Mathematical Finance

Lecture 31: Greeks, Put-Call Parity, Change of Measure

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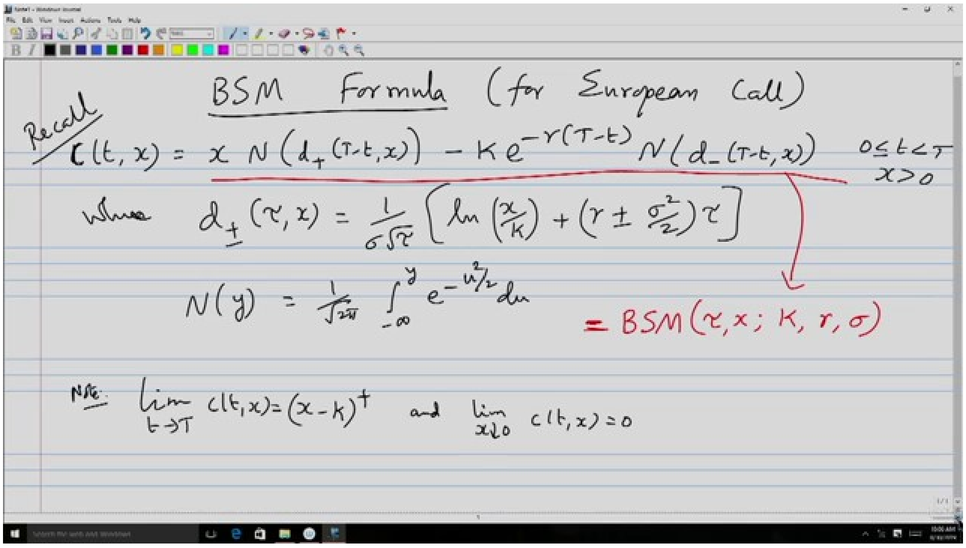

Hello everyone. In the previous lecture we saw the Black-Scholes-Merton partial differential equation and we gave the solution of that Black-Scholes-Merton partial differential equation as and we call that as BSM formula. This is for the particular case of the European call and we call that as some $C(t,x)$ which is what this expression is, where d plus minus is given by this expression and $N(y)$ is the Cumulative Distribution function of a standard normal random variable evaluated at the point y and we call this as the BSM formula.

And sometimes we also denote this as BSM(τ,x), where τ essentially means the time to maturity. Which is actually that remaining time, which is $T-t$ if you look at the expression inside in the BSM formula that we have given, this is what it is. So, an x is the value of the underlying asset price process at that point. K,r,σ are parameters so we may denote it in this form and τ,x are really the variable that is why it is the function of t,x.

t,r,τ it is one in the same. because τ is $T-t$ it could also since capital T is fixed. So, it is essentially small t if I have to write in that form, so it is one. Remember, this formula is given for t lying between 0 and t, 0 included and T excluded and for all $x \geq 0$. Now, what at those corner points, when $t=T$ and x is equal to 0, they are defined by continuity.

So, they do satisfy in that particular case that means that at those corner points what we have is, limit t tending to T of $C(t,x) = (x-k)^+$ and $\lim_{x\to 0} C(t,x) = 0$. So, this claims you can verify as an exercise. So, this is what we have seen as the BSM formula.

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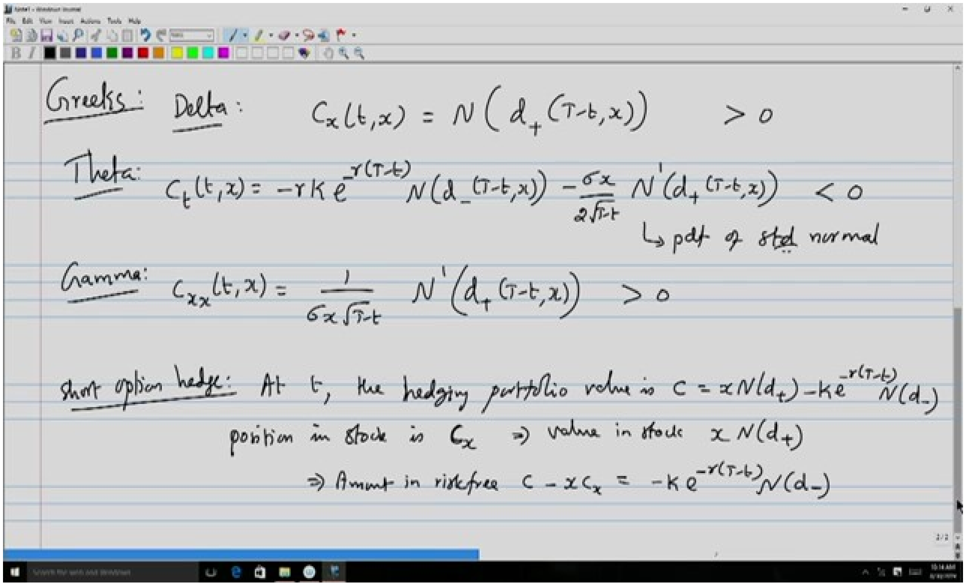

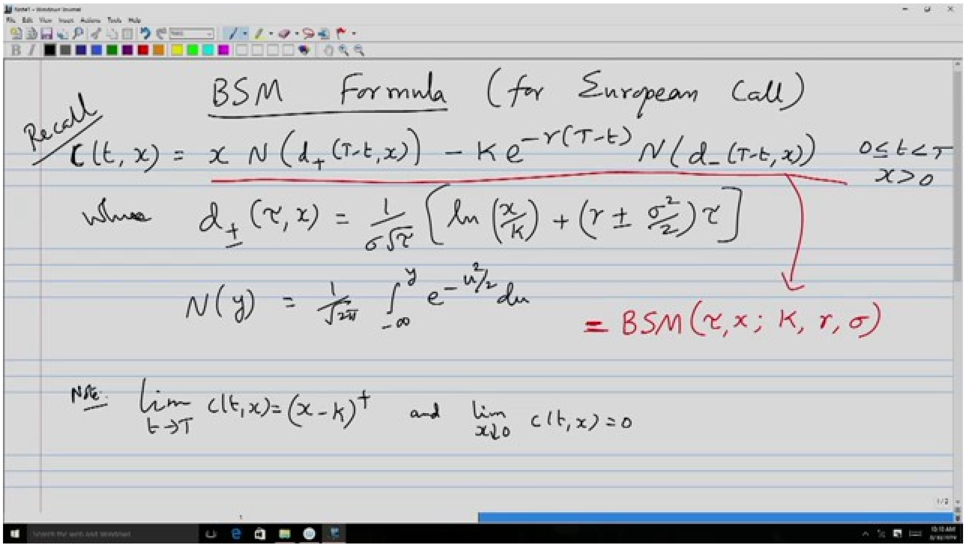

Now, what we will do? Associated with such formulas we also have few quantities, which are generally refer to as the Greeks, which play an important role in the risk management are constructing portfolios which is sort of insensitive to the changes in the underlying quantity. So, this basically the derivative of this $C(t,x)$ or BSM formula with respect to various quantities. Recall, BSM formula is τ is there, x is there, K is there, r is there, σ is there and mainly we will be concerned with t and x, r, τ whichever it is.

