Mathematical Portfolio Theory

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Module 02: Basics of Financial Markets
Lecture 01: Financial markets

Hello viewers, welcome to this next lecture on this MOOC course on Mathematical Portfolio Theory. So, far we had looked at some mathematical prerequisites necessary for the course namely the probability theory and it is basics along with the first and second moments, namely, the expectation and the variance and we talked about linear regression. And in particular, we discussed about two different distributions, one in discrete time and one is continuous time that will eventually be used when you talk about the asset pricing model. In this class, we will start talking about the financial aspect of the course and we will talk today about financial markets well we will emphasize on three specific things, namely, the different types of markets this will be followed by a discussion on the different kinds of financial instruments and then we will essentially look at the different kinds of traders. And please note that this discussion will essentially be focused on the financial derivative markets which I will explain in a little more detail.

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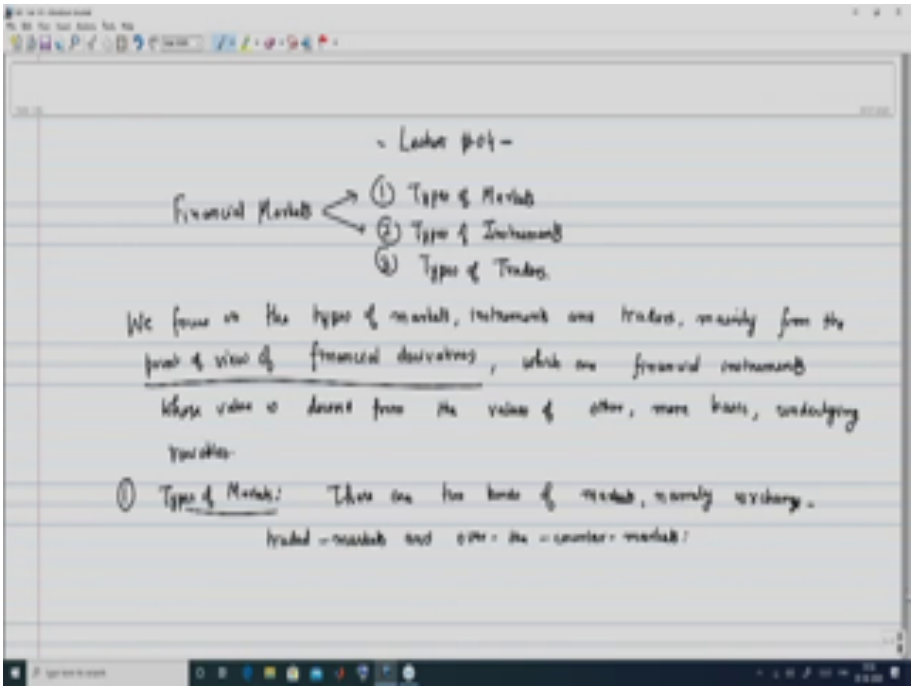

So, let us start this lecture. So, the emphasis of this lectures as I said just now this is going to be on financial markets and in particular, we will look at three aspects of financial markets namely the types of markets and will then talk about types of instruments and finally, we will talk about the types of traders. So, to sum this up I mean the we write that we focus on as I pointed out the types of markets, instruments

and traders and we will do this discussion mainly from the point of view of financial derivatives. So, this is important to emphasize on this and what are financial derivatives. So, these are financial instruments whose value is derived from the values of other more basic underlying variables. So, this essentially means that the word derivative here is from the point of view of the fact that the value of a financial derivative derives it is value from some more basic underlying asset. So, for example, you could have an asset and on that asset you define a derivative. So, the valuation of the derivative will move a dependent on how the value of the underlying asset moves. And so, consequently we say that the value of the derivative is quote unquote derived from the value of the basic underlying asset. So, now, let us start of with types of markets. So, in this case in the context of the discussion. So, we will talk about that there are two kinds of markets and this is a very broad classification. So, namely exchange traded -markets and over the counter markets.

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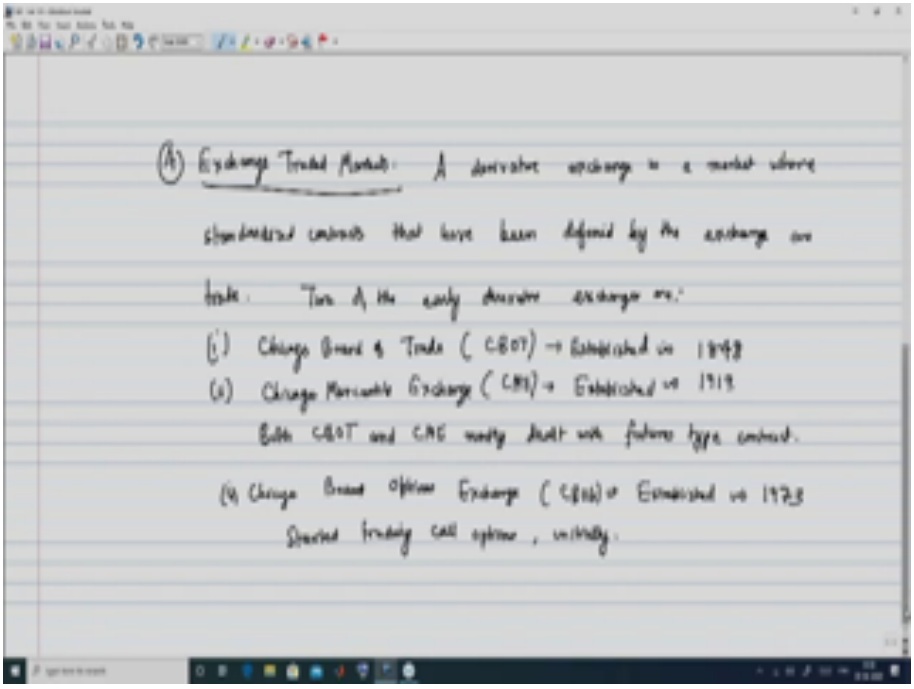

