Mathematical Finance

Professor N. Selvaraju
Professor Siddhartha Pratim Chakrabarty
Department of Mathematics
Indian Institute of Technology Guwahati

Module 3: Time Value of Money and Risk Free Assets
Lecture 1: Markowitz Theory, Return & Risk and Two Asset Portfolio

Hello viewers, welcome to this course on Mathematical Finance. We will begin lecture today on a new topic and this will be Module 3, where we will talk about a broader area known as portfolio theory and primarily we will be looking at two main topics in portfolio theory, namely, the classical portfolio theory due to Markowitz and then capital asset pricing model. So what is a portfolio?

Portfolio is essentially a collection of assets which can be both risky and risk free and question that we want to answer when we are discussing portfolio theory is to see that what is the optimal way or the best way by which we can allocate the money that is available to us for investment amongst various assets and then see that we are achieving a reasonably high rate of return while ensuring that we have minimized the risk.

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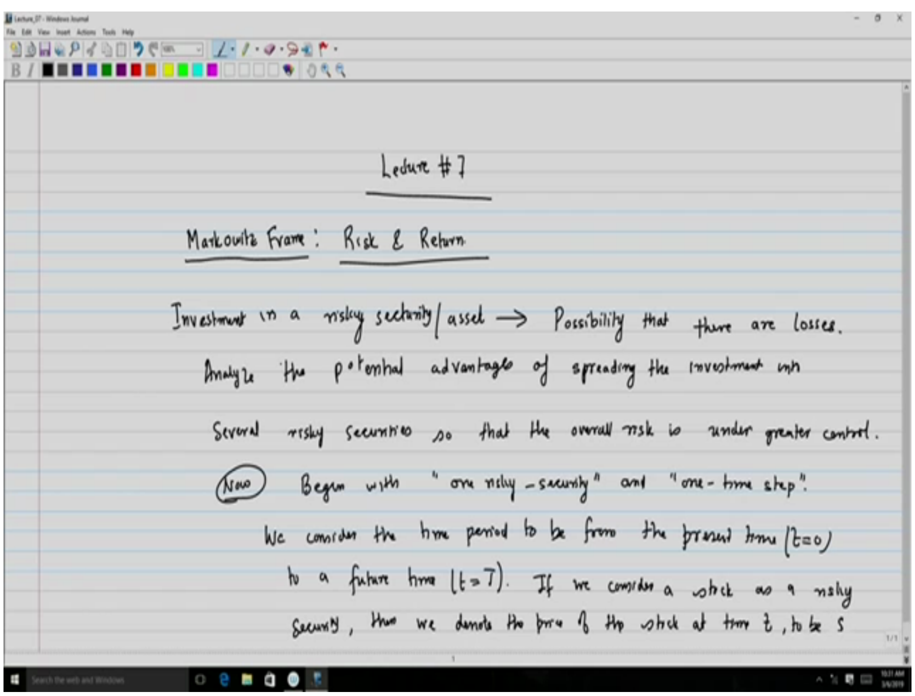

So, accordingly let us begin this lecture which is the Lecture 7 of the course. So first of all, we will essentially look at a Markowitz framework and for that we have to look at the concept of risk and return. So the concept of Markowitz framework was given by Harry Markowitz in early 50s when he essentially recognized that when deciding on an investment strategy one need not always look at just the return, but also they need to take into account what the risk is. So prior to that there was no appropriate or justifiable or scientific methodology by which one could decide on the choice of investment. And a lot of times it happens that people went into an investment into a particular security driven by history of good returns without recognizing the fact that high returns typically are associated with higher risks.

And Markowitz for the first time reconcile these two concepts of risk and return and so to strike a balance between these two, namely, that while ensuring that you get a good rate of return you must also factor in the risk that are applicably associated with higher rate of return. And these two concepts of risk and return forms the foundation or the two basic pillars of modern portfolio theory. So, let us first begin by talking more about risk and return. So if you consider an investment in a risky security or asset then there is always the possibility as you have seen in case of stocks that there are losses.

Now, we will analyze the potential advantages of spreading the investment into several risky securities so that the overall risk is under greater control. So by this we mean that as we have seen in the case of stock which is one of the basic examples of what risky asset is, if you invest into one stock, then your essentially your entire savings or investment is at high risk, and if the stock is doing well, then there is no problem but if the stock is doing reasonably badly and if the company is incurring losses, then obviously you are unlikely to be getting dividend and as well as capital gains by selling of the stock and so that means that you are at risk of wiping out a significant part of your investment.

So it is advisable that you do not put all your money in one particular risky asset if you decide to invest in a risky asset, because it opens up the possibility that a slight adverse situation affecting that one particular asset could in turn wipe out your entire investment. A more pragmatic way of handling this would be for you to make the investment in multiple numbers of such securities so that overall the risk that you are facing is to some extent is at least mitigated. So it means that instead of one security if you decide that you put money in multiple number of risky securities, even if something does not go well with one or two securities, then the effect to your overall investment in terms of adverse situation or losses is reduced as compared to just exposure to only one risky security which is not doing well.

And this is why they say that do not put all your eggs in one basket because if something goes wrong with one then you need to have protection that you get from the others and so this is basically what it forms the backbone of the modern portfolio theory. And the question that we will answer is that when we have a choice of multiple numbers of securities how do you allocate our wealth among those securities in terms of percentages of money that you take out and invest in each of those. Now, what we will do is that we will first begin with one risky security and one time step.

So even though I mentioned here that in a portfolio theory we will be considering multiple number of assets, just to introduce the notation I will first begin by considering only one asset and that too for a single time period by which I mean there will be an initial time period $t = 0$ and a final time period $t = T$. And once the basic notations are in place, we will then go ahead and extend this in case of multiple assets. So therefore, we consider the time period to be from the present time which is $t = 0$ to a future time which is $t = T$. Now, if we consider a stock for example as a risky security, then we denote the price of the stock at time t to be $S(t)$.

And this is for $t = 0,\ T$, since you are just considering a single period model. Therefore, note that under this notation the current stock price $S(0)$ is known but the future stock price is unknown. So this means that you have two time points 0 and T, the price of the stock at $t = 0$ is $S(0)$ and price of the stock at time $t = T$ is $S(T)$. So this $S(0)$ is something, that is known and $S(T)$ is something that is unknown. So having said that $S(T)$ is unknown we can accordingly make the statement that the return on the stock will be $K = \frac{S(T)-S(0)}{S(0)}$, so this is the definition of return that you have already observed that you know is the extra amount of gain that you make or losses in case of adverse movement.

So $S(T) - S(0)$ is the additional amount of money that you make which could be negative by selling of the stock at time $t = T$ divided by the original investment at $S(0)$, and you multiply this by 100, then you basically get the return in terms of percentage. So, from now on we will be making use of the notation K for return for the purpose of discussion on portfolio theory. So, that means that in this discussion, we

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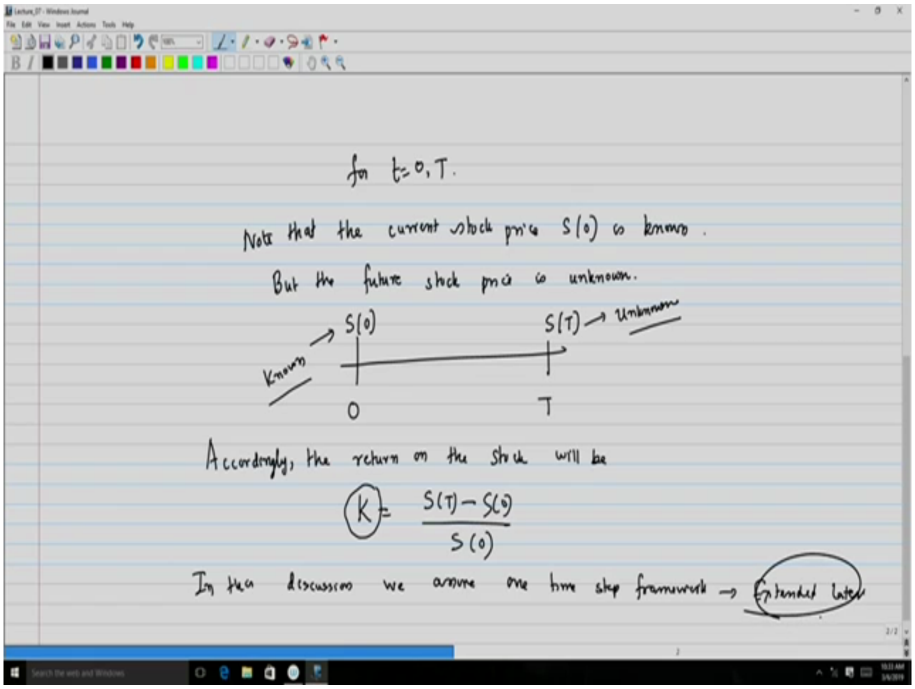

assume one time step framework and this will be extended later.

