Mathematical Finance
Lecture 29: Ito Formula Ito processes

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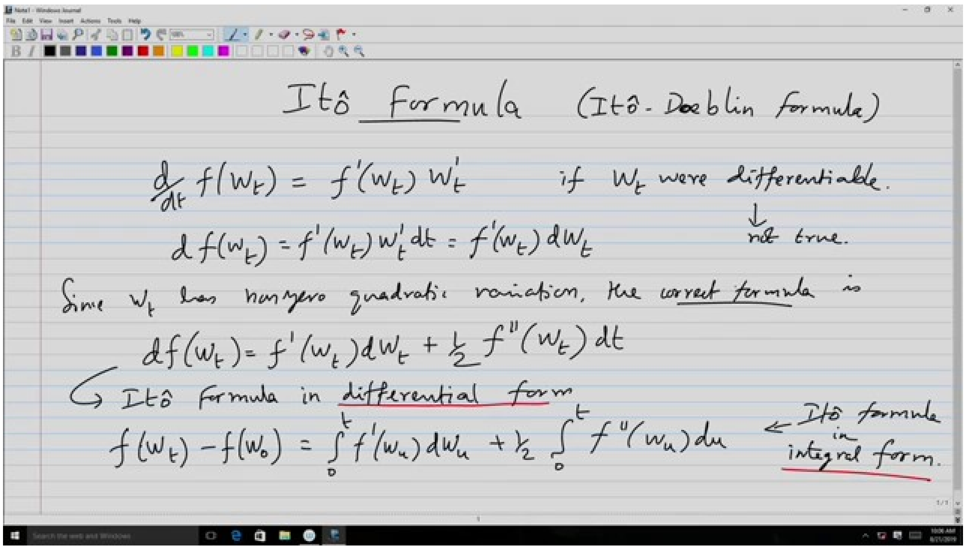
Hello, everyone after having gone through the Ito integrals. Now, the next topic that we will be going to look at that is Ito's formula. (Refer Slide Time: 00:39)



Also called as Ito Doblin formula, we will simply call this as the Ito formula. So, basically what you want? We want a rule to differentiate some functions of the form $f(w_t)$, where f is some differentiable function and w_t is brownian motion. So, basically the question is if w_t was also differentiable. Then the chain rule of ordinary calculus would give something like

$$df(w_t) = f'(w_t)w_t'dt = f'(w_t)dw_t.$$

This is was what you would have, if w_t were differentiable and the ordinary calculus results, if you use basically the chain rule. Then what you would have, is this differential of this $f(w_t)$ would be given by this quantity. But, this is not true and because since W_t has non zero quadratic variation. So, the correct formula is essentially

$$df(w_t) = f'(w_t)dw_t + (1/2)f''(w_t)dt.$$

So, this is what called as Ito's formula in differential form. Now, this you can integrate it over a certain region 0 to t. And, you can write this as

$$f(w_t) - f(w_0) = \int_0^t f'(w_u)dw_u + (1/2)\int_0^t f''(w_u)du.$$

So, this is the Ito formula in integral form. So, we need to be comfortable with both and what we mean by each other. Just like in the case of Ito integral we just wrote something as in the differential form and the integral form.

Even this formula has a differential form. So, this is differential form this is the integral form that we have here. And, this differential form has an imprecise meaning because, this is just written for in formal way; as an imprecise meaning. But, more intuitive to what happens to $f(w_t)$ then you make a small perturbation in t. So, it is given by the two terms as given in the right hand side. But, the integral form has a precise mathematical meaning, which is basically the difference of the function evaluated at time 0.

And the time $df(w_t)$ is the function w_t is given by the sum of this two integrals one integral is Ito integral the other is ordinary integral. And, since all these integrals have a precise meaning, so this is the precise meaning. But, both are equivalent and for pen paper calculation for all analysis that we might do. We might take the differential form itself and we will do. But, the meaning of the differential form (is given only into) has to be understood in terms of the integral form the differential form again.

As you see is does not have the precise meaning that is the only reason that you have it, so this is what it is. So, what we said that this is the Ito's formula in differential form. So, which has an extra term in the half of which, is actually an ordinary integral and it is added to the function. And, this extra term is due to the none zero quadratic variation. Remember recall when if wt were differentiable the quadratic variation is 0. So, obviously this term will vanish and because this is as a non zero quadratic variation this term would remain. And, that is what the difference between in the ordinary calculus and the stochastic calculus that you have here.

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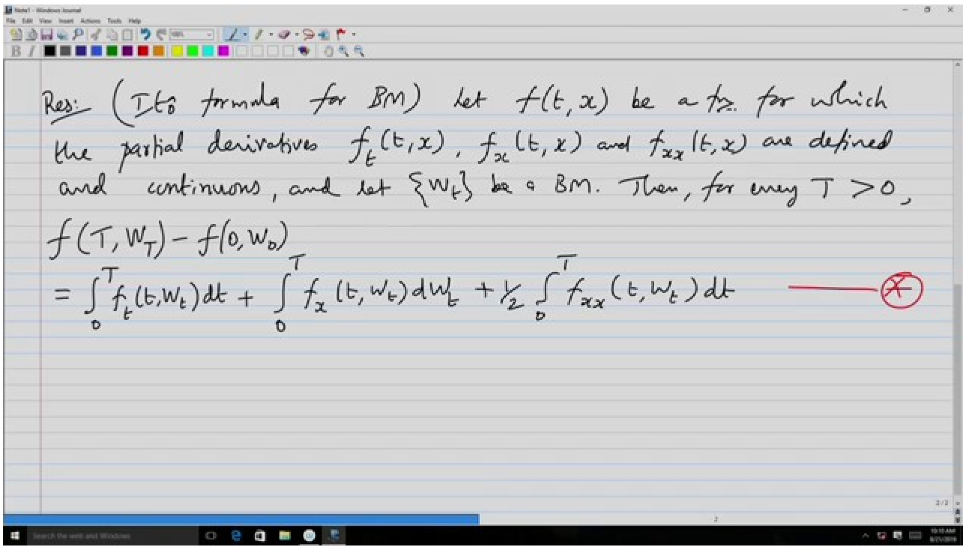

Now, we will see now the Ito formula for a brownian motion scenario. So, what you have a probability space and brownian motion defined under probability space. And, you also have a function. Now, this is slightly general then the first form of the Ito integral that we wrote just now. Here, we are considering as a function of not just x but, this f as a function of both t and x to make it little it bit more general. So, that the previous case becomes a special case of this.

So, be a function for which, what happens the partial derivative say $f(t,x)$ and $f_t(t,x), f_x(t,x)$ they are defined and are continuous. And, let w_t be a brownian motion. Then, what we have for every capital t which is greater than 0. What we can say is the following. This difference is equals

$$f(T,W_T) - f(0,W_0) = \int_0^T f_t(t,w_t)dt + \int_0^T f_x(t,w_t) + (1/2)\int_0^T f_{xx}(t,w_t)$$

So, this is the claim or this is the Ito's formula for a brownian motion function that we have so $f(t,x)$.

Just like that the previous case it is $f(t,x)$ is just of a function of f of x. So, the first ordinary integral in this equation does not appear because ft is not there. So, f_x and f_{xx} which is what we saw f' and $f'(w)$ is what we wrote it. So, you see that this is exactly same as that but, it is slightly more general. Now, the proof of course it is quite involve so what we will do is that we will just give a sketch as to how and how this can be seen true by giving a sketch of the proof. The proof involves little bit of conversable results in probability theory which we will not worry. But, we will just understand like why this has to be true and simple cases we already know.