So, let us first talk about what is an exchange traded market. So, remember that our discussion is in the context of derivatives. So, will you we will say that a derivative exchange is a market where standardized contracts that have been defined by the exchange are traded. So, basically in an exchange traded market any derivative that is traded on such a market the terms and conditions of such a derivative is basically specified by the exchange and is binding on both the parties who actually go ahead with this transaction of this derivative under the supervision of the exchange. So, then two of the early derivative exchanges are one is the Chicago Board of Trade or CBOT which was established in 1848 and secondly, the Chicago Mercantile Exchange CME which was established as a rival to CBOT, but in 1919 and both CBOT and CME are mostly dealt with a future stripe contract. And we will explain this in detail when I talk about the various types of financial instruments in the next section. Now once the Black Scholes morton framework came in 1973 for which they won the Nobel Prize the options market got a very big boom because it was for the first time that mathematically justifiable way of pricing of options were introduced which give a far greater confidence to the market when it comes to pricing of options and this gave a great thrust to the development and very fast and rapid expansion of the options market. So, accordingly the another market exchange came up and this was the Chicago Board Options Exchange or CBOE and this was established soon after the Black Scholes paper came out in 73. So, this was establish in 1973 and they essentially started trading call options initially. So, again when I talk about options at that point I will discuss in a lot more detail as to exactly what a call option is ok.

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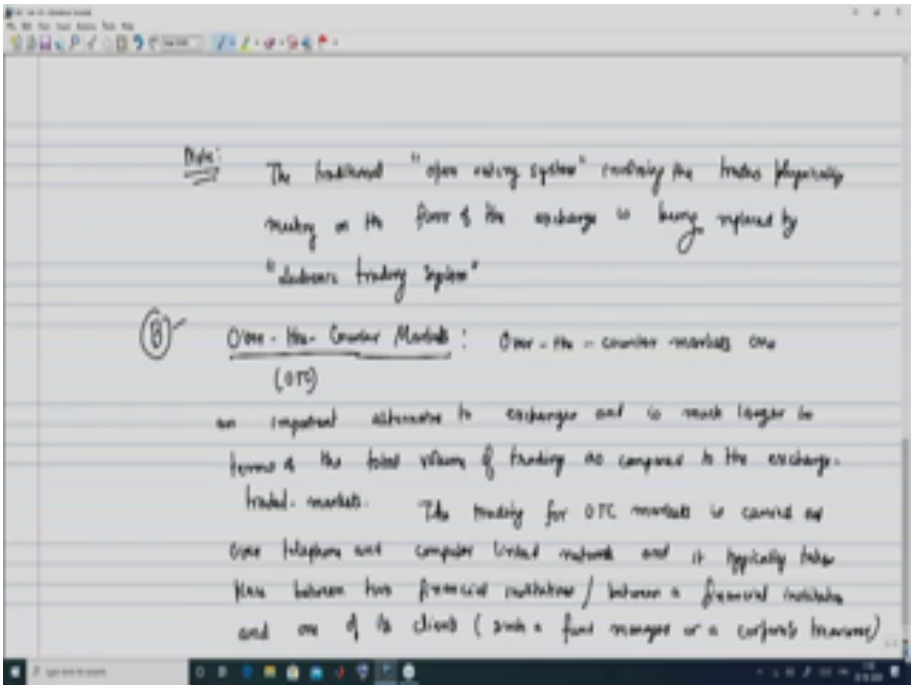