So, first of all, when you are talking about returns, recall that is written that we had $K = \frac{S(T)-S(0)}{S(0)}$, this K is unknown this qualifies as what is known as the random variable. So when we talk about random variable, so naturally the next question that arises is what is going to be the expectation of the random variable. And in this case, the expectation of the random variable is going to be the expected return or in real terms it means that the return that you expect to get in a single period based on this investment.

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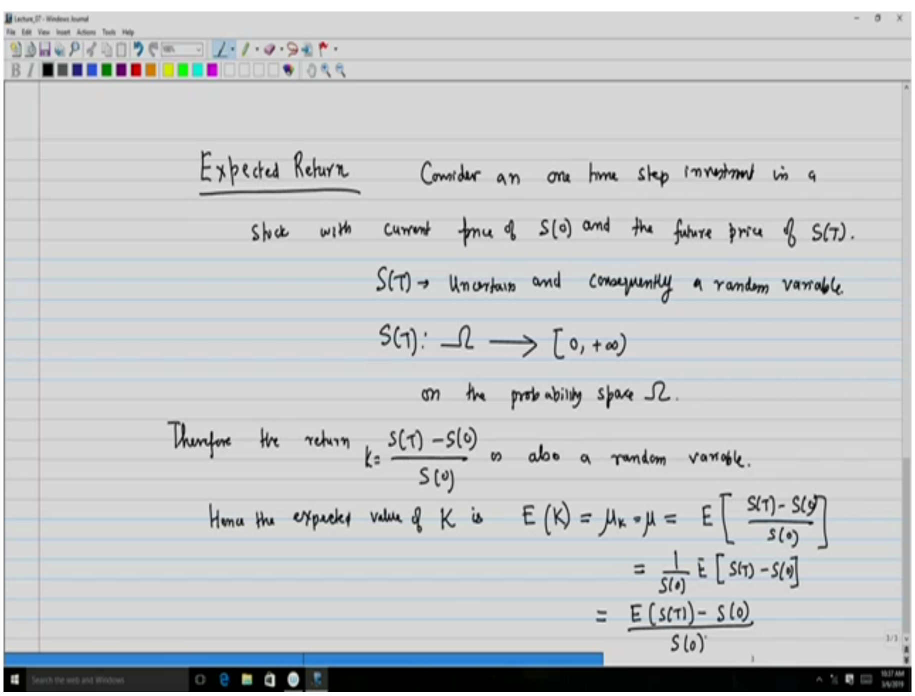

So, start off by talking about first of the two periods of the Markowitz model, which is the expected return. So, accordingly, consider one time step investment in a stock with current price of $S(0)$ and the future price of $S(T)$. Now, this future price of $S(T)$, this is uncertain or unknown and is consequently a random variable. What is a random variable? The random variable $S(T)$ is going to be a real valued function from omega to the set $(0, \infty)$ and where omega is the probability space. Therefore, the return $\frac{S(T)-S(0)}{S(0)}$, this is also a random variable. Hence, the expected value of K is $E(K)$, this is sometimes denoted by μ_K or even μ and this will be what?

This will be the expectation of $\frac{S(T)-S(0)}{S(0)}$. I can use the properties of expectations to get the $\frac{1}{S(0)}$ out, since it is a constant so this becomes $\frac{1}{S(0)} E\left[S(T) - S(0)\right]$ and this can now be written as $\frac{E[S(T)]-S(0)}{S(0)}$.

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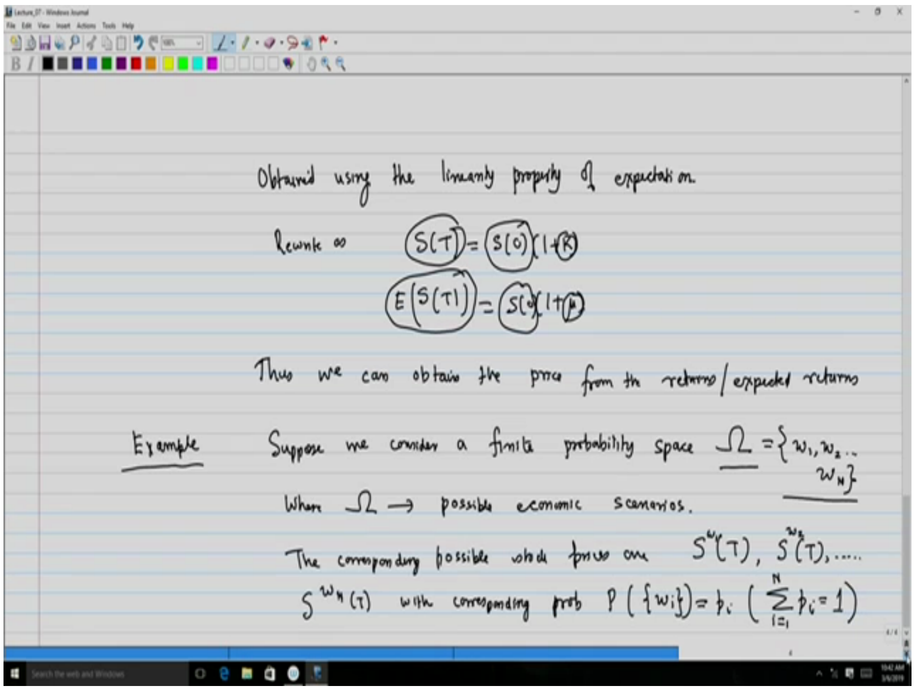

And this was obtained using the linearity property of expectation. So once you have opted this, so let us look at how you can make use of this, so you look here that the expected value of the returns is going to be the expected value of $\frac{S(T)-S(0)}{S(0)}$, so this can be rewritten as follows. Therefore, from here we look at this, we get $S(T) = S(0)(1 + K)$ and from here we will get $E(S(T)) = S(0)(1 + \mu)$. So, we can obtain the price from the returns or expected returns. So by this I mean that we can find price $S(0)$ if we know the return and $S(T)$. Likewise, we can find $S(0)$ by making use of μ and $E(S(T))$.

So, in order to illustrate this, let us now consider a simple example. Suppose we consider a finite probability space, recall that you have denoted this as ω with the elements $\omega_1, \omega_2, \ldots, \omega_N$, where ω could be possible economic scenario. That means that the samples space ω or the probability space ω could comprise of N number of different economic scenarios or states of the economy. So we will look at a simple example later on in the lecture of what I actually mean by economic state or state of the economy. So coming back to this, we have this finite probability space with $\omega_1,\ \omega_2$, all the way to ω_N representing the n number of possible economic states.

Therefore, let us say that the corresponding possible stock prices are when state are $S(T)$, now when I am in the state ω_1, so I will write $\omega_1,\ S^{\omega_1}(T)$ which is going to be the price of the stock at time T if the state of the economy is ω_1. Likewise the state of the economy is ω_2 so then the stock price $S(T)$ will be $S^{\omega_2}(T)$, all the way to $S^{\omega_N}(T)$. And the probabilities of each of those states of the economy and the consequent stock price will be so with corresponding probabilities $P(\{\omega_i\}) = p_i \left(\sum_{i=1}^{N} p_i = 1\right)$.

