Mathematical Finance
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Module 1: Introduction to Financial Markets and Instruments
Lecture 3: Introduction to Options

Hello viewers, welcome to Lecture 3 of this course on Mathematical Finance. You would recall that so far we have looked at the definition of financial derivatives and also had an overview of financial markets and in particular, we have looked at stocks and bonds followed by discussion on forwards, futures and swaps.

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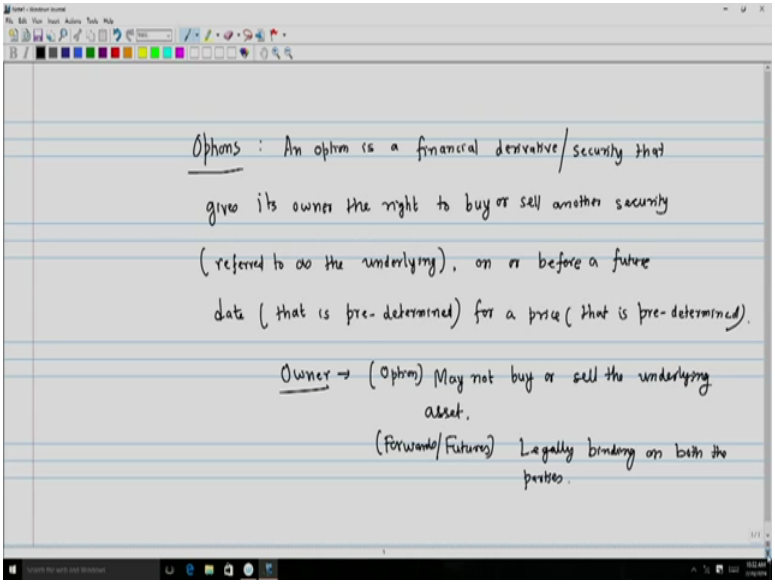

In today's Lecture, we will discuss about one important class of financial derivatives, namely, options. Let us begin with what is the definition of an option. An option is a financial derivative or security that gives its owner the right to buy or sell another security and we will refer to this security as the underlying and this right is bestowed to be exercised on or before a future date and this date is pre-determined, that means decided ahead of time, for a price that is pre-determined. So, as the definition suggests, it basically is a contract between two parties where one of the parties agrees to sell some underlying asset to the other party and the price of the asset that is to be paid at a future time point is decided at the current time period or is pre-determined, also this future date on which this transaction will actually take place is

also pre-determined. This might give you the impression that this is very similar to forwards and futures that we have already discussed.

However, one important distinction between forwards, futures and options is that a forwards or futures is a legally binding contract on both the parties where the transaction actually has to take place, irrespective of the behaviour of the underlying asset which we typically take a stock in the context of our discussion. However, in case of options this legally binding aspect is not applicable to both the parties, but rather it gives one of the parties the right but not the obligation and the other party has to fulfil that particular obligation. So, the situation is slightly different in case of options in that one of the parties has to take a position of certain weakness, if I must say that wherein they are obligated and consequently the other party enjoys a certain amount of leverage. So what is the distinction between this two? The distinction between them, as I pointed out is that the owner in case of an option may not buy or sell the underlying asset which again we will consider in the context of a discussion, to be a stock, but you can of course include a whole class of different kinds of assets like you know bonds, swaps and so on. Now in case of forwards, futures it is legally binding on both the parties, so what do I mean by this?

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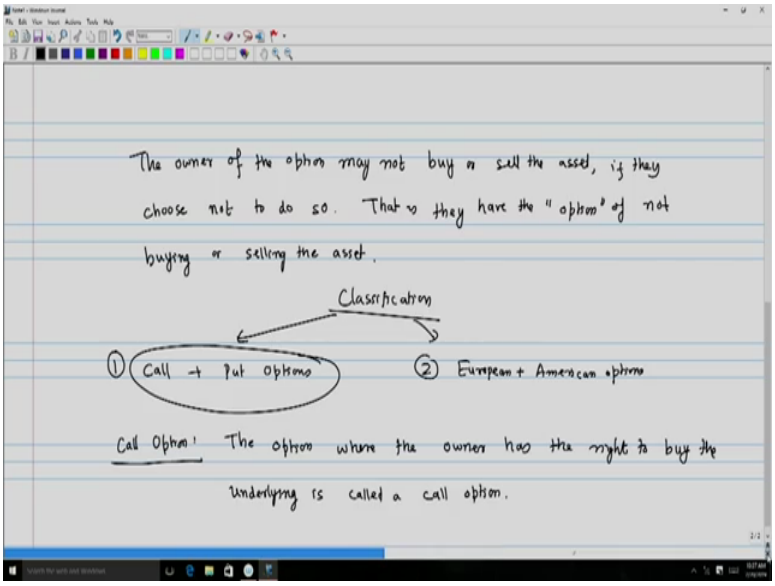

So let us actually look a little deeper into what we mean by actually the right to buy or sell, so the right to buy or sell here means that the owner of the option may not buy or sell the asset if they choose not to do so. So that is they have the option and in this context option means the choice of not buying or selling the asset. So let us now start to look at some of the

classifications of options so that this particular statement regarding right to buy or sell the underlying asset can be understood with greater clarity.