Now, I want to make a note about exchanges. So, our typical impression of an exchange is that you have a lot of traders on the floor of the exchange and they trade through a set of complicated hand signals. However, the evolution of the computer network we have seen that there is this shifting of this mechanism of functioning of exchanges where instead of traders physically trading on the floor slowly the trading process is being switched to the digital mode. So, accordingly we make the following note that the traditional and a process of the traders trading on the floor is what is known as the "open outcry system" which involves the traders physically meeting on the floor of the exchange is being replaced by "electronic trading system" alright. So, these are the two main points that I had to discuss about the exchange traded markets. And now I want to move on to what are known as the over the counter markets or this is what is known as OTC markets. So, over the counter markets these are an important alternative to exchanges and is more importantly much larger in terms of the total volume of trading as compared to the exchange traded markets. And now as this over the counter market the modus operandi of this is that the trading for OTCs. OTC markets is carried out over telephone and computer linked network and it typically takes place. So, any trading on an OTC market typically takes place between two financial institutions or between a financial institution and one of it is clients and the client could be such as a fund manager or say a corporate treasurer. So, here I want to emphasize a couple of things see the exchange markets is open to small investors; however, as I have noted the over the counter markets is just a place where large financial institutions trade among themselves or a financial institution trades with one of it is larger clients in terms of net worth. So, the advantage of an exchange traded market is that the exchange specifies the rules and acts as an intermediary. So, this means that there is a far greater safeguards for the both the parties that are involved in trading in the exchange market. However, the disadvantage of this is that becausee the contracts are standardized. So, it offers little flex very little flexibility in terms of how the contracts are designed. On the other hand the advantage of the exchange traded markets becomes the disadvantage of the over the counter market and vice versa. So, this means that because there is a greater protection in exchanged in market it offers less flexibility; however, since the over the counter markets they are a place where individualized contracts can be negotiated they are far more flexible. However, because they are over the counter markets and; that means, that there is an absence of an intermediary in the de form of the exchange; that means, that there is far lesser protection that is offered by exchange as compared in case of the OTC market. So, OTC markets are more flexible, but offers a lesser protection to both the parties where as the exchangeable market even though they are less flexible they offer far greater financial protection to both the parties that are involved because of the

presence of the exchange as an intermediary ok. So, this concludes our discussion about the markets.

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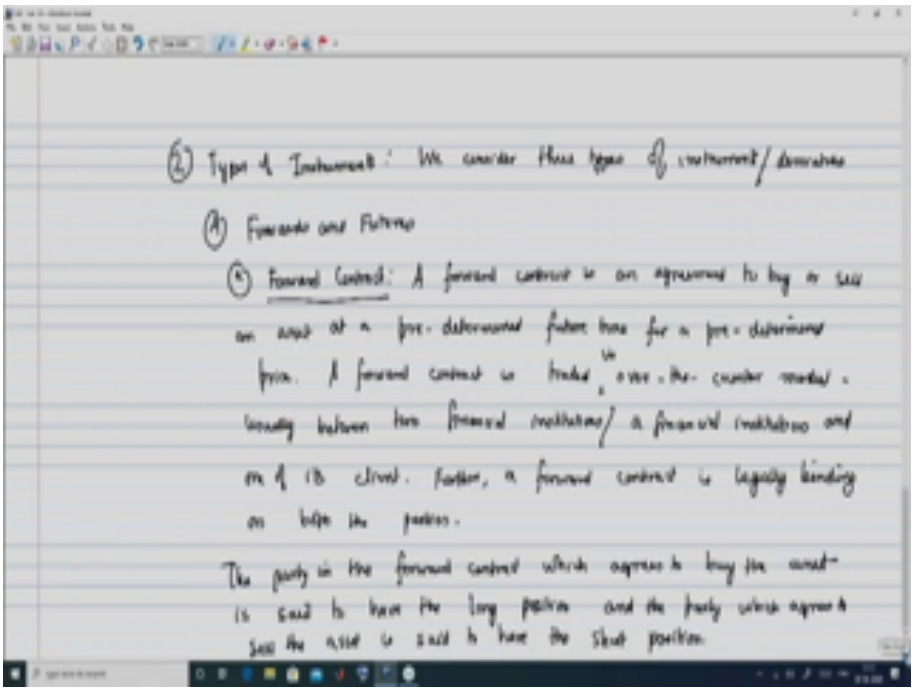

So, next we move on to financial instruments or types of instruments and we consider three types of instruments right and by this I finished I mean financial instruments or derivatives. So, the first one that you will consider is what is a forwards and futures. So, we begin with talking about what is a forward contract. So, a forward contract is an agreement to buy or sell an asset at a pre-determined future time for a pre-determined price. And another characteristic of a forward contract is that it is traded in over the counter market usually between two financial institutions or between a financial institution and one of it is client. Further forward contract is legally binding on both the parties ok. So, when I say that what I use the statement that the forward agreement is legally binding on all both the parties it means that whenever both the parties get into a forward contract that one party is going to buy the asset and the other buy is going to sell the asset and the time at which the purchase or sale will take place it is a future time and some price is fixed, but the future time and the price at which this transaction will take place is decided at the present time point. And so, this means that when I say that it is a legally binding contract it means that both the parties are legally obliged to fulfill their commitment so; that means, the party which has agreed to buy has to buy and the party which agree has agreed to sell has to sell the underlying asset and then there is no good possibility of any default at least from the legal point of view. Now at this point it might not be very clear as to why this is an important statement, but once we talk about options we will see that options are similar to forwards and features except that the options are legally binding on only one of the two parties which is the main feature they are distinguishes options from forwards and features. So, coming back to forwards as I said there are two parties and there is an agreement to buy an underlying asset for a pre specified price. So, at this point we will introduce two more terminologies to identify the party which buys the asset and the party which agrees to sell the asset and receive money for it in lieu. So, the party in the forward contract which agrees to buy the asset is said to have the long position and the party which agrees to sell the asset is said to have the short position.