Alright, so what is going to be the expected price at time $t = T$, so in order to observe this, let us sort of tabulate all the states of the economy, so I had $\omega_1,\ \omega_2, \ldots, \omega_N$ with the stock prices being

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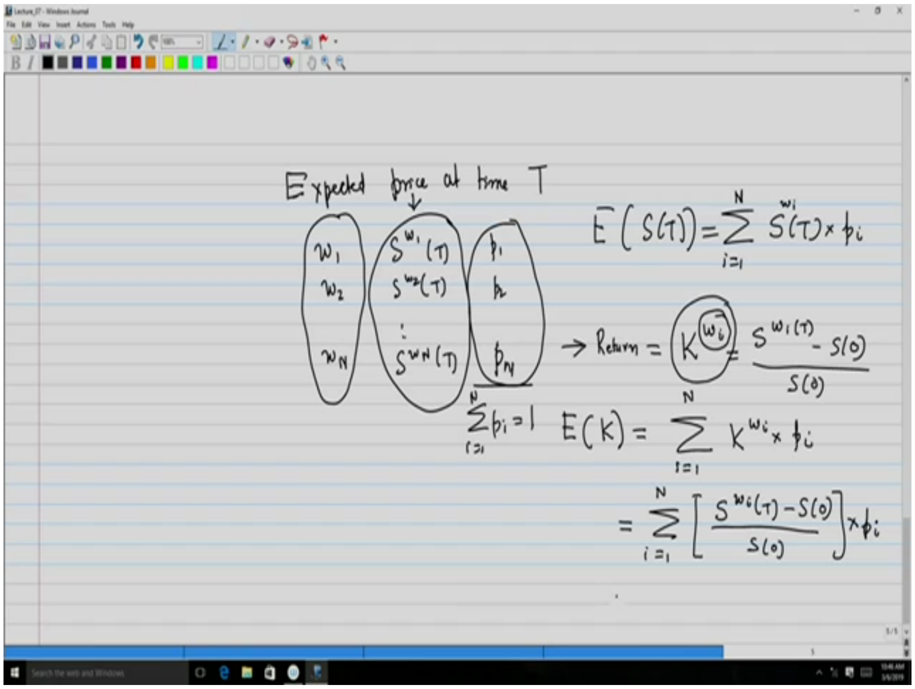

$S^{\omega_1}(T),\ S^{\omega_2}(T),\ S^{\omega_N}(T)$, with the probability $p_1,\ p_2, \ldots, p_N$ such that $\sum_{i=1}^{N} p_i = 1$. Then the expected price $E(S(T))$ will be obtained by using the usual definition of probability, when we take the random variables and the corresponding probabilities. So remember that the expectation of a random variable is going to be the sum of the values of the random variable can take multiplied by the probability of that random variable occurring.

Accordingly, what is the random variable? My random variable is $S(T)$ raise to ω_i multiplied by the corresponding probability p_i and summing this over $i = 1$ to N. So, in this case, the return is going to be, I denote this by K_{ω_i}, this return is going to be as before $\frac{S^{\omega_i}(T)-S(0)}{S(0)}$. So that means corresponding to each of those states of the economy ω_1 to ω_N, since there is going to be a price which I have denoted by $S^{\omega_i}(T)$, therefore for each of them I will have a return K^{ω_i}. Therefore, in this case what is going to be the expected value of the return?

The expected value of the return in this case is going to be summation, again I use the definition of expectation. Here K^{ω_i} is the random variable where I am considering the return, now this is going to be the summation K^{ω_i} multiplied by p_i, $i = 1$ to N. Now, I can write this, you can substitute $\sum_{i=1}^{N} K^{\omega_i}$, you can substitute K^{ω_i} with $\left[\frac{S^{\omega_i}(T)-S(0)}{S(0)}\right] p_i$.

And this can be rewritten as $\frac{1}{S(0)}$, I separate out the term and got $\sum_{i=1}^{n} \left[\frac{S^{\omega_i}(T)-S(0)}{S(0)}\right] p_i$. Now, what is this term? This term is nothing but expected value of $S(T)$, so this becomes $\frac{1}{S(0)} E(S(0)) - 1 = \frac{E(S(0))-S(0)}{S(0)}$, as we have seen previously.

So, let us now come to a little bit of discussion on what is going to be the risk. Remember that we have had begun by talking about return and risk being the two pillars of the modern portfolio theory and we had justified that just by looking at the return one should not make the decision and one need to factor the risk that are associated with each of the investments. So accordingly, we now look at what is going to be the risk measure or how to actually quantify risk based on the market data or standard deviation. So the question that you want to answer here is going to be what would be a suitable measure of risk, so this is something that we are going to address now. So accordingly, let us look at the motivation that it is important to understand the scatter of returns around some reference value. So by this what I mean?

For example, if you are considering a bond with a simple interest and it says that you are basically getting

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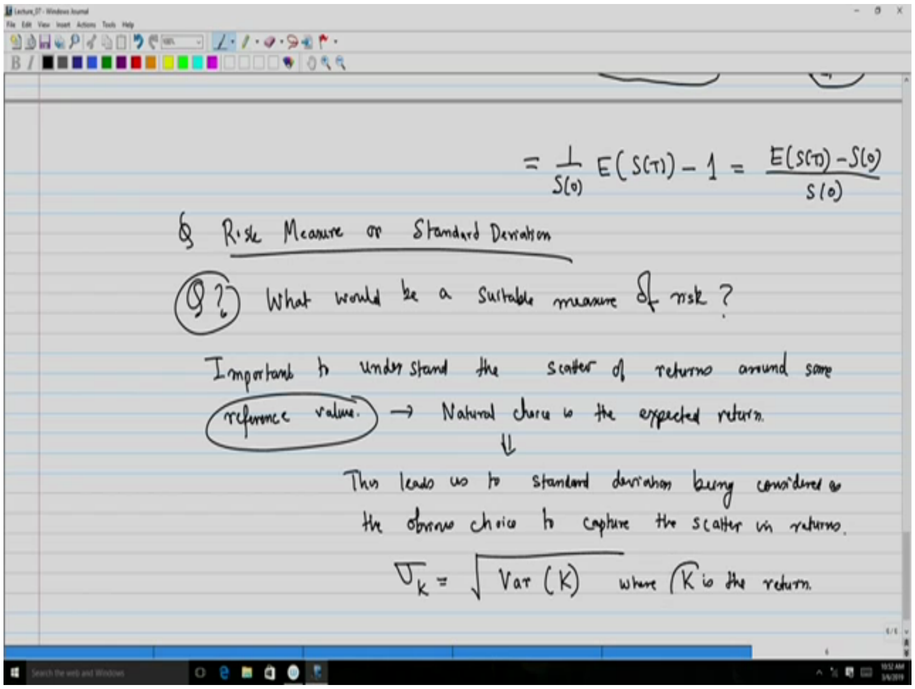

10% so this means that on an average you are getting 10% return (say) every year but with no risk. Assuming that you are holding onto your position until the expiration of the bond. However, in case of stock you might have returns something like -20% , 0% and 50%. So. in this case, you see that $-20\% + 0\% + 50\% = 30\%$. This divided by 3 gives you a return of 10% per year. So, apparently this seems to be identical investment, but in case of bond there is no risk, but in case of stock which moves from -20% to 50%, you see that there is a great amount of fluctuation in terms of risk.

So, this is a simple example where the risk are playing important factor in deciding where you should invest in because for both the bonds and this particular investment your average return is going to be 10%, but in case of 2nd the risk is significantly higher because of the unpredictable nature and extreme fluctuations in the returns of the particular asset. So, this means that while the average return was 10 percent, in the 1st case you were 30% away from the average return at -20% and this means that the extent to which you are fluctuating around this average value plays a very very critical role in ascertaining the instability as far as the returns are concerned, and this is what motivates the usage of standard deviation as the easiest one that one can think of as a measure of risk.