These are really the variable and even with respect to K, r, σ also the derivative, essentially they are constants but were the derivative, which we mean that the error in the estimation of K, r, σ suppose if there was an error what would happen? Which is essentially in the optimization literature or somewhere else it should be known as sensitivity analysis.(So, to do that one takes a) but we really interested with respect to t and x and see like what is the quantities that we have.

So, at least three derivatives which will be of interest towards to C and one of them is, we have already noted down, which is Δ, which is $C_x(t,x)$. which you can derive to be the quantity. which is given by this and the other which is called θ, which is basically $C_t(t,x)$, which is the derivative with respect to t.

And you can see that this quantity C_x is positive and this is negative. And by the way this N dash means essentially the pdf of standard normal at this point. Actually the pdf and as denotes the pdf of standard normal means probability density function of the standard normal random variable and that density is evaluated at this point, this is $N'(d_+)$. And there is another one which is what known as gamma, which is also of our interest, which is the second derivative of with respect to x of this function $C(t,x)$ which essentially the C_x is given here you just have to see how did we arrive at this formula?

You simply look at this BSM formula that we have written in the top line. What we have seen here is, we are just differentiating with respect to t wherever t is occurring and you just have to differentiate and simplify. That is all we did, there is nothing. And C_x we already know it is the Δ_t position when we are trying to replicate the European call option that the portion that we take is what delta t, x which is C x t, x which is what is N of t plus.

So, which means that, it is greater than 0. which means you take a positive position in the underlying asset when you trying to replicate the short hedge. Now, C_{xx} in the same line if you differentiate this $N(d_+)$ once more, you get that. Like Δ this is also greater than 0. so you can always see that it is always positive.

Now, in the following discussion that we are going to make you know let us suppress the arguments $T-t, x$ and so on we may simply called d_{+-}, which essentially means that it is a function of this T minus t and so on. Now, if at time t if the underlying stock price is x, then the short hedge calls for holding C_x number of stocks to replicate the short position in your option. So, the hedging portfolio value is basically given by the BSM formula, and since out of which you hold C x amount of stocks, which is invested in the stock then the amount invested in the money market would be the remaining.

So basically what? Short option hedge we are looking at the European call itself. So, at t the hedging portfolio value is what? It is basically C given by C, which is what $C(t,x)$ we just said that we are suppressing the arguments. You take how many the position in stock is given by C_x.

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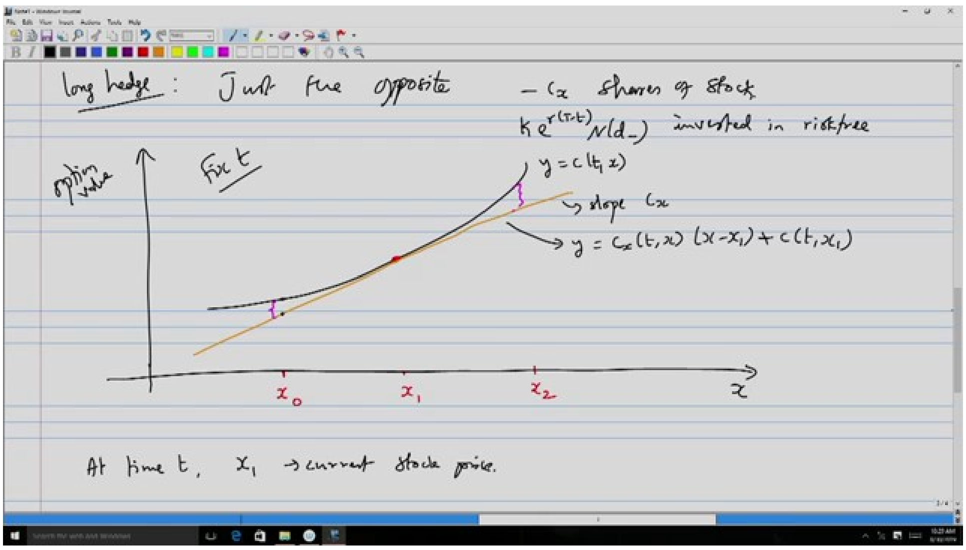

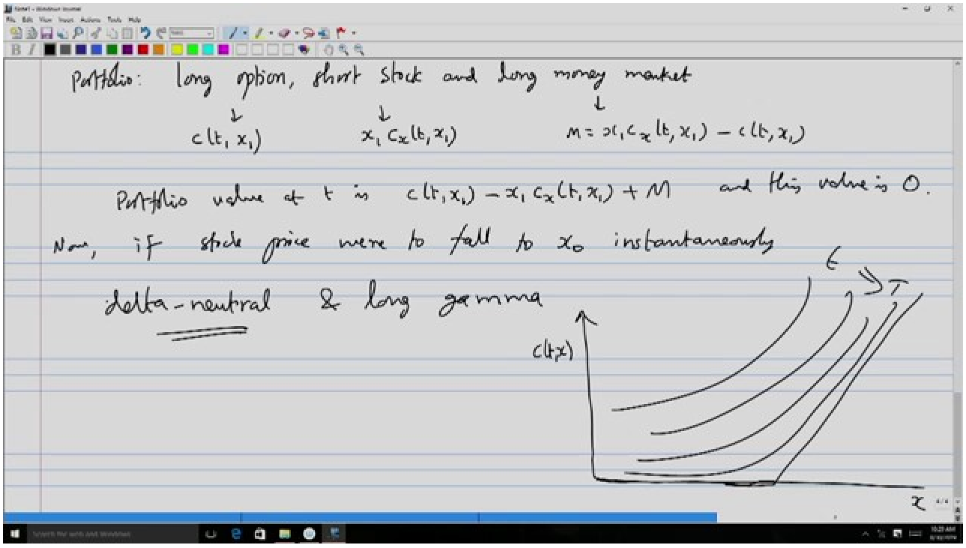

Now, for long hedge just the opposite as you already familiar with. In that case you will hold short position in the stock and you will invest in the risk free for holding the long positions that is what it will be.

Now, because the Δ and γ both of them are positive, if you fixed a t and if you look at the function $C(t,x)$ is a function of x alone, and because your delta and gamma are positive so this will be a Convex function. So, I can draw that and see, because that is what it is going to look at now. So, essentially what will happen is you have this, and you will have a curve which will go like sorry not of this nature.

So, it will be a little bit straight like this nature and then I will have to draw. So, this is the x, which is the stock price and this is the option value and this curve is essentially my y is equal to $C(t,x)$ for a fixed t and let me draw one more line. So, this is the point, at which this is tangent and let me call that as my x =1 and let me pick another point here which is x= 0 and let me pick another point here which is x= 2.

