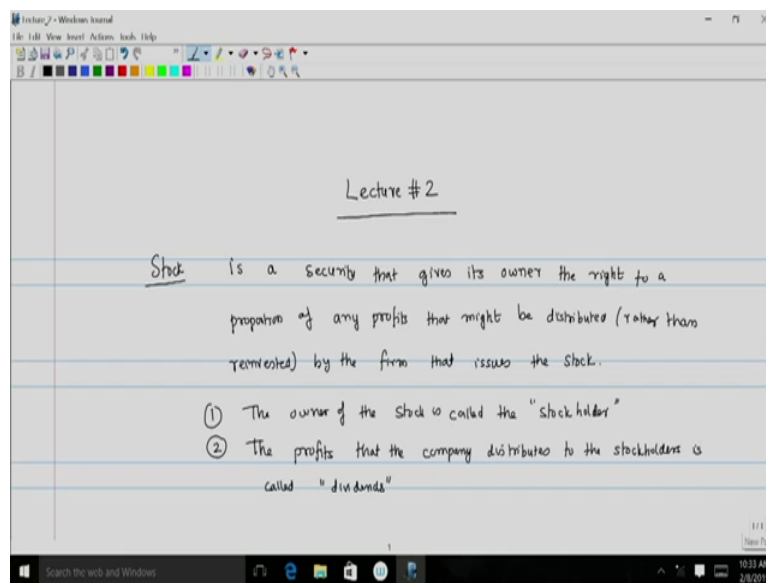


**Mathematical Finance**  
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**Module 1: Introduction to Financial Markets and Instruments**  
**Lecture 2: Introduction to Stocks, Futures & Forwards and Swaps**

Hello viewers, welcome to the second lecture on Mathematical Finance. You would recall that, in the previous lecture, we introduced financial markets, in general and in particular, we talked about bonds and some of the basic properties of bonds and the reason why people purchase bonds. In today's class, we will talk about the another kind of security, namely, stocks which are considered as risky securities and also we will introduce a little bit about financial derivatives and discuss about forwards, futures and swaps.

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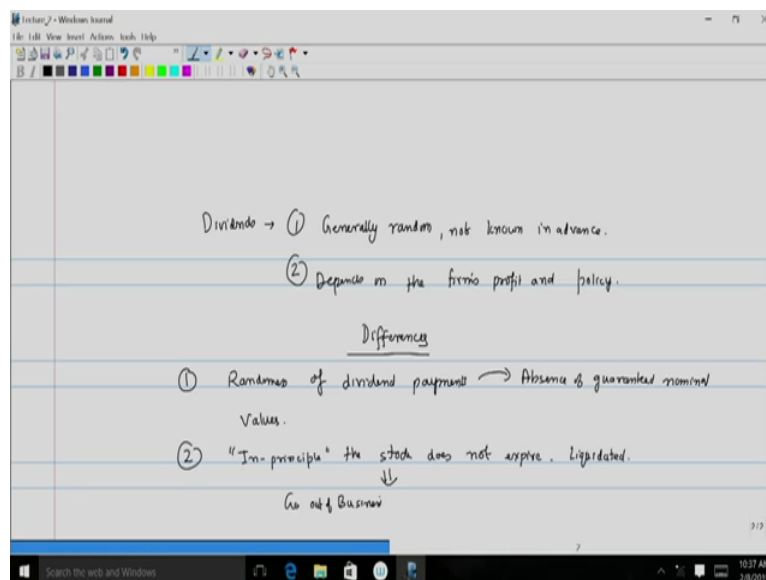
Let us begin with our lecture. What is a stock? A stock essentially is a risky security. So, stock is a security that gives its owner the right to a proportion of any profits that might be distributed (rather than reinvested) by the firm that issues the stock. What does this mean by distribution of profit and also what does it mean by distribution of the profit rather than reinvestment?

Typically, what will happen is that a firm or a company would like to raise money and the two typical ways that they raise money is issuing bonds which we have already discussed and

by issuing stocks. So, essentially stock certificates are going to be issued to individuals upon receipt of payment and this money is invested in the business of the firm and once the firm sees profit, they can do one of the following two things with the profit, either they take the profit and distribute a share of the profit to all the people who had purchased the stocks in which case, we say that basically a dividend has been paid and in other case, they might not actually give the share of the profit, but rather invest the entire amount of money or a part of it in furthering their business in which case the gain to the owner of the stock would be through the increase in the price of stock.