The most basic classification of options are the following. In one classification you have what are known as call and put options, this is, one form of classification that means every option is going to be either call or a put option and the second one is the classification is European and American options. Let us first begin with the first set of classifications, namely, call and put options. I will first discuss call options. What is the call option? A call option is an option, so this option where the owner has the right to buy the underlying stock in our case, is called a call option. Suppose there are two parties A and B and the owner of the call option is A. This means that here the party A has the right to buy the underlying asset from party B, so that means if the underlying is a stock, then party A under this option if they want, they can buy the asset or the stock from party B if they want to, but they need not buy the stock from party B, if they do not want to do so.

On the other hand, for party B, if party A demands that they sell the stock, then party B is obligated to sell the stock to party A at the predetermined price. However, if party A is not interested in purchasing the stock from party B, there is no way that party B can compel party A to buy the stock. This means that in this arrangement, party A is in an advantageous position over party B and party B is at a disadvantages position as compared to party A.

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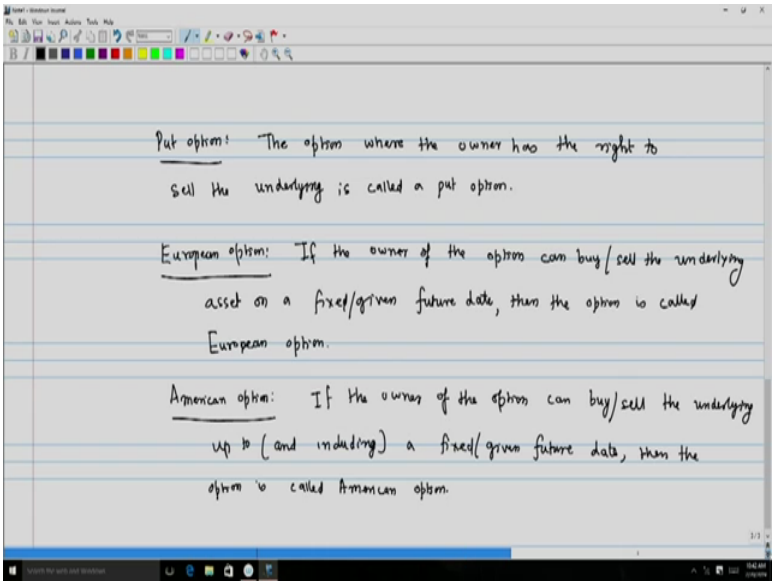

Next, we look at what is a put option. This is the kind of option where the owner has the right to sell the underlying is called a put option. Let us discuss this in the context of the previous

example, that again there are party A and B, but this time party A is the owner of the put option. This means that party A has the stock, so if party A wants, they can sell the stock to party B and if party A does not want, they need not sell the stock to party B. On the other hand, if party B is asked by party A to buy the stock, then party B is obligated to buy it but if party A does not want to sell the stock to party B, then there is no way party B can compel them to actually sell them the stock. Again, here party A is in an advantageous position over party B. Now, in case of call options, this advantage is in terms of the right that party A, if they want, they can buy the underlying asset or they can call the underlying asset and that is why it is called a call option and in case of put option, party A the leverage that they enjoy is that they can sell the asset or the stock to party B if they want to or sell or they can put the asset to party B and that is why this is known as a put option.

Next, we will look at the second classification that we had said. So you would recall that we looked at the classifications as put and call option, and European and American. Let us now define what is the European option and what is an American option. So we begin with European option. What is the European option? If the owner of the option can buy or sell the underlying asset on a fixed or given future date, then the option is called European option. In contrast American option is a kind of option where you can basically make the transaction or exercise your right at any time on or before a pre-assigned date, in contrast to European option when the transaction can only happen at one particular pre-determined future time point or a future date. So, in the American option, we can write this as that if the owner of the option can buy or sell the underlying up to and including a fixed given future date, then the option is American option.

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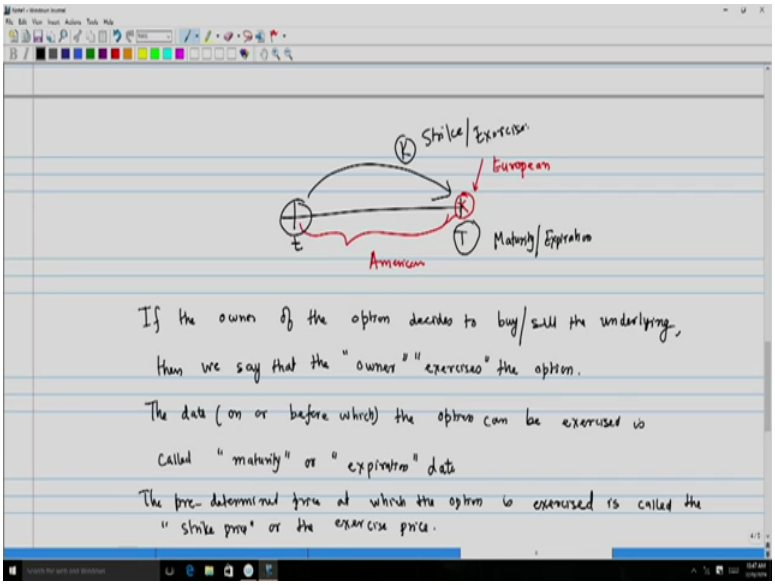

Let me explain this in a little more detail. So suppose today is a time t and there is a future time T. So, in case of European option, you can make the transaction at any time or rather at the fixed time T that you decide today so at time T, you decide that this is a European option so that means that the future time T that you have decided on as the date of transaction is the date on which the actual purchase and sale of the underlying asset can takes place.