So, now what happens? So, this is actually the tangent, if it is not the case, so you just look at what is happening there. It is actually tangent to this curve that is what you know we are trying to draw. So, this is the long hedge that you have here. Now, in this particular case so what happens? If you are trying to replicate the long position rather than the short position this is just for convenience and if you have the at time t. So, at time t_1, x_1 is the value of the current stock price.

Now, the hedge that we are setting it up and the portfolio, that what is the hedging portfolio now? So, now for long option hedge requires that you have to the total portfolio (rather than you say that). So, your portfolio now is long option short stock and long risk free or money market. I will call this as money market because risky and risk free. So this is what your portfolio.

Now, what is the value of the option? Value of the option is $C(t,x_1)$ short stock. So, the value so this is all what we are talking about this for the stock the value is x_1 the together current value and $C_x(t,x_1)$ and the value of the money market account is say M. So, the initial portfolio value means? The portfolio value at t is essentially that.

Now, let us see what happens? Now, if stock prices were to fall to x_0 in an instantaneously, so what happens now, is the quantity that you look at here. (Suppose) I mean you are looking at instantaneous moment, suppose if the stock price were to fall to x_0, then the hedging portfolio value, so, by the way this is the line which has the slope C_x and this line is actually

$$y = C_x(t,x)(x - x_1) + c(t,x_1).$$

why we write here? Now, you look at here, what we have written down here, so you have to understand that you will see.

So, this is the hedging portfolio value. So, the orange line is the hedging portfolio value and y is the option value. So, you are trying to hedge, which means you are trying to construct a portfolio. Which mirrors the orange line which follows the orange line path and whereas the corresponding option price is as per the dark black line, the value of this portfolio. Now, if the stock price from x_1 if it were to fall to x_0 instantaneously, then the stock position or the value of the option at that point of time would be somewhere here, but the value of your hedging portfolio would be somewhere here.

So, what you are having now is this difference and (similarly is your) actually this is your value of the portfolio and that is the option. Now, since you are holding a long option you will get this profit and similarly at x_2 the values would be this two and if it were to instantaneously increase then the difference will be this pink line, so that is the profit that you get.

So, this is what will happen and such this hedge is actually what we call? This portfolio, that we setup is actually what we call a delta neutral and long gamma. Delta neutral because the hedging portfolio is tangent to the curve c at the point t, x_1 rather if the underlying asset stock price is x_1 at that point of time. So, this is the setup as a tangent to this curve.

So, that is what a delta neutral portfolios objective is and that is what a delta neutral portfolio will setup a hedge in such a manner only. And long gamma because it is trying to make profit or it trying to get advantage of the convexity nature of this $C(t,x)$ and you setup because of that. So, this is called as long gamma. So, because this long gamma, it is essentially as you see if there was some instantaneous falls in the stock prices, then the long gamma position gives you profit when you hold a long option hedge.

So, when it will that happen? When there is high volatility in the underlying stock prices. So, a long gamma position would be helpful to take advantage of the instantaneous fall as well as increase in the underlying stock price process. But otherwise in the normal scenario this will not give you, if there is no such instantaneous fall, so what is the delta neutral portfolio does? You can see from this graph and we can understand. For a small moment along the x axis what you have is that, this curve $C(t,x)$ as well as the straight line, their values are very closed to each other. So, which means that for small moment in x both are equal and there is nothing else and remember this is we have fixed t and then so this is you remember this is for fix t. Now, for a fix t this curve is there, now as t goes this curve shifts downwards essentially.

So, in that case all these combinations, all these positions will get neutralized that is what expect in a case and where else you can use this kind of idea. So, this is what happens ,when actually in the previous discussion when we formed the BSM formula and we computed and we continuously hedge change the hedging portfolios position then what happens is this. You may recall that or you can plot the $C(t,x)$ and C that as you will see suppose if what is the pay of at maturity of in European call option? It is be like this.

Prior to maturity what will happen is, this is how this curve will move towards this. So, as t goes, so this is with respect to x and $C(t,x)$, so this is how this behavior will be, you can plot this function and see that this is actually the behavior. So, it moves towards this as t approaches capital T, (so t approaches capital T this move). So, this is the capital T part, this is the piece wise continuous line is what the capital T ,which is the payoff of the European call option that you would have seen.

Now, this is what happens when you setup the long hedge or short hedge equivalently. but if you have a position, if you know that stock price is going to go up and down or something like that then you can change. Suppose, this curve if you make this as a higher slope than C_x then what will happen? This curve, this orange curve will shift upwards for x greater than x_1. This will be higher value, so in that case what will happen is that.

So, by holding in a short appropriately, short or long option and short or long on this hedging portfolio so you will be able to make profit in one way or other way. So, to do such kind of hedge risk management is what the main role of this Greeks that you will use. And if all these things are assumptions based upon that if the underlying asset follows a Geometric Brownian motion, then this is what will have and as long as we continuously rebalance our portfolio using the delta hedging strategy then we will be able to replicate the option in the underlying asset.

And at maturity we will have the hedging portfolios value would be equal to the underlying assets, underlying option price value, which is payoff that is we required to pay that you will have with them.

So, this is a brief about the Greeks. So, what we have seen is these three Greeks. at least you could derive these three for yourself and see and it works when you want to setup a portfolio, which is insensitive to the short change or in the moments of the prices or the underlying.

Suppose if, it is stock then you make a delta neutral, if you want to make suppose time then you make is a theta neutral. Now, you can see like this formulas actually like these three quantities that we have given delta, gamma and theta is you can easily plugin to the BSM equation and see that this satisfies that, so you can verify those points.

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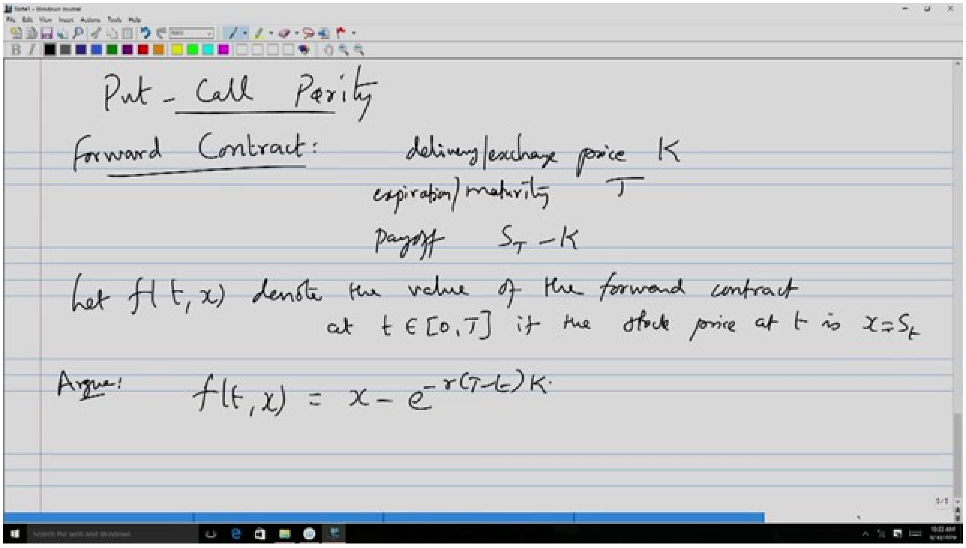