So, this reference value that you are talking about here for this as I have already mentioned, the natural choice is the expected return. And this leads us to standard deviation being considered as the obvious choice to capture the scatter in returns. Accordingly, sigma K will be used to denote a standard deviation of returns or the measure of risk, so this is nothing but square root of the variance of K, where K is the return. So please take note of the fact that here K is the return and not the price of the asset, alright.

So by definition, $\text{Var}(K) = E(X - \mu)^2$, remember what was μ? mu was nothing but $E(K)$, so this should be actually K. And this can be written as $E(K^2) - \mu^2$, using properties of variance. So, what we have defined now, so far we have defined what is going to be the returns, which is the $E(X)$ and now we have defined what is the risk given by σ_K or the standard deviation. So with this $\mu = \mu_K$ and $\sigma = \sigma_K$. We now have the return and risk definition in place for us to move ahead.

So, at this point, we will start now looking at a portfolio with two securities and by this I mean two risky securities, alright. So, now once you start talking about two securities, we are essentially moving into the portfolio framework. So in this framework we consider investment in (say) two stocks, so when you are talking about investment in two stocks, it is time for us to introduce the concept of weights to identify the proportional allocation of funds to various securities instead of specifying the number of shares of the

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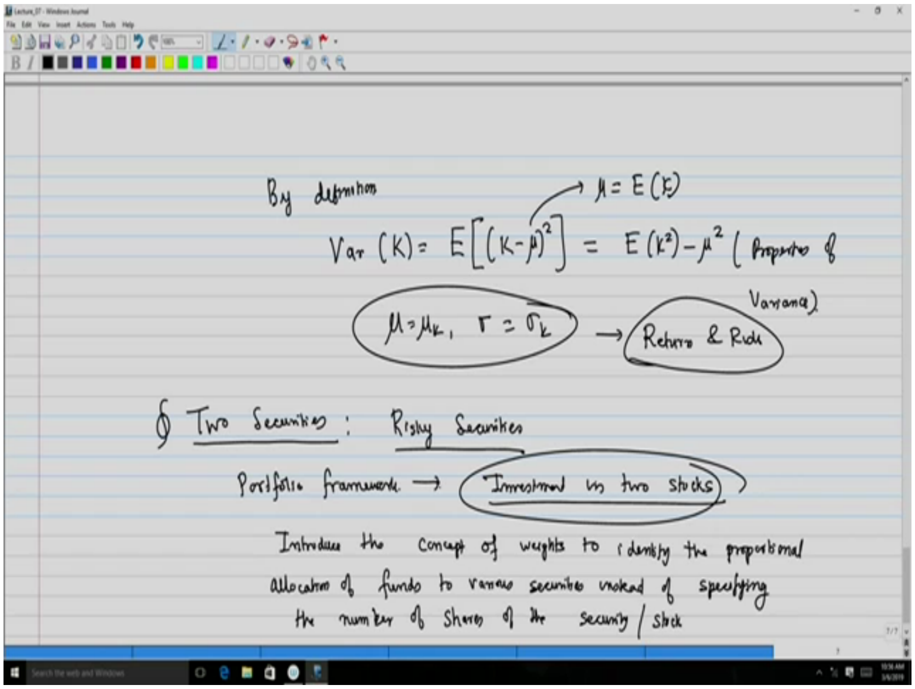

security or in our case stock.

So, by this we mean that when you are looking at a portfolio intuitively it occurs to us that we are specifying the portfolio and we can go ahead and specify this in terms of number of units of the stock that you are purchasing. However, it is more customary in portfolio theory and actually more convenient as we will later see, to identify the portfolio not by the number of units of the stock that you are purchasing for each of those stocks, but rather by the weights of each of them. And by weights I mean that weights are nothing but a proportional investment of the total initial investment that you have allocated to each of those particular assets notwithstanding what are the number of units of the asset. So let us now start to give mathematical formulation for what the weights are.

Therefore, for two securities, we define the weights as $W_1 = \frac{x_1 S_1(0)}{V(0)}$ and the $W_2 = \frac{x_2 S_2}{V(0)}$. So what are the notations that we have used here? Here the only thing that we are family with is $S_1(0)$ and $S_2(0)$, these are going to be the stock prices for the first and second stock at time $t = 0$. Now, let us now go to the notations that we have introduced one by one. So, first of all, let us start with $V(0)$. $V(0)$ is basically the total amount available for investment at time $t = 0$. Next, $x_1,\ x_2$, these are number of shares or units of stock 1 and stock 2, respectively. So what does this give you? That means that if you have x_1 number of stock 1 that you have purchased and if you have x_2 number of second stock that you have purchased, then the term $x_1 S_1(0)$ is going to be that the total amount of money that you have invested in the first stock.

So, this is going to be the total investment in stock 1 and $x_2 S_2(0)$, this is going to be your total investment in stock 2. So this brings us to the definition that W_1 and W_2, these are going to be weights of stock 1 or security 1 and stock 2 respectively. So, by this I mean, you observe each of the term. Here I have said that $x_1 S_1(0)$ is going to be the total investment in stock 1 that means this $x_1 S_1(0)$ is the total money that you have invested in the first stock and $V(0)$ is the amount of money that was available with you initially. So that means out of an amount of $V(0)$ that is available to you, you have decided to invest an amount of $x_1 S_1(0)$ in the first term.

So that means $\frac{x_1 S_1(0)}{V(0)}$ is the fraction of your total amount of money that you have invested in the first stock and this is what is defined as the weight of the first stock. And likewise $x_2 S_2(0)$ is your total investment in the second stock which as a proportion of the total amount that you have invested will give you the weight of the second asset in the portfolio, alright.

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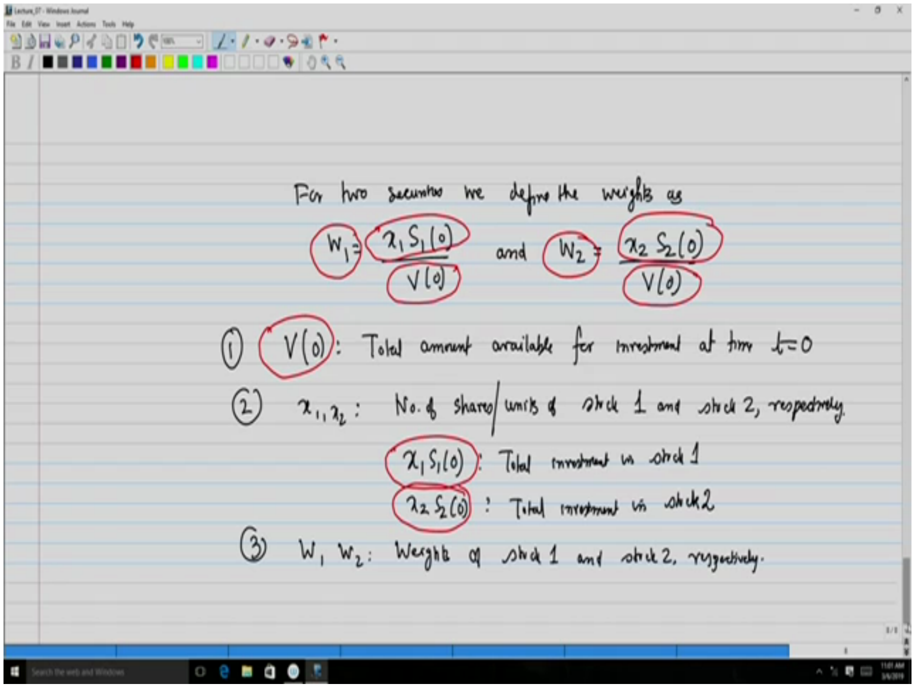