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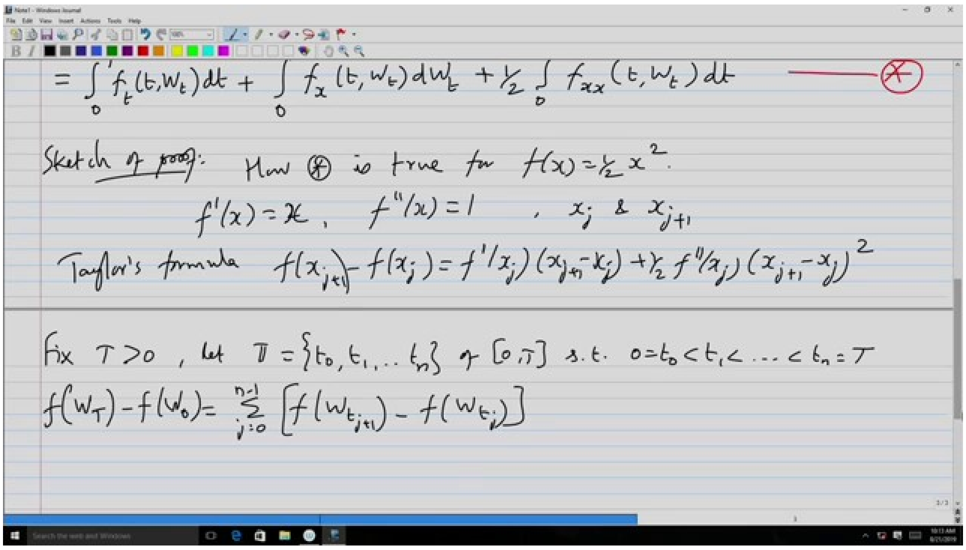

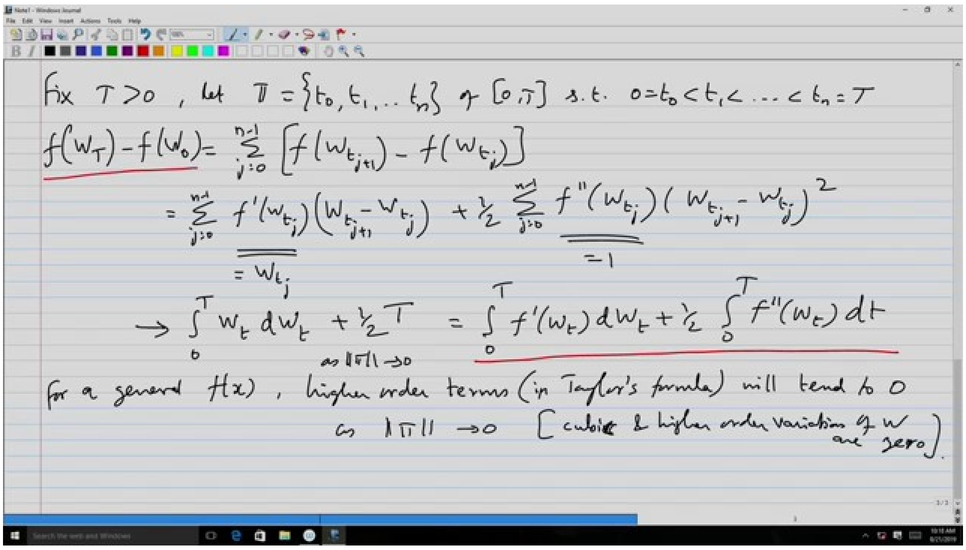

Just look at the sketch. (See why this is) let us see how start rather than why. We will see how star is true for the function $f(x) = (1/2)x^2$. So, in this case my $f' = x$ simply and $f'' = 1$ and you pick x_j and x_{j+1} two real numbers. Then, your Taylor's formula implies the given expression for $f(x_{j+1}) - f(x_j)$.

So, what this is, suppose this function is you take, for which is the first and secondary which is as given here. And, you pick two real number x_j and x_{j+1} and if you write the difference then by Taylor's formula this is given by this. And, since this is a quadratic function the Taylor's formula is exact and there is no reminder term here. Now, you fix t and let $\Pi = \{t_0, t_1, ..., t_n\}$ of $[0, T]$.

Now, what we are interested, we are interested in the $f(W_t) - f(W_0)$. So, this change in the overall $[0, T]$ can be written as the sum of changes of this function over each of this subintervals. And, so and hence what do you have $f(W_t) - f(W_0)$ can be written as the sum of differences. And, now this difference can be written for this function using the Taylor's formula above what we have written.

If you write that here, so what you can see is that, this summation can be written as above. So, if I take the limit on both sides the left side limit as $||\Pi|| \to 0$, the left side does have any impact. But, the right side would converge to right so you have 0 to t, So we get $\int_0^T w_t dw_t + (1/2)T$.

Now, for instead of having this quadratic function for a general function $f(x)$ in the Taylor's formula you would also have higher order terms quadratic cubic terms and higher. But, all those terms, the cubic terms are higher order terms in that Taylor's formula. When, you apply the Taylor's formula to this function f. They all the higher order terms, which is in Taylor's formula will tend to 0. As this is happens conversions here is basically as $||\Pi|| \to 0$.

As $||\Pi|| \to 0$ since, you have seen the cubic variation and the higher order variation. Cubic and higher order variations of W or 0. So, all this higher order terms would tends to 0. So, that will not change our final answer as far as a function of this form is concerned a function of f of x is concern. So, if you have a function of a W_t alone. Than the Ito's formula will be this, is equal to this in the integral form. The red underlined question is what is the Ito and, this you can write it in the differential form plus la. So, this is what happened, if this is the function of x alone so, now here, we are not looking at the complete course. But, we are just like another sketch.

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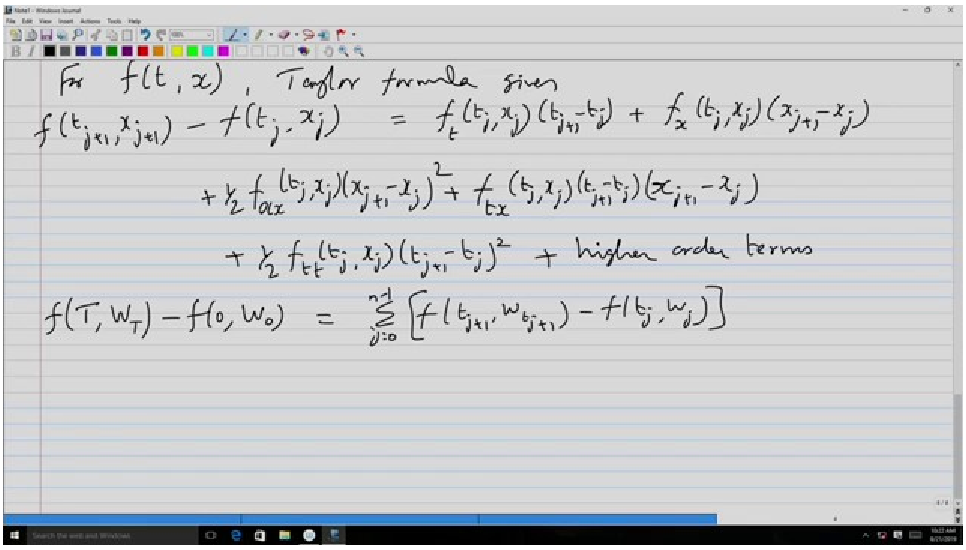