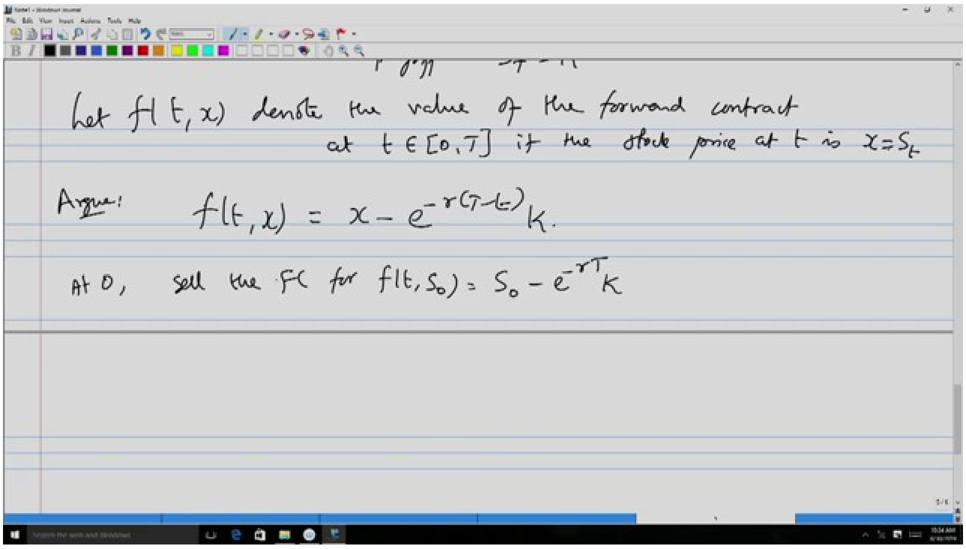

So, the next that we will look at is something called which you already know is Put-call parity. So, what we will first consider is, so we have considered so far a European call option. Now, we can consider a forward contract and there is nothing peculiar about the European call option there. In the BSM formula if I take forward contract what is going to change is only the terminal condition instead of $x-k$ at the positive part of it. I would simply have $x-k$ and I will adopt the exact same procedure, I will solve that then I will get the corresponding price for a forward contract. but for the time being what we will do, that we will arrive at through arbitrage arguments the forward prices. So, what we have? So, usual the forward contract with a delivery price K or exchange price delivery or exchange price say K, maturity or what we call which expiry, expiration, expiration time or maturity time t.

So, what this is? Suppose if, you are a long position holder in the forward contract then you will buy the underlying asset at time T for a price K. So, the payoff would be actually $S_T - K$. Let $f(t,x)$ denote the value of the forward contract at t belonging to the earlier time if the stock price at t is x, which is what is basically S_t in our underlying.

So, now what we will do? We will argue that my $f(t,x) = x - e^{-r(T-t)K}$ is the value of this such a forward contract. So, how do we do? So, suppose you assume that sell this, so arbitrage argument is what you do is at 0. you sell the forward contract for $f(t,S_0) = S_0 e^{-rT} K$.

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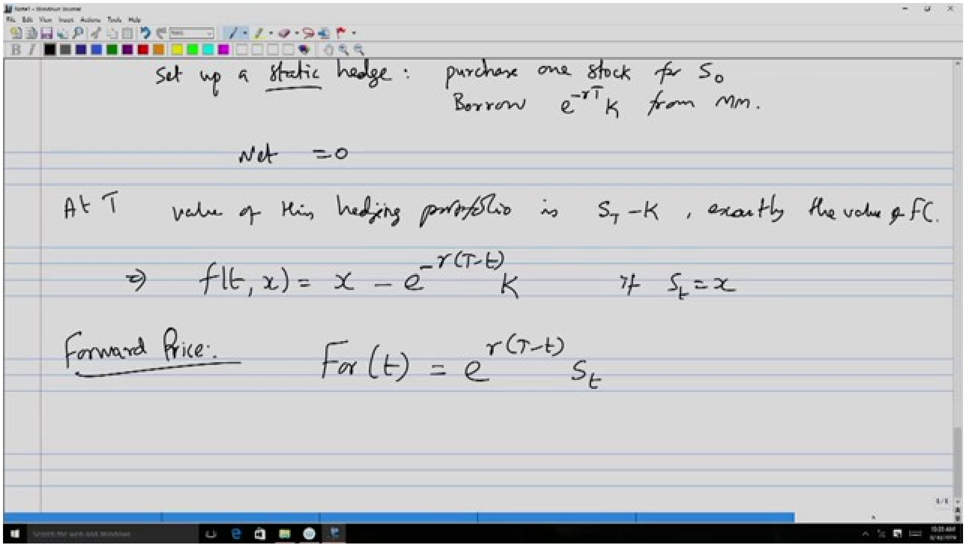

Now, setup a static hedge (this word we will just in a moment will). Static hedge means it is the hedge that does not trade except at the initial time. Now, what we say? What is the hedge? Hedge is purchase one stock or one share of stock for S naught amount. So, what you need to do by for doing that? Borrow e to the power minus r T times K from money market or risk free asset.

So, makes no further trade if you recall for (you know) replicating the call option, we had used what we call dynamic hedge. which means a hedge that changes at every time instant whereas to replicate your forward contract you do not need the dynamic hedge, you can setup a hedge which is static, which means you trade at only at one time, which is a time 0.

So, in any situation one would prefer a static hedge over dynamic hedge because you do not need to them hedge continuously. So, borrow this much so that net value is 0 at time 0. Because you sold a forward contract for this much amount and you borrowed this, added to this so you had S naught amount use that S naught to buy one stock done so you had. Now, at T, so the value of this hedging portfolio is basically what? You have long one forward contract which requires pay off of $S_T - K$. Now, what is the hedging values portfolio? This is essentially now what you will have this borrowed amount would have grown to K and the current stock price is S_T, so the hedging values portfolio is $S_T - K$, which is equal to the long position pay off for the forward contract.

So, this is the exactly hedging portfolio. Which is exactly the value of the forward contract, that is what you have seen. So, because the agent has been able to or you are able to setup a hedge which replicates the payoff of a forward contract at all intermediate times also it must replicate the corresponding value of the forward contract. otherwise there will be arbitrage opportunities in between and hence f t, x this implies that my $f(t,x)$ given above, because at any intermediate time this is the value when underlying asset price is, stock price is S, this is what the value where x is replaced, if $S_t = x$, so this is what it is.