However, in case of American option this exercise can take place any time from the time t up to time T. Next, let us talk about this time, future time point T that you are talking about and also the pre-determined price that we have decided at time t and introduce a couple of terminologies that are in very common usage when we are discussing options.

Accordingly, if the owner of the option decides to buy or sell the underlying that means the party with the position of leverage decides that they want to purchase the asset or the stock in case of call option, or they decide that they are going to sell the stock in case of put option, in that case, we can say that the owner exercises the option or basically the owner has made use of the right or the leverage position that he or she has, by virtue of being the owner of the option.

Next, the date on or before which the option can be exercised is called maturity or expiration date. In this case, this T that we have here, this is what is known as the maturity or expiration. Next, we take note of the fact that there are two key aspects to option that date on or before which we can exercise and which I have already said is the expiration date or the maturity

and the second key aspect or the key parameter in case of an option is the price which has been predetermined at which the underlying stock is going to be purchased or sold.

So, this brings us to another terminology or the second key terminology in case of options, namely, this pre-determined price. We decide that the pre-determined price at which the option is exercised that means the purchase or sale is taking place is called the strike price or the exercise price. This means that sitting at time t, we decide that we will buy the asset for some price K and this is what is known as strike or exercise.

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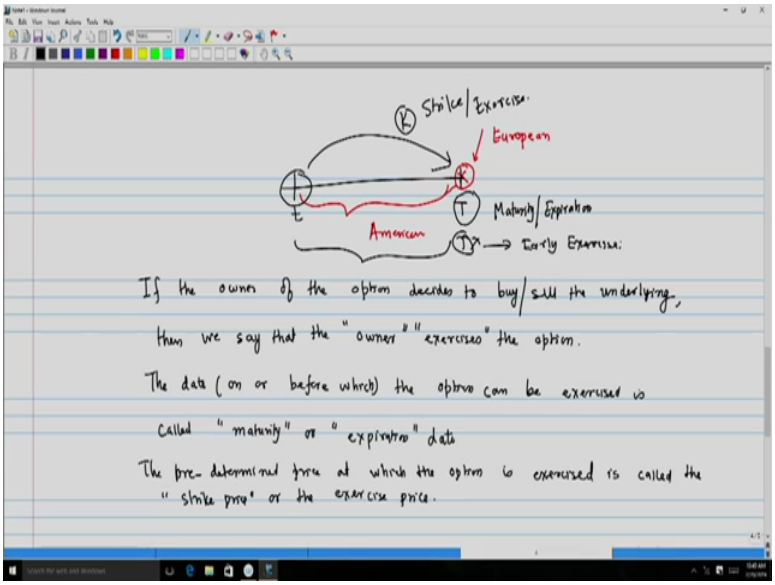

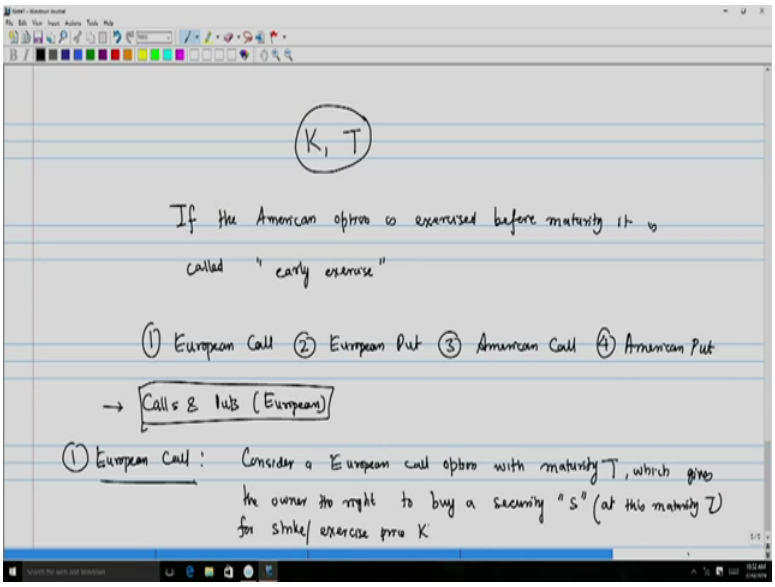

These are the two key parameters, we must remember whenever we are talking about options, namely, K agreed price at time T on or before which the exercise can actually take place, it has to be exactly on that day in case of European option, but it can happen anytime between t

and T including T in case of American options. Next, I just want to make an observation regarding American option and that is if the American option is exercised before maturity it is called early exercise. This means that if the purchase or sale is taking place at any time in this period, but not T, then this is what is known as the early exercise. Early exercise means that you are basically doing the transaction not at the final time, but sometime prior to that. Next, now we have decided to look at these classifications, namely, the European call and put depending on whether it is right to buy or right to sell and another classification that we can look at or we have looked at is European option and American option which is dependent on the time at which the exercise can actually take place. This basically opens up four different combinations of options, namely, European call, European put, American call and American put. As a result of this, what do we get? We get European call, secondly, we get European put, thirdly, we get American call and fourthly, we get American put. In case of European call, the owner of the option has the right to buy the stock only at time T for (a fixed time) a fixed price of K.

In case of a European put option, the owner of the option has the right to sell the stock only at time T for the price K. In case of an American call option, the owner has the right to sell the stock for price K at any time between t and T included. And in case of American put option, the owner of the option can sell the asset for the price K at any time between t and T included.