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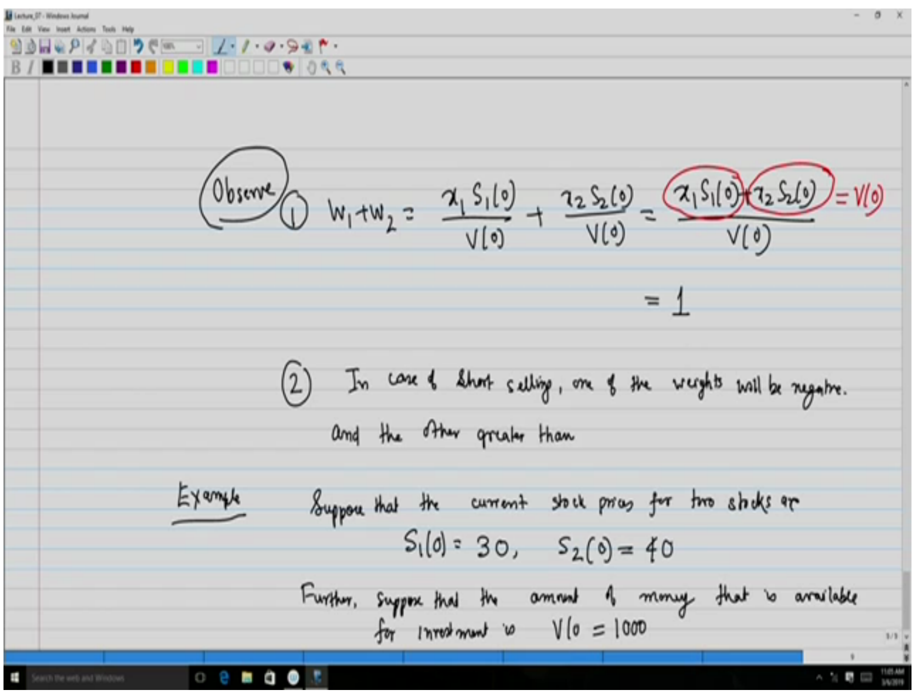

Now, let us make an observation. What is W_1? $W_1 = \frac{x_1 S_1(0)}{V(0)}$ and $W_2 = \frac{x_2 S_2(0)}{V(0)}$, and this is $W_1 + W_2 = \frac{x_1 S_1(0) + x_2 S_2(0)}{V(0)}$. Now here as you recall, this $x_1 S_1(0)$ this is your total investment in your first stock and $x_2 S_2(0)$ is your total investment in your second stock so obviously the sum of this must exactly match the initial amount of money that you had available for investment. So, that means this sum is going to be equal to $V(0)$, so that means this becomes equal to 1. So what are the key properties of weights of assets in portfolio; is that the sum of the weights are going to be always equal to 1. So this was the 1st observation. Second observation is that typically you expect that W_1 and W_2 are going to be positive.

However, there is no restriction on W_1 and W_2 and it is possible either of them could actually end up being negative. So, in case of short selling, one of the weights may be negative or rather in case of short selling this will be negative, and since some of the weights must be equal to 1 and one of the weights is negative, so obviously the other weight is going to be greater than 1. Then you can say that the other is going to be greater than 1. So, what we have done so far is we have looked at the return of an asset in a single period and the risk for it, and then we have introduced the definition of weights.

So, now let us look at an example which will show the importance of weights, and in particular illustrate how weights can actually change over a period of time in our case over a single time period model when the number of units of the assets in the portfolio can actually remain unchanged and this will illustrate why this is more logical to make use of the weights to identify certified the portfolio instead of using the number of units of each of those stocks of assets in order to precisely define what the portfolio is. So, we begin with this little example, so suppose that the current stock prices for two stocks are $S_1(0) = 30$ that means one stock or one share is 30 for the first stock, and for the second one, this is 40. Further, suppose that the amount of money that is available for investment is 1000 and remember that we denote this by $V(0)$.

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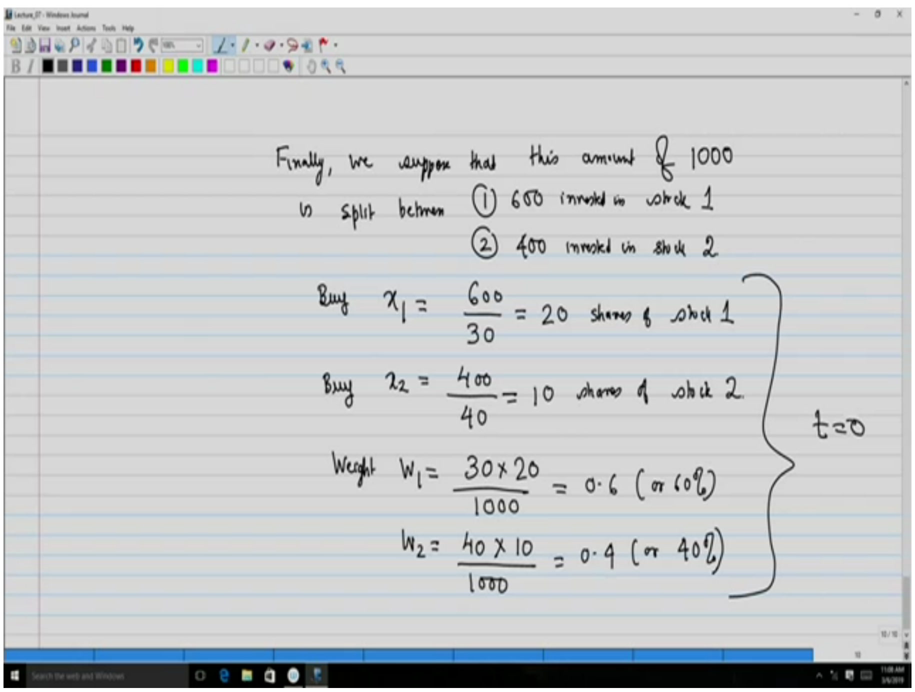

Then finally we suppose that this amount of 1000 which is our initial investment is split between 600 invested in stock 1, and the remaining 400 invested in stock 2. So that means what? If I invest an amount of 600 in stock 1, what was the price of the stock? The price of the stock was 30. Therefore, we can buy x_1 where investment is 600 divided by the price of each of those stocks which is 30, so accordingly we can buy $x_1 = \frac{600}{30} = 20$ shares of stock 1 and $x_2 = \frac{400}{40} = 10$ shares of stock 2.

Then from this we can get the weights. So, the weight of the first asset $W_1 = \frac{30 \times 20}{1000} = 0.6 = 60\%$. And the weight for the second stock is going to be $W_2 = \frac{40 \times 10}{1000} = 0.4 = 40\%$. So, now we have essentially the setup at time $t = 0$.

Now, let us see what happens, so the question is what happens at time $t = T$? So, suppose that at time $t = T$ the price of the first stock $S_1(T) = 35$ and the price of the second stock $S_2(T) = 39$, then what is going to be the value of the portfolio? So the value of the portfolio, remember what was your portfolio? Your portfolio had 20 shares of the first stock and 10 shares of the second stock, so if you sell them off now, the amount of money that you get for the first stock is 20×35, so that is the money that you can get from the first stock. And for the second one, the 10 stocks you can sell them off at 39, so this is 39. So I add them up so that means the total value of the portfolio or the amount of money that I can make if I decide to sell of all the 20 and 10 stocks respectively, this turns out to be equal to 1090.