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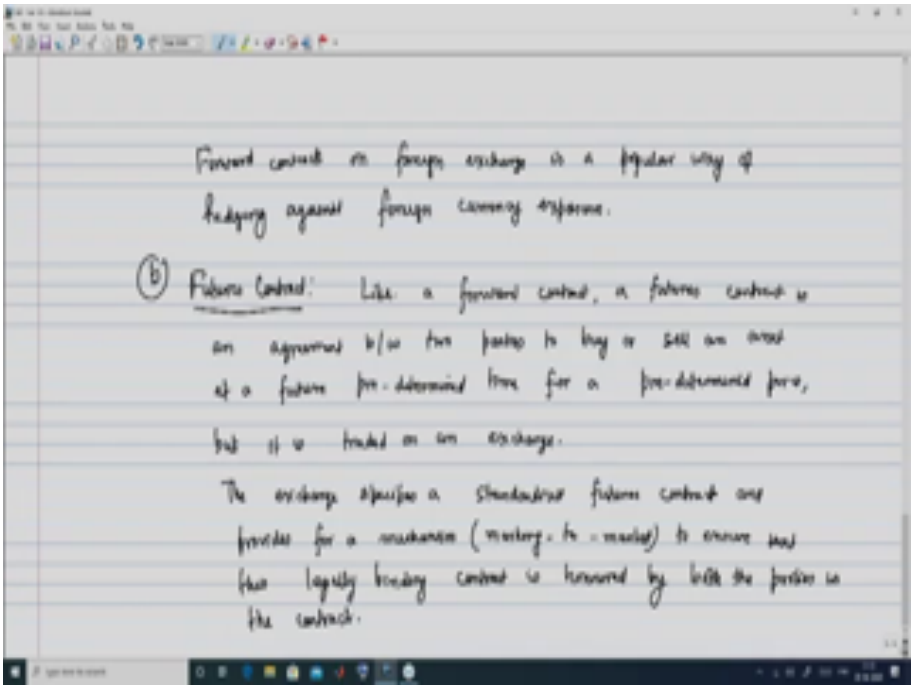
Now forward contracts one piece were forward contracts are popular is that forward contracts on foreign exchange is a popular way of hedging against foreign currency exposure. So, let us now move on to what is the futures contract I will come back to the issue of payoff later on. So, for futures contract. So, the futures contract is very similar to a forward contract with only distinction being that it is traded on exchanges. So,



unlike a forward contract, a futures contract is an agreement between two parties to buy or sell an asset at a future pre-determined time for a pre-determined price, but it is traded on an exchange. So, I should actually say this should be like. So, just to wind it up so, basically like a forward contract a futures contract is an agreement to buy or sell the underlying asset at a future pre-determined time for a future pre-determined price, but it is traded on an exchange. So, it is in principle it is the same as the forward contract with the only distinction being is that the forward contract is over the counter while this is an exchange traded derivative. And now since there is an exchange involved so; obviously, as I said that there is a mechanism to ensure that neither of the party defaults and both the parties they are protected. So, accordingly now I can make the statement that the exchange specifies a standardized futures contract and provides for a mechanism and this mechanism is known as marking to market to ensure that this legally binding contract. So, like forward this a legally binding contract is honoured by both the parties in the contract. So, let me just wind up with one last observation that is regarding the payoff.

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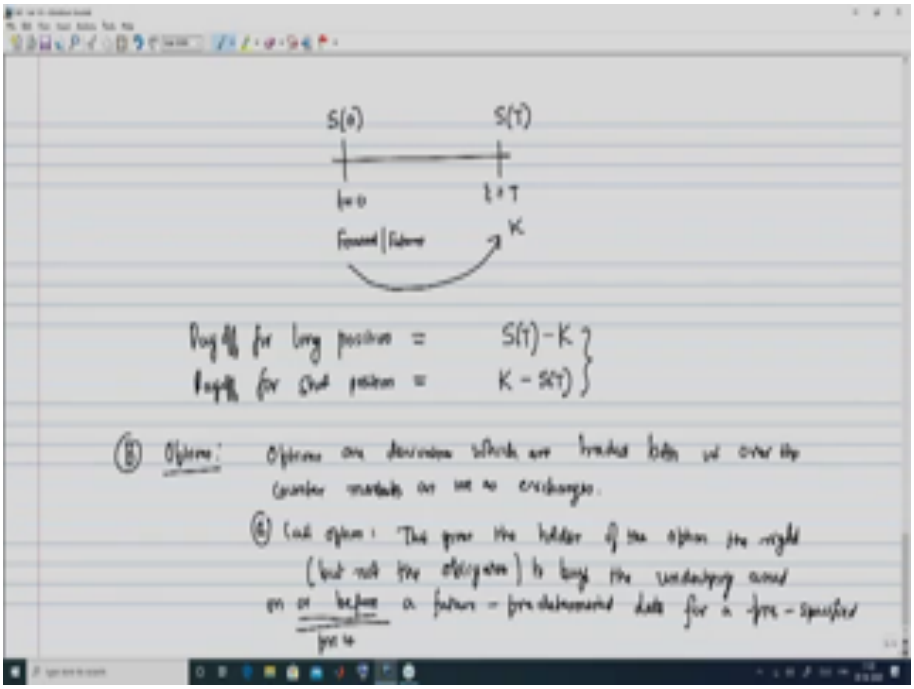
So, in this case suppose that I have some time t equal to 0 at which I get into a forward or a futures agreement and suppose the price of the underlying asset at time $t = 0$ is denoted as $S(0)$. Now suppose that the price that we agree so, we agree that the asset S will be sold at time $t = T$ for some price K and suppose that at that time the price of the asset in the market which is known as the spot price is $S(T)$, then the payoff for long position is going to be given by this. So, pair for the long position is going to be the following that ah. So, please observe that the party which is agreed to by the underlying asset at time t equal to T, they are going to sell there and they are going to pay an amount of K and buy the asset so; that means, an amount of K is spent by them. So, I write $-K$ and then they can immediately sell it in the market in which case they will receive an amount of $S(T) - K$. So, essentially is the difference between. So, another way of looking at it is basically is the difference between the price that the party with the long position would have paid at time $t = T$ if they had not got into the contract with the price that they are paying because they got into the forward or futures contract. So, if $S(T) > K$; obviously, the party with the long positions tends to gain because again an amount of $S(T) - K$ because they are paying price K for an underlying asset which otherwise would have cost them $S(T)$ at time $t = T$. And on the other hand if $S(T) < K$, then they end up paying a higher amount of K as compared to the prevailing market price $S(T)$ and which wish that they have basically incurred and loss. From the counterparty point of view; that means, from the point of view of the short position the payoff is going to be. So, pay off for short position this is going to



