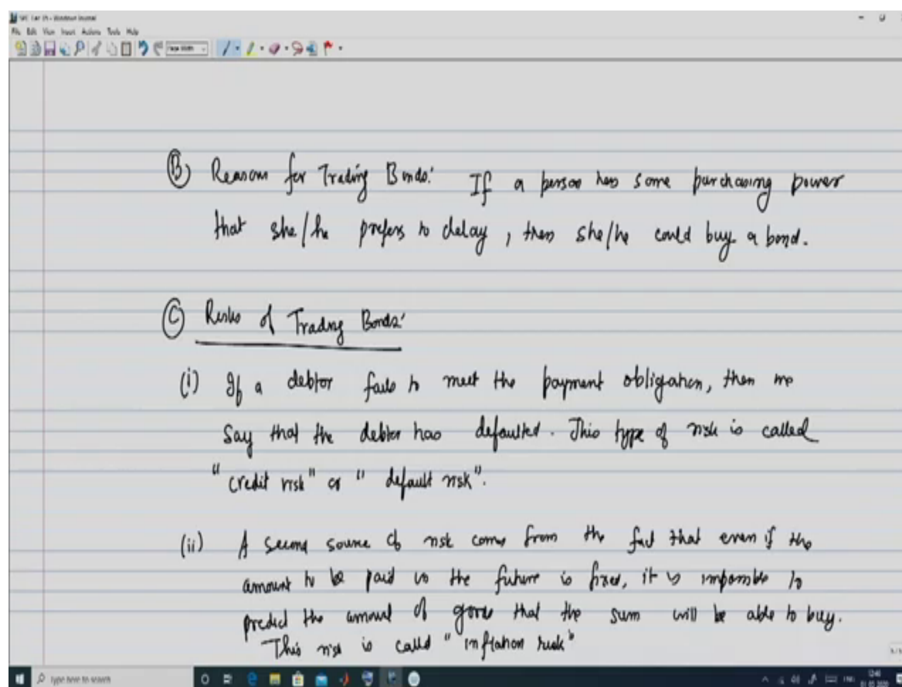

So, I will just briefly talk about reasons for trading bond and I will identify one a key aspect. So, the reasons for trading bond from the point of view of the bond purchaser is that if a person has some purchasing power; that means, some extra cash or disposable income and because this is extra cash that the person is holding that there is no need to spend at that point of time that he or she prefers to delay then she or he could buy a bond, ok. Now, we started off you recall that we started off looking at the broader classification of securities and we identified bonds as being representative of a risk free asset. However, irrespective of the fact that bonds are considered risk free in the paradigm of a fixed and predetermine returned at maturity, there are some inherent risks that are associated with bonds and such risks are more pronounced in the case of corporate bonds as compared to government bonds. So, government bonds are bonds that are issued by a government and so, in the case of any difficulty the government can always print currency and then they can meet the obligations under the bond. However, in the case of corporate bonds they do not have this luxury and secondly, corporate bonds are issued to raise money for a certain project and that once that project if the event that it fails then the company which has issued the bond faces difficulty in meeting its obligations under the bond payment setup. And so, then they are likely to default. So, taking all these things