Let us now start talking about a little bit about the various terminologies that are associated with stocks and stocks owners and firms that issue the stocks. So, first of all, the owner of the stock is called the stockholder or sometimes they are called the shareholder and secondly the profits that the company distributes to the stockholders is called “dividends”. So, essentially dividends are the proportional right or the proportional profit that the buyers of the stock or the stock holders receive depending on the percentage investment that they have made in terms of the total number of stocks that are actually issued.

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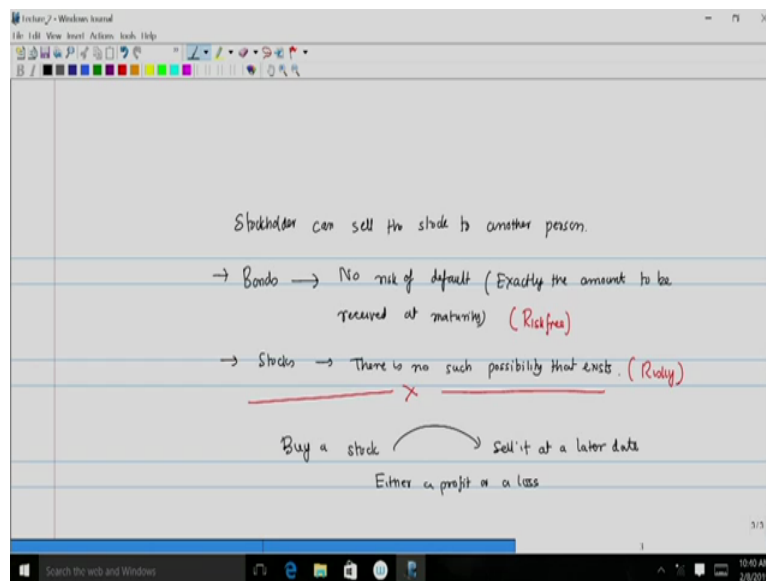
Now, let us look at a couple of observations about dividends. First of all, dividends are generally random, that is, not known in advance and you notice that this is in contrast to bonds where basically at the time of investment, you know exactly the amount of money that you are going to receive upon maturity. This means that there is a greater amount of uncertainty as to what returns you are going to get upon your investment in the stock.

And the second point about dividends is that, it depends on the firm's profit and policy. So why it depends on the firm's profit is that, I mean the question of dividend arises only if the firm is making a profit and upon making the profit it depends on the firm's policy whether they want to hand out dividends to the stockholders or whether they want to actually go ahead and reinvest the amount of money in order to expand their business, you know in the existing business or whether they want to further their business in other diversified areas.

Now, we talk essentially about the two kinds of securities, bonds and stock. Let us just briefly dwell over the some fundamental difference that exists between bonds and stocks. What are the differences? First of all, the first difference is that the randomness of dividend payments and this basically means the absence of guaranteed nominal values. So, in case of bonds, as you have seen previously, there is always a guarantee of a certain return which we have called the nominal, but no such nominal return is guaranteed in case of the stock. So this is one key difference that exists between stocks and bonds.

And secondly, in-principle the stock does not expire, so why do I say in-principle? In-principle basically means that unlike bonds which have a fixed maturity and we get that amount of promised money upon maturity, stocks do not expire unless the company goes bankrupt or something seriously goes wrong with the functioning of the company resulting in company being no longer in a position to run their business. Basically, in that case, the company has to be liquidated and the shareholders may get or may not get, depending on the circumstances, proceeds of the company if it is eventually liquidated. This means that one of the key things is that basically in-principle means that the company may go out of business.

