Mathematical Portfolio Theory

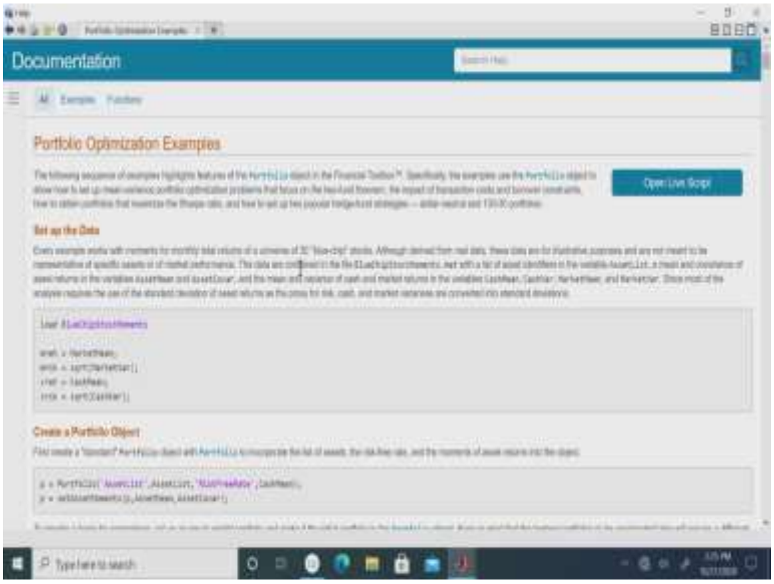
Dr. Siddhartha Pratim Chakrabarty
Department of Mathematics
Indian Institute of Technology, Guwahati

Module – 08
Applications with market data
Lecture – 02
Portfolio optimization

Hello viewers. Welcome to this next lecture on NPTEL MOOC course on Mathematical Portfolio Theory. Recall that in the previous lecture, we started talking about using the financial toolbox of MATLAB in order to look at Portfolio of optimization problem and in this lecture, we will continue our discussion on that and we will look at today's topic on two-fund theorem being used in order to derive the optimized portfolios.

So, for this, we look at the derivation of an efficient frontier and the tangent portfolio and with the point of tangency will later be identified as the point, where the sharp ratio is maximum and we look at this problem not just in terms of the basic mean variance optimization setup; but also, by incorporating some practical realistic constraints that we see in actual market.

(Refer Slide Time: 01:20)



So, we start off with portfolio optimization examples. So, we will look at a few examples which mainly uses this portfolio object that is a critical part of the financial toolbox and the specific examples that we will look at is how to set up a mean variance portfolio optimization problem, with an emphasis on a few things, namely the two-fund theorem and also will incorporate the notion of transaction cost and turnover constraint into the problem.

And we will look at the problem of maximization of the sharp ratio which was an indicator of effectiveness in the management of the portfolio and we will briefly mention two popular hedge fund strategies; namely, the dollar-neutral and 130-30 portfolio. So, we will just mention those since there they were not a part of our regular discussion in the preceding lectures. So, first thing we do is we will have to set up the data and we will see that how this example works for the monthly total return.

So, we are just looking at the monthly returns and by total returns that we mean that we are talking about the returns without bringing into picture things like transaction cost and turnover constraints and we will focus on composition of 30 “blue-chip” stocks.

So, blue-chip stocks are the ones of companies which, have a reputation for quality, reliability and most importantly, the profitability both in good and in stressed market conditions and this term blue-chip comes from the game of poker where you have chips with the blue-chips having a high value.

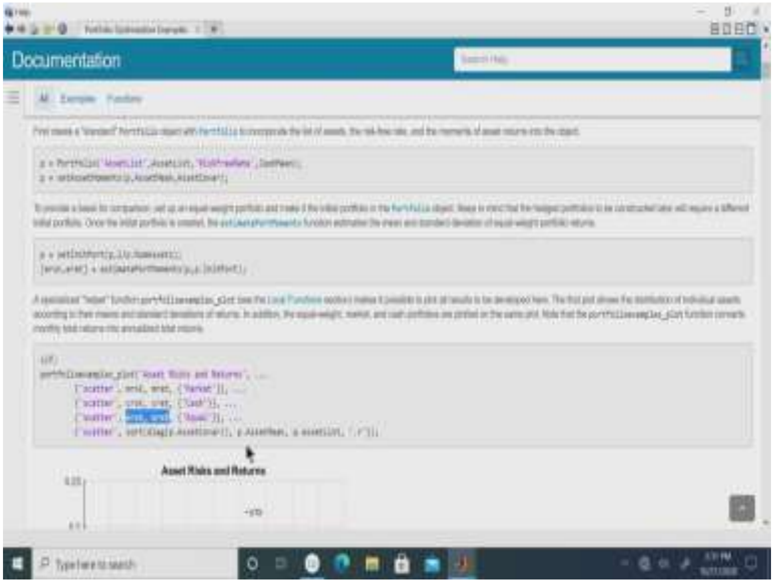
So, for this purpose, the MATLAB toolbox here uses data which are real data, but this data have been chosen for illustrative purposes and is not intended to actually talk about or giving investment advice. So, instead of this data, which is included in MATLAB which is called the Blue-ChipStockMoments dot mat. We essentially, which comprises of this blue-chip stock, so, you can use your own files for the asset parameters.

So, accordingly, we first load this, the Blue-ChipStockMoments. So, this will give the moments of this blue-chip stocks and remember that these are just the monthly total returns and once, so, once we have loaded this, then we will take the we will define mret that is the market return to be equal to the MarketMean and market risk is will be taken to be the square root of the market variance.

So, accordingly, and similarly, we will take what is known as the CashMean and identify is at cret and we will talk about the square root of CashVar which will be taken as crsk. So, let me just elaborate what this CashMean, this concept of CashMean and CashVar and MarketMean and MarketVar. So, here the cash term indicates the cash component or this uses a proxy for cash.

So, you can view this as some sort of risk free investment and the MarketMean or the market variance, you can view this is that these are akin to equities or a portfolio of equities or particularly, the market portfolio. Now, accordingly, for that purpose, we essentially look at the average return from the MarketMean and we look at the average mean of the cash. And for the counterpart risk, we look at the square root of the variances for the market as well as the cash.

(Refer Slide Time: 05:14)



So, the next thing that we do is we create a portfolio object and the purpose of this creation of a portfolio object is to bring into the picture the asset list. So, we will incorporate the asset list and we will bring in the risk-free rate which is taken to be the CashMean and the moments of the asset return, so we will basically incorporate the individual AssetMeans as well as the asset covariance.

So, this is just like a starting point, where you set up the basic essential components that are required for the portfolio object. So, then what you do is for the purpose of comparison, what you do is that we set up an equal-weight portfolio and by equal-weight portfolio.

I essentially mean that the weights assigned to the portfolio are taken to be all identical to each other and we make it the initial portfolio. So, accordingly, you see that we set the initial portfolio to be 1 over p number of assets and so that means, the e risk and e return. So that means the e here stands for the equal-weighted portfolio.

This will be essentially these values will be return by the estimation of the portfolio moments of the initial portfolio that was set up with the equal number of weights or the equal level of weights being assigned to all the assets that are part of the portfolio. So, the next thing we do is we will now look at how to plot this. Now, in order to plot this we what we are going to do is that we are going to look at three components.

So, the first two are the market and the cash components and for that we recall that the market risk and market return and the cash risk and cash return and so, for the equal-weighted portfolio, we take the risk and the return for the equal-weighted portfolio.

(Refer Slide Time: 07:09)

And for these three components namely the market, cash and the equal-weights, we essentially, then have this plot for all of this in the mu sigma diagram. So, this is nothing but on the y axis, we have the mean of returns and on the x axis, we have the standard deviation of returns.

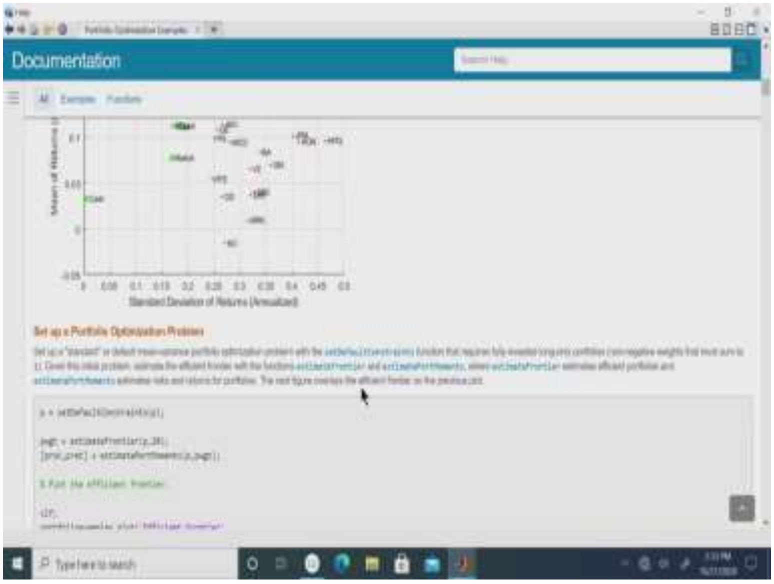
So, let me just identify particular terms here. So, in this case, what you are doing here is that we are doing a scatter plot. So, the first thing that we are doing is a market portfolio. So, this is where the market portfolio lies. Then, you have the cash portfolio which lies here.

So, as you can see that the risk is effectively 0 because it is a cash position and the portfolio with all equal words lies here and then, in addition to that we also do the scatter plot which will plot the individual mean and variances for each of the individual assets that are a part of the database with these blue-chip stocks.

So, for example, here you will see there are several databases. So, for example, if you observe here HPQ that is for a Hewlett Packard and then, you have GM that is for General Motors. For example, there is JNJ; so, that is Johnson and Johnson. So, each of those corresponds to a whole bunch of these companies.

And so, this is just a mu sigma scatter plot of all the companies that I have been considered in the database, their expected return and risk as given by the standard deviation along with the risk and return of the cash, the market and the equal-weighted portfolio.

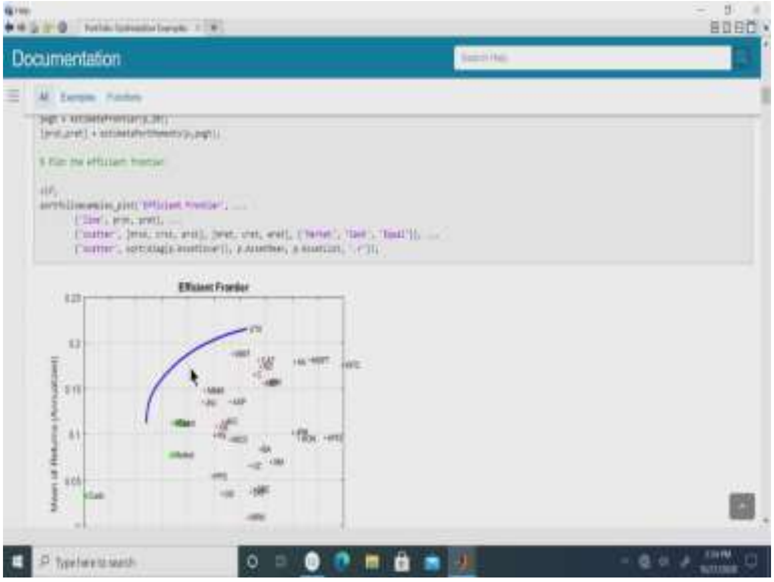
(Refer Slide Time: 08:49)



So, this is the basic plot that we have for this database of blue-chip stock. Now, we need to set up the optimization problem one by one and then, solve for those. So, what you do is that we will; so, we set up a mean variance portfolio optimization problem and it is set up default constraints are set up for that.