into account let me look at the three main type of risks that are associated in the bonds notwithstanding the fact that it is risk free from the point of view of no uncertainty about the final payment that you will receive at maturity. So, let us look at risks from so, risks of trading bonds. So, the first one that we look at is that if a debtor and these as I pointed out happens in the case of corporate bonds. So, if a debtor fails to meet the payment obligation; that means, the coupon and the nominal payment obligation, then we say that the debtor has defaulted. This type of risk is called credit risk or default risk, ok. So, next we consider another type of risk that is associated with the actual value of money. So, let us look at it in the following point of view of another example. Suppose, you invest an amount of 100 and then we are making an investment at the rate of 10% with maturity 5 years as before. So, this means that at the end of 5 years I receive an amount of 150. However the amount of a certain commodity that we can purchase today at 100 that may not be available for purchase at 150 at the end of 5 years, but may instead cost 200. So, this means that a certain amount of commodity that you are able to buy today for an amount of 100, but instead you decide that you will invest it in a bond and purchase it after 5 years. After 5 years you receive an amount of 150 on the bond which is the nominal, but you find that this amount of 150 is no longer sufficient to purchase the same quantity of the commodity and you have a shortfall of 50. So, this has result of the fact that the rate at which the price of the commodity has gone up is more than the rate of interest that you have received on the bond. And this is what is known as inflation and consequently, this risk that you have a shortfall of 50 this is what is known as the inflation risk. So, a second source of risk comes from the fact that even if the amount to be paid in the future is fixed, it is impossible to predict the amount of goods that the sum will be able to buy and this risk is called inflation risk.

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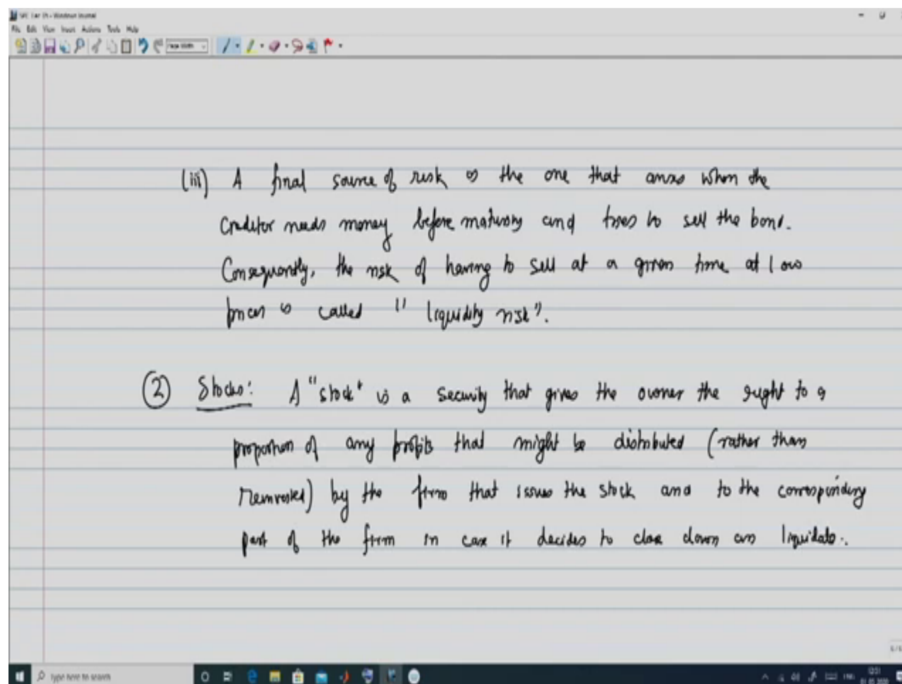

So, next we come to the last of the risks. So, a final source of a risk is the one that arises when the creditor needs money before maturity and tries to sell the bond. Consequently, the risk of having to sell at a given time at low prices is called liquidity risk. So, let me explain this concept of liquidity risk again in the context of the previous example. So, you have purchase the 5 year bond with an amount of 100 and interest rate 10%. Suppose that after 2 years you need the money and you decide to sell off the bond and basically then the party to whom you sell the bond they become the new creditor for the debtor. So, what do you expect is that since you are selling it after 2 years then you should get 100 which is an original amount plus the interest for 2 years namely 120. So, then you will expect that the price of the bond that you receive at by selling it to another creditor at the end of 2 years that is going to be 120. But, that price need not