be nothing my, but $K - S(T)$. So, this means that they are receiving an amount of K and $-S(T)$ is the amount. So, suppose that the party they where they were holding on to the asset if they had sold the asset in the market they would have received an amount of $S(T)$. However, instead of that they will have to sell this asset for a price of K because they have already got into a forward or a futures contract which is legally binding. So; that means, that their gain or loss will be given by $K - S(T)$ that is the difference between the for price at which they agreed to sell under the forward or features agreement and the price they would have got had they are not got into this agreement and you notice that the sum of this two this is going to be equal to 0. So, this is what is known as the 0 sum game. So, whatever is the gain of the party with the long position becomes the loss of the party with the short position and vice versa. So, if $S(T) - K$ is positive; that means, the long proposition party they gain an amount $S(T) - K$ which is same as the loss of the party with a short position and if the party with the short position has a positive gain then that gain is the same as the loss of the party with the long position and these are what are known as the payoffs for the long and the short position respectively, ok. So, this concludes our discussion on forwards and futures and the next thing that we move on is what is known as options. So, options are one of the most mathematically well studied financial derivatives. So, these are so, also options are derivatives which are traded both in over the counter markets as well as exchanges. So, before I move on to the specifics of options what I want to point out here is that while forwards and futures was a legally binding contract on both the parties options are actually legally binding only one party. Now both forwards features and options are an agreement to buy or sell the underlying asset for a pre specified price at a future pre specified time. However, in case of forwards and futures both the parties have the obligation to honour the contract; however, the options are designed in a way that only one party which has there is always the obligation and the other party has the it does not have that obligation. So, what actually happens is that they are the party which has the obligation that is in a weaker position as compared to the party which does not have the obligation. And so, consequently what happens is that the party which does not have the obligation is in a position of leverage over the party which has the obligation for which the party with the obligation being in a weaker position will demand an upfront amount to be paid at time t equal to 0 from the party which is an advantageous position which is known as the price of the option which is a very very important area in mathematical finance. Now in order it so, now when you talk about the purchase or sale of an underlying asset under options where one party has the obligation and the other has the right to sell the asset it immediately brings us to two different ways in which this can happen. In one case the one of the two parties of the option agreement will have the right to buy

the underlying asset, but not the obligation and the other possibility is the one of the two parties will have the right to sell the asset, but not the obligation. So, according to this brings us to the basic classification of options namely call and put options. So, accordingly we start off with the call options. So, the first one let us talk about call option. So, this gives the holder of the option; that means, the party with the position of advantage. So, holder of the option the right, but not the obligation to buy the underlying asset on or before a future pre-determined date. So, please see that we now allow for this purchase to happen before a few future pre-determined date for a prespecified price, ok.

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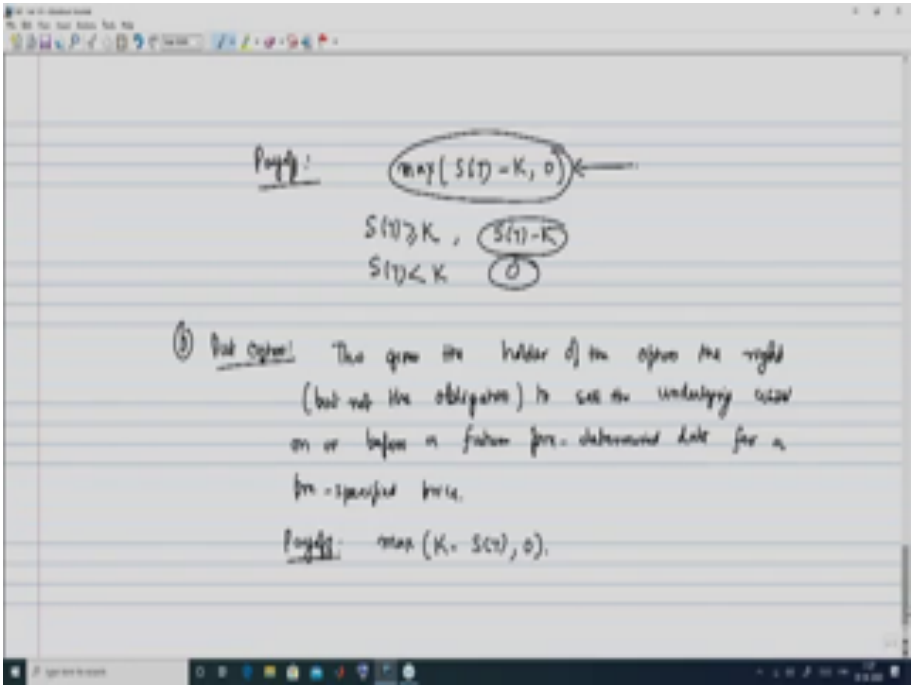