Let us now discuss this a little more in detail. Let me first start off with call and puts European type. So, we will basically now look at European call and put options. First of all, we will start off with European call. For European call, we consider a European call option with maturity T as before which gives the owner the right to buy a security S at this maturity T for strike or exercise price K.

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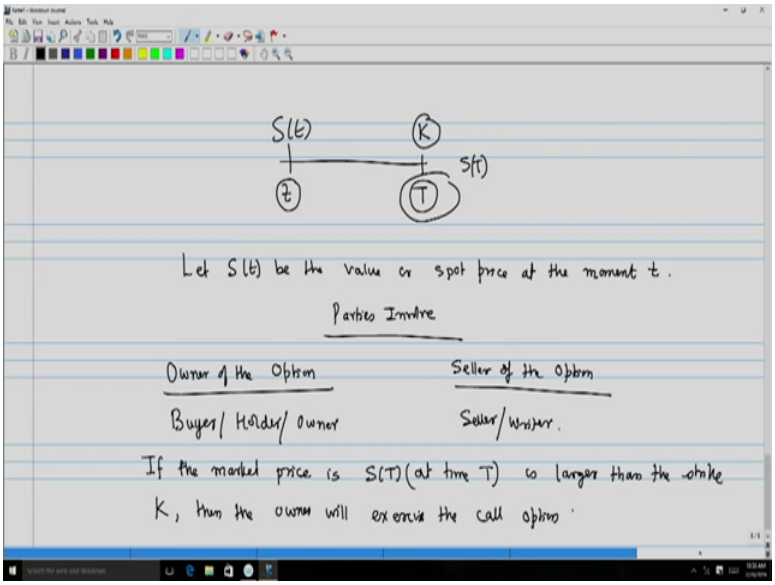

Recall that we had chosen this timeline that are t, this is T. At time t, we basically write the option or we basically get into the option and T is the exercise time or expiration and K is the exercise price. Then let me introduce the notation. Let S(t) be the value or spot price at the moment t, which is the current moment and that is why it is called the spot price, because this is the price at which I can buy the stock immediately on the spot, which is why this is referred to as the spot price.

Denote S(t) as basically the value or the spot price at this moment t and in general, S(t) will be used to denote the price of the stock at any generic time t. Then what are the parties involved? Let us look at the position of the parties involved here. We have the owner of the option and the other party is who was known as the seller of the option. So, in the parlance of option pricing, the owner of the option is the person who has the leverage and this leverage could either be the right to buy or the right to sell and correspondingly the seller of the option is the party which is under obligation and the obligation is the obligation to sell or the obligation to buy, respectively.

So, owner of the option sometimes are known as the buyer or holder or as usual they might be called simply the owner and the seller of the option. Typically is referred to a seller or the writer. Please understand that in case of a European call option, there are two different ownerships, one is the owner of the option and the other is the owner of the underlying asset. This means that if the party A is the owner of the option and party B is the seller of the option, but because it is a call option, party A has the right to buy the stock from party B at

the price K, so that means the owner of the stock is actually party B and the buyer of the stock is party A if the exercise actually takes place.

The owner of the option is in this case the buyer of the underlying asset, namely, party A and the seller of the option, namely, party B is actually the owner of the underlying stock. Next, to put it in some mathematical formulation, now I am talking about the market price. Suppose that at time t, the spot price was S(t) and S(T) is the price of the stock at time T (expiration), so if the market price is S(T) at time T is larger than the strike K, then the owner will exercise the call option.

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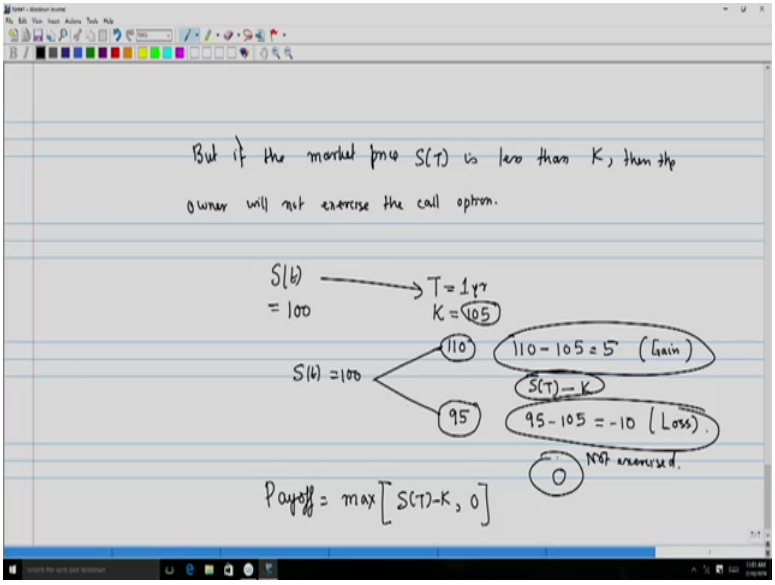

But if the market price S(T) is less than K, then the owner will not exercise the call option. Let us illustrate this through an example. Suppose S(t), that is, the spot price today is 100 and then I agree on an option with expiration T (equal to one year) and the strike price or the exercise price is 105. Now suppose that this stock price today which is 100, goes up to 110 or another possibility, it goes down to 95. So let us examine what is going to happen in case of the stock price going up to 110. Then party A which is the owner of the option, they will consider two positions, they see that they can purchase the stock for 105 which is the strike price and they can immediately sell the stock at the current market price of 110, so that means they are able to make a profit of 110 – 105 = 5.