necessarily be 120 and you will depend on the interest rate that is prevailing for 3 years that is from time t equal to 2 to time $t = 5$ years at that point of time. Now, if the price of the bond is higher than 120, then you have benefited, but if the price of the bond that is 115 say for example, then you lose an amount of 5. So, basically you by selling the bond you are getting a lesser amount of money, then what was accumulated by the bond as a result of the two interest that has happened at the end of first and second year. But since you do not have a choice as you need the money you have to sell the bond for an amount of 115 which is lower than 120. So; that means, you are facing what is known as the liquidity risk and this means the liquidity risk here means that inability to get an amount of money which you had anticipated that you will get at the end of 2 years sitting at time t equal to 0, ok. So, this concludes our discussion on the risk free asset namely the bond. Now, we look at the second asset that we will discuss in this class and that is stocks. Stocks as I pointed out is a risky asset. And it is risky in the sense that unlike bonds the return on it or the future payment that you receive on it as a result of your investment which was fixed in the case of a bond in the case of a stock this is actually uncertain and that is what qualifies it as a risky asset. So, we start off our discussion on stocks. So, let me first define what is a stock. So, a stock is a security that gives the owner the right to a proportion of any profits that might be distributed rather than reinvested by the firm that issues the stock and to the corresponding part of the firm in case it decides to close down and liquidate. So, what is stock? The stock is essentially it is another documentary security where the owner of the stock has the following two rights. First of all is a right to any proportion of profits that the firm or the company that has issued the stock decides to disperse among the people who have purchased the stock rather than reinvesting. And secondly, it is the corresponding part of the firm in case it decides to close down and liquidate. So, let me illustrate this through an example. Suppose that a company or a firm issue stocks to 10 individuals each of them paying an amount of 1000. So, the company raises a total amount of 10000. So, suppose I have invested I am one of the people who have invested an amount of 1000. At the end of one year the company makes a profit of 2000 on this 10000 and then what so, what am I do to do with this is that there are two things the company can do. Either they will take an amount of 2000 and then they will reinvest in the company in which case I do not get anything, but my valuation of stock improves, but if the company decides to disperse the amount of 2000 to all those people who have invested in the stock, now since I have owned 10% of the company; so; that means, I will get 10% of this amount of 2000, which is 200. Now, the other scenario is that if the company for some reason is not doing well and decides to liquidate and then the valuation at liquidation is only 5000, then again as the owner of the stock I am entitled to 10% of that 5000, which means 500. So, just to sum it up, you are entitled to a proportion of the profit that is not reinvested and also in the event of the closure of a company you are entitled to the proportional valuation of the company that is received as a result of the closure and consequent selling of the assets of the company, ok.