So, let us look at this payoff. So, in this case the payoff will be given by $\max\{S(T) - K, 0\}$. So, the reason for this is the following that the party which has the right to purchase, but not the obligation. So, when they. So, if $S(T) > K$, then obviously, the party which has the right to buy the asset is going to buy the asset for a price of K and then sell it in the market thereby making a profit of $S(T) - K$. However, if $S(T)$, so, this can be equal to sign and if $S(T) < K$ this means that the party which has the right to buy will not exercise or execute their right the reason being that if they execute the right they have to pay a higher price of K while they have they can always purchase the particular asset for a lower price and the prevailing market for a lower price of $S(T)$. So, it does not make sense for them to buy the asset for a price of K which is higher than the prevailing market price at time $t = T$. So, in this case basically they will get an amount of 0, because no transaction actually takes place. So, this gives us that either you receive an amount of $S(T) - K$ or you receive an amount of 0. And so, this means that you basically get $\max\{S(T) - K, 0\}$ and this is from the point of view of the party which purchases the option or which agrees to buy the so, which basically retains the right to buy the asset, but not the obligation to buy it and consequently their payoff is going to be given by this, recall that in case of a forwards or futures we had $S(T) - K$. So, here there is no possibility of you having a payoff that is going to be a negative, it is either going to be 0 or it is going to be positive, but the payoff for the long position in case of the forward or futures it can become negative. So, there is a possibility that you will actually incur losses. So, this means that one of the parties in the call option that the party which is the right and no circumstance are they going to make a loss which causes a disadvantage to the other party. And this is the reason why this party which has this done a non negative payoff that party has to make an upfront payment of premium to the party which has the obligation in this case the party which agrees to sell the underlying asset for a price of K an amount and that amount is what is known as the price of the option that I had mentioned earlier ok. Now let us talk a little bit about

put option. So, here this gives the holder of the option the right, but not the obligation to sell the underlying asset on or before a future predetermined date for a pre specified price and in this case the payoff is going to be $\max\{K - S(T), 0\}$, ok.

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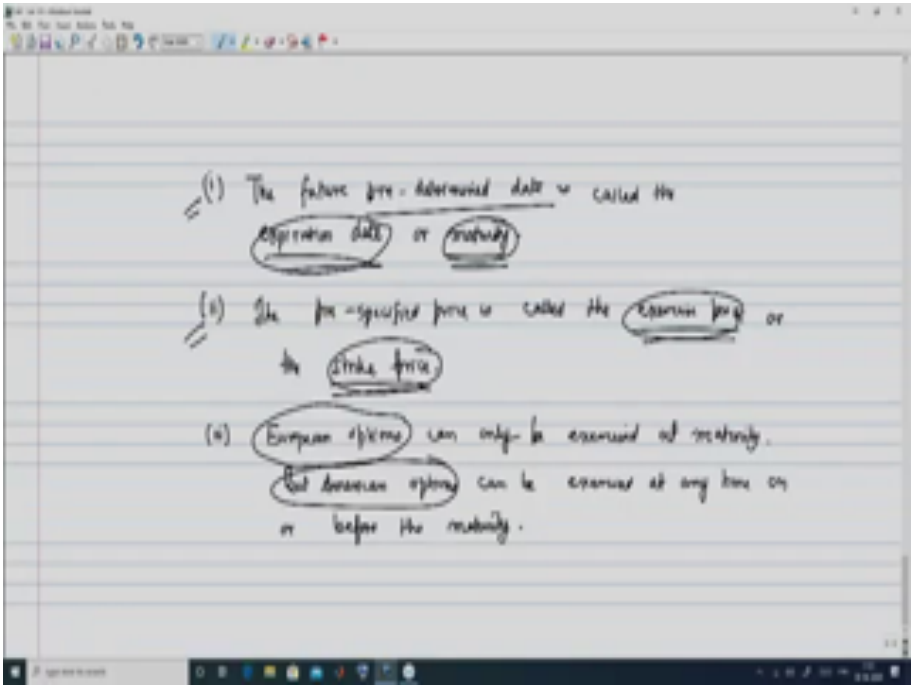