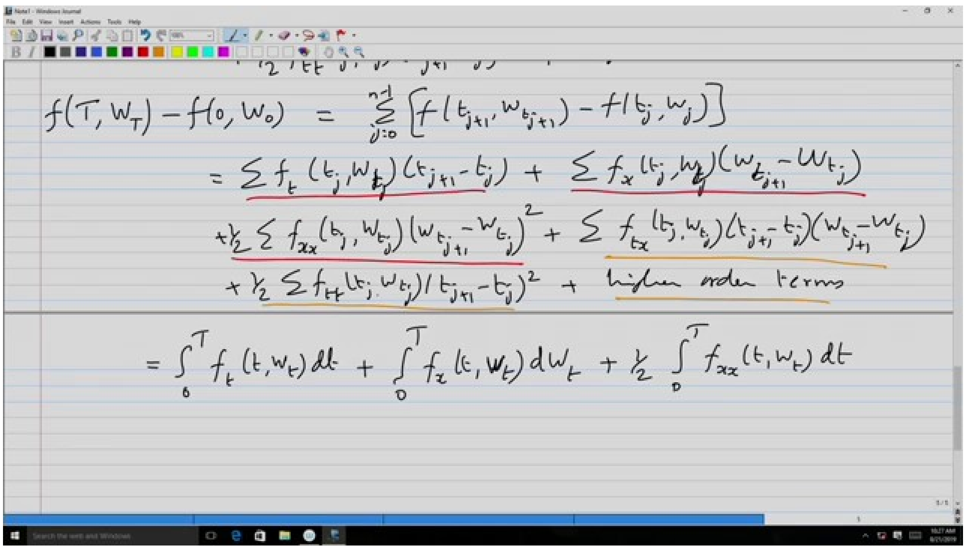

Again we will look at what will happen, if this is the function of both for $f(t,x)$ both as a function of t and function of x. If you have to consider what is the Taylor's formula in this case would look like? See the figure for elaboration.

I mean precisely you can write if you are assuming of up to some nth order can agree with it exist and up to continuous. Then you can write an reminder term and what happen to the reminder term, that is what one would want as, when you have precise proof if you want to do. But, since you want to get an idea about how this is true. So, for us just we can look at it simple manner. Now, again so what you are trying to look at is this quantity $f(T,W_T) - f(0,W_0)$.

Which, you can write it as sum of just like in the earlier case you can write it as sum of the displacements in this smaller intervals. Now, for each of this sum you can apply the Taylor's, so this is the Taylor's formula gives this. So, this Taylor's formula term you can plug it inside and you can look at essentially what happens to each of this terms. Where x_j replaced by W_j and so on. So, now if you do that you will have terms. (You will) suppose if I write that for that moment, though I will write briefly.

So, the first one, so if I look at the first one, second one and the third one would only be contributing to the Ito's formula. This term I mean we have seen earlier in the when we are looking at the quadratic variation and the cos variations. So, this term would tend to 0 and so also for the higher terms. (So, this terms will tend to 0) so this quantity this orange coloured underline terms would contribute nothing to the right hand side. Whether the first three terms is what would contribute and that contribute would be in some sense as given in the formula. So finally we get

$$\int_0^T f_t(t,w_t)dt + \int_0^T f_x(t,w_t)dw_t + (1/2)\int_0^T f_x x(t,w_t)$$

And, that gives us this term in some sense dt and rest of all terms one can show easily that this is tending to 0. So, this is what will be left out and this is what is the final answer as far as this is concern. The proof mainly like these terms are straight forward the first two in the terms f_t and f_x terms are straight forward implications with the third would involve little bit that you can not simply replace sense here that this two sum. But, in the limit when you replace this by $f_{j+1} - f(t_j)$.

And submit and then take the limit then that limit would be the same as this limit one can show. And, that is what is the major part of the proof of this result would be. And, that contributes to the third term in the case. So, this is how the sketch of the proof of this Ito formula and essentially this is what it means. So, this is what the basics idea of the proof if you are interested you can look at in any stochastic book for a proper precise proof of this.

But, in this for us, since the use of this formula is more important, we will go ahead with the idea that you have gained here. Now, for example that of this orange terms what that we have given or example this $t_{j+1} - t_j$and this $W_{t_j} - W_{t_j}$ is 0. You know you can also be gauzed from the, your earlier idea of dt for a term dt and dw_t is 0. So, it is basically the informally recorded in that manner can also be used and similarly for the $(t_{j+1} - t_j)^2$term can also be seen from the earlier informal language that we have used as the tt times dt is 0.

The quadratic variation of t with itself is 0 of course this is a differentiable functions. So, obliviously this is to take the function as f of t equal to t this is true. Now, we can write this terms in simple way quadratic form. So, one can do is that, in the following way, which is more general and can be applicable even in a more general situation.

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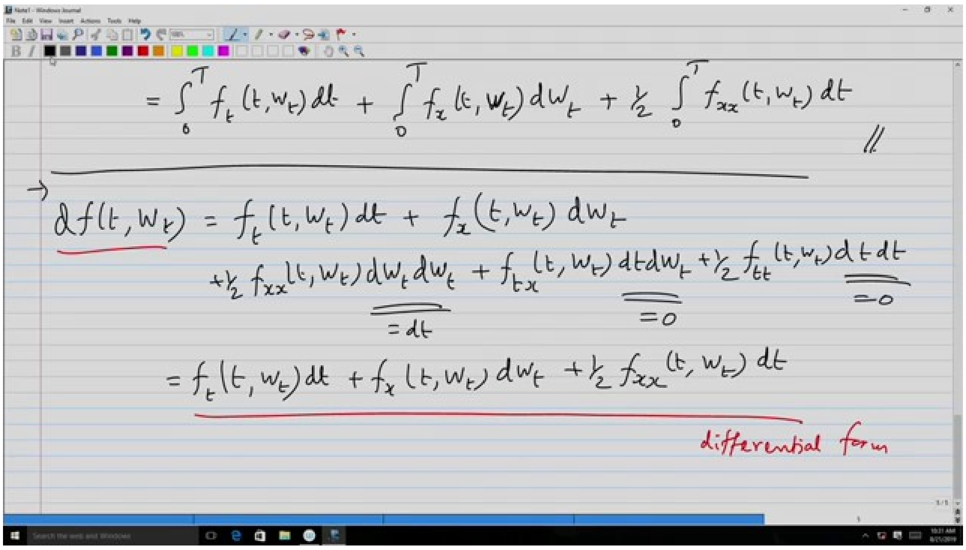

What you can do when you have a function of t and wt you are looking for the differential. So, you can write that using, keeping in mind the Taylor's expansion. Now, you know that this quantity is dt, this quantity is 0, this quantity is 0. So, this leads to the formula. This and this is the differential form of in this particular case dW_t equals this is the differential form of this Ito's formula. And, how did we obtained?

That you apply the Taylor's expansion up to two terms. Because, higher other term anyway by the same logic if you extend. The dW td W_t in to dW_t is essentially because, is cubic variation of W, which is 0. And, hence all cos variations and everything also will go. So, this is what one would end up so this is what is the differential form of this Ito formula that we will use. So, this I mean we can go back to the ordinary calculus to understand why and how this Taylor's series approximation is required and in what terms.