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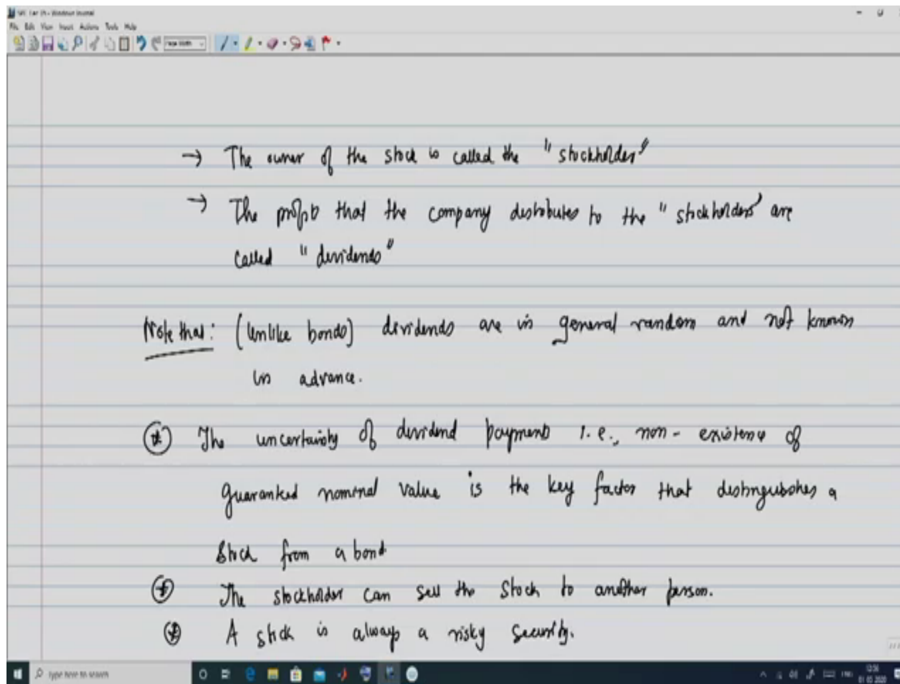
So, now we introduce a couple of more terminologies. So, the first terminology that we use is that if we have purchased the stock of issued by a firm then the owner, so, we have the ownership of the stock and the owner of the stock is called the stockholder. And secondly, the profits that the company distributes to the stockholders as defined in the previous slide are called dividends. And note that unlike bonds, dividends are in general random and not known ahead of time or not known in advance. And the uncertainty, so, this uncertainty of dividend payments that is non-existence of guaranteed nominal value; remember that in the case of bonds we had guaranteed nominal value, but it does not exist in the case of stocks is the and accordingly this is the key factor that distinguishes a stock from a bond.. And the second observation that I want to make is they are just like bonds can be sold to another creditor, likewise, the stockholder can sell the stock to another person who of course, recognizes that this is and the third point that I want to make is that a stock is always a risky security. Now, the last observation that I had made is that the stock is a risky security. So, this means that if you are selling the stock to another person you are likely to either incurred a profit or a loss. So, this means that if you have purchased a stock for a certain price and you are able to sell it to another person at a higher price then; obviously, you have gain, but if that is not the case and you are



compelled to sell it at a lower price then you have incurred a loss.

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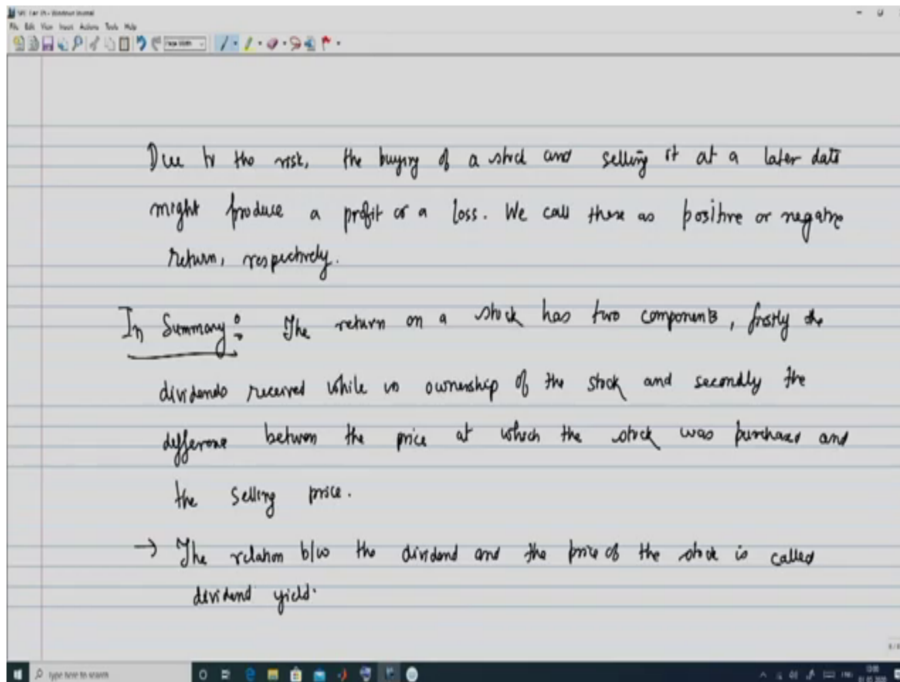

So, to summarize this we make the following statement that due to the risk that I just mentioned the buying of a stock of a stock and then selling it at a later date might produce a profit or a loss. So, both of them are a possibility. And we call this profit or loss this has positive or negative return, respectively. So, in summary just to paraphrase the discussion so far on stocks. So, the return on a stock has two components. So, there are two kinds of returns that you can experience in a stock. Firstly, the dividends received while you are in ownership of the stock. And secondly, the difference between the price at which the stock was purchased and the selling price. So, the relation between the dividend and the price of the stock is called dividend yield.