Obviously, this is a beneficial situation for them, so they are obviously going to exercise the option, pay 105 and immediately sell it in the market at the current prevailing market price of 110 thereby making an immediate profit of 5. However, in the case when the price goes down

to 95, what are they going to do? They are going to do, they see that okay if I purchase by exercising the option and I purchase the stock for 105, then I will be able to sell it in the market only at the current prevailing price of 95. This means that I will purchase this for an amount of 105, but I will receive only an amount of 95. Now, 95 – 105 = – 10, so this means that in the first scenario, there is a gain and in the second scenario, there is a loss. Obviously, whenever there is a loss, then the option will not be exercised, because there is no reason for party A to purchase the stock for 105 when can get it in the market for simply 95 which is a lower price.

However, in the other case, they know that they can purchase it for 105, sell it for 110 and then make a profit of 5. So this means that here either they will get an amount of 5, this is, $S(T) - K$ and in this case, second case, there will be no exercise, that is, going to happen so that means here the payoff or the gain is going to be equal to zero, because the exercise actually has not taken place and no transaction takes place.

So, this brings us to the concept of what is known as a payoff. Payoff is basically the gain or loss to the owner of the option. So, in this case, since they have the right not to exercise in the event of a loss, so it is basically either the gain that they are going to make or simply zero, because they have decided not to exercise the option.

This means that we have payoff which is going to be the $\max\{S(T) - K, 0\}$. So, it is going to be the larger of this. So, that means if your $S(T) \geq K$, then $S(T) - K$ is going to be your gain and if $S(T) < K$, that means you do not exercise and there is no loss or no gain.

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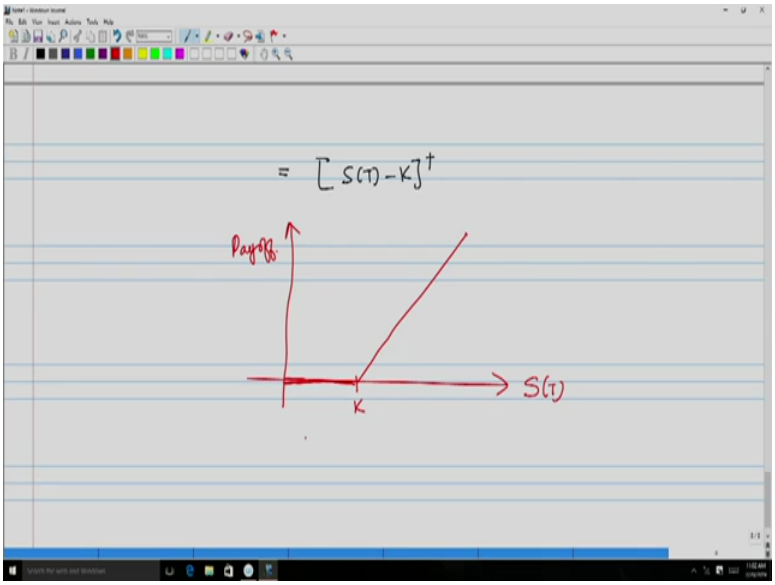

So, sometimes this is also written as (S(T) – K)+, so this is max{S(T) – K, 0}, sometimes is put in this particular notation. Graphically, let us see how this looks like. So, if I take on my x-axis the price of the stock at time T and on the y-axis, I take the payoff and this is the strike price K, then the payoff which is max{S(T) – K, 0} will be given by this graph.

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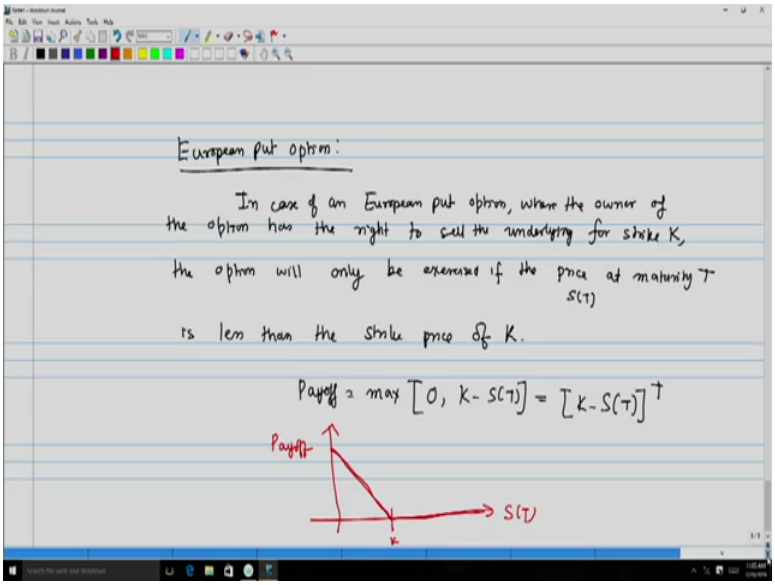

Next, we move on to the European put option. Remember in case of European put option, the party A who is the owner of the option is also the owner of the underlying asset and they have the right to sell the underlying asset. Let us see what is the analogous situation in case of put option extending what you have already seen just now in case of the European call option.