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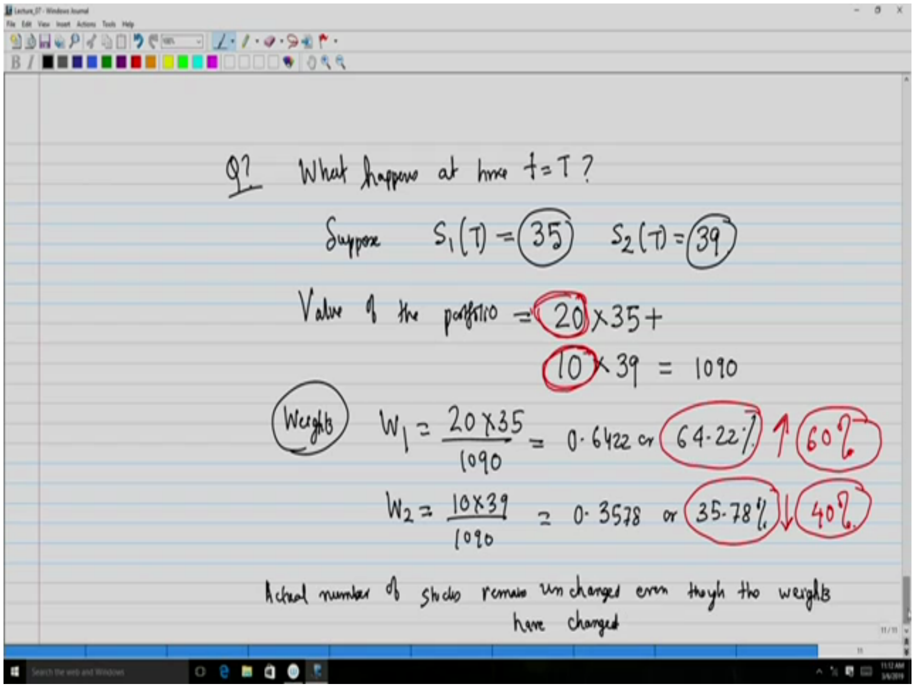

So that means in this case the weights, what are the weights going to be? The weight $W_1 = \frac{20\times35}{1090} = 0.6422 = 64.22\%$ and notice that earlier it was 60%, so the weights have actually changed. And for the second case, the weight is going to be $\frac{10\times39}{1090} = 0.3578 = 35.78\%$. So that means here this has gone up from 60% earlier and in the second case the weight has gone down from 40% earlier. So you see this is the point that I was making earlier that when you started off, you bought 20 units of the first stock and 10 units of the second stock, which gave the weights to be 0.6 or 60% and 0.4 or 40%.

At time $t = T$ you still have 20 stocks and 10 stocks respectively, but in this case the weights have changed to 64.22% and 35.78% respectively, so you see this is the illustration of why weights play such an important role when we are discussing portfolio theory. So this means that I can conclude this by making a statement that the actual number of stocks remains unchanged even though the weights have changed.

Now, what we will do is that, let us reconsider this example or review this example again from a slightly different context, we reconsider this example. Now, as before we start off with an initial investment $V(0) = 1000$, but this investment now will mean a long position in stock 1 and short position in stock 2. And suppose we take the weights for the first stock to be 1.2, so naturally the weight for the second asset is going to be -0.2. Then x_1 will be $\frac{W_1 V(0)}{S_1(0)}$, I am just rewriting the expressions for weights, remember it was $W_i = \frac{x_i S_i(0)}{V(0)}$ for $i = 1,\ 2$ (so I am just rewriting this). And what is this? This is going to be $1.2V(0) = 1000$, and remember what was $S_1(0)$?

$S_1(0)$ was 30, so this is equal to 40. And x_2 is going to be $\frac{W_2 V(0)}{S_2(0)}$, this is going to be $-\frac{0.2\times1000}{40} = -5$. So, basically you have borrowed first stock and you have short sold and you have used that money to purchase additional units of the first for whatever reason. Okay, now what we do next is that so when the stock price is changed to $S_1(T) = 35$ and $S_2(T) = 39$ as before, we are revisiting the example, then what is going to be the value of the portfolio? The value of the portfolio $V(T) = x_1 S_1(T) + x_2 S_2(T) = 40\times35 - 5\times39 = 1205$.

So, you see the interesting thing is that earlier you choose your $x_1 = 20$, and $x_2 = 10$ and now you are choosing $x_1 = 40$ and $x_2 = -5$. In both the cases you had $V(0) = 1000$, but here in the first case you end up getting your $V(T) = 1090$, and in the second case, you end up getting your $V(T) = 1205$ and you see this is higher. So this means that you start off with an initial amount of 1000, if you decide to buy 10 and 20 units of the 2 stocks then at the end of time $t = T$ your value of the portfolio is 1090. Instead of that, if we had decided to choose to buy 40 units of the first stock and short sell 5 units of the second, then you end up

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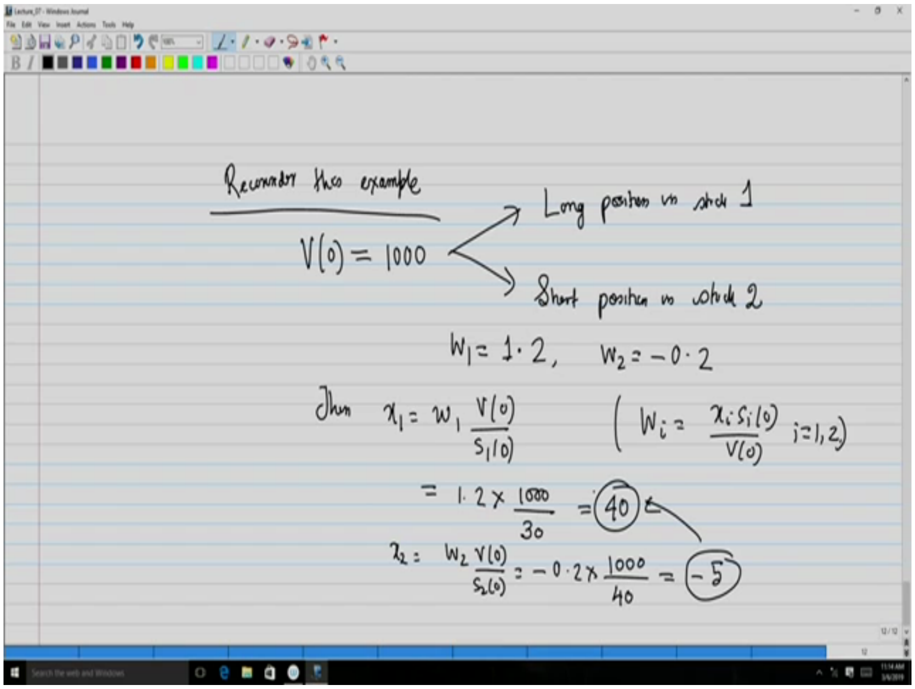

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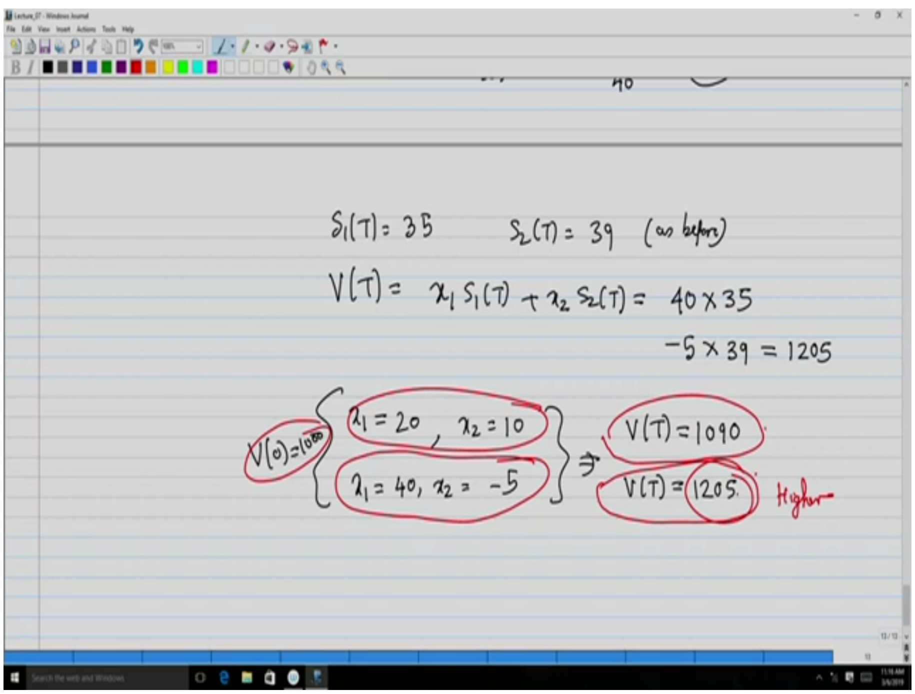

getting final value of 1205 which is the higher amount.