And, extra thing is far more accuracy, so in the normal scenario this term the first two terms only would remain. But, in case of brownian motion as an argument to this function f. Then, the more accurate form is what given by this Ito formula. Because, it has the none zero quadratic variation. The reason why this third reminds here is because of the non zero quadratic variation that you have here or infinite first other variation that you have. Now, this formula often simplifies the computation of Ito integrals. We recall Ito integral be define an $\int_0^t w_t dw_t$ we have obtained using the basic principle using the definition.

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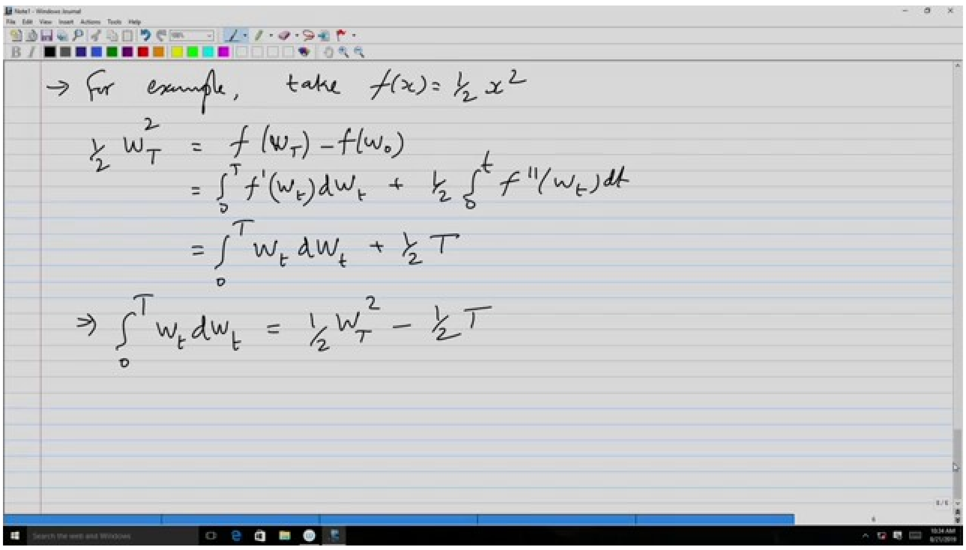

But, one can see easily that it can simplify greatly the computation of Ito integrals. For example, suppose if you take so take $f(x) = (1/2)x^2$ as function. Then what this formula says is the following,$(1/2)W_T^2 - (1/2)W_0$, which is 0 which is what this $f(W_T) - f(W_0)$. Which is nothing but $\int_0^T w_t dw_t + (1/2)T$. So, if you want to integrate rearranging here is in the terms what is you get is

$$\int_0^T w_t dw_t = (1/2)W_T^2 - (1/2)T$$

.

Because, that this is what precisely we have obtained. So, if you can identify a function that will give us this Ito integral than you be able to evaluate. Because, you know that this formula involves the left side you have the function evolution and the right side you have the Ito integral plus an ordinary integral. So, this Ito integral can then be in written in terms of those. So, this is what you have seen, so really did not go for any approximation and so on. But, directly by usually Ito's formula we could obtained this. This is the Ito formula for brownian motion.

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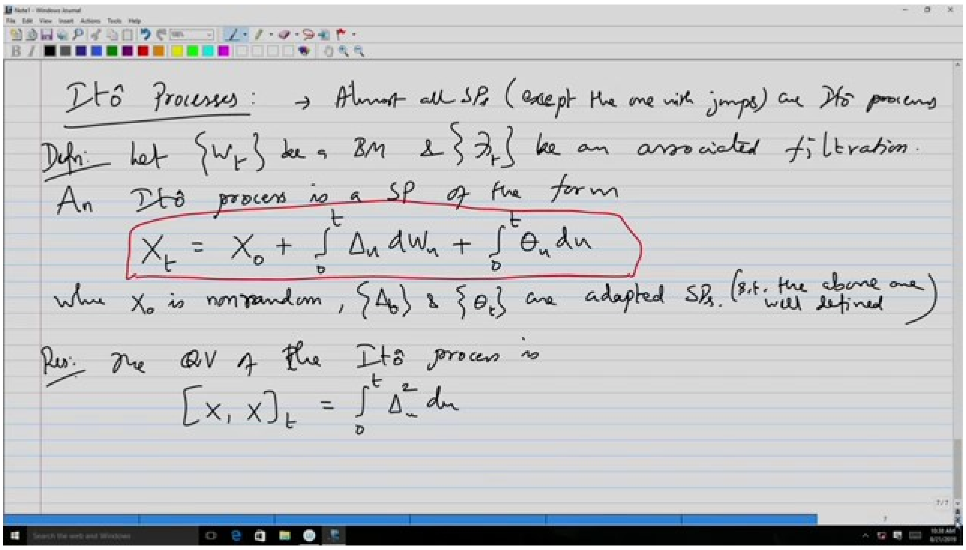

Now, we can extant the Ito formula for more general process. Which is what we call it as Ito processes, (almost all stochastic processes except those that have jumps). Almost all stochastic process except the one with jumps are Ito processes. But, what is that Ito processes? So, this is the reason why we might consider this is more generic model. So, the definition of an Ito processes is the following. So, you have to brownian motion BM and ft be an associated filtration.

And, Ito process is a stochastic process of the form what is this? So, this is

$$X_t = X_0 + \int_0^t \Delta_u dw_u + \int_0^t \Theta_u du$$

,where X_0 is non random and Δ_u or Δ_t and θ_t are adapted stochastic process. And, also you have to keep it in the mind is Δ_u or Θ_2 are such that this quantities are out of the process. Such that the above are well defined so whatever condition required to be imposed and delta u. So, that this Ito integral exist and also what are condition that is required to be imposed on this theta u.

So, to this is imposed, which is means that the require integral condition is what we mean, that you know they all and delta u and the integral t condition, is what imposed on this theta u process, so that we are assuming. So, such a process is what we call it as the Ito process, essentially it is given by this that there is a sum of ordinary integral and Ito integral is what plus a non-random quantity is if you want to add. So, that is what is given as an Ito process.

Almost all stochastic process except the one that have jumps can be represent in this form. So, that is why this process makes a integral. Now, the results is that the quadratic variation of the Ito process is essentially what the claim, the claim is this process has delta square u du as a quadratic variation is this, is what we are claiming. And you know from the idea of the Ito integrals. So, this is exactly the quadratic variation of the corresponding Ito integrals that we have define. So, what happens to the other one? So, this of course we will not to the proof. But, this can be easily remembered by the first writing the Ito process this written in the integral form.