So, by default constraints, it means that of course, you know you will have the sum of the weights being equal to 1 and in addition to that you are not allowed any short selling; that means, your weights must all be greater than or equal to 0. So, with this initial problem, the goal is to figure out what is going to be the efficient frontier.

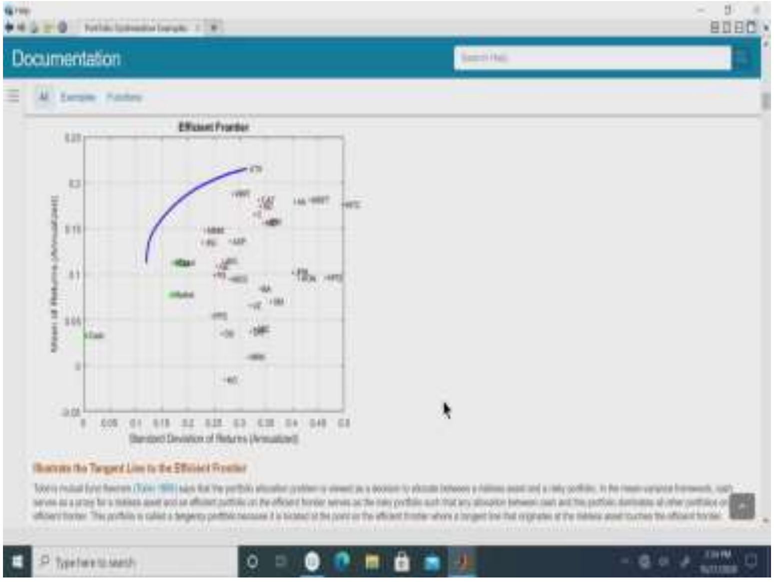
(Refer Slide Time: 09:25)



So, accordingly, we set the default constraints as I mentioned that all long position. So, the sum of weights being equal to 1 and then, what you do is that we estimate the efficient frontier. And then, we get the estimate the portfolio moments of the efficient frontier and store them as the p risk and prsk and p return.

So, in this case, what does the efficient frontier look like? The efficient frontier looks like you know it is pretty much the same diagram that you have here with the you got earlier the scatter plot; but on top of that, so in this case, you observe that it is the scatter plot of the market, cash and the equal-weighed portfolio as well as the individual assets.

(Refer Slide Time: 10:12)



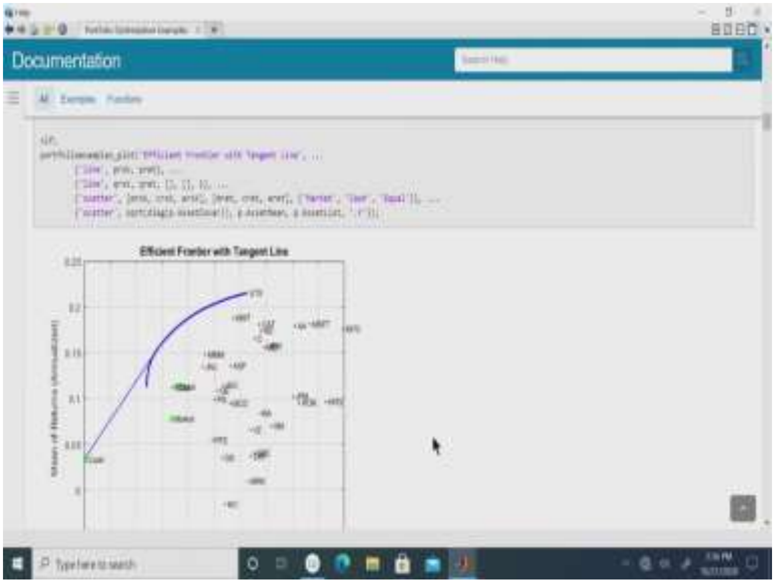
But on top of this, we have this blue curve which is going to be the efficient frontier of this database of individual assets of all the blue-chips stock companies.

(Refer Slide Time: 10:17)

So, now we were we will talk about the tangent line to the efficient frontier. Remember that we will eventually make use of the goal; the first problem that we look at is going to be the two-fund theorem. So, the two-fund theorem will basically make use of combination of investment in cash as well as a risky portfolio. So, now what you do here is the following that the portfolio that has been already been set up, the object that was set up earlier, it already has the risk-free rate.

So, what do you do is the following. So, we can generate the tangent line and what is the tangent line? So, the tangent line is nothing but it is basically a collation or a collection of all those points on the mu sigma diagram, as you vary your risk free rate from 0 percent to 100 percent or that is from 0 to 1.

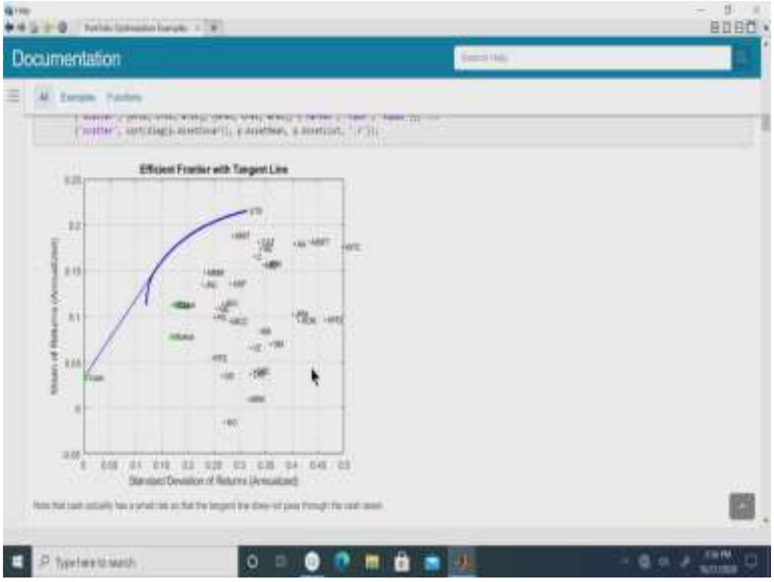
(Refer Slide Time: 11:18)



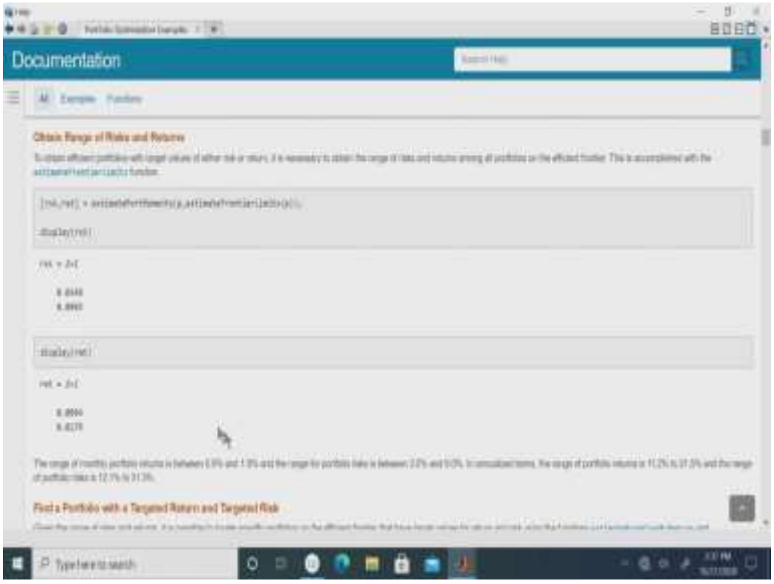
So, accordingly, we set the budget of this risk-free asset from 0 to 1 and then, what you do is that we essentially what we now have is we will have the scatter plot, as before we have the efficient frontier and then, now will essentially generate this line.

So, here you observe that we will make use of the alphabet p risk and p return for in or for the portfolio itself and then, when you talk about in order to generate the efficient frontier and the q here will indicate the tangent line which is tangent to the efficient frontier.

(Refer Slide Time: 11:51)



(Refer Slide Time: 11:53)



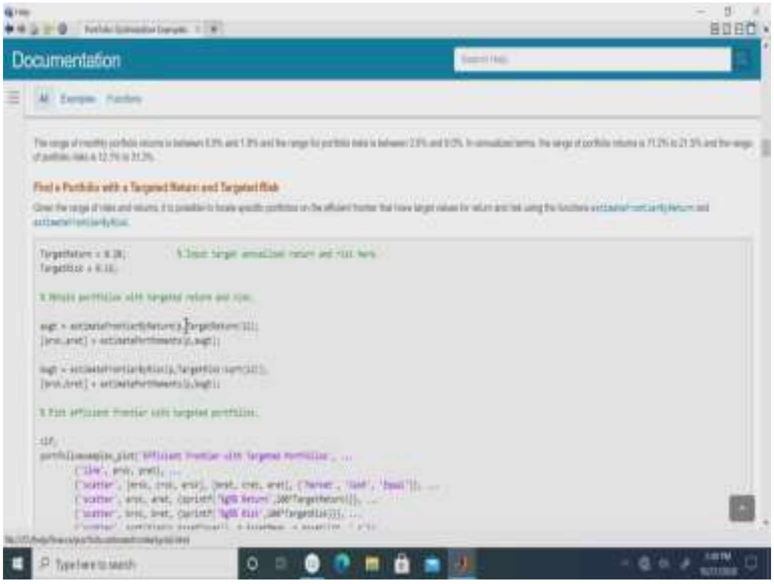
So, now, what we do is that in this case, so now, what you have to do is now that we have set up this mu sigma diagram, for each of the individual assets along with these three assets and we now have the efficient frontier as well as the tangent portfolio. So, we now need to what you do is that we want to obtain efficient portfolios with a pre-specified value of either risk or return.

So, what you do is basically that means, that you want to figure out an efficient portfolio for your either where you either pre specify what is the level of risk that you are willing to take or you pre-specify what is the return that you are expecting and it is necessary to obtain the range of returns and risk amongst all the portfolios. So, accordingly, what you do is in order to accomplish this we talk about what is known as the estimate frontier limits function.

So, what is this estimate frontier limit? So, estimate frontier limits basically what it does is that before you can actually set your target for your return or you want to set a maximum tolerance for risk level, you first really need to know that what are the values of return and risk at this end point and at this particular end point, so that you do not end up choosing a return that is not even anywhere on the efficient frontier.

So, accordingly you get this. So, if you observe carefully; so, here we say that the monthly portfolio returns are between 0.9 percent and 1.8 percent. So, this graph is remember that this is based this efficient frontier is based on the monthly returns of the individual assets.