So this means for different combinations of x_1 and x_2 initially or equivalent different combinations of W_1 and W_2 you will end up getting different values of $V(T)$ at different points. And so this brings us to the natural question and I am wondering because I have really a wide range of choices of x_1 and x_2 and the question that we will subsequently address in the course of this discussion on portfolio theory is that what would be the best choice in terms of choosing your x_1 and x_2 so that my $V(T)$ is going to be as large as possible.

Okay, now let us come to a couple of theorems to illustrate whatever we have done so far and this is

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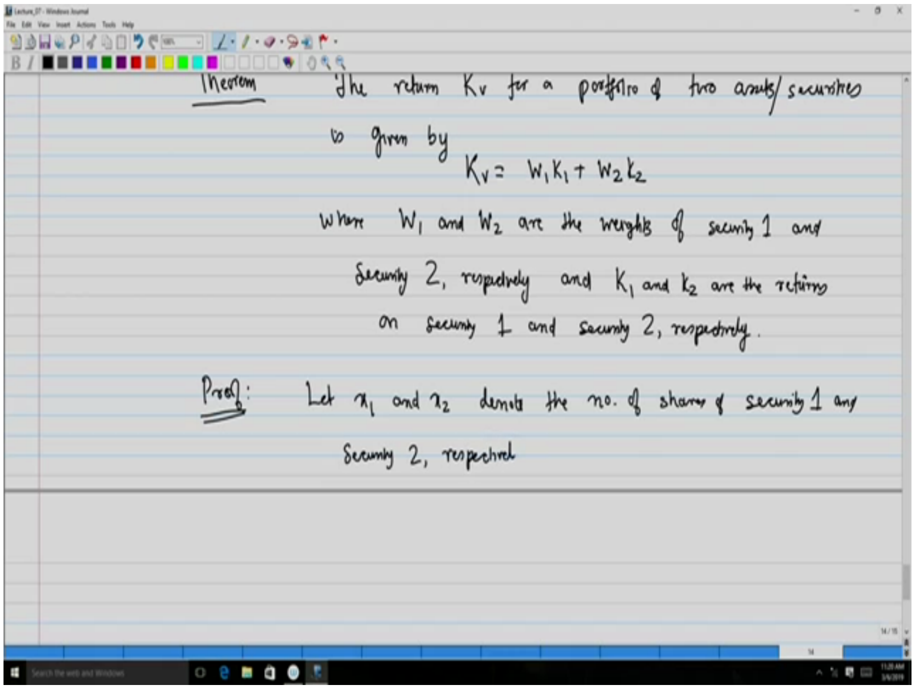

going to be expected returns and risk for a two-asset portfolio. So, so far we have only looked at the weights and what is going to be the returns. So, let us now give it a more general framework when it comes to two assets. So I will state this with the theorem, so the return K_V for a portfolio of two assets or securities, remember we are using these words interchangeably is given by $K_V = W_1K_1 + W_2K_2$, where W_1 and W_2 are the weights of security 1 and security 2, respectively and K_1 and K_2 are random variables which gives you, so these are the returns on security 1 and security 2, respectively, so these are random variables which gives you the returns on the first and the second asset. So, the proof of this is very straightforward and can be done by just making use of the definition of weights. So, let x_1 and x_2 as before denote the number of shares of stock or security 1 and security 2, respectively.

Then at time $T = 0$ the value of these two assets portfolio $V(0)$ which is the initial amount of money that is available to you for investment and this is going to be $X_1S_1(0) + X_2S_2(0)$. Since this is a single time period model, so similarly, at time $t = T$, the value of this two asset portfolio is $V(T)$, and what is this going to be? This is going to be x_1 and x_2 units of stocks multiplied by the current price of the stock which is $S_1(T)$ and $S_2(T)$, respectively.

Now, you recall that $S_i(T) = S_i(0)(1 + K_i)$ for $i = 1,\ 2$. So accordingly,

$$V(T) = x_1S_1(0)(1 + K_1) + x_2S_2(0)(1 + K_2) = W_1V(0)(1 + K_1) + W_2V(0)(1 + K_2),$$

or,

$$V(T) = V(0)[W_1 + W_2 + W_1K_1 + W_2K_2] = V(0)[1 + W_1K_1 + W_2K_2].$$

Therefore, the return K_V, what is this going to be? This is going to be $K_V = \frac{V(T)-V(0)}{V(0)} = W_1K_1 + W_2K_2$. So, that means the written on the portfolio V is equal to the weighted sum of returns on security 1 and 2. So this brings us to straightforward consequent theorem is that the expected return $E(K_V)$ of a portfolio of two assets or securities is given by $E(K_V) = W_1E(K_1) + W_2E(K_2)$. So the proof is obvious. I replace this K_V with this expression for K_V, so this is going to be $W_1K_1 + W_2K_2$ and this is going to be $W_1E(K_1) + W_2E(K_2)$ using linearity of expectation.

Finally, we become to one last example. So let us consider the probability space omega with $\omega_1,\ \omega_2$ and ω_3, where as I had mentioned earlier these are going to be the states of economy. So, we identify ω_1 in

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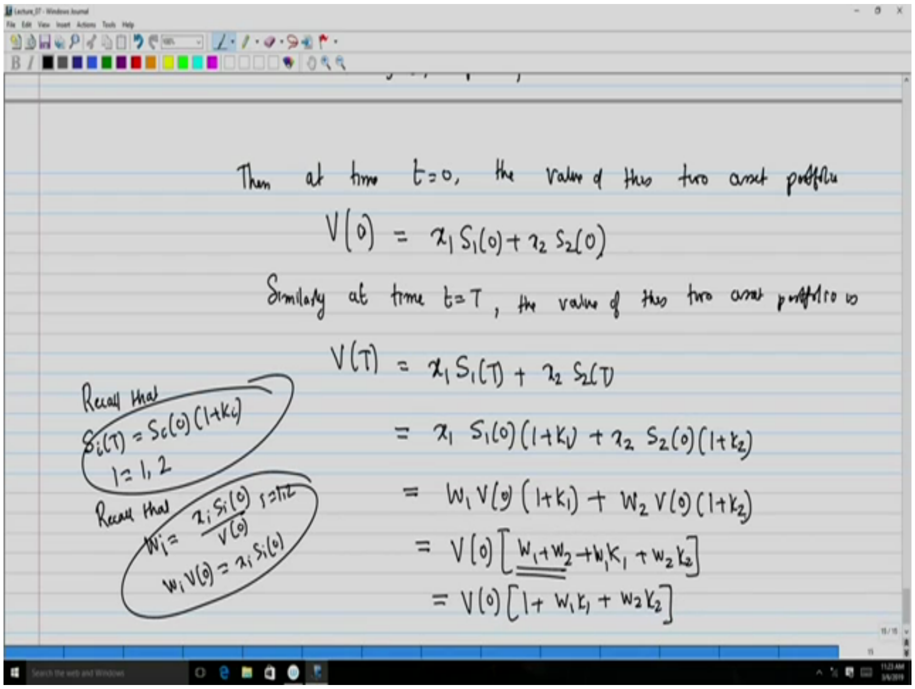

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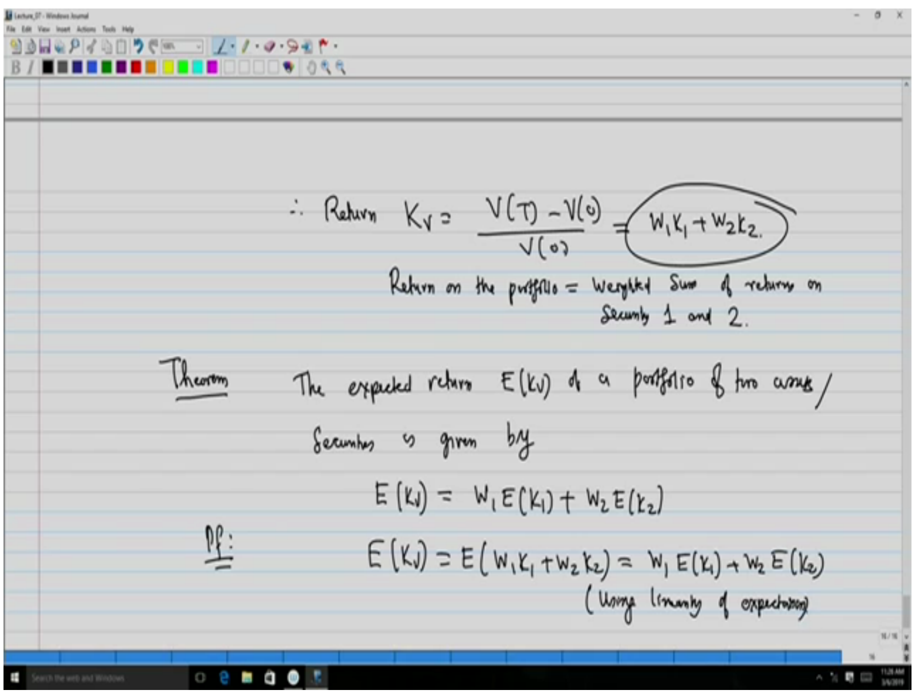

situation where there is economic recession in the market, ω_2 is going to be economic stagnation and ω_3 is a situation when there is economic boom.