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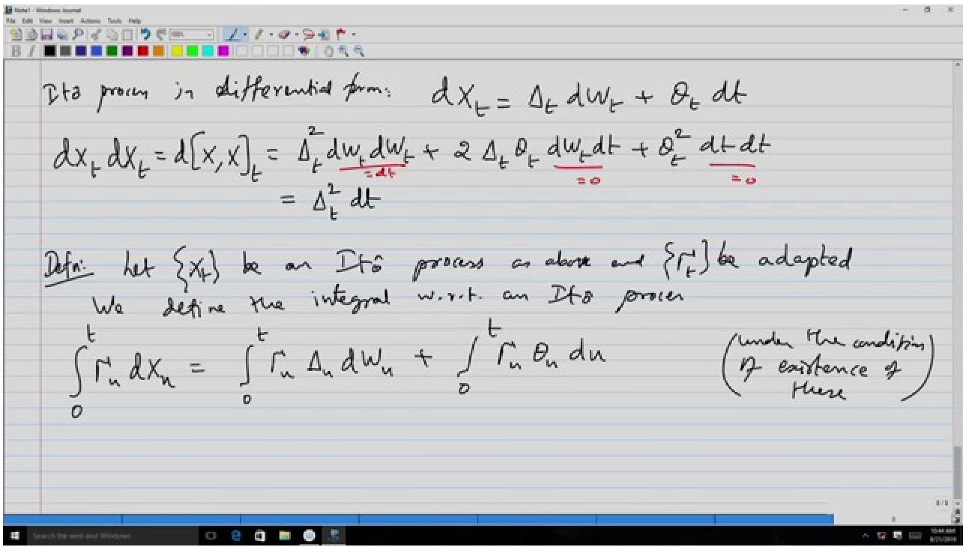

So, Ito process in differential form, if you write in the differential notation, you will end up with

$$dX_t = \Delta_t dW_t + \Theta_t dt$$

, is what then you are showing. Then if you want to compute quadratic variation you can consider $dx_t dx_t$, which is sometimes is also written as in this form the differential of this is nothing but you have to twice multiply this. So, finally we get Δ_t^2.

And this quantity is also. so this two quantities are 0 and this quantity is dt, so you will end up with this. So, this is what X here. So this what is the quadratic variation of the Ito process. This is if you write differential form then you can easily write down the quantities that you have. So, in this particular case the ordinary integral which is there in the definition of Ito process does not contribute to the quadratic variation. And, the Ito integral there contributes to the quadratic variation of the Ito process.

So, this does not mean that there is no randomness there. There is randomness it because theta d is random but, that is in some sense less random than the randomness which is inside the Ito integral. So, that is why a model which will have assets depicting the evolution of risk free asset say bond or money market will have only the dt term as we see it later. And, the model use to depict the asset price connecting to a stock will have a dW_t term compulsorily be present.

So, that is what it would mean. So that, it is in some sense, it is like more random than the other one, that is what the contribution towards the quadratic variation of the process will happen. Now, so far we have defined the Ito integral with respect to the brownian motion. Now, we can also define the Ito integral with respect to n or Ito process. So, let X_t be an Ito process as above the previous discussion. And, you have the another process a gamma t be adapted process.

Now, we define the integral with respect to an Ito process as following

$$\int_0^t \Gamma_u dX_u = \int_0^t \Gamma_u \Delta_u dW_u + \int_0^t \Gamma_u \Theta_u du$$

So, that we are assuming under the conditions, under which you are assuming that this quantities exist. Then this is what the definition of an integral with respect to an Ito process again you can write down in the form of differential notation for ease of understanding. Now, once we define this once you have this integral then we can also look at Ito formula for an Ito process similar way.

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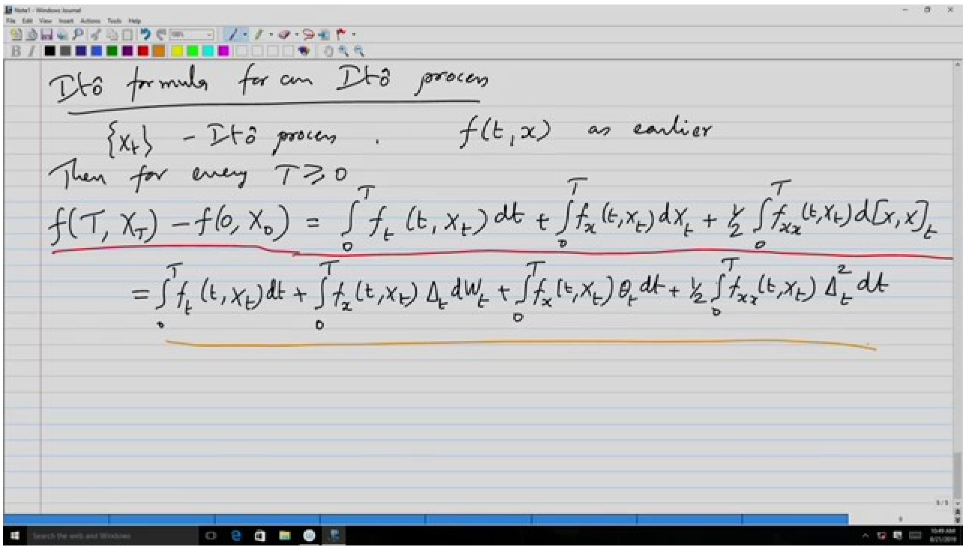

Ito formula for an Ito process earlier you saw that Ito formula for a brownian motion process. But, now you can work through the Ito process exactly in the same way as earlier and you can write down this in the following form. So, you have X_t as an Ito process and you have this function $f_{(}t,x)$ with the condition as earlier. With a conditions that $f_{,}t f_x, f_x$ are defined and continuous. Then, for every $t \geq 0$, so what you have your $f(T,X_T) - f(0,X_0)$.

Much like the case that we have written earlier and this now, if you plug for dx_t the integral form of the Ito process 1. The differential form of this Ito process 1, if you plug it here you can segregate this even further to write it in terms of dt and dW_t as an Ito integral based on brownian motion rather than this and quadratic variation and everything. So, which means it can be further written down to mean the following this is basically 0 to capital T of this would the first one would remain as the same.

But, in the second one you will have two quantities, return of the second one dX_t the moment you substitute the differential form of Ito process, it will fit in to two. So, this is what Ito the proof or this sketch of the proof rather goes along similar line as you have done earlier. Like you take this function you take that as an expansion of up to second order and for all terms.

Then, you apply the results that we are connected just now that quadratic variation of the process X is given by that (see pic). That you apply and the rest everything you follow in similar line. So, this is what the Ito's formula for an Ito process? Is what then you will have. So, this is in terms of x_t itself and the second term is in terms of d_t and dw_t. So, you will have three ordinary integrals and one Ito integral is what would apply when you apply the Ito's formula for an Ito process this is what you end up here.

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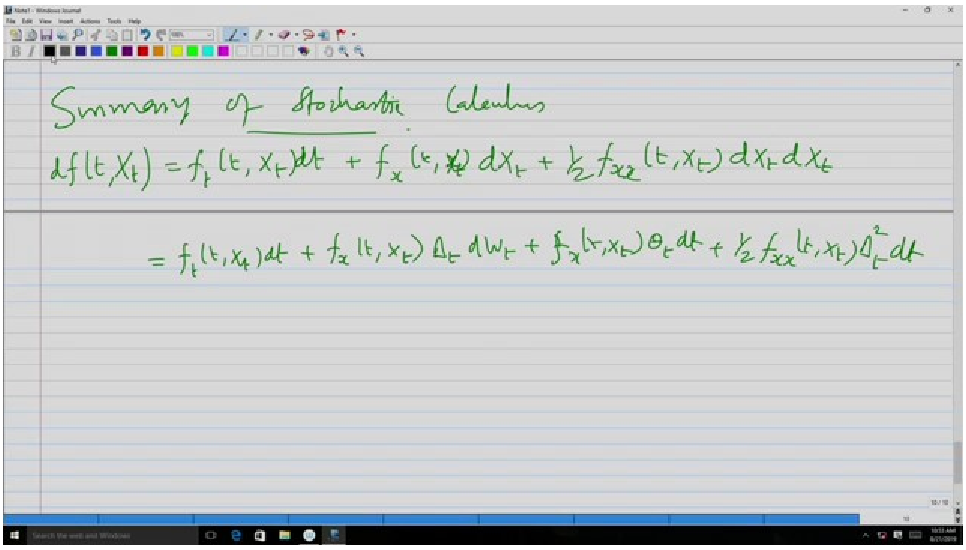

Now, once we see this is now we can make a summary of how do we do given any process that you have here. So, basically the summary is the following. Let me use a different colour to mean that. Summary of the stochastic calculus part is essentially the following. So, what we have is if you want to write the differential form, if you have, it is how to remember formula that is what you know how we mean when we say this quantity. So, df_t rather than we can write X_t itself because X_t would also be considered.