(Refer Slide Time: 13:38)



So, accordingly, you observe that for the returns that means, from the return at the bottom most end of the efficient frontier to the topmost end, it ranges from 0.9 to 1.8 percent. Similarly, if you observe carefully and from 3.5 to 9 percent, so, this; so, in annualized terms, this means that it will range from 11.2 percent to 21.5 percent. So, let me just be more specific that if you look carefully here that this value that you have here, this is somewhere around 0.11 and then, the top value is somewhere 0.21.

So that means, the range is from 11.2 percent to 21.5 percent and this is what translates to 0.9 percent and 1.8 percent on a monthly basis and likewise, you have 12.1 percent to 31.3 percent. So, if we observe carefully, the variance that we have here is probably 0.12 and this is around 0.31.

So that means, the annual rate risk percentage is from 12.1 percent to 31.3 percent and this manifests to an equivalent monthly value and in order to move to the monthly value, you just cannot divide by 12; but you have to factor in a square root of capital T which is the square root of 12 ok.

So, now that we have this entire setup of the assets along with the efficient frontier and the tangent portfolio, we now can talk about we once this has given us an idea as to what range of returns that you are looking at and what is the risk range that is available to us, so, now, we are in a position to answer our prior question of determining a portfolio with a targeted return and a targeted risk.

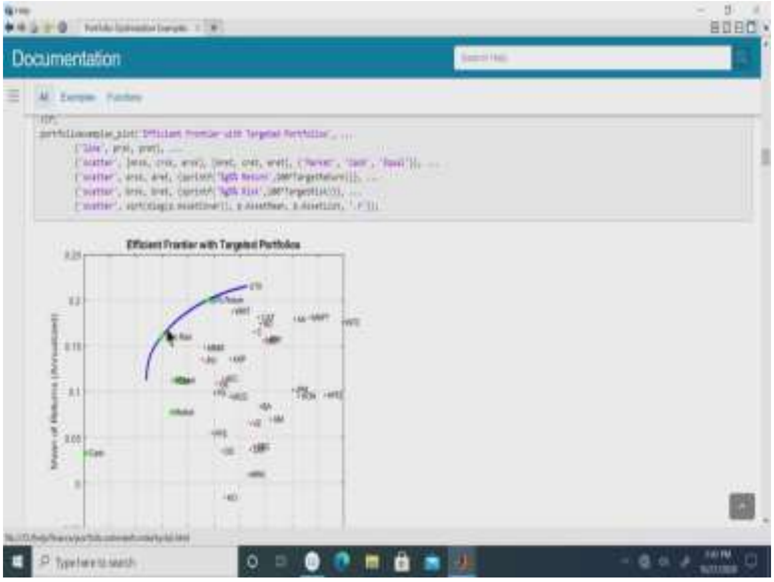
So, by targeted, we mean here that there is a specific target that we have in mind as far as our return is concerned and there is a level of risk that we are not willing to exceed as far as our investment strategy is concerned. So, accordingly, we say that, so, what you do is that for example, you know for illustrative purposes, we will input a target return of 0.2 or 20 percent.

And this means that we want a expected return of 20 percent and we will target the risk at 0.15. So, that means, we are willing to accept up to 15 percent of the risk. So, accordingly, what you do is that we will estimate the efficient frontier by return and we will essentially take the target return by 12.

So, this is what is going to be this is the will input the annualized target return and risk and then, we will convert this to the monthly return and risk and so, this gives us the estimated. So, in this case, arsk and aret, these are going to basically give us the target return on a monthly basis. So, that means, that it is going to give us a portfolio or the efficient portfolio with the targeted return of 20 percent per annum.

And in the second case, you see that we are taking the target risk and dividing by square root of 12. Remember that I said that we have to divide this by square root of capital T. So, in this case, we get the efficient frontier for the second case when our target risk has been set at 15 percent.

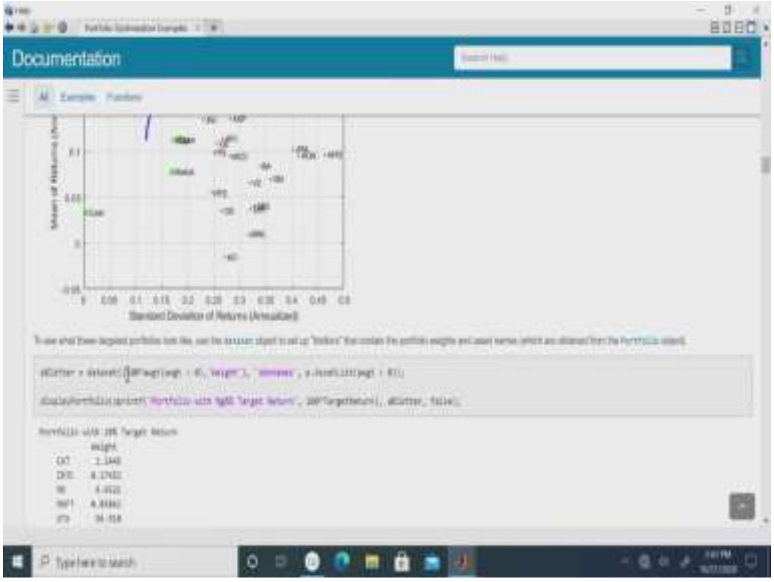
(Refer Slide Time: 17:05)



So, once you do this what you do is that we now keep extending our plot that was done earlier. So, remember that so as before, we have all the return and risk of the individual components along with whatever we had seen earlier in terms of the market equally and cash return and we have the efficient frontier that we have we see here.

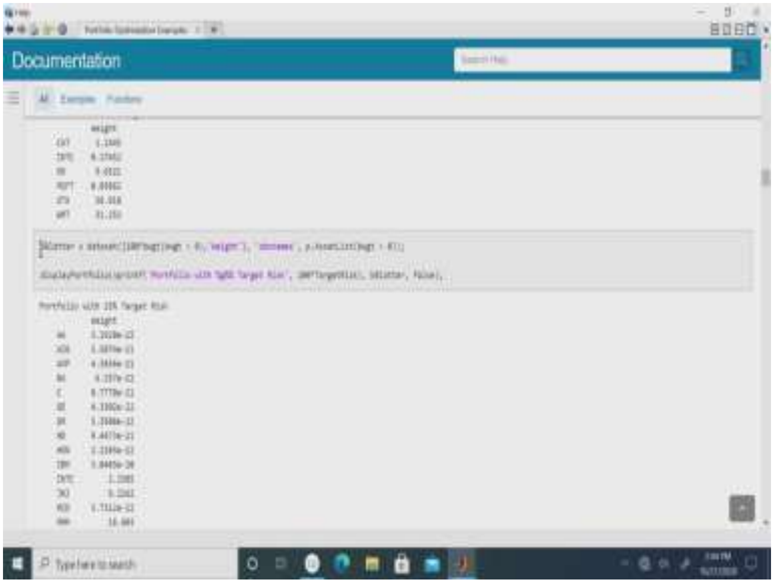
So, then what we have here, we observe that in addition to whatever we have seen earlier, we now have identified the two specific portfolios. Now, this is a portfolio which has the expected the portfolio on the efficient frontier with mu p or the expected return to be 20 percent and this the other point here, this is going to be the 15 percent level of risk for the portfolio and it is that portfolio which runs lies on the efficient frontier with a 15 percent risk.

(Refer Slide Time: 18:02)



So, now we need to actually look at determining the exact portfolio for the 20 percent return and the 15 percent risk and so accordingly, we will use Blotter so that you know or a table that will consist of the portfolio weights. So, first of all we will use a Blotter which essentially takes a 100 percent into awgt. Remember awgt, this is going to be the one corresponding to the targeted return.

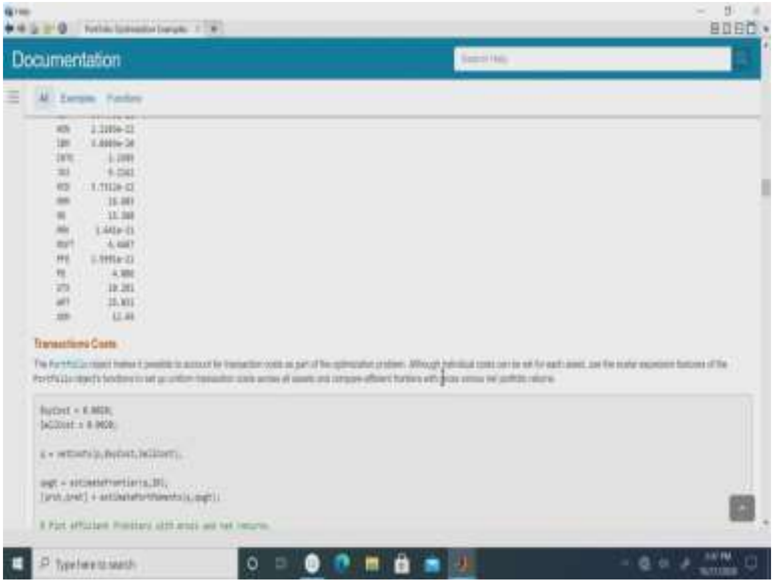
(Refer Slide Time: 18:34)



And so, it turns out that the portfolio with a 20 percent target return will comprise of these 6 individual assets. Observe here that amongst this the highest weight adding up to about 80 percent of the portfolio are for UTX which is 56 percent and WMT is 31.253. So, this UTX is Raytheon technologies and WMT is Walmart ok.

Now, this was what was identified as aBlotter corresponding to a weight for the x targeted return of 20 percent and the next thing we do is that we will consider the bwgt which is for the targeted risk of 15 percent and will store the portfolio in terms of percentage as this Blotter which you call as bBlotter.

(Refer Slide Time: 19:22)



So, this of this as you can see has a much larger number of assets that are in it. In particular, I want to identify the ones which actually have more than 10 percent. So, the first one, you can identify is triple M which is 16 percent. So, this is the company 3 M. The next is you have MO and this MO which is 15 percent, this is Microsoft and then, we have UTX which is 10 percent.

So, as already mentioned this is going to be Raytheon technologies and you have 25 percent for WMT that is Walmart and 12.69 percent that is XOM which is Exxon Mobil. So, you have been able to actually now identify that the portfolios that you want as per your requirement.

Now, you are now free to actually set whatever target return you want and whatever target risk you want, of course, within the range that was obtained in the previous step. So, once you pick up any return or risk within the range that has been obtained, you can essentially immediately through this command, you can estimate the weights of the specific portfolio as per your personal preferences.

So, this is you can see is a very useful tool because it straight away gives you the weights of the portfolio who satisfies your constraints. Now, so far what we have been doing here is that we have been essentially looking at only the cases that of a mean variance portfolio optimization by looking at these this assets without even actually accounting for the practical reality of transaction costs and turnover.