So, to wind up the discussion on a options I will just quickly describe some terminologies. So, the future predetermined date is called the expiration date or maturity and the pre-specified price is called the exercise price or the strike price. And finally, I want to make an observation that European options can only be exercised at maturity, but American options can be exercised and at any time on or before the maturity. So, here I want to identify three things which has led me to make these three observations say I said that it is said agreement where there one party has the right and the other has the obligation to buy or sell the asset or before a future pre-determined date at a future at a pre-determined future time. So, there are three things one of them is that there is a future time and one is that there is a pre specified price and finally, once I have talked about our future pre specified future time then I have said that you can do the exercise at any time on or before that future pre-specified time. So, first what I talked about is that I talked about this future pre-determined date with and we call this as the expiration date or maturity and the future pre-specified price we call it the exercise price or the strike price and I have said that the option can be executed or that a transition can happen and at any time on or before this future date which I call the expiration date or maturity. If the contract is such that the purchase or sale of the underlying asset can take place only at the maturity and not anytime before it then we call it as a European option. But on the other hand if the option has the flexibility that it can be exercised and I and the purchasing cell can take place at any point either at expiration on which and which or slash maturity or any time before it such kind of more flexible options are what are known as the American options. So, that way you can see that we have seen that there are two broad classifications of options, one is the call in the put where the holder has the right to buy and or the right to sell respectively and then whether the execution can take place only at maturity or it has the flexibility of any time on or before the maturity results in what are known as European and American options respectively. So, this gives us a 4 4 combinations namely European called European put as well as American call an American put. So, when come to the last of the three derivatives that we wanted to talk about and the third one derivative that you talked about is what are known as Swaps.

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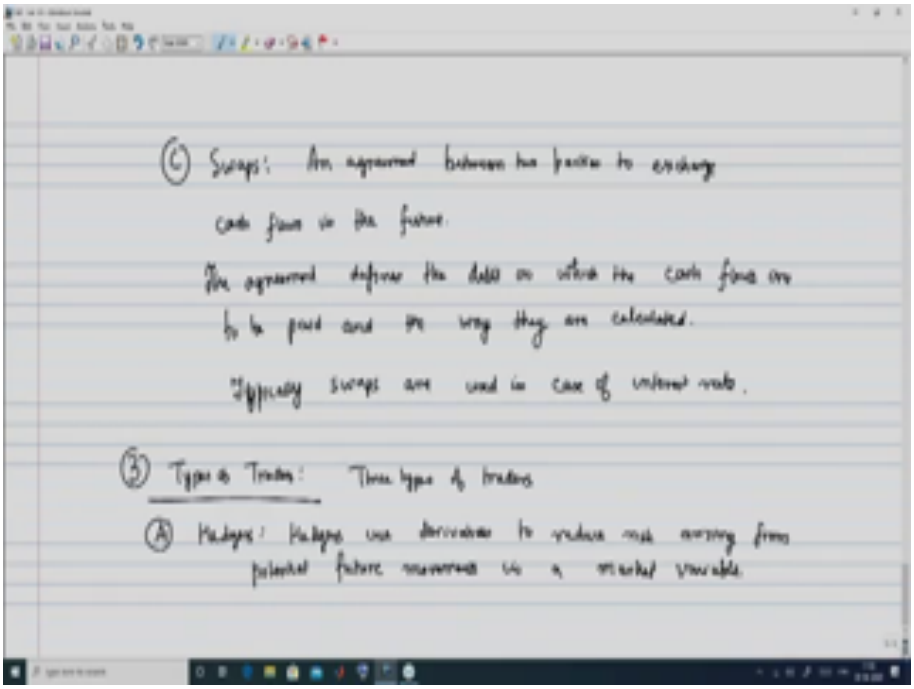
So, here what are swaps. So, it is again it is an agreement between two parties to exchange cash flows



in the future and this agreement defines the dates on which the cash flows are to be paid and the way they are calculated. So, typically swaps are used in case of interest rate. So, let me explain what do I mean by exchange of future cash flows, suppose you decide to take a home loan and you know that the home loan has a floating interest rate; that means, if we start today with say 12% interest rate, a year from now the interest rate might be different maybe 8% you become 13% or 11% for example. Now even you have taken the loan from say a bank A and you are not comfortable with the idea that your interest rates keeps floating or fluctuating and so, that there is a certain amount of uncertainty. So, what you do is that, you go to bank B and the bank B agrees to your proposal and says that alright, what we will do is that we will pay the floating interest rate to bank A and in lieu of it you basically give us a flat interest rate say of 14%. So, this means that instead of playing a floating rate to bank A or a continuously shifting rate to bank A you are paying and a fixed interest rate to bank B and bank B in turn takes care of the floating interest rate which it pays to bank A. So, what you have done essentially is that you have exchanged the cash flows. So, instead of paying to bank A you are now paying this to bank B. So, what you have done is that, you have basically swapped your interest payments a from bank A to bank B with bank B taking care of your interest rate payments to bank A ok. So, this concludes our discussion on derivatives. So, finally, we come to the last topic for this class and that is on different types of traders. So, there are broadly three types of traders. So, the first one are what are known as hedgers. So, hedgers use derivatives such as forwards futures options and swaps to reduce risk arising from potential future movements in a market variable.