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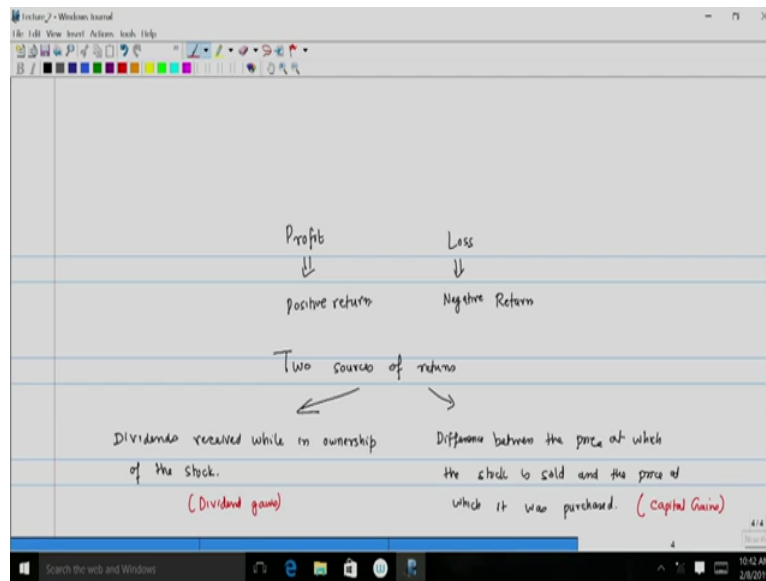


Now, when we were discussing the kind of risk associated with the bond and the liquidity risk, we talked about how bonds can actually be transferred to another person. In a similar way, stockholders can sell the stock to another person. Now, in case of bonds, what we had was that there is no risk of default and when there is no risk of default, for example, government bonds, then we know exactly the amount to be received at maturity.

However, in case of stocks, there is no such possibility that exists and the reason being that you do not know basically how much dividend you are going to get and secondly, it cannot be predicted right at the beginning, the way the company is going to perform which is eventually going to affect the price of the stock. This means that because of this reason, a bond is always considered or typically it is considered risk-free assuming that there is no default and stocks is always considered as a risky asset.

Now, let us talk about the gains or losses that can actually happen from a stock. Suppose that you buy a stock today and then you sell it at a later date. Now the consequence of this action could either be a profit or a loss.

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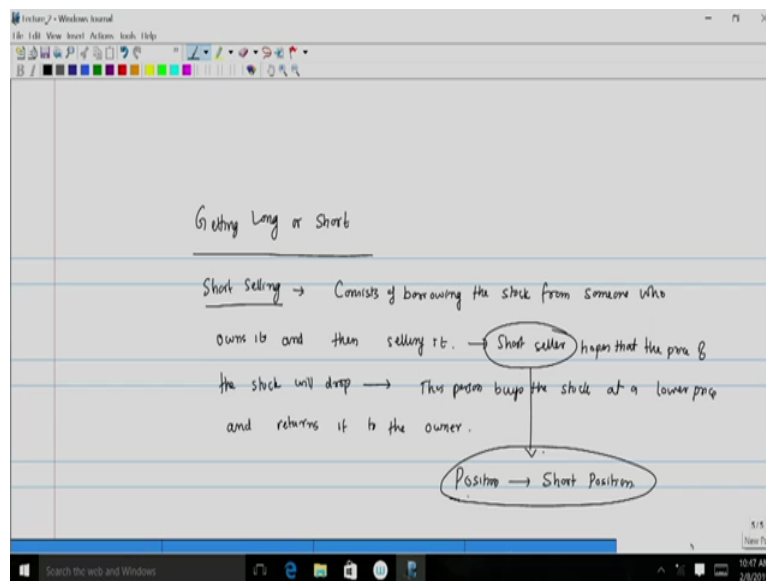


So, when there is a profit, then we call this to be a positive return and when there is a loss, we call this to be a negative return. Now, there are two sources of returns, what are the two sources? The first source is dividends received while in ownership of the stock. This means that during the period where you own the stock that means from the time period that we have purchased the stock till the time period you decide to sell of the stock you are said to be in ownership of the stock and then if any dividend is paid during this period of ownership that is what is known as basically the dividend return or the dividend income.

The other kind of returns that you can actually get is what is known as the difference between the price at which the stock is sold and the price at which it was purchased that means suppose you decide to buy the stock at 100 and after 6 months you decide to sell the stock for the prevailing price at that time, for example, 120 then basically your gain is going to be an amount of 20.