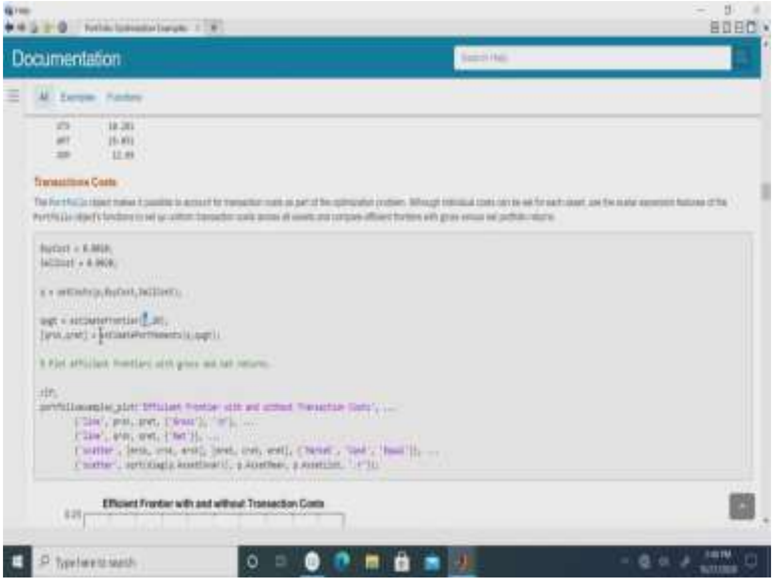
So, now we look at an extension of the problem to make it more realistic and accordingly, we incorporate the transaction cost. So, again you know the portfolio object that is there in the MATLAB financial toolbox, it is amenable and has the provision for accounting for the transaction cost as a part of the individual’s problem.

So, accordingly, what you can do is that we can set up individual costs for each asset. So, I mean that this is the transaction cost for example, you might have to give a brokerage and you can so, as an illustrative purpose you set up a uniform transaction cost across all assets and once you do that, we can calculate the efficient frontiers of gross versus net.

So, the gross efficient frontier is nothing but the efficient frontier without this cost being included and the net portfolio is the portfolio, where these costs are actually being included. So, here for the illustrative purpose we say the buying and selling cost are both 0.0020 or 0.2 percent.

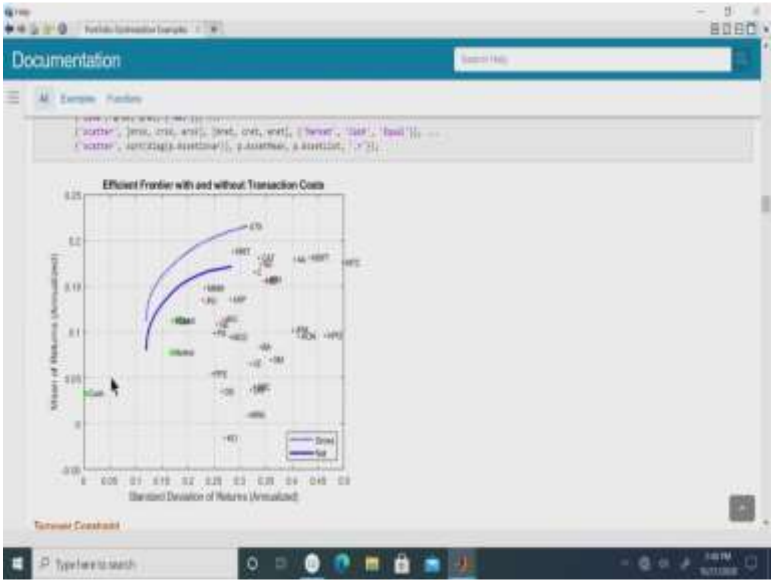
But of course, you know you are free to choose other values depending on what exactly is the cost involved and instead of having uniform cost, there is of course, the room for setting up individual cost for each individual asset separately. So, this you know the set cost is now assigned to this variable q and now, we are going to make an estimation of the efficient frontier after having incorporated the cost in terms of this q.

(Refer Slide Time: 22:52)



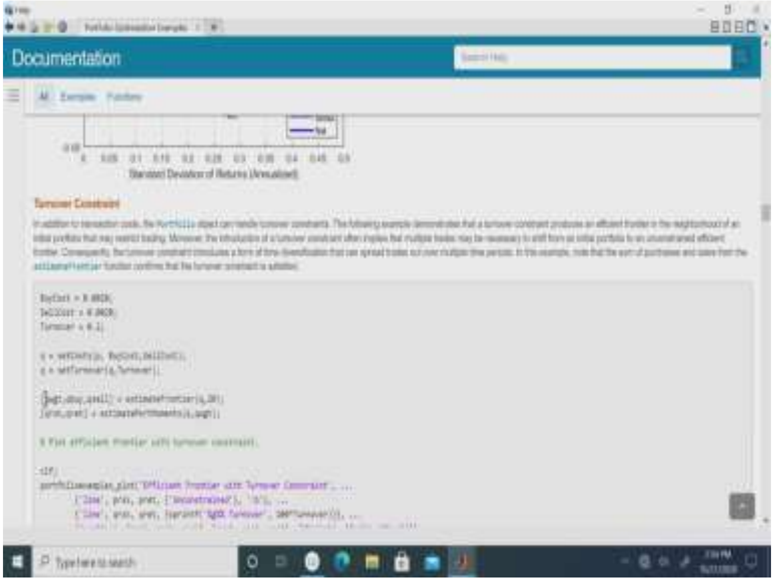
And then, we get the estimation of the portfolio moments using this q and the q weight; that means, q weight is the weights of the efficient frontier after having incorporated the cost in as q.

(Refer Slide Time: 23:07)



So, again, we now look at the plot. So, here we have the efficient frontier without the transaction cost. So, observe carefully here. Here everything is pretty much the same as before. You have all these individual assets market and a cash and this equal portfolio and as I said that you know here you earlier, we had looked at what is the gross efficient frontier which is this case, which is the dotted line and this bold line that you have here this is the net efficient frontier.

(Refer Slide Time: 23:45)



So, this is one constraint that had to be included. The second constraint that we consider for inclusion is what is known as the turnover constraint. So, you would recall that from the previous lecture. We talked about that the portfolio object in addition to transaction cost.

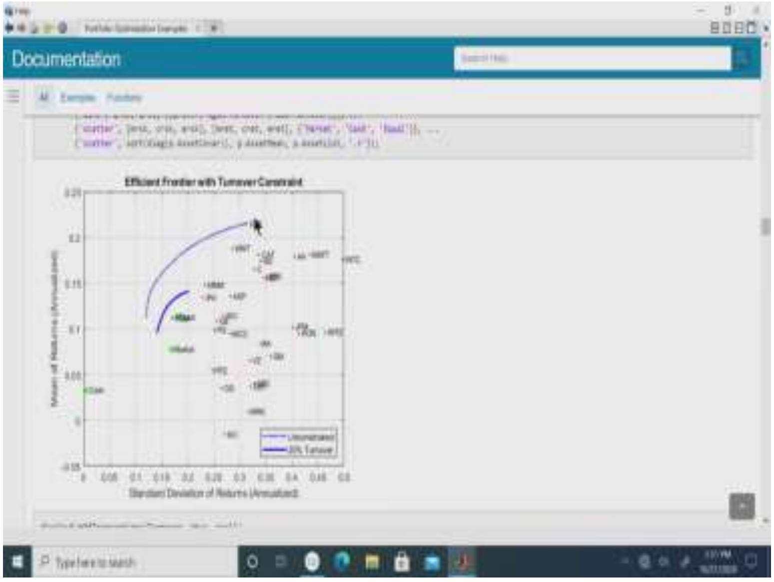
It also can accommodate turnover constraints and remember that the turnover constraint was basically some sort of a restriction on the extent to which you can do your trading because you have to reshuffle your portfolio from time to time in order to obtain the best optimized portfolio.

But in order to ensure that there is no not too much of volume of transaction because that also incurs cost. So, accordingly, we will demonstrate in this example. The next example that you are considering here that a turnover constraint it produces the efficient frontier in the neighbourhood of the initial portfolio.

So, accordingly, what we do is that we now you know in addition to the cost of 0.0020 of buying and selling that is 0.2 percent that is applicable in case of the cost, we set the turnover at 0.2 or 20 percent. So, accordingly we set q. So, now, this q, that we have here this, of course as before the buying and the selling cost are included.

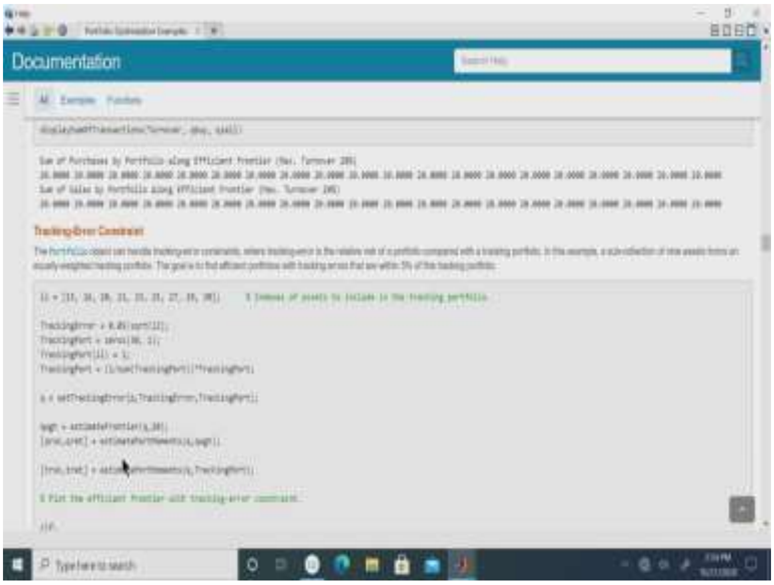
So, this is what you have done, when you consider only the transaction cost and on top of it now, we add to qth turnover as per our specification which in this case is 0.2 or 20 percent. So, accordingly, what you do is that we will now have the efficient frontier that is being estimated and the output being set to give you the weights and the quantity of buying and selling.

(Refer Slide Time: 25:31)



So, once you have this of course, again you will have a similar kind of efficient frontier. So, here if you observe carefully that here, we had this unconstrained efficient frontier and here in this bold line, we have the efficient frontier which has accommodated for a maximum of 20 percent turnover ok.

(Refer Slide Time: 25:54)



So, now let us now talk about a tracking-error constraint. So, let us now talk about again we go back to the portfolio object that is inbuilt into MATLAB and we can handle tracking-error constraints where tracking-error is the relative risk of a portfolio compared to the tracking portfolio.

So, here we do is that we have a portfolio that is being actively managed and in order to track the errors, we introduce a tracking portfolio and the goal is to ensure that you have to find those efficient portfolios with tracking constraints with tracking errors that are within 5 percent of the tracking portfolio.

So, essentially, it is some sort of you know controlling portfolio and the efficient portfolios is that you want is the one that is not too far from this tracking portfolio and in this case, the specific example that is being considered here is that these errors are within the 5 percent limit of the tracking portfolio. So, now how do we set up the tracking portfolio?

So, here is a; so, a tracking portfolio in this example and there are different ways in which you can do this. So, in this case, the tracking portfolio is being collected as a sub collection of 9 asset from an equally weighted portfolio. So, accordingly, we will identify the index of the assets that you want to include in the tracking portfolio.