Therefore, in case of an European put option where the owner of the option has the right to sell the underlying for strike K, the option will only be exercised if the price at price S(T) at maturity T is less than the strike price of K. So, in this case, the payoff is going to be max{0, K – S(T)}, which is (K – S(T))+. Graphically, this will look something like this, this is your S(T) and this is your payoff, so then if K is your strike, then this is basically going to be the graph of max{0, K – S(T)}.

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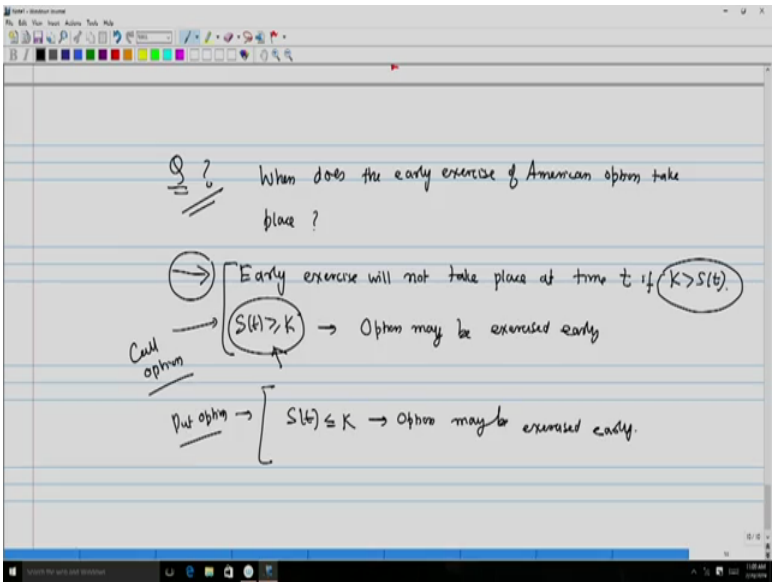

Now that we are done with the European call and put option, let us just discuss briefly about the American call and put option. In case of American call and put options or a American options, in general, the following is the question that you want to answer and this is a key question, because in case of American option, you have the right to exercise the option anytime between t and T included. Then there is an additional question as to what would be a good time to exercise at every point and every time point, you have to decide whether you want to exercise or not.

So, this is the key question that when does the early exercise of American option take place? To answer this question, let us just see the motivation. What is the motivation that one should exercise that American option? Let us just view this in the context of a call option, one will exercise the option only when there is a gain. That means only if the owner of the option sees that they can purchase the stock for a price of K and sell it in the market for a higher price of S(T), then only there is an incentive for the owner of the stock to actually exercise the option in order to make the profit. This means that early exercise will not take place at time t if K > S(t). So, this means that basically, if we find that the current stock price is S(t) and the excess

price is K, then there is no reason for you to purchase the stock for K and sell it at a lower price. So, this is the case where there obviously any rational owner of the option will not exercise the option.

However, let us look at the case when $S(t) \geq K$, what happens? In this case, the option may be exercised early and please remember that this statement that I am making this in case of call option. In case of put option, if your $S(t) \leq K$, then option may be exercised early. This means that if your $K > S(t)$ and it is an American call option, you definitely know that the exercise will not take place.

However, if it is a situation where $S(t) \geq K$, then the owner has a reason to consider exercise, but may decide not to exercise and hold on to the option for a future time point.

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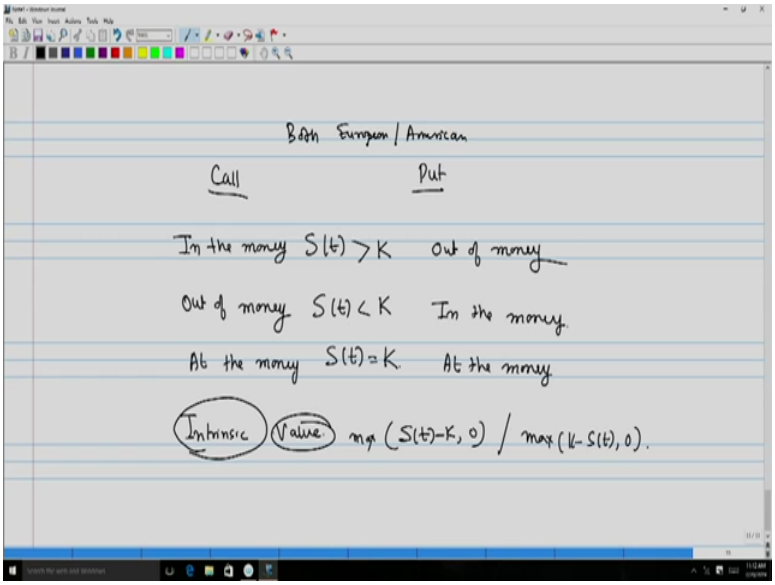

Next, we will introduce the notion of in the money, at the money and out of money. So, we will discuss this in case of call and put option and this is true for both European and American. So, there are three possibilities if your $S(t) > K$, you have $S(t) < K$ and your $S(t) = K$. So, when $S(t) > K$, in case of call option, it is called in the money and in case of put option, this is called out of money. When $S(t) < K$, then it is out of money in case of call option and in case of put option this will be in the money and when $S(t) = K$, this is called at the money for both the cases.

What is this mean in the money? In the money means that in case of a call option, if your $S(t) > K$, then you are obviously going to have an incentive to exercise, so you are in the money.