So now we consider investment in two securities and we specify the properties. So, let us just have this in a tabular form. Scenarios are ω_1 for recession, ω_2 for stagnation and ω_3 for economic boom. The probability of $\omega_1 = 0.2$ that means there is 20% chance that there is economic recession, $\omega_2 = 0.5$, so there is 50% chance of stagnation and $\omega_3 = 0.3$, so that means there is 30% chance that there is an economic boom.

So, under these three scenarios what are going to be the returns of each of those two assets? So, for

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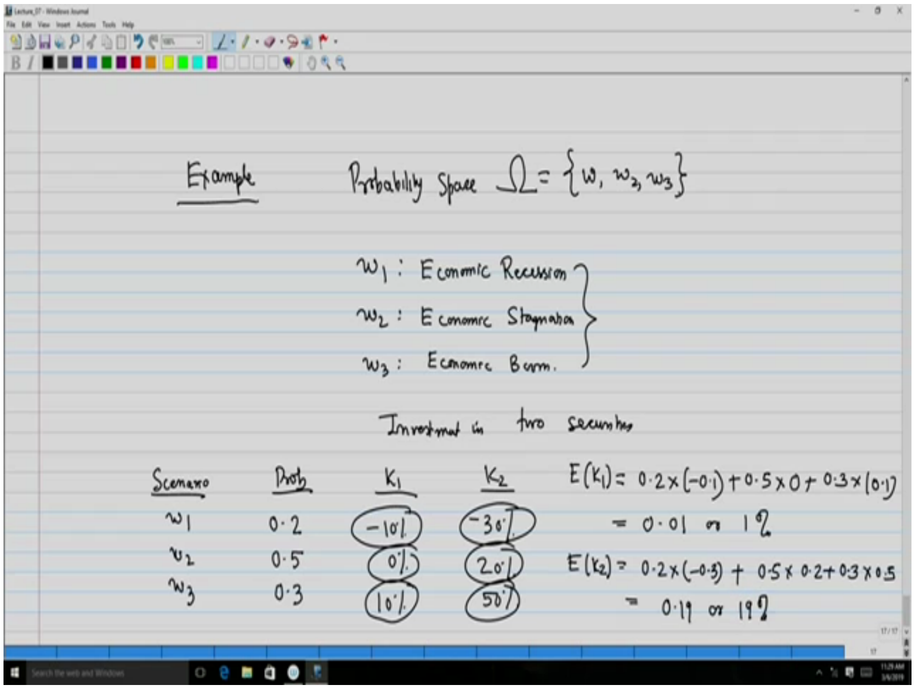

the first asset the written K_1 is (say) -10% percent, for the second case, it is going to be 0% and for the third economic state, it is going to be 10%. For the second asset the written K_2 this is going to be -30% in recession, 20% in stagnation and 50% when there is an economic boom. So, what is going to be $E(K_1)$? $E(K_1)$ is going to be nothing but the random variables multiplied by the probabilities summed up.

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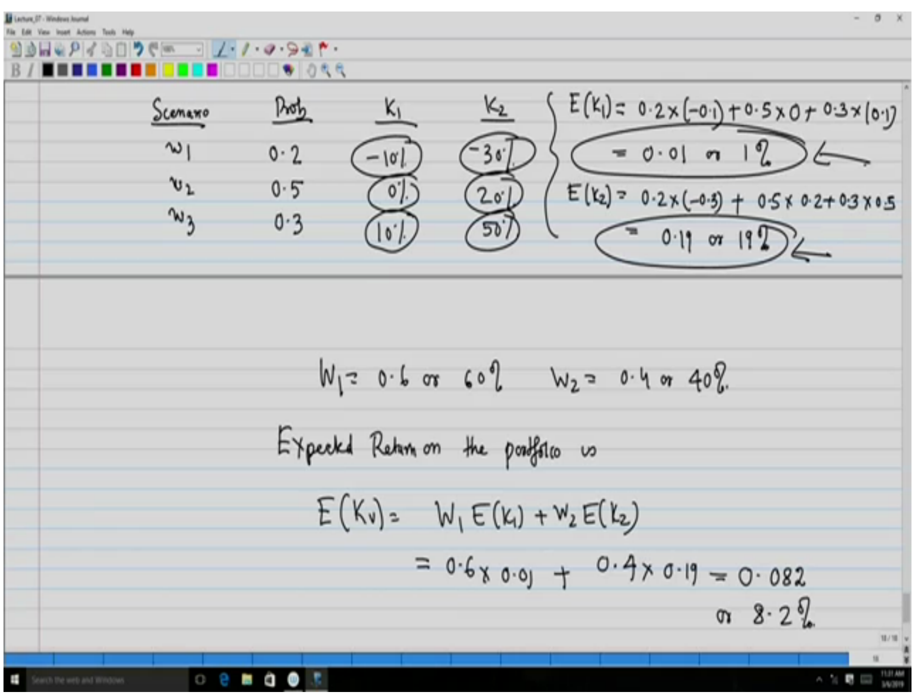

So, what are the probabilities, probabilities are $0.2,\ 0.5$ and 0.3. We multiply by random variables of returns so the first case it is -10%, so this is going to be -0.1, in the second case, it is going to be 0%, so I multiply this with 0 and in the third case, this is going to be 10% so I multiply this with 0.1 and we add them up to get the expectations as 0.01 or 1%. Similarly, for $E(K_2)$, I get this as $0.2,\ 0.5,\ 0.3$ multiplied by

-0.3 for this 30%, multiplied by 0.2 and multiplied by 0.5 for the 3rd case. I summed them up to get 0.19 or 19%.

Alright, so I have got the expected returns. So now suppose we decide to invest in these 2 assets. So, we specify the weights, the weights being $W_1 = 0.6$ or 60% and $W_2 = 0.4$ or 40%, then the expected return on the portfolio is you make use of this formula, the expected return is going to be the weighted sum of the individual expected returns, so accordingly the expected return of the portfolio $E(K_V)$ which is the weighted sum $W_1E(K_1)+W_2E(K_2)$. So $W_1 = 0.6,\ W_2 = 0.4,\ E(K_1) = 0.1$ from here and $E(K_2) = 0.19$ from here, plus I add them up and I end up getting 0.082 or 8.2%.

So, to sum it up, what we essentially looked at is that we first recognized what Markowitz had worked on, predicted in early 50s that just by looking at the return of an asset it would not be judicious to make a decision as far as investment is concerned and accordingly he introduced a notion of risk and gave the idea that the return and risk both need to be considered in conjunction to make a rational decision about how to make your portfolio choice.

We next spoke about what a portfolio is which an essential collection of assets, we recognize that it is more logical to make use of weights as a way of specifying a portfolio as against the number of units of each of the assets being used to specify the portfolio. And finally, we defined what is going to be the return in case of two asset portfolio and what is going to be the expected return.

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So, in the next class, we look at other components of Markowitz framework on the other pillar of Markowitz framework, namely, the risk and then proceed ahead to look at Markowitz framework in general. Thank you for watching.