So, the value is given by f_t, S_T and this is the formula function $f(t,x)$ which gives the value of the option prices. Now, from this you can also compute the (what is the) forward price. if you are interested to do. So, which we can call this as forward price at time t is given by $e^{r(T-t)} S_t$. Now, how do we get this forward price? It is that value of K, which makes the value of the forward contract at time t is 0.

So, you pick a value of K such that this quantity is 0, I mean you can simply get from $f(t,x)$ given this previous expression you equated to 0 and you get that value of K, which is this, S_T forward price that you get. But all intermediate times, of course, so which means that at suppose if you are locking this forward price at the time 0 and you enter into your forward contract at that forward price at time 0.

Then its value is 0 at time 0 and hence you do not pay anything for that forward contract but as it progresses it will assume some positive value or negative value, depending upon the behavior of this S_T and that value you can easily write down into this case.

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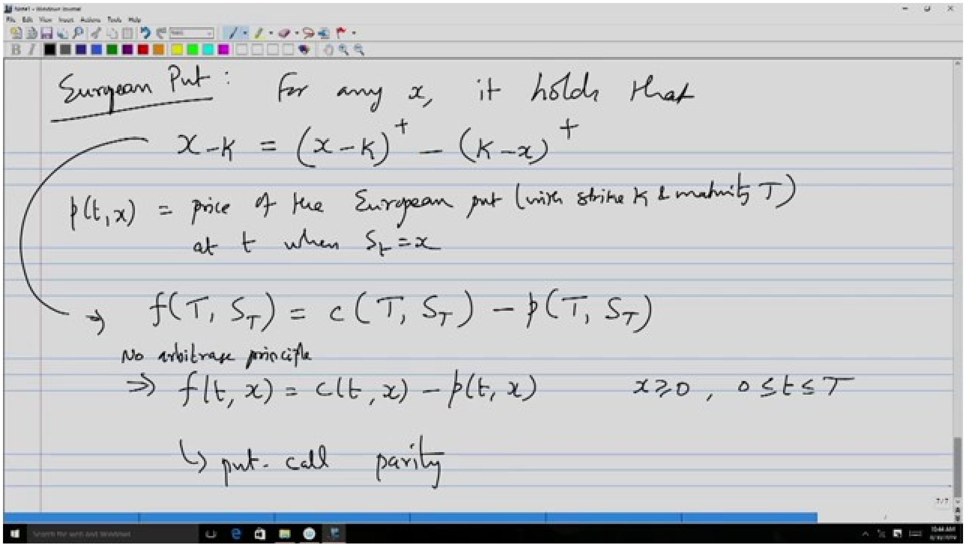

Now, we have the forward contract price. Now, let us look at the European put case. Now, what we have here? What is the payoff which we know? Which is $(K-S_T)^+$. So, but we know for any number x we have this equality,

$$x-k=(x-k)^+-(k-x)^+.$$

We denote for European put $p(t,x)$ as the price of the European put with strike K and maturity T like the similar case with the other case, price of the European put at time t when my $S_t = x$. T

So, similarly we have the $C(t,x), f(t,x)$. Now, what this signifies is essentially your f. So this means, I can write this one as $f(T,S_T) = c(T,S_T) - p(T,S_T)$. So, with the corresponding payoffs as given by these two, I can write this expression means this. So, which means that the payoff of the forward contract agrees with the payoff of a portfolio which is long one call option and short one put option on the same underlying asset. All of them have the same underlying assets, same maturity and call and put and forward everything has the same strike or exchange which is the K.

Now, since the value of these two agrees at maturity, and by the No Arbitrage principle, they also must agree at all time prior to that which is t, x which is $C(t,x) - p(t,x)$ for all x greater than or equal to 0 and for all time T. Because, unless this is true, there would be arbitrage in the intermediate times and one can take advantage and to make profit.

So, the No Arbitrage principle implies that this price must be equal to must be equal even at all prior times starting from time 0 and this is what we call it as the Put-call parity, so it must be same as this. Note that, this Put-call parity we have derived without any assumption on the or without appealing to the Black-Scholes-Merton model of Geometric Brownian motion.

What we need is without any assumption in the price except sufficient liquidity where you can buy and sell, so that you can these long and short calls and puts they are available to construct such a portfolio that is all you need. So, if you have that assumption then the Put-call parity holds. It is much more general without any model assumption that the Put-call relationship holds on the underlying asset. The underlying asset behavior you do not need actually.

But if we make the additional assumption, for example the constant interest rate region is what then you have then probably in the earlier part of the course you would have seen this $f(t,x)$ as given in the previous expression that expression would have replaced here. So, call minus put is equal to that is what then you would have shown, but that is actually the far corresponding forward price. So, if you put in this framework is much more general that what you are seeing here. If r is constant then $f(t,x)$ is given by this.

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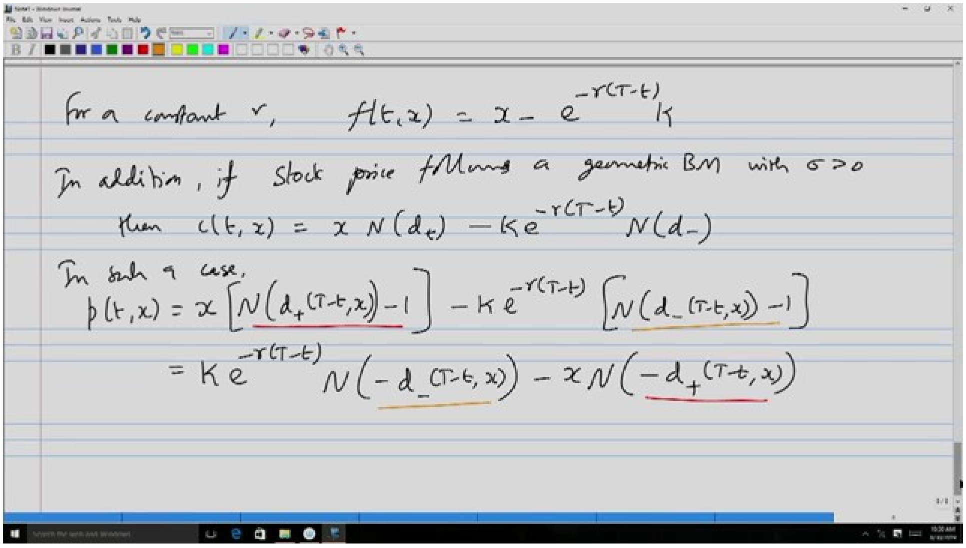