However, if your $S(t) > K$, then as an option holder or a holder of a put option, there is no reason for you to actually go ahead and exercise and that is why it says that you are out of money.

And likewise when $S(t) < K$, we say that you are out of money in case of a call option and in the money in case of a put option and when both the prices coincide that means the price of the stock at that given point of time t coincides with the strike price K, then we say you are basically at the money. So, this brings us to the notion of intrinsic value. What is the intrinsic value? Intrinsic value is basically $\max\{S(t) - K, 0\}$, or $\max\{K - S(t), 0\}$.

Now, we let us come to a very important question as we have discussed in detail that the price of an option or rather the position of an option offers leverage to one party and puts an obligatory situation in case of another party. So, obviously the party which is the holder of the option enjoys a certain amount of privilege, so since this privilege ends up being that the disadvantage for the other party, the question naturally arises what is the incentive for the other party in the context of a discussion, party B to get into an obligatory position.

And the answer to this is that party B will get into this obligatory position because they are provided an additional incentive in terms of an upfront fee or premium to compensate for the weakened position they are getting themselves into.

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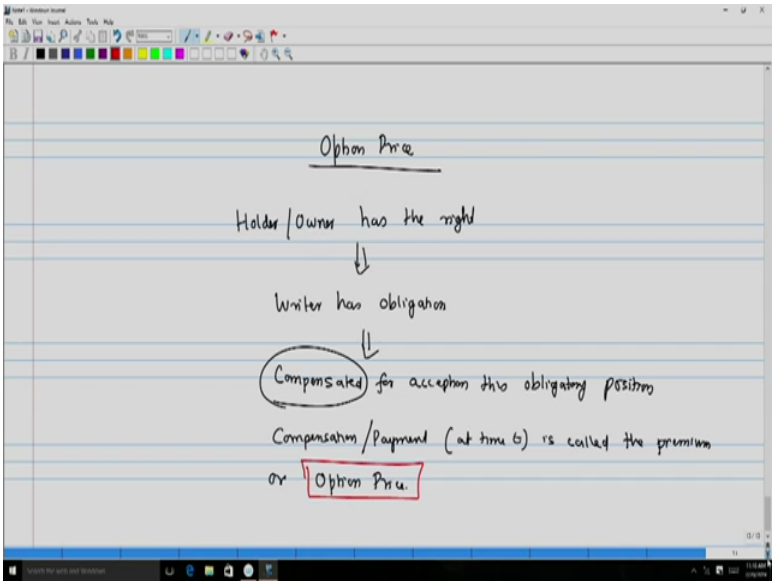

This brings us to a very important question of option price. Remember that in case of the option price the holder or the owner has the right and which means that the writer has obligation and because the writer is in an obligatory position they have to be compensated for accepting this obligatory position.

In this case, because party B have decided to take a position where they are at a disadvantage, so they expect that party A who are the owner of the option and have a position of advantage will provide them with certain amount of compensation right at the beginning at time t.

And this particular compensation that you are talking about this compensation or the payment and this will be made at time t is called the premium or option price. So, in the parlance of mathematical finance option price is the term that is actually more commonly used.

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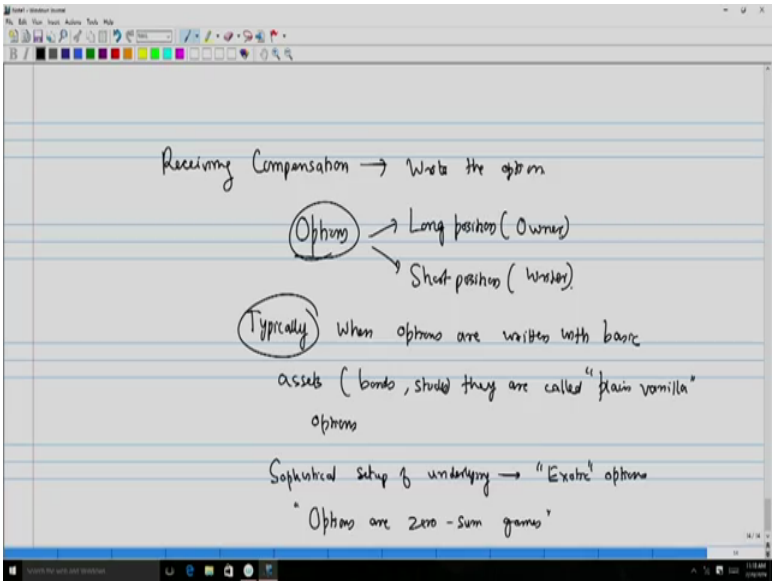

And as a result of the compensation, after receiving this compensation, the party B basically write the option. Remember that when you are talking about forwards and futures and stocks and bonds we have talked about long and short positions, likewise in case of options also. So, for options the long position is the one that is held by the owner and the short position is the position held by the writer and typically when options are written with basic assets such as bonds, stocks, they are called "plain vanilla" option, but when you have more sophisticated setup of underlying then they are called "exotic" options. So, one final observation is that again options are zero-sum games, that means whatever is the gain of one party is going to be the loss of the other party.