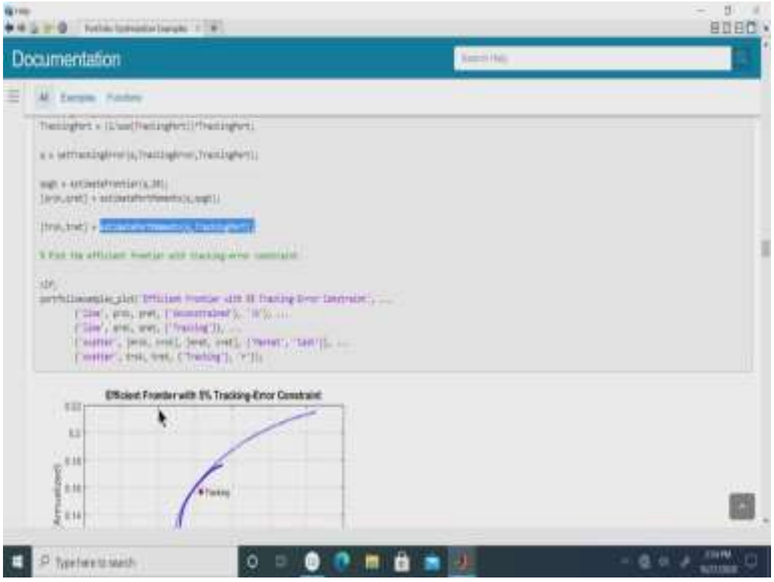
So, out of you know say 30 assets, you have identified that you will include the those particular assets which are indexed 15, 16, 20, 21 all the way to 30. These are the ones that you are going to include in the tracking portfolio. So that means that while the main portfolio has a larger number of assets for inclusion, the tracking portfolio will have a subset of those collections of those assets.

And the tracking-error is then set to 0.05 or 5 percent divided by square root of 12 for the monthly tracking and the tracking portfolio here is said to be zeros 30, 1. So that means, what you do here is the following that the tracking portfolio here initially is said to be have weights all zero and what you do is that then you assign the weight of one to those points in the those components of the tracking portfolio which are there for inclusion.

So, this will mean that for the 30 entries that in the portfolio in the tracking portfolio only 9 have non-zero values which are all set to be equal to 1. And obviously, then the sum of those weights is going to be equal to 9. So, then you divide, then by the sum of this tracking portfolio which is 9.

So that means, that the all the 9 entries of one that are there will now be converted to 1 by 9. So, again, we estimate the efficient frontier and this is the efficient frontier of q which incorporates the tracking-error and takes care of the tracking portfolio.

(Refer Slide Time: 29:11)



So, here we will estimate the portfolio moments and for the weights and then, the one with the tracking portfolio. So, what you do now is that we will calculate the efficient port efficient frontier in both the scenarios.

(Refer Slide Time: 29:27)

So, in the first one what we will do is that the one with the dotted line, this is going to be nothing but the unconstrained efficient frontier as before with no constraints being imposed and this red one is the tracking portfolio. So, this is the expected this point here represents the expected return and risk of the tracking portfolio, which comprises of those 9 assets that have been identified with the equal amount of weight being assigned to each of them.

And then, you know on the basis of that, what you can do is that without tracking this dotted line would be the efficient frontier; but with tracking which only allows an error of 5 percent, the efficient frontier obviously gets reduced significantly because it cannot be too far from the tracking portfolio.

So, it gets significantly curtailed as is evident from the length of the efficient frontier and it is that part you know it is essentially that part of the efficient frontier is closest to it which as essentially satisfies the condition that the tracking-error should not be more than 5 percent ok.

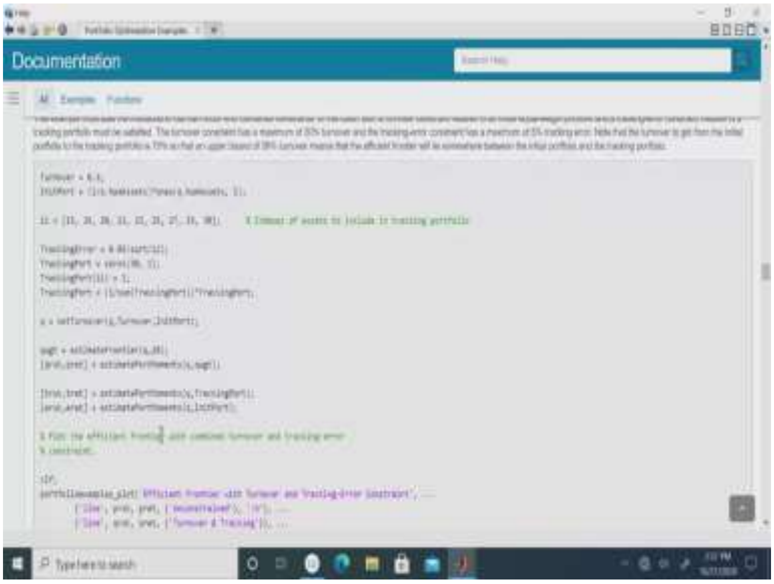
(Refer Slide Time: 30:40)

Now, we will look at an example of the combined turnover and the tracking-error constraints. So, suppose that we set the turnover to a maximum of 30 percent and we say that the tracking-error can be 5 percent. So, now, we are looking at the constraint problem which brings into picture the constraint of the tracking-error that was there in the preceding example and along with it the combined turnover.

So, accordingly, here we set that turnover to be 0.3 and we set the initial portfolio that the portfolio to be this and then, we have this tracking portfolio as given here. So, the initial portfolio is said to be the portfolio to be an equal-weighted portfolio. So, there are three aspects here that needs to be taken care off.

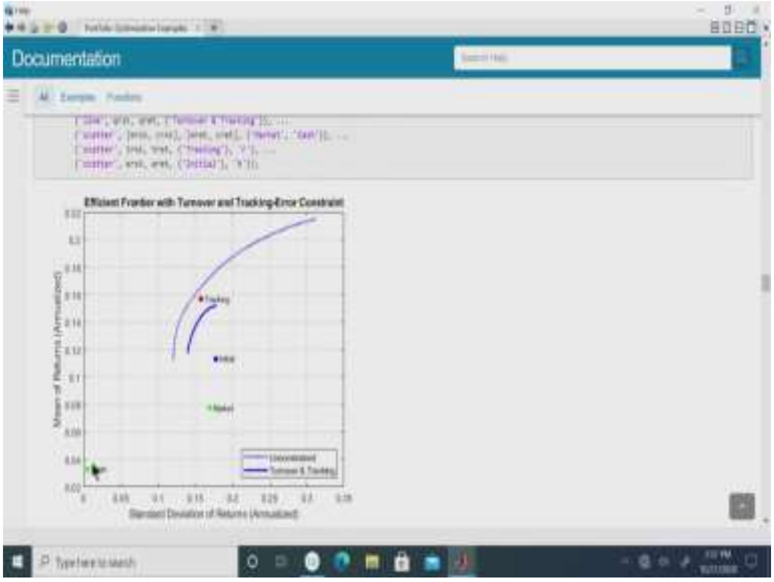
The first is that the turnover is set at 30 percent. Now, in order to start your generation of the efficient frontier portfolio, you need to have some initial portfolio. So, the initial portfolio is set to be an equal-weighted portfolio and the tracking-error portfolio is the same as that has been said before. So, there are two portfolios here; one is the initial portfolio and one is the tracking-error portfolio.

(Refer Slide Time: 31:58)



So, accordingly, what you do is that we. So, here q is said to be you set the turnover and you have this initial portfolio that you have here.

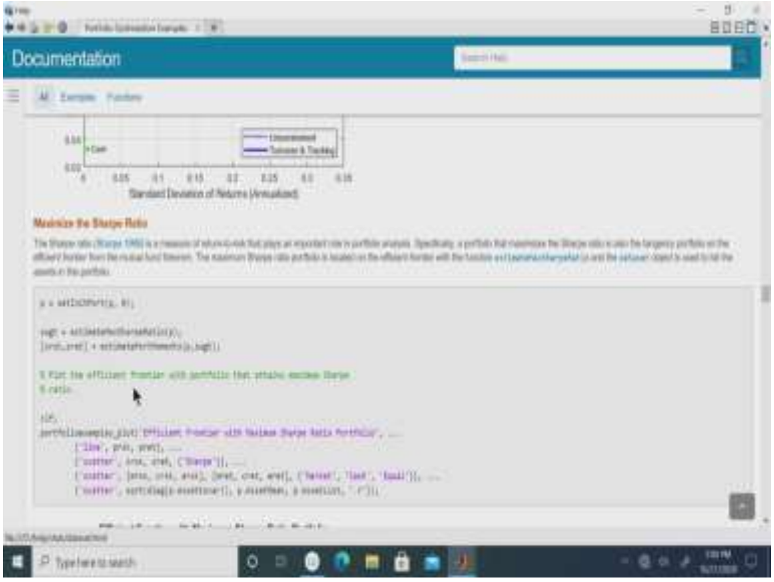
(Refer Slide Time: 32:15)



And you estimate the portfolio moments for q and you calculate what is going to be the you determine what is going to be the efficient frontier. So, for if you observe carefully here this is a cash in the market. So, the reason why we are only identifying cash and market because these are the two important portfolios and then, we have this unconstrained efficient frontier that we have in the top and this is the initial portfolio and this is the tracking portfolio.

So, starting off with this initial portfolio, you are able to reach the tracking portfolio that is tracking efficient frontier which is determined by a constraint of being the tracking-error being no more than 5 percent. So, start from here and with this initial constraint and with this tracking portfolio, the those portfolios which satisfy the condition of 5 percent tracking-error that will constitute this significantly truncated efficient frontier that is given here in bold ok.

(Refer Slide Time: 33:15)



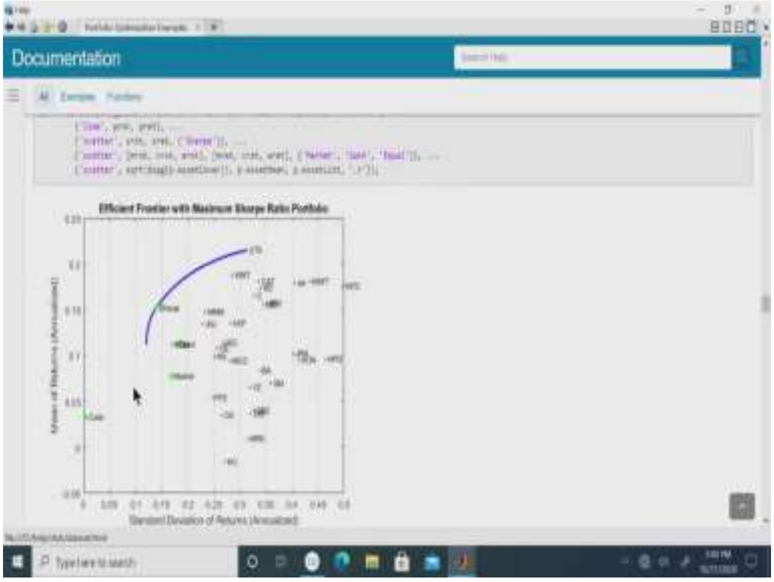
So, now, the next thing that we want to look at is what is known as the maximization of the sharp ratio. So, remember that the sharp ratio is a critical a measure of portfolio performance analysis and it is given by the return to risk. So, it is the excess return to risk and particularly, we are interested in a portfolio that maximizes the sharp ratio and it turns out that this is the tangency portfolio that we had discussed in the class. S

o, in this case we know that the reason why you want to maximize the sharp ratio is because the sharp ratio does is that it looks at the excess return over risk. And the larger this value is the larger, it indicates that you are able to achieve a larger amount of excess return per unit value of the risk that is given by the standard deviation.