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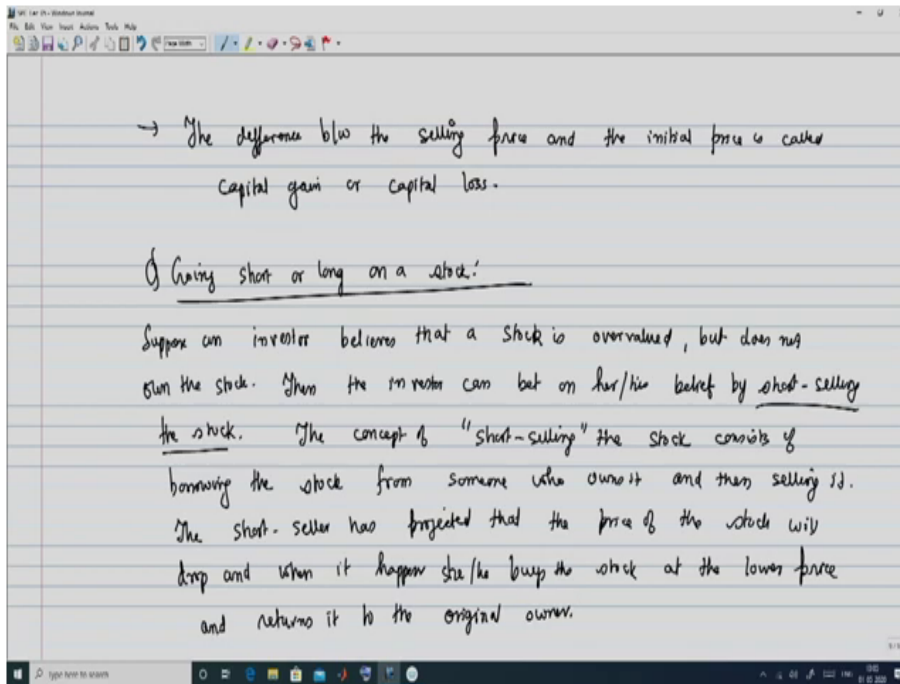

And secondly, the difference between the selling price and the initial price at which it was purchased is called capital gain or capital loss. So, to sum this up basically there are two kinds of returns that you can get from the stock one is basically a proportional share of the profits that you get and this proportional share in terms of the price of the stock is what is known as the dividend yield. And secondly, when you decide to sell the stock either you can sell it at a higher price or a lower price from which you had purchased. And these two cases respectively are termed as capital gain and capital loss respectively. So; that means, the returns can either be a dividend yield and in the second case it can be a capital loss or a gain. So, we now move on to the last topic for this class that the concept of long and short position. And in particular I want to emphasize the concept of short selling and you will see the importance of this later when we talk about modern portfolio theory, where the concept of short selling will essentially be equivalent to the setup where you own negative number of assets. So, when you first encounter this idea of folding negative number of assets, it might seem counterintuitive and, but it is justified in the sense that this kind of situation actually exists in the market and this particular situation is what is known as the short selling. So, let me start the last topic which is going short or long on a stock. So, suppose so, I begin with a motivation why this concept of short selling is exist in the first place. So, suppose that an investor believes that a stock is overvalued that means, its price is higher than it should be, but does not own the stock. So, then the investor can bet on her or his belief and the belief that this is the stock is overvalued by short selling the stock. So, what is this concept of short selling the stock? So, the concept of short selling the stock consists of borrowing the stock from someone who owns it and then selling it. The short seller has projected that the price of the stock will drop and when it happens he or she buys the stock at the lower price and returns it to the original owner.