So, you write down this $f_x(t,x)dX_t + (1/2)f_{xx}(t,x_t)dX_tdX_t$, this is what it is precisely. So, this is what the Ito formula for an Ito process. Since, an ordinary Ito integral brownian motion also can be considered as an Ito process, so this formula holds true in general. And, if you want to write in this, so the guiding principle when we write out this is, that you take Taylor's series terms up to second of all. And, use the basic formulas dW_tdW_tinvolving these terms and to arrive at this expression.

This can further be written down in terms of dt and the label two for the sake of completeness we will write it down that portion also dX_t dt plus $f(x,t)$. So, when we write here this is Xt we should write, $X_t\Delta dW_t + f_x(t,x_t)\Theta_t dt + (1/2)f_{xx}(t,X_t)\Delta^2 t dt$. Stochastic calculus is little more than repeated use of this Ito's formula in various contacts. But, whenever you have to remember you are involving brownian motion or an Ito process in general.

When we have this, see the correct formula for the differential for anything is basically given by Ito's formula you should not use the chain rule or formulas form the ordinary calculus, that is what you know you need to keep that it in mind.

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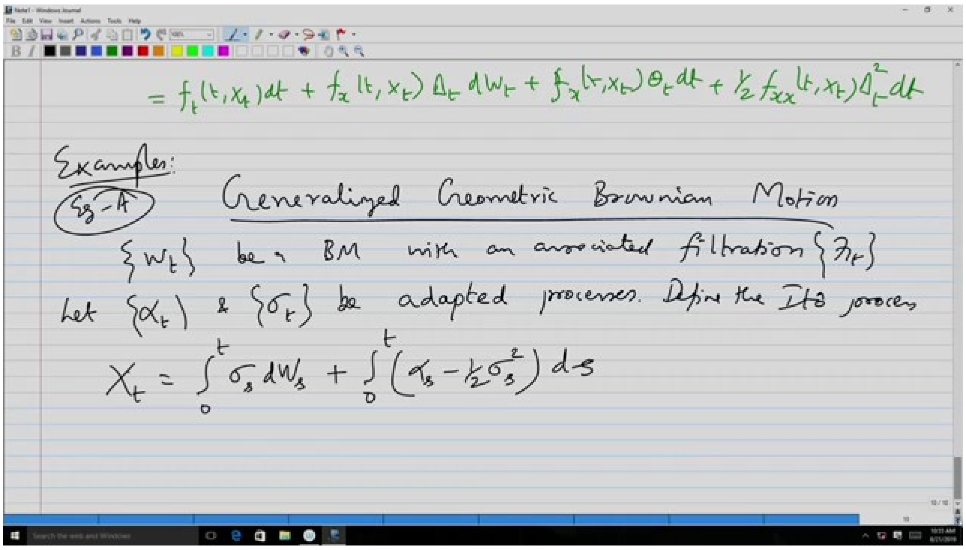

Now, let us look at some examples, the first one is more important, as far as in this course whenever is concern. So, this is example A) which is we call as generalized geometric browinian motion, is what we call, the reason why we the word generalized, it will be clear in a moment. So, what we have the ingredients are you have a Brownian motion B_m Brownian motion with an associated filtration say f_t and let α_t and σ_t be adopted. Now, define the Ito process say

$$X_t = \int_0^t \sigma_s dw_s + \int_0^t (\alpha_s - (1/2)\sigma_s^2) ds$$

is what the Ito process that you are picking.

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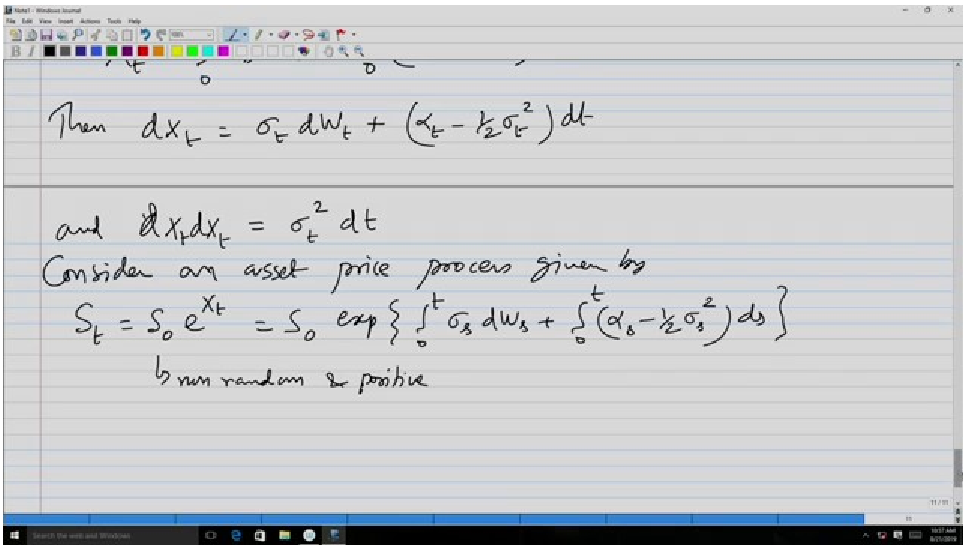

Now, so then, what we have as $dX_t = \sigma_t dw_t + (\alpha_t - (1/2)\sigma_t^2)dt$, We use $dX_t dX_t = \sigma_t^2 dt$ Now, consider an asset price process given by $S_t = S_0 e^{X_t}$. The renaming expression is easily follows. So, this is what you consider that this is an asset price of process, which is given by this Ito process.