So, the maximum sharp ratio portfolio is lies on the efficient frontier. So, accordingly, what you do is that we set; so, in this case, we plot the efficient frontier with the goal of maximizing the sharp ratio.

So, in this case, we use the estimate max sharp ratio command to get the corresponding weights which results in the maximum sharp ratio and then, we generate we determine what is the corresponding risk and return by again by making use of the command of estimate portfolio moments.

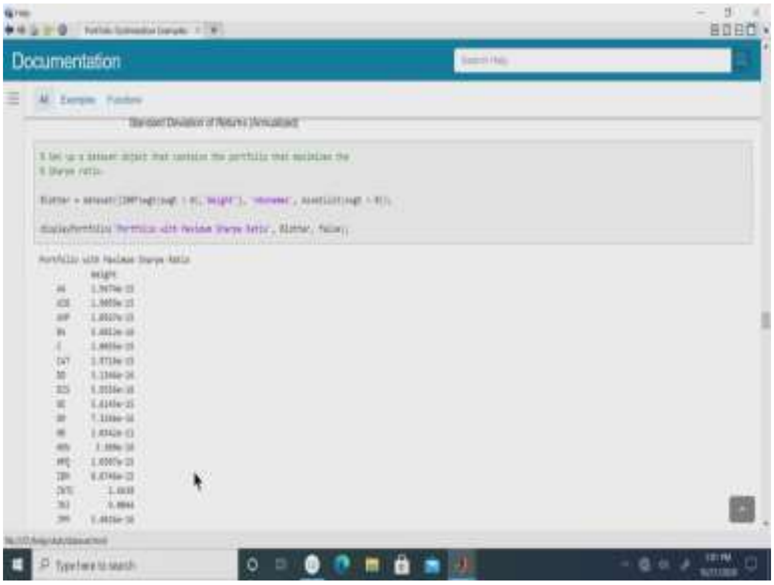
(Refer Slide Time: 34:53)



So, if you look carefully here in this particular case, again we look at the mean of returns and you look at the standard deviation. So, and you see that here it we have the scatter plot along with everything else that was there earlier. Of course, you know here I have not put in any constraint.

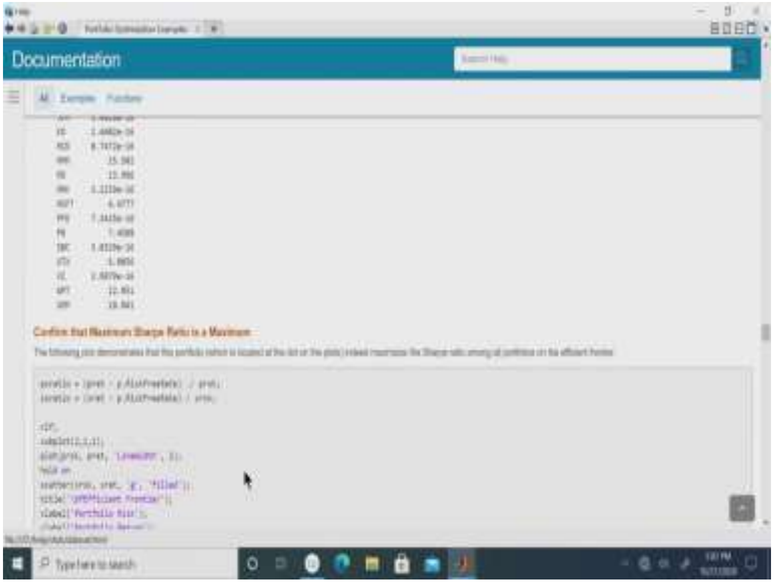
So, this is my efficient frontier and the only thing over and above the basic efficient frontier setup that we have done at the beginning of the class is the identification of this particular point on the efficient frontier, which maximizes the sharp ratio and this is the tangent portfolio.

(Refer Slide Time: 35:29)



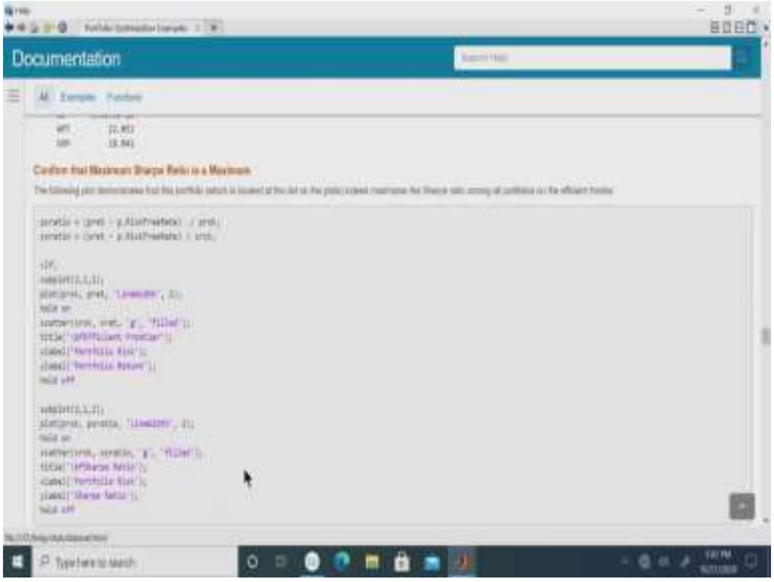
So, interestingly, it turns out again if you do a Blotter of this. So, the Blotter of this is going to be again, we look at the Blotter here is going to be the weights of all the portfolios that maximizes the sharp ratio. So, this will give you our weights in terms of percentage of the individual assets that constitute the portfolio with the maximum sharp ratio.

(Refer Slide Time: 36:02)

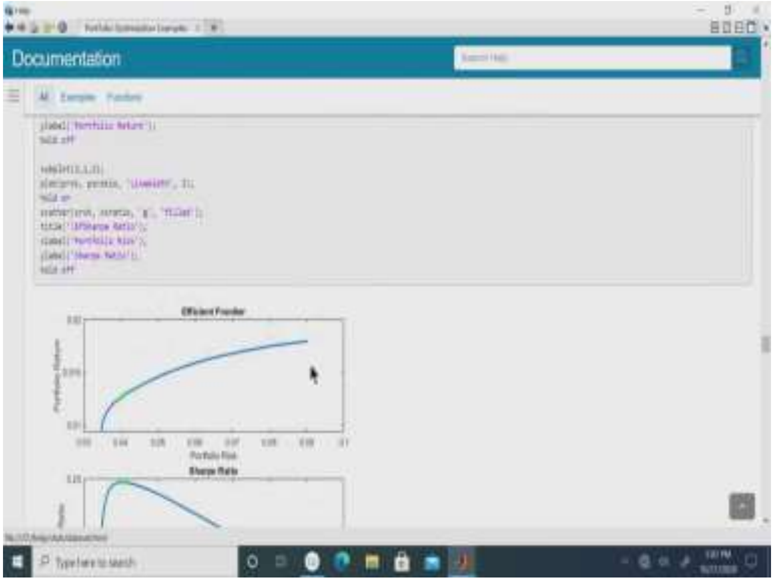


And here, if you observe carefully while most of the weights are extremely low, the some of the important words that you again have you observe it is going to be those similar kinds of names that are showing up here. So, 15 percent here for 3 M; 13 percent here for Microsoft; 22 percent here for Walmart and 18 percent here for Exxon Mobil.

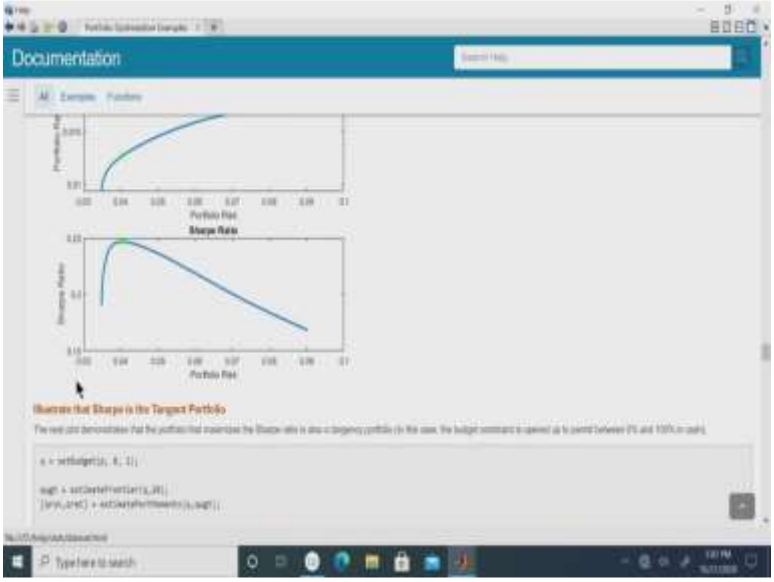
(Refer Slide Time: 36:32)



(Refer Slide Time: 36:33)

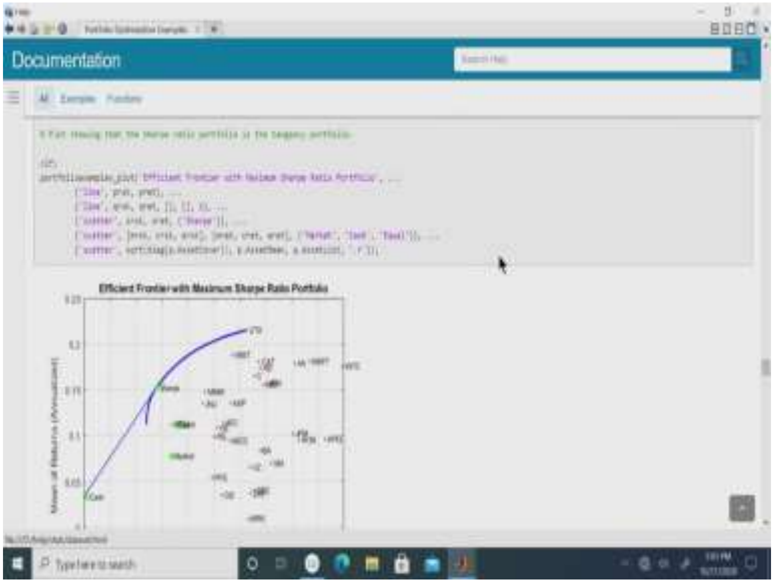


(Refer Slide Time: 36:35)



So, here you can actually do a cross check and verification that the maximum sharp ratio is indeed maximum by this. So, we now come to the illustration that the sharp ratio is the tangent portfolio.

(Refer Slide Time: 36:39)



So, if in order to obtain that, we again just look at the previous problem and remember that in order to prove that this that the sharp ratio is indeed the tangent portfolio, what you have to do is that we have to then generate the tangent line.

And only when we generate the tangent line that the point where it touches the efficient frontier is going to be the tangent portfolio and if it turns out that the point from a visual point of view, if it turns out that the point where the tangent line touches the efficient frontier is the same as the portfolio with a maximum sharp ratio. Then, you know the illustration of this problem actually has is accomplished.

So, accordingly, we just look at this particular previous figure that we had here and to it we will now add the tangent line. So, in order to do that, again you know we set the budget where the risk-free investment can be from 0 all the way to 100 percent. So, we have estimated the efficient frontier for this.