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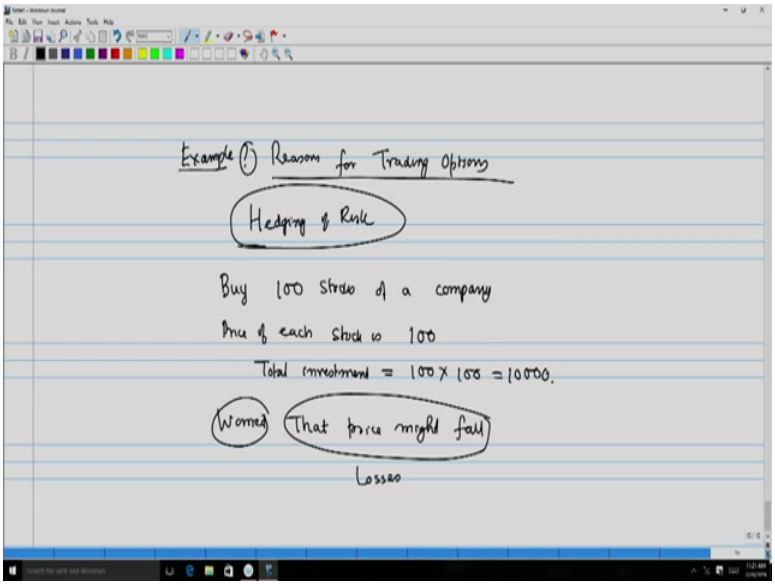

Let us conclude this discussion with a one example and that example is designed with the idea of addressing the question as to the reasons for trading options. So, options as you have seen gives one party advantage and put disadvantage on another party and of course, then they get compensated for taking up this position of disadvantage. However, the question is that what purpose does trading of options actually serve? So, whenever you are actually introducing a financial derivative in the market, there has to be underlying example or underlying motivation as to why such a financial derivatives needs to exist in the first place.

In particular, we will look at an example which will give you an insight into why options are traded in the first place. In this example, we will talk about something which is known as the hedging of risk, so hedging of risk basically means, as the name hedging suggests, it is trying to protect yourself against future unpredictable market movements for the position that you are actually holding.

In particular, let us consider an example suppose that we buy 100 stocks of our company, price of each stock is 100 and total investment in this case will be $100 \times 100 = 10000$, but we are worried that price might fall. Suppose the price falls from 100 to 60, that means our investment of 10000, then gets reduced to 6000 giving us a loss of 4000. So, in order to take care of this genuine concern that the prices of the stock might fall and that you might end up incurring losses.

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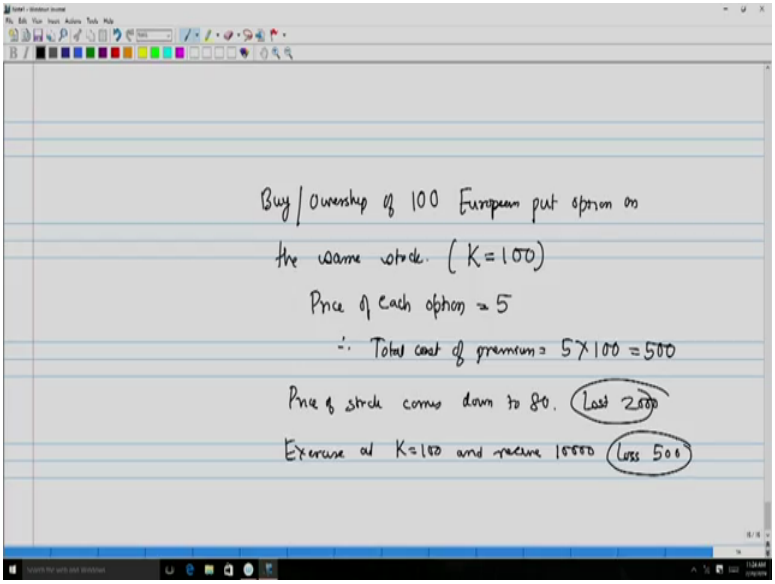

What we do is the following is that we buy or take ownership of 100 European put option on the same stock with say strike K = 100. So then suppose the price of each option is 5 that

means this is the premium I have to pay to another party to take the obligation and the obligation is that if I want, I will sell them the stock for an amount of 100 at some expiration at a future time, maybe say 1 year.

So, since the price of each option is 5, therefore total cost of premium is going to be $5 \times 100 = 500$. Now, let us say what happens in case the price of the underlying stock goes above 100, then there is no problem, we will sell it at a higher price, say, it is 105 and on each share we make a profit of 5, so overall profit is 500. However, why did we get into the put option? It was to take care of any possible losses.

Now, suppose that a loss has happened. For example, suppose that the price of stock comes down to 80, if I had not taken the put option, then what would have happened is that I have already purchased the stocks for 10000 and I have to sell it for 8000 resulting in a loss of 2000. However, since I have purchased the put option, now that I observed it in the market the price is 80. I basically then decide that okay I am going to exercise at K = 100 and receive 10000.

This means that in absence of me getting into the put option, what I would have done is that I would have lost 2000. However, because I got into the put option, I am able to recover the entire investment of 10000 and my only loss is simply the premium of 500 that I have paid. So, that means I have been able to cut my losses down from 2000 to 500 in the event of the price moving in the downward direction from 100 to 80.

So, this brings us to a conclusion of the discussion on options, so what we have done so far is that we have looked at different kinds of markets. In particular, we looked at exchange-traded markets and we looked at over-the-counter markets and then we looked at different kinds of securities, the basic securities that we discussed are bonds which are risk-free and stocks which are risky and then we moved on and discussed about forwards, futures, swaps and most importantly options which are the three most basic kinds of financial derivatives. Thank you for watching.