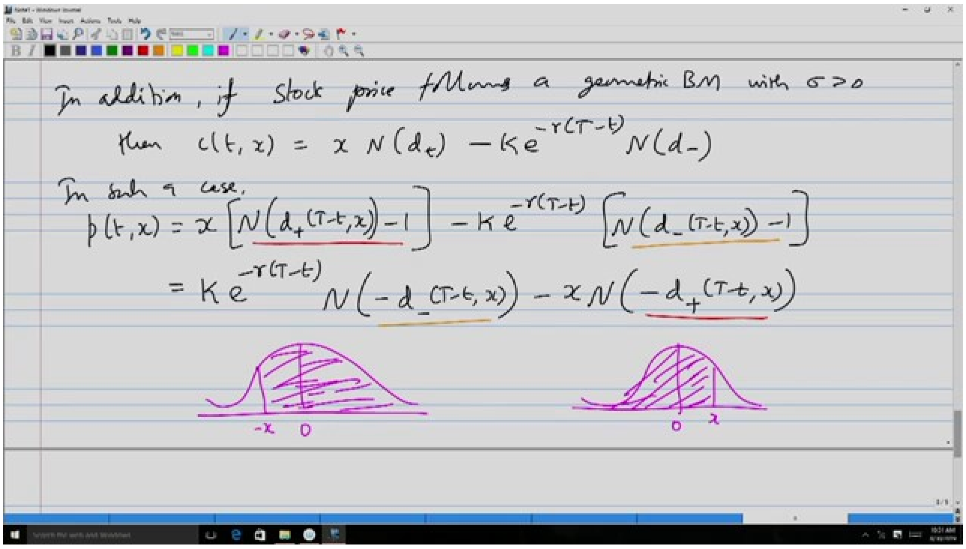

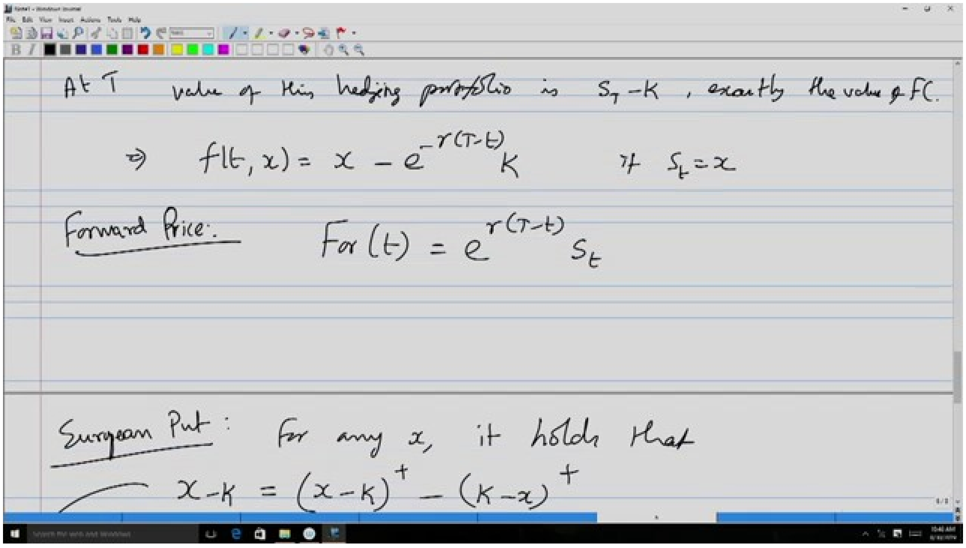

So, with additional assumptions, for a constant r my $f(t,x)$ is given by this expression. Now, in addition, if the underlying asset, if f the stock price follows a Geometric Brownian motion with sigma greater than 0 concern vitality then we know what is then my $c(t,x)$ is, essentially we know that, and in that case and in such a case I can write the $p(t,x)$ which is the Put-call parity.

Now, Put-call parity $p(t,x)$ would be what? $c(t,x) - f(t,x)$. The expression is given in the figure.

So, this form is easier to remember because if you co-relate this with $c(t,x)$ and $p(t,x)$ you can see the terms, first one terms or second terms are multiplied by both interchanged and there is a minus inside N of minus. So, you see this quantity what we have made is essentially this quantity is what is equal to this quantity and a minus of it essentially rather. So, this quantity is equal to this quantity with a minus, minus often.

So, I take one minus out then 1 minus of N minus d minus will be exactly N of minus d minus. So, that you can get from the properties of the standard normal variable. So, because of the symmetry of the standard normal random distribution that you have it here. You can easily recall, remember that something, suppose if I have the standard normal property, so I will see this so this is 0. Suppose if this is my x so this area, this area would be same as if I do this so this is 0 this is assume that this is standard normal.

So, this is actually minus x here and this area so both these areas are equal. The total under the curve is 1 and this area or this area both are equal so from there easily you can draw this conclusion that we have here. So, this is what is the Put-call parity by which you can also determine the put price. Again Greeks can also be written or determine for both put also as well as forward of course it may not be that much important but still one can have that but for put also this is important that you can construct that.

So, what we have seen is the call price and then we constructed the forward price by using No Arbitrage arguments and the put price. The Put-call parity holds in much generality but if you assume constant interest rate regime and the Geometric Brownian motion for stock prices then the put price can also be written in this as given in this expression either of them one can utilize to get the put prices that you have. So, you can derive this, otherwise you can also start from the scratch and then you can derive just like we have derived for $C(t,x)$ exactly same, again you will have a same Black-Scholes-Merton equation with a different condition which is given by for the corresponding Put-call sorry put option that you see over here. So, this is what the replication part and the corresponding call and put, Put-call parity you can relate and then you can see like how the hedging works in this particular case.

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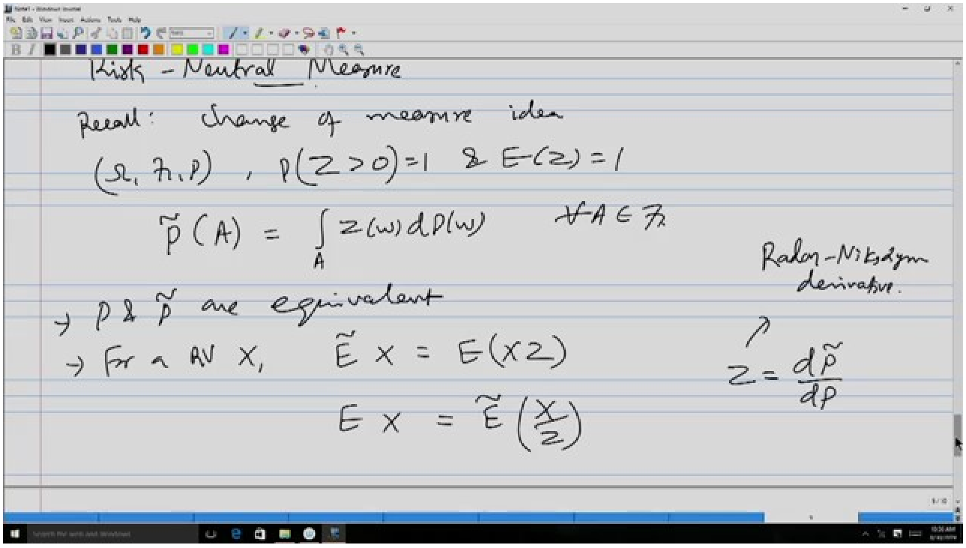

Now, what we can do is that we can look at the risk neutral measure or risk neutral pricing how do we get the risk neutral measure. So far what we have done is, we have replicated the first part, which we did in the discrete time, which is through arbitrage price we can obtain in the prices through arbitrage, No Arbitrage argument or using arbitrage pricing theory argument we obtain the prices of the derivative. Which essentially leads to the Black-Scholes if the underlying asset price is the GM, GBM then it will lead to the BSM formula with a terminal condition appropriate to the derivative, boundary conditions of course you need to determine if you want to solve through PDE you will have another method of solving soon, so that, you will not worry about it here.