So, remember that again we have the scatter plots of the individual assets; mrsk, crsk, ersk correspond in the market, cash and equal weights portfolio and this srk and s return this is going to be generate this particular point which is the point portfolio with a maximum sharp ratio and of course, you know here the prsk and qrsk.

So, prsk, it will generate the efficient frontier curve here and qrsk will generate the tangent line. So, you observe very carefully that here this tangent line, the it touches the efficient frontier at this point and as you can see visually this is going to be the same as the point which has the maximum sharp ratio ok.

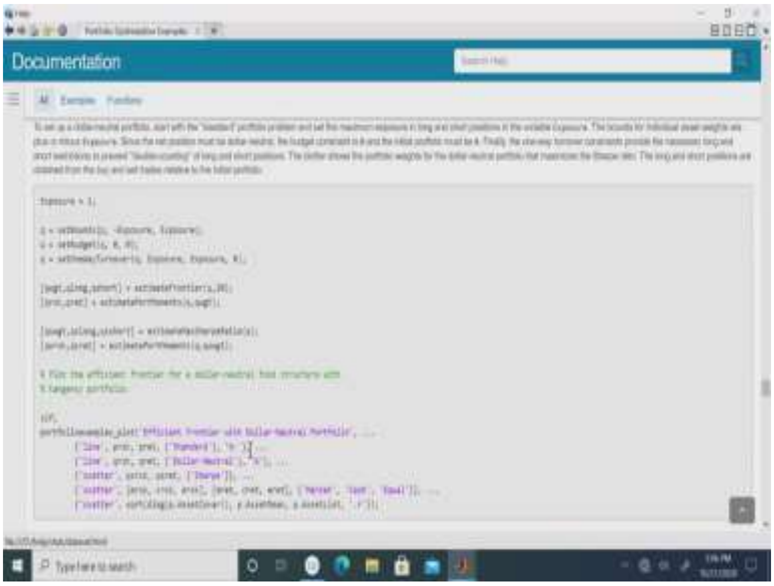
(Refer Slide Time: 38:32)

So, let us now just come to the last topic. So, I will just explain this in a narrative fashion because this is something that we actually have not done in the lecture; but nevertheless, it is a it gives you an interesting insight into more sophisticated approaches of financial investment and these are what are known as a dollar-neutral hedge-fund structure and the second one is what we call as the 130-30 fund structure.

So, here this commands that will be done here, it basically makes use of the portfolio optimization tools in hedge fund management. So, hedge-funds essentially are funds with a large amount of assets and typically, assets under its management and typically, the as they will be handling assets on behalf of high-net-worth individuals or people who have a lot of disposable cash for investment.

Now, two popular strategies that are adopted for portfolio optimization by the hedge-fund management, these are what are known as dollar-neutral and what are known as 130-30 portfolios.

(Refer Slide Time: 39:48)

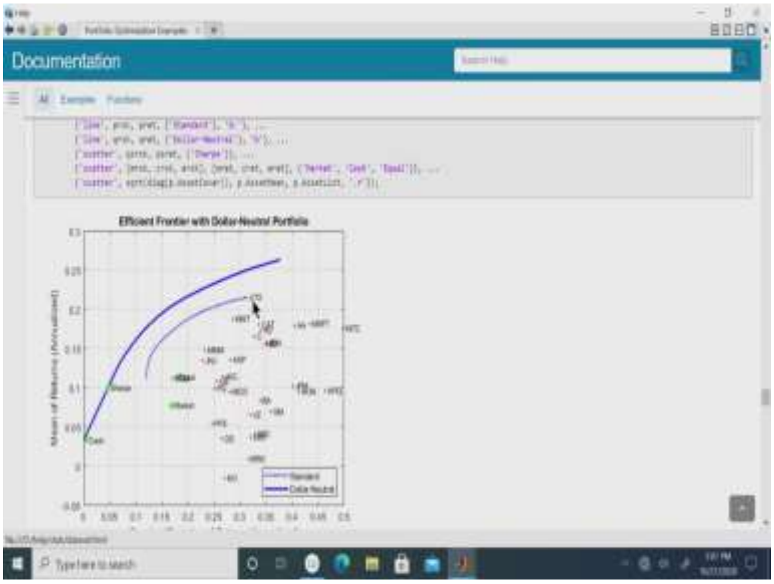


Now, let us now first talk about what is the dollar-neutral strategy. So, the dollar-neutral strategy as the name suggest involves some sort of neutrality. So, more specifically this means that the dollar-neutral strategy is the one, where one invests equally in long and short position such that the net portfolio position is 0.

So, that means, that you get into a long position in a certain number of assets and you get into a short position in certain other assets and the total valuation of the long position is equal to the total valuation of the your short position. So, that means, that it is something like you are adding and subtracting the same number, giving you the net valuation of your portfolio to be equal to 0.

So, you can actually go through this example at on how to set up either dollar-neutral portfolio. So, if you observe carefully here that here you know the exposure is said to be exposure and minus exposure. So, this is the one that corresponds to the long position and this is the one that is said to be the short position.

(Refer Slide Time: 41:00)



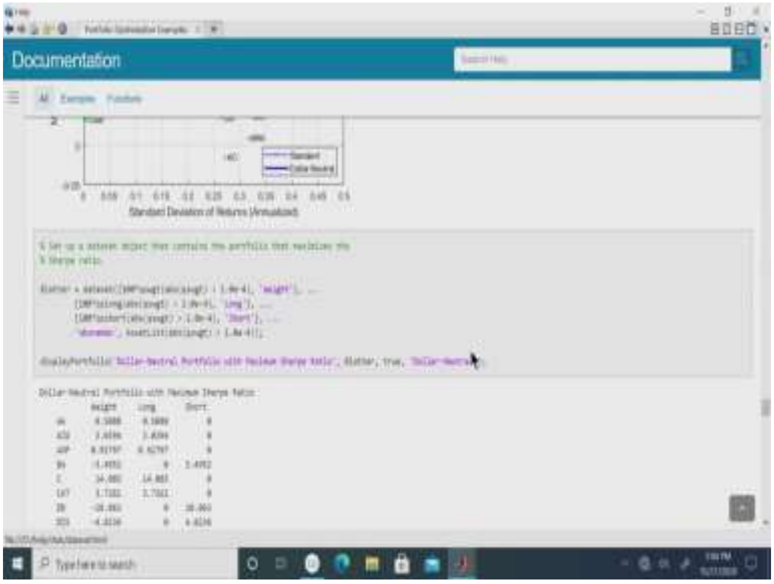
And here also you end up getting an efficient frontier of the dollar-neutral. So, if you observe carefully here that this is the efficient frontier that we have here and this is the line this is the tangent line and this point of tangency is the one with the maximum sharp ratio and I and here.

So, the so accordingly, this bold curve here, this is what is the dollar-neutral efficient frontier and this dotted one is the standard or the usual efficient frontier and if you observe carefully. So, here you are you have set for fourth the setup for the dollar and neutral portfolio and you have estimated the efficient frontiers.

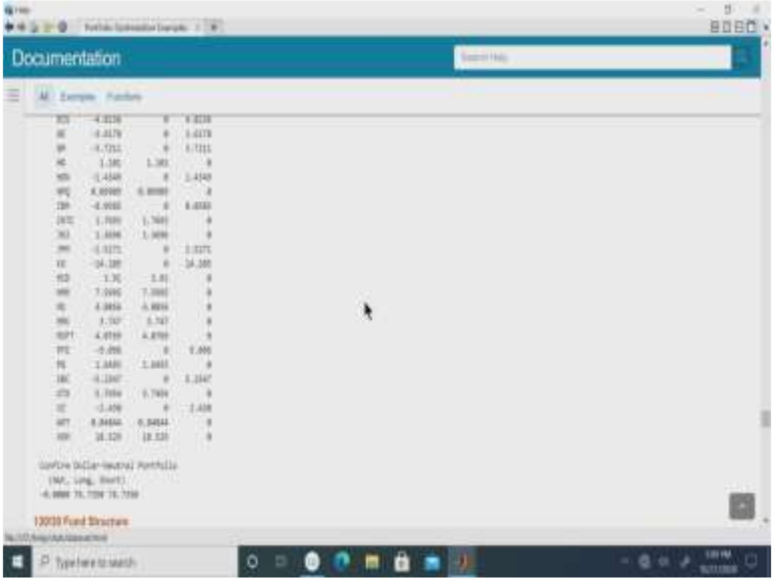
And so, accordingly, these two efficient from these to efficient frontier, you see that your dollar neutral portfolio that you have that particular efficient frontier is at a higher level than what is given by standard efficient frontier. So, this means that obviously, that you have been consistently getting a better return as compared to us the standard efficient frontier. But of course, you know you have to recognize that to obtain this obviously involves a higher level of risk.

So, accordingly because mainly because you have a indulge in a short position and that is the reason why typically these are strategies that are used by hedge-funds on behalf of individuals who actually are in a position and physically and financially well of enough for in order to take such risks.

(Refer Slide Time: 42:37)



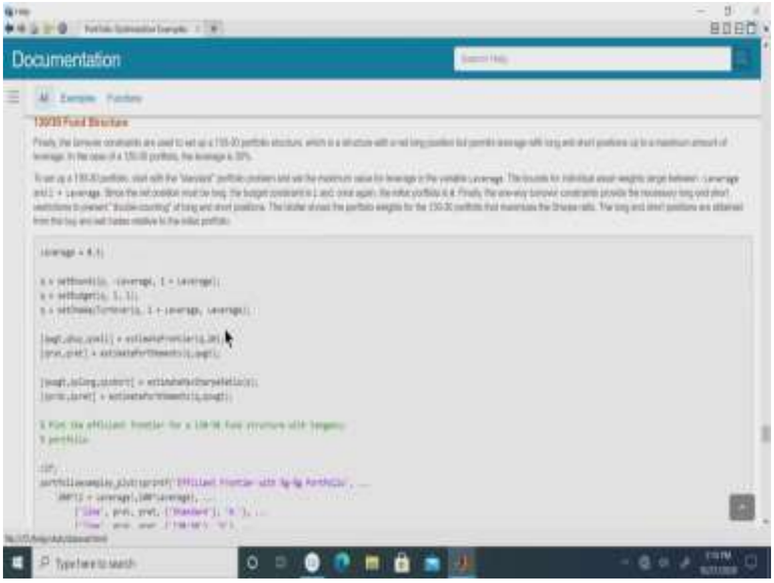
(Refer Slide Time: 42:38)



So, accordingly, you actually see here that we have a dollar-neutral portfolio with the respective weights. Now, as I said that your total position must be equal to 0. So, you observe here that the weights are specified here. So, for example, if you look carefully here J and J, there is Johnson and Johnson saying this is positive value.

So that means, you have this is the position, your long position and 0 is your short position. On the other hand, if you look at JPM that is JP Morgan here, then it is minus 2.5271. So that means, this long position is 0 and this short position is 2.5271 and then, you confirm the dollar-neutral portfolios and you observe that the long position is the same as the short position of the negative sign and consequently, the net is going to be equal to 0 ok.