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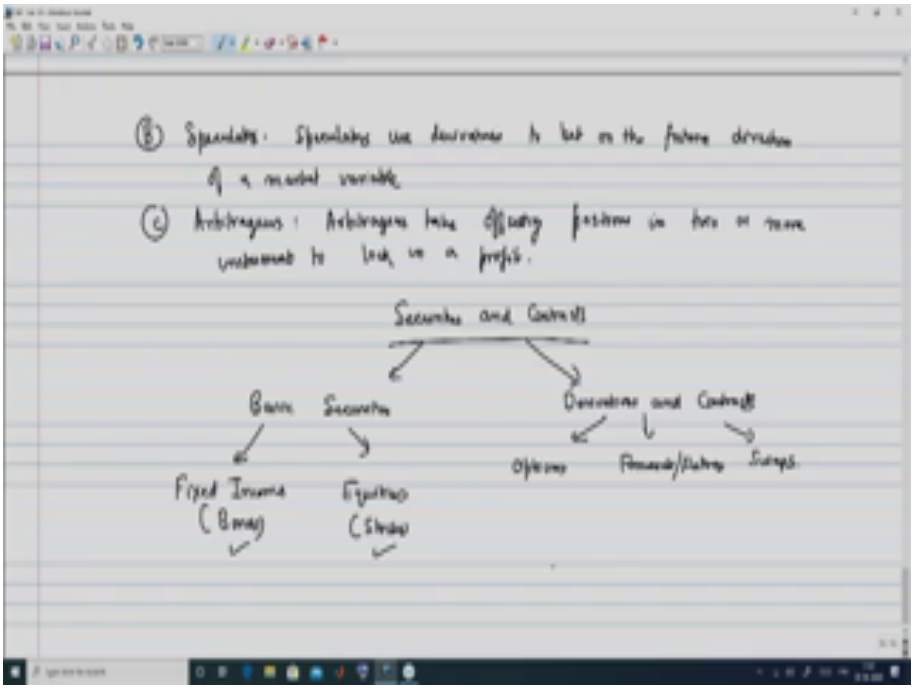
The second one is what are known as speculators. So, a speculators use derivatives to bet on the future direction of a market variable. And finally, we have something called arbitrageurs. So, arbitrageurs take offsetting positions in two or more instruments to lock in a profit. So, let me explain this a little more detail who are the hedgers. So, the hedgers use derivatives to reduce risks. So, hedgers are essentially risk managers who will use these derivatives to hedge against or take preventive measures against unfavourable movement in case of the movements of the underlying asset. So, a simple example could be that a hedger could buy a futures to lock in the price of a certain underlying asset at a future time point. So, an example for this is the as I said is the foreign exchange rate. So, a company which is expects to receive some foreign exchange at say 6 months down the line is worried about that the exchange rate might move in an unfavorable direction to them. So, they get into a futures agreement with a bank or a foreign agreement with a bank under which the bank agrees to pay them a fixed exchange rate decided at time $t = 0$. So, that



way they have basically hedged or protected themselves in the sense that they know certainly sitting at time $t = 0$ how much is going to be the foreign exchange rate that they will receive at the future time point of 6 months. Speculators on the other hand there are people who are willing to take risk and they basically go and speculate about the direction in which a particular asset or the market will move and accordingly they take up the positions to reflect their speculative opinions. And finally, arbitrageurs are people who basically take advantage of the price difference between two financial instruments or two more than two financial instruments. So, what they do is that, they are always looking for to financial instruments in the market which are under priced and they buy it and then they basically sell it in another place for a higher price. So, effectively to sum it up arbitrators are nothing, but individuals who are on the lookout for a price mismatch and taking advantage of them. So, an example of this is that there could be two markets A and B, where a asset in market A is selling for 20 and that in market B selling for 30. So, they can buy from one place at 20 and then sell it in the other place for 30 thereby pocketing a profit of 10. However, arbitrage opportunities do not last for long because once the market notices this price mismatch then an automatic mechanism will ensure that the prices eventually match and the market will correct it iself. So, finally, what I want to do is, I want to do a broader classification and this will act also as a prevail for my next lecture. So, this classification will be of securities and contracts. So, we have basically discussed the contract part of it today. So, contracts they can be. So, the contracts I includes basically derivatives and other contracts and we have looked at three types of this we have looked at options forwards and futures and swaps which we have already discussed in todays class, but we are here to discuss on securities. So, we will basically look at the two basic securities, one of the basic securities is the what is known as the fixed income securities and the most common example of this is bonds which we will discuss in the next class. And the other one is equities and the most common example of equities are stocks. So, bonds and stocks are something that we will discuss in the next class. So, just to sum it up what have we done today we have basically looked at financial markets with an emphasis on the derivatives market and we identify the three components of this markets. First of all one is the kind of market where we talked about exchange traded and over the counter markets and what are the advantages and disadvantages with these two markets. Next we talked about financial derivatives and we talked about the three most important financial derivatives namely forwards features, options and swaps. And finally, we briefly outlined on what are the different types of traders namely hedgers, speculators and arbitrators and then we looked at a broad classification of securities and contracts. And contracts are something that we have discussed it in detail in the next class and

the securities part we will discuss in the next class with a particular emphasis on the basic securities namely the fixed income securities and equities, the two examples of which are bonds and stocks respectively.

Thank you for watching.