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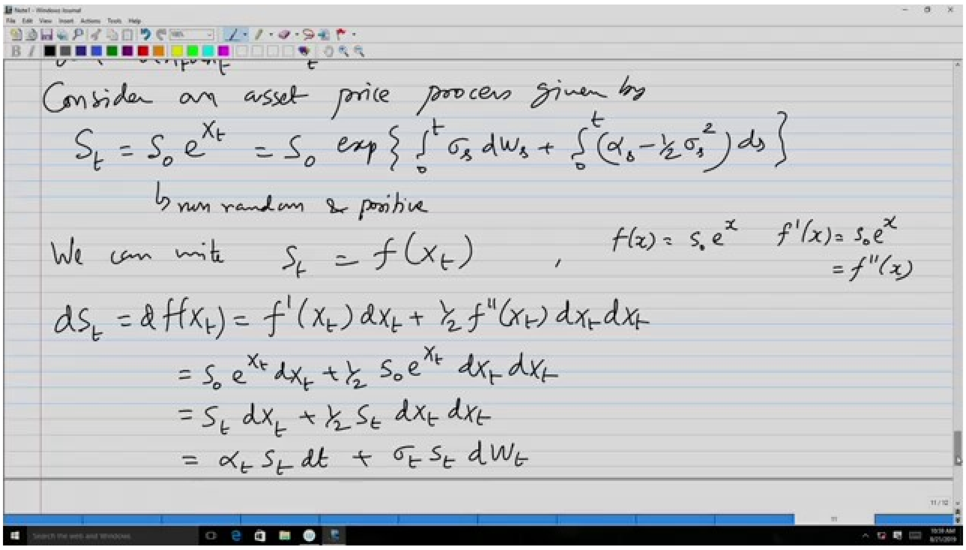

So, we can write $S_t = f(X_t)$ with $f(x) = S_0e^x, f'(x) = S_0e^x = f''(x)$. Now, we can easily derive the expression for dS_t.

Now, this is an important in the sense that this example, whatever this generalized geometric brownian motion that we have considered it includes all possible models of asset prices, that is always positive has no jump and driven by a single brownian motion. So, any asset price process, which involves, which has this characteristics can be represented in this particular form. So, that this is what this important of this.

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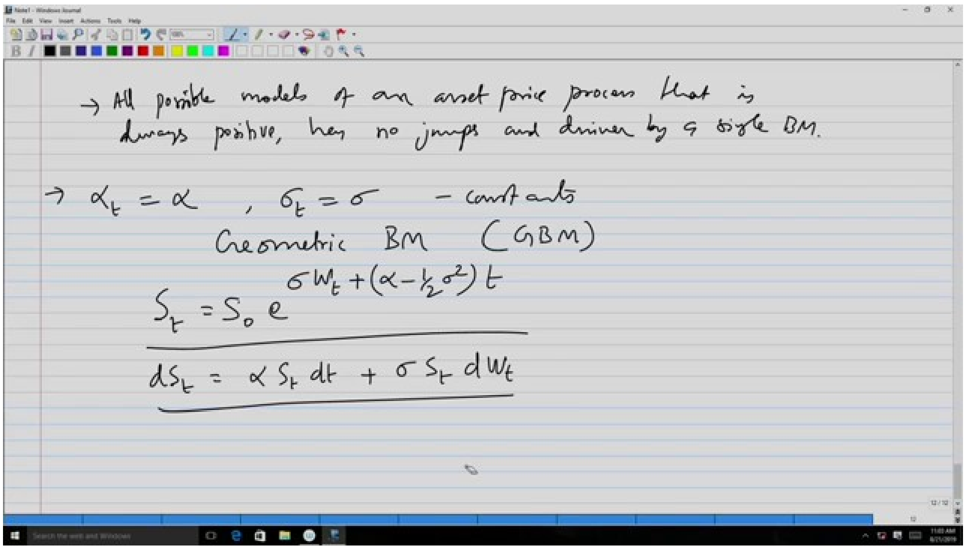

So, this is basically involves, this denotes all possible includes all possible models of an asset price process, that is has this three properties always positive. And, has no jumps and driven by a single brownian motion so any suggestive price process can be return in terms of this appropriate alpha and sigma process. Now, so this in general, since alpha t and sigma t are allowed to be random means may be assumed that to be adopted processes but, they can also be non-random

And, when you have alpha t equal to alpha and sigma t equal to sigma some constants, then what we have is what we called as geometric brownian motion. We may abbreviate as GBM going forward. so this is what the case in this particular case my St is simply e to the power sigma W_t minus alpha plus half sigma square times t is what the price process then what you are looking at it. And it is a differential form would be $dS_t = \alpha S_t dt + \sigma S_t dW_t$ is what the differential form that you will look at this is integral this is the solution.

Now, if I have to give one the other, so this is a special case of what we will going to describe, that stochastic differential equation. And the solution is what given by the previous line. And, if the previous line if it is solution and it if you want to verify that this is actually satisfies this or what is the differential form of this St, then what you have to apply is the Ito's formula to this process St to arrive at the corresponding equation that you will have here.

So, this is what as (you say this) we say repeated use of the case that we have here Itos formula, that is what in a way when we mean we say. Now, this also gives this example gives a rise to the following result, which may be useful. When we are looking at little bit more examples or anything.

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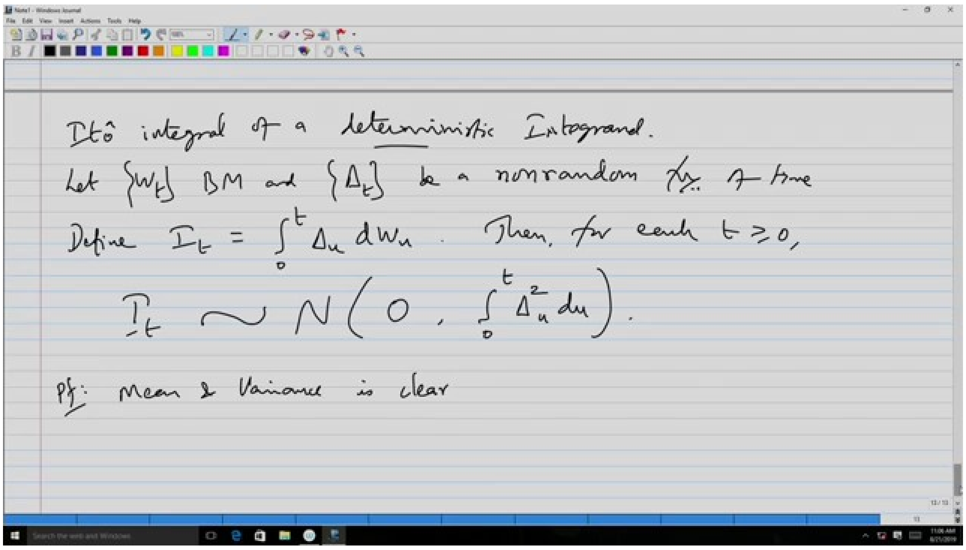

Which is basically, what we say Ito integral, or what can you say about the Ito integral of a deterministic integrand. In this case what happens? So, what is the result it says is that, so you have a brownian motion and delta t be a nonrandom function of time or function t. So, now you define this Ito process

$$I_t = \int_0^t \Delta_u dW_u.$$

,my I_t is normally distributed with expected value 0 and variants $\int_0^t \Delta_u^2 du$, this is what the result is.

So, again if you proof is simple the mean and variants you can easily obtain. Because, we have already obtained, it essentially the mean and variants is simple. Because I t is an Ito integral were we find it for a general integrand process. But, this is a nonrandom function of time. So, this is again falls in the category for which is martingale we have shown. Martingale at $I_0 = 0$, so the mean is 0 is clear. variants also we have arrived at as expected value of $\int_0^t \Delta_u^2 du.$

But, since this delta square u is non-random the expected value is same as this so mean and variance is clear. But, only thing is now like what we want to see is the normal distributor, is what then we have to prove. So, we can show that. Since, mean and variance is known, we can assume that this I_t as a normal and show that is more its mjf is what then we are looking at which is correct. So, we will just do this by establishing that the mjf of a normal random variable with this mean with this variance.