(Refer Slide Time: 43:35)

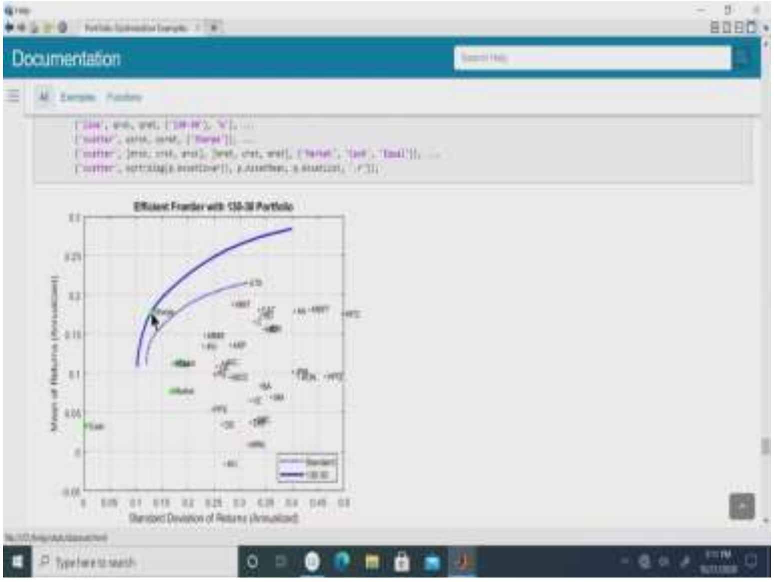


So, let us now come to the second example of hedge-fund strategies and this is what is known as the 130-30 fund scheme. So, as the name itself again just like dollar neutral the name itself in this case of 130-30 fund structure suggests that it is going to involve some sort of a proportion of 130 and 30 and since 130 is a larger number and 130 minus 30 is equal to 100.

So, you can immediately guess that it is a structure, where you have a net long position. But you have you can involve in short positions. So, in this case your long position is 130 and your short position is 30. So, what you do here is accordingly in this case, what you do is that your sum of the weights is equal to 1 and then, you have a leverage of the 0.3 weightage and accordingly, so that means, that it is something like you already have a portfolio in the usual way and then, you short 30 percent of that amount and you will generate the funds.

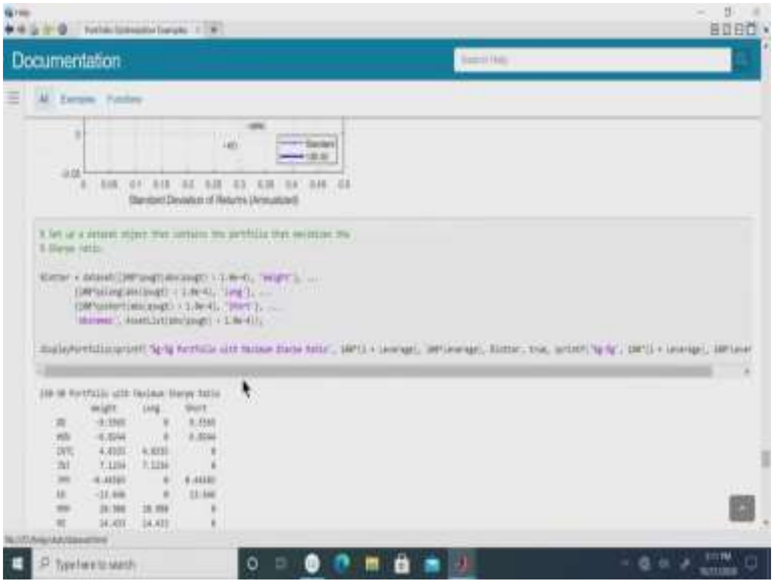
So, essentially you have a 130 percent of a long position and then, you have a 30 percent of a short position. So, the difference between them which gives you 100 percent of the short position.

(Refer Slide Time: 44:55)



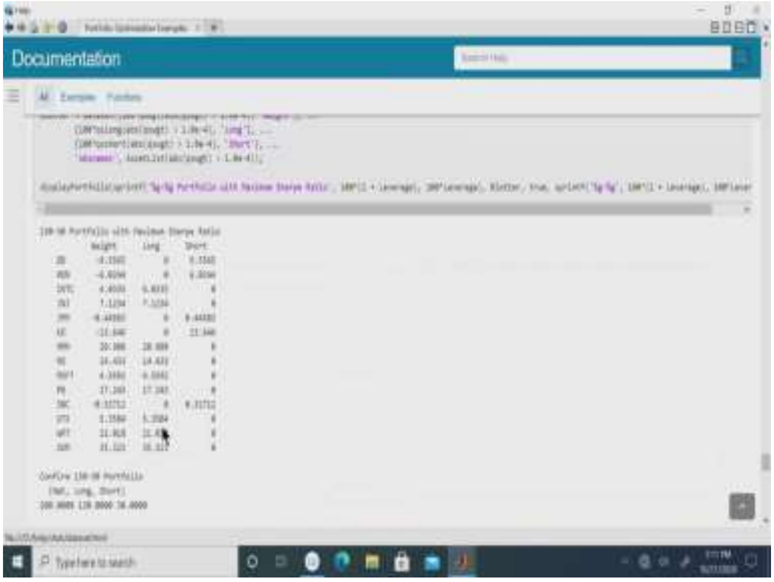
So, here this is an example, where you can see that this is the dotted one is the standard efficient frontier and the one above it in the bold, this is going to be an efficient frontier in involving this 130-30 portfolio strategy and here is the sharp ratio for that or rather the sharp ratio the point or the portfolio with the maximum sharp ratio.

(Refer Slide Time: 45:23)



And even though, the name suggests that it is 130-30, but it is not a very rigid structure that you have to involve 130 percent and 30 percent. All that you need is that the sum of these two should end up being equal to 100 percent. So, it could be it could start from a 120-20 structure; that means 120 percent long and 20 percent short and go all the way to 150-50 structures of 150 percent long and 50 percent short.

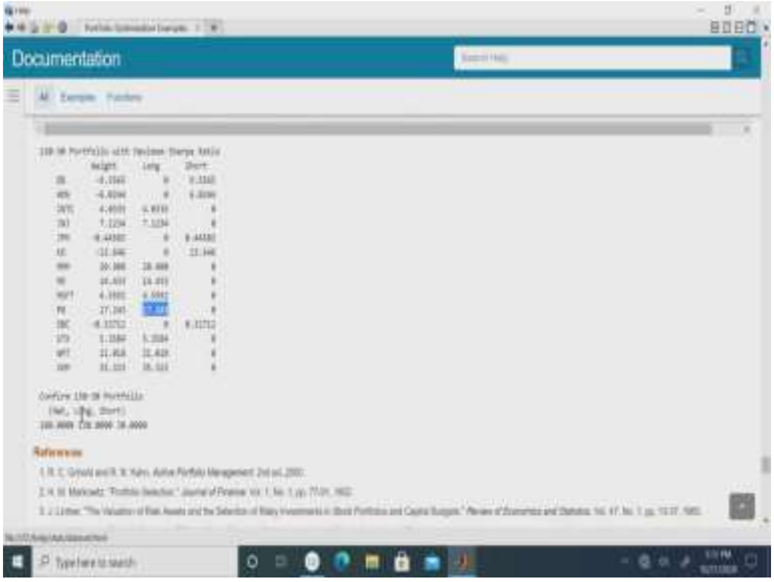
(Refer Slide Time: 45:58)



So, if you observe carefully that in this particular efficient frontier, so you can see that the one. So that means, the particular weights that for the 130-30 portfolio that have been enlisted here that is nothing but the portfolio corresponding to the maximum sharp ratio on the efficient frontier of the 130-30 portfolio.

So, again, here you see that you know there are certain weights which are negative for which the position is a short position and then, there are certain assets like say PG which is Procter and Gamble for which the weight is 17 percent and so that means, your long position is 17 percent and the short position is 0.

(Refer Slide Time: 46:44)



And again, you see here you see that if you confirm this positions. So, you can confirm this position and it turns out that this position, the long position is going to be 130, you just add this up and the short position is 30 and consequently, the net position is 100. So, this takes care of the several examples in case of portfolio optimization problems.

So, let us just do a recap of what we have done in today's class. So, our class today was mainly motivated by the two-fund theorem. So, we started off by looking at a database of some blue-chip stock companies.

And then, we look at their moments and you looked at the efficient frontier of those along with the scatter plot of each of the individual assets which constitute that particular database, namely there are 30 assets that are there. And then, we looked at what is the efficient frontier and we brought about a tangent line to that efficient frontier.

Now, once you have done that, once you determine what is the efficient frontier, we brought about into constraints such as the constraint the first constraint that you looked at was trying to figure out the points on the efficient frontier or that portfolio for a specified level of expected return on the part of the investor and this was followed by the optimal portfolio on the efficient frontier for a pre-specified tolerance level of risk of the investor.

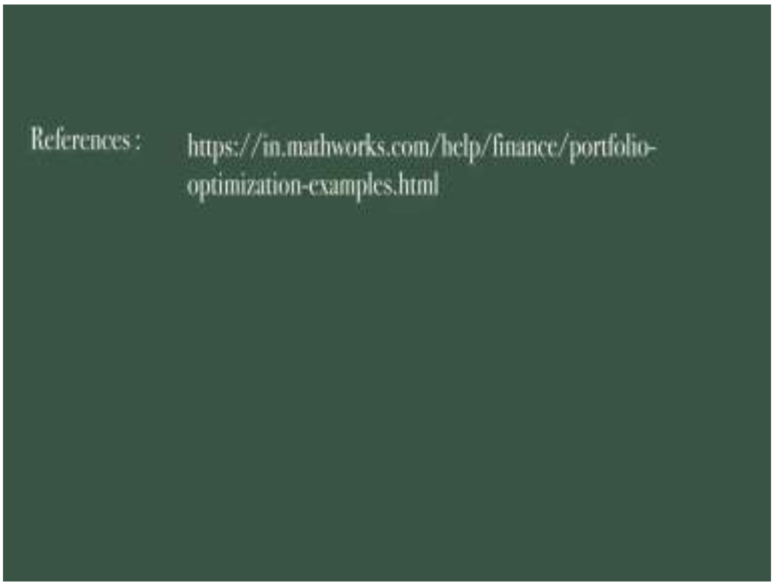
Now, next we looked at the efficient frontiers with the targeted portfolios and we looked at the efficient frontiers by incorporating two constraints, namely one which involves the buying and selling or that is the transaction cost and the other was the turnover constraint. In addition to that, we looked at the portfolio optimization and we use the maximization of the sharp ratio as the goal.

And it turned out that we could visualize that the maximized sharp ratio portfolio is the one which is the point of tangency of the tangent line and the efficient frontier and then, we looked at a couple of interesting applications from the more sophisticated setup of hedge-fund management; namely, the dollar-neutral portfolio evaluation, where you actually have that the long and short positions in the portfolio are exactly matching each other.

So, that the net value is 0 and the other setup which is called 130 setup, where 130 percent of the assets are invested in the long position and the 30 percent are invested in the short position or equivalently, some 100 plus x percent of the portfolio is in long position and x percent is the short position resulting in a net portfolio position of 100 percent.

So, this brings us to the end of this lecture and in the next lecture, which will be the concluding lecture of the course, we will talk about a few more examples again making use of the financial toolbox of MATLAB.

(Refer Slide Time: 49:45)



Thank you.