Now, for this risk neutral measure now recall again the discrete time discussions that we had, that we had a two measures P and $\tilde{P}$ and they are connected through the Radon-Nikodym Derivative and when this stochastic calculus in the beginning when we discussed also like the change of measure idea. Recall the change of measure idea, how one would do a change of measure.

So, what we had? So, we had, recall change of measure idea. So, what we have? We have a probability space where P is the probability measure and random variable Z which is greater than 0 with probability 1 and $\mathsf{E}(Z) = 1$. Then we define

$$\tilde{P} = \int_A Z(\omega) dP(\omega)$$

for all A in $\mathcal{F}$, then we said that P and $\tilde{P}$ are equivalent.

And we also had the expectations for any random variable. For random variable x you will also have $\tilde{\mathsf{E}}(X) = \mathsf{E}(XZ)$ and $\mathsf{E}(X) = \tilde{\mathsf{E}}(X/Z)$ Since we already assume Z to be strictly greater than 0, so this too straightaway holds. And where what is our Z? So, this $Z = \frac{d\tilde{P}}{dP}$, which is what the Radon-Nikodym Derivative.

So, this is somehow a ratio of two probabilities, which is what this Radon-Nikodym Derivative and Radon-Nikodym Derivative guaranteed that, there will be if we have equivalent probability measure then there is always an Z with the required properties and which you can use that Z to define a new probability measure $\tilde{P}$ this is important.

So, this is what we said and we showed that in the earlier case ($\tilde{P}$ is not) we did not show actually so we just take it the result, $\tilde{P}$ is actually a probability measure and they are equivalent, equivalent means what? Again you recall that both agree on which sets have probability 0 and which sets have probability 1, what they disagree on it intermediate cases where P may assign some 0.5, $\tilde{P}$ may assign 0.7 for a particular set, that is what you recall for the equivalent definition so that is what we have seen.

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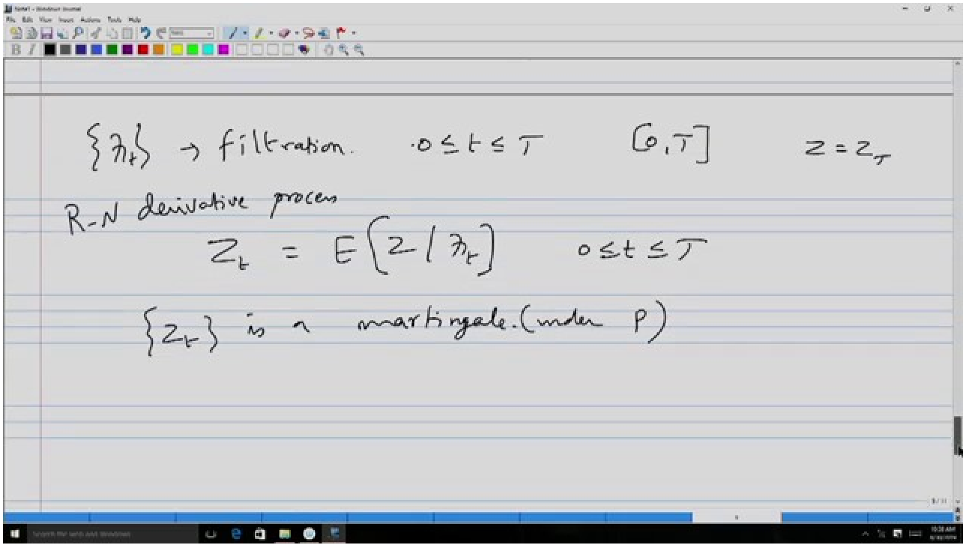

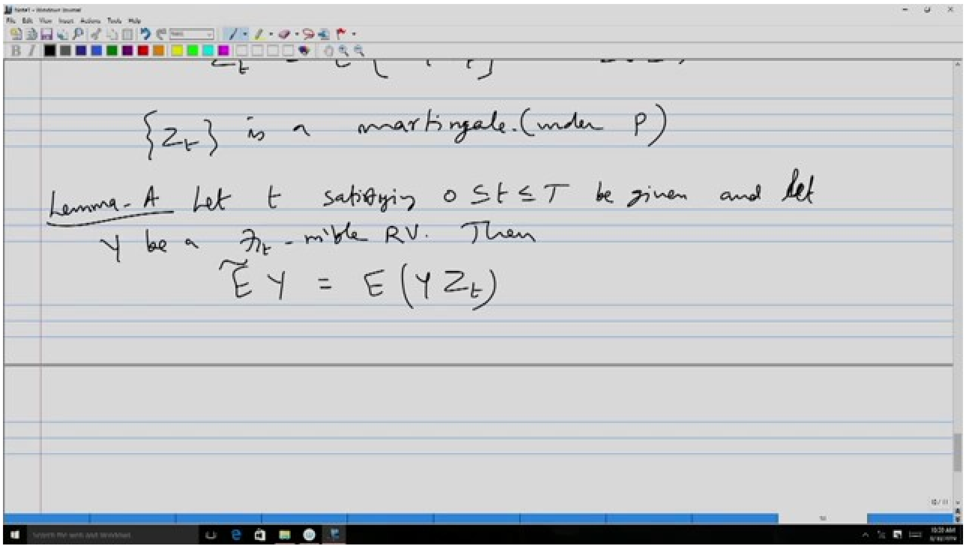

Now, what we do? We have to do a similar change of measure idea for a whole process, that also we have seen the RN derivative process. So, again we have probability space. So, assume that there is a filtration. So, let $\{\mathcal{F}_t\}$ is a filtration with $0 \le t \le T$. So, my Z would be in that particular case would be Z_T. Then we can define the R-N, Radon-Nikodym Derivative process as $Z_t = \mathsf{E}[Z|\mathcal{F}_t]$.