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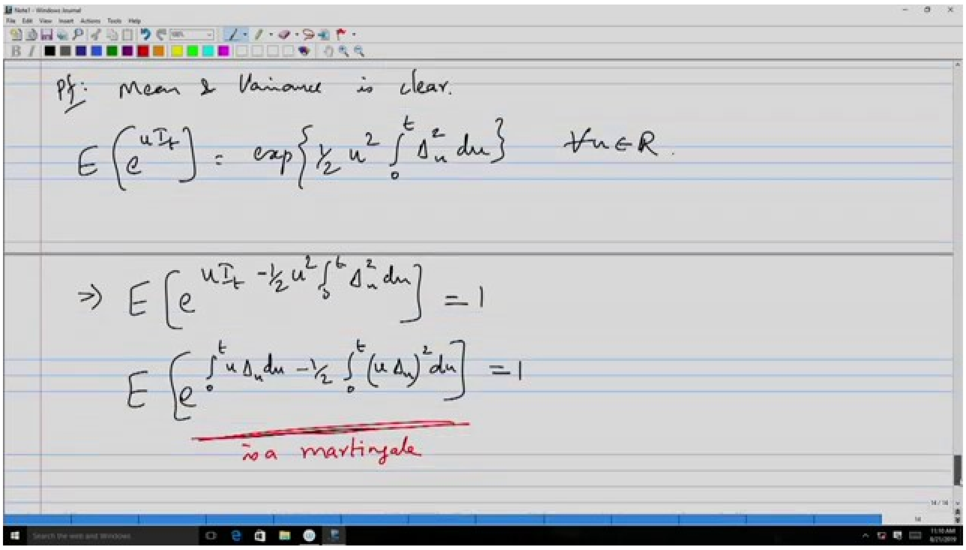

Which is basically,$E(e^{uI_t})$ if I pick, which is basically $e^{(1/2)u^2 \int_0^t \Delta_u^2 du}$. which is this for all u $in\mathbb{R}$. Now, because this is nonrandom that is where you know the point comes here. Because, this is nonrandom you can right hand side also you can think if a dig expectation it is going to remain the same. So, you can bring inside this expectation and you can write this whole thing as as above.

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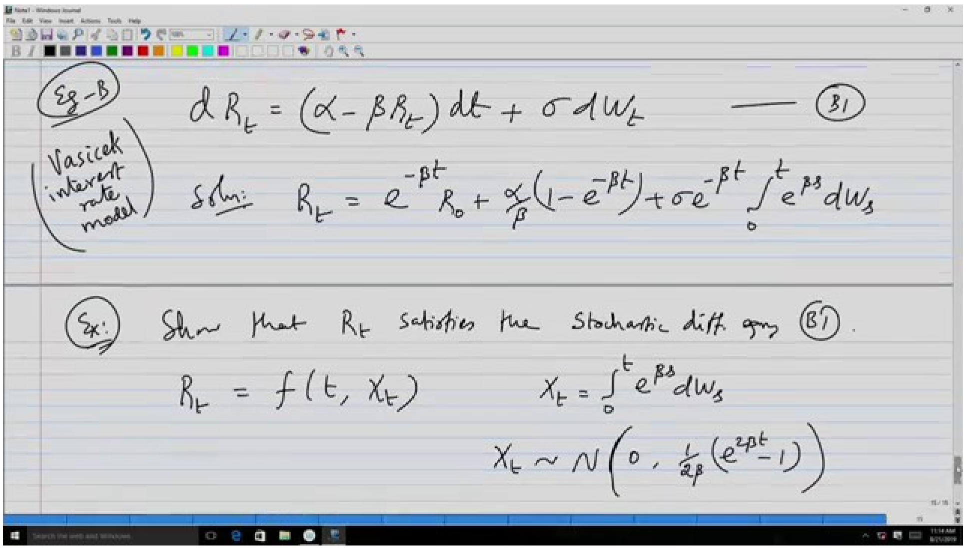

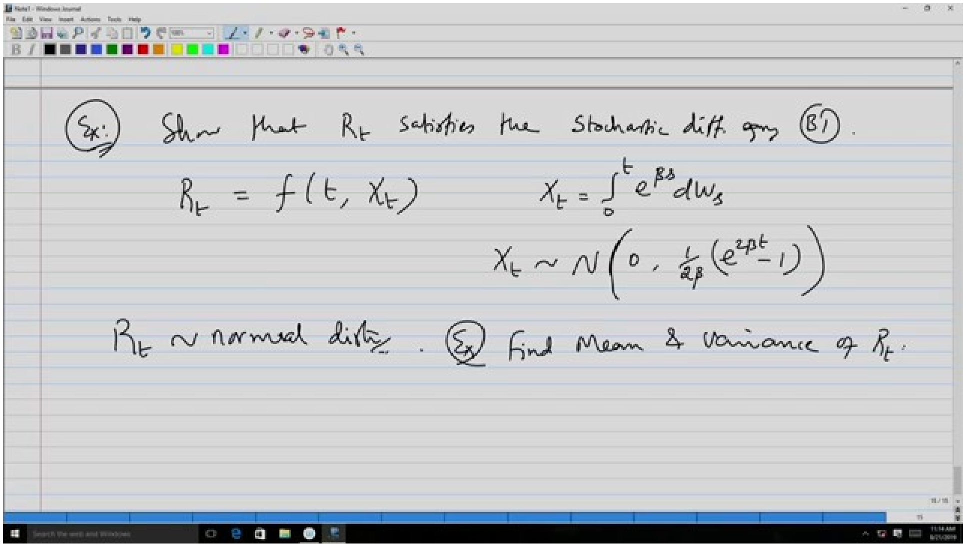

So, this we can quickly see that gives useful, this is becomes useful in our second example. We will just see the second example, where we have, what we are going see, is that we have a process $dR_t = (\alpha - \Theta R_t)dt + \sigma dW_t$. This is what in the literature is called as Vasicek interest rate model. So, this is used as the model for determining the interest rate with some α, β, σ as parameter. Again this is a stochastic differentially equation about which we you see little later.

An example of a stochastic different equation because, in the right hand side it involves $R_t dW_t$,so both it is involve. So, it is a proper stochastic differential equation. The solution is given above. So, this is a solution. now your exercise shows that R_t satisfies the stochastic differential equation, say suppose if I call this as some B1 say B1.

Now, do you show, you take this R_t (so you how do you show is the following. So, you take this R_t) as a function of t and some X_t , where my $X_t = \int_0^t e^{\beta s} dW_s$. So, you can see that this Rt is this now apply Ito's formula to arrive at this expression. So, you can employ the Ito's formula to do this exercise. Now, more you can say about it because of the previous result. Now, this X_t is you can see that it is an Ito integral with a deterministic or nonrandom integrand.

So, this X_t will follow a normal distribution with mean 0 and variance written above. This is the mean and this is the variance of this X_t. And, hence R_t itself you can for say that is normal distribution. We can find out now appropriate what is the mean and variance. Again exercise is find mean and variance of R_t. So, this as you see here you are given a sde you are given some expression as its solution.

Now, you have to verify you are using the Ito's formula. or other way later we will see given an sde by solving this you can arrive at this expression. Now that we will see in a later here. So, this how one can use in various situation and as I said. Stochastic calculus is nothing but a repeated application of this Itos formula over and over in various context. To verify that some given expression is a solution of an sde, used as are given something to obtain the differential form of that process you used Ito's formula. So, you should not use the ordinary calculus thing but, you should use the Ito's formula. So, let us end here and then we will see more later thank you.