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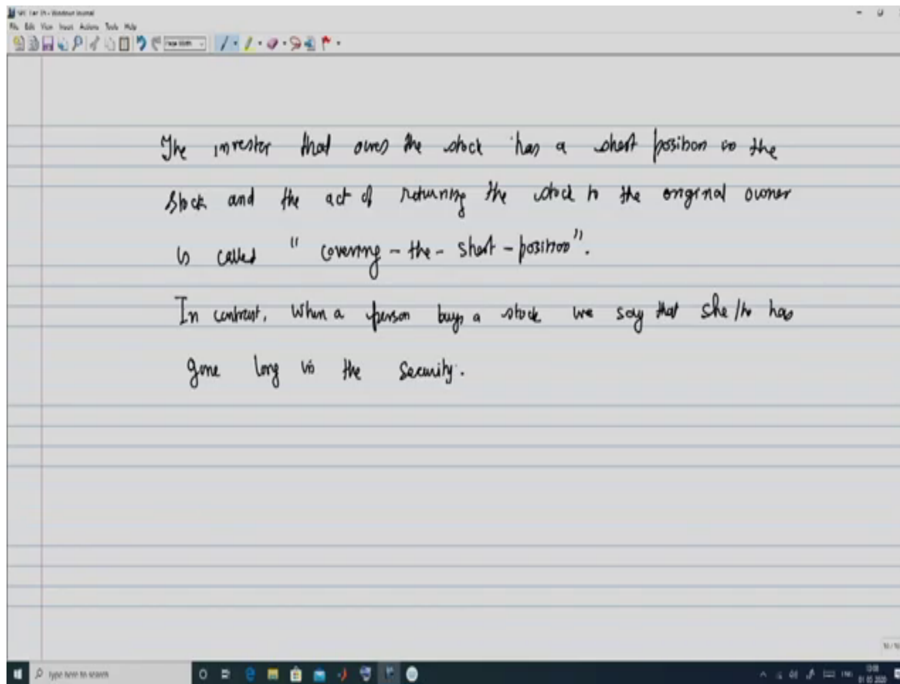
The investor that owes the stock has a short position in the stock and the act of returning the stock to the original owner is called covering the short position. So, let me elaborate little bit on this. Suppose that I am an individual investor who decides that observing the market pattern I find that a stock is overvalued; that means, its priced higher than it should be and then I expect that the market will correct itself and its price will come down. Now, if I own the stock that obviously, I am going to sell it at a higher price. However, if I am not in ownership of the stock, I would like to stick take advantage of getting this higher price. Now, if I do not have the ownership of the stock what I go and do is that I asked the owner of the stock and I borrow it from them and I sell it in the market and keep the cost of the stock that I have the value of the stock that I have received by selling somebody else a stock. After some time when my projection if it comes true and



the stock prices fall then I take out a part of that money that I had got by selling the stock and then I buy the stock and return it to the owner thereby keeping the difference between the two. And this process of returning the stock to the owner after some time this is what is known as covering the short position. So, once I talk about the short position then; obviously, I need to talk about something as the long position. So, I will mention that briefly. So, in contrast, so, on the other side when a person buys a stock, we say that she or he has gone long in the security. So, basically the ownership of the stock essentially means a long position in the stock. So, this concludes our class for today. Just to do a recap, we looked at bonds and stocks and we look at the characteristics of those we identified in detail that bonds are risk free in the sense that their final value that you receive or the nominal value that you receive at maturity is fixed, but in the case of stock there is uncertainty. And having said so, we also looked at the point of view of riskiness that can actually exist in stock; particularly, when it comes to corporate bonds which results in a credit risk or a default risk and then there is a liquidity risk and as well as a risk related to inflation. And in case of stock we talked about the risky nature of the stock and we particularly emphasized the concept of short selling of a stock which will be of great importance later on when you talk about portfolio optimization.

Thank you for watching.