So, this is first thing that you can observe, that this Z_t is a martingale. Which is what by construction it take any random variable, take any filtration and then you take conditional expectation of that martingale under (now there are two measures will be playing, so we will need to look at under) which measure this is a martingale. So, Z_t is a martingale. Now, recall the two results that we have written down for connecting the expectation of E and E tilde in the discrete time case.

So, the continuous time analogs of which we call as lemma A and lemma B. what is lemma A and B? So, it is what this is. So, let $0 \le t \le T$ and T be given or be fixed and let Y be a $\mathcal{F}_t$ measurable random variable then what we have $\tilde{\mathsf{E}}(Y) = E(YZ_t)$ So, this is the first part which is again, is not so difficult to prove.

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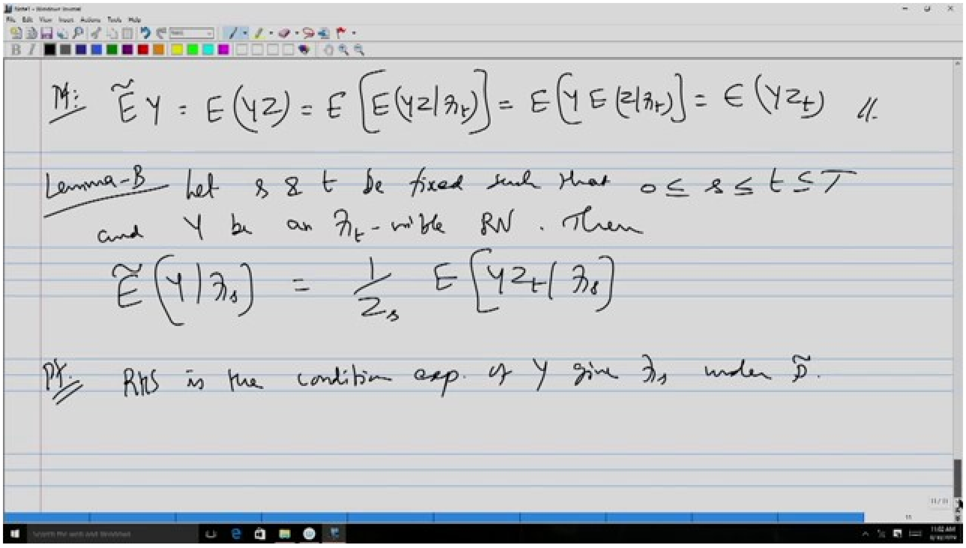

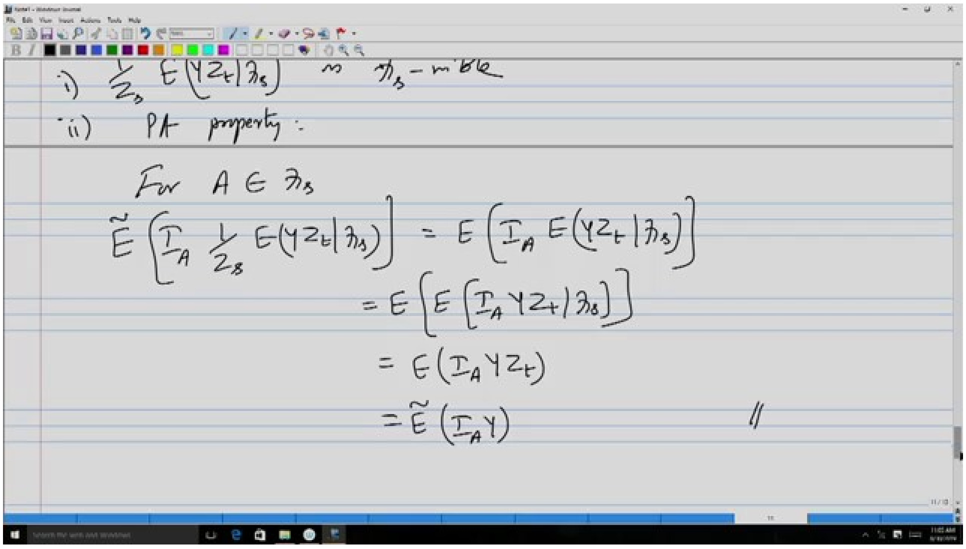

So, what we have is, the proof is simple. It is given above.

But now Y is given to be $\mathcal{F}_t$ measurable so you can take this Y out and then what you would get is expectation of Z given $\mathcal{F}_t$, which is $\mathsf{E}(YZ_t)$ and hence the proof.

So, this is lemma A part A. Part B is we call that as lemma B, which is let s and t be fixed such that your 0 less than or equal to s less than or equal to t less than or equal to T and Y be $\mathcal{F}_t$ measurable random variable.

Then, what we have is the following. Now, proof we have to show that RHS is actually; the RHS is the conditional expectation of Y given $\mathcal{F}_s$ under $\tilde{P}$. that is what we have to show. So, now what are the properties that, so this RHS means that some quantity is given you want to show that, that is the conditional expectation of some random variable given $\mathcal{F}$ sigma field, then what you have to show? You have to show the two properties of the definition, which is it must be measurable with respect to that sigma field and it must satisfy the partial average property.

So, $\frac{1}{Z_s}\mathsf{E}[YZ_t|\mathcal{F}_s]$, there is nothing that here so complex , because one is conditional expectation of something given $\mathcal{F}_s$. so that is $\mathcal{F}_s$ measurable and Z is $\mathcal{F}_s$ measurable. So, this is $\mathcal{F}_s$ measurable this is property 1. The property 2 partial averaging property we have to verify. We have to verify the partial averaging property that the right hand sides satisfies actually the partial averaging property.

So, what we have to pick? We have to pick for A in $\mathcal{F}_s$ so what we have to pick is right side that is $\tilde{\mathsf{E}}$ of indicator function of A, $\frac{1}{Z_s}\mathsf{E}[YZ_t|\mathcal{F}_s]$. this quantity we must ultimately equal to we have to show that it is the equal to $\tilde{\mathsf{E}}[I_A Y]$ is what we have to show. Let us see how we can proceed.

Now, this if I use the previous result or lemma A essentially, I can write this as E of A of indicator function of A $\mathsf{E}[YZ_t|\mathcal{F}_s]$. this is from the previous one. This must be then equal to expectation of this indicator function of A, A is in $\mathcal{F}_s$ so I can take this inside. So, $\mathsf{E}[I_A Y Z_t|\mathcal{F}_s]$, expectation of this, is nothing but now my $\mathsf{E}[I_A Y Z_t]$.

So, this is nothing but $\tilde{\mathsf{E}}[I_A Y]$ and Y that is what we wanted to show. You are using the properties of conditional expectation and part A lemma to show that this is the case. So, these two are the two lemmas, which has continuous analogs to the discrete cases that we have done here. Now, this gives, so we using these two now what we want to show is basically an important result, which is known as the Girsanov theorem, which is what will tell us how we change the measure from one to the other. Of course, then that we will see in the next lecture, thank you.