So, then the dividends that is paid this is what is known as dividend gains and the one that you get as a result of selling the stock this is what is known as capital gains. So, please note that once you have sold off the stock then you are no longer entitled to dividend and the dividend or any subsequent dividend will be passed on to the new owner of the stock.

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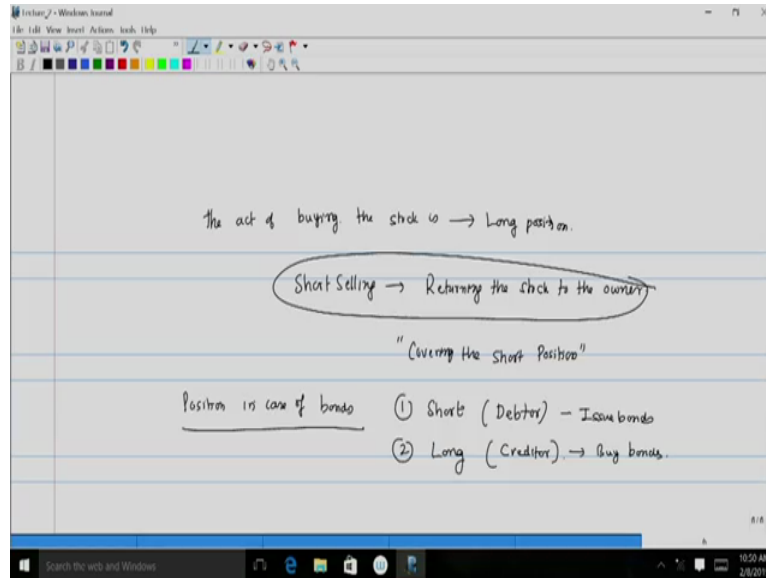
Now, let us look at a couple of more points or in some sense you know key factors in terms of positions of stock and this is mostly defined in terms of the ownership and the way you buy and sell stocks. So, this is what is broadly known as getting long or short. What do you mean by getting long or short? Here we will introduce the concept of short selling which at the beginning might seem to you to be slightly counterintuitive. So what basically happens in a short selling is that it is mostly individuals who assess that the current price of the stock is high and then in the near future it is expected that the stock price is going to fall.

Now, in that case what this individual who holds this belief does, is that they decide to borrow the stock from an owner of the stock and sell it at a price which is high at this point of time and when the stock price eventually goes down, purchases it at a lower price and returns the stock to the owner thereby keeping the profit resulting from the differences of the price at which he sold somebody else's stock and the price at which he bought the stock so as to give it back to the original owner of the stock.

Let us first start talking about this concept of short selling. Short selling of a stock essentially consists of borrowing the stock from someone who owns it and then selling it. Now this short seller and this means the person who has borrowed somebody else's stock and sold it, this short seller hopes that the price of the stock will drop in the near future and so when this drop actually happens, this person buys the stock at a lower price and returns it to the owner (owner means the person from whom he or she had actually borrowed the stock).

Next, the investor that owns the stock, so this person who short sells this particular stock, this person is what is referred to as short seller and that this position that I have here is known as a short position.

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Now, I have talked about the short position. Short position basically means that you are short in the sense that you own the stock to somebody. This naturally leads to the nomenclature of what is or who is defined a person who has a long position in the stock. So essentially this means that the act of buying the stock is basically this is what is known as the long position. So just one more point we are to add, that this process of short selling and then subsequent returning the stock to the owner, this particular act of doing so or basically meeting the obligation that you had to the original stockholder, this is known as what is known as the covering the short position.

Now, from this context, how do you actually extrapolate this in case of bonds. I mean what would be a long and a short position in bond. Basically, in case of a bond, you are said to have short position is the one that is associated with that debtor, that means the person who has basically borrowed the money from you and basically a long position is the one who is the creditor.

This means that the person of the short position in case of a bond is that is the debtor because they have borrowed the certain of money and issued you a bond and the party which has lend out the money to the debtor that party is said to have a long position. So that means if you

want to go into a short position in case of a bond all you have to do is basically issue bonds and long position means that you have to buy bonds.

Now, as I mentioned before, one of the fundamental differences that exist between bonds and stocks. The two key differences are or one is obviously that, it is with respect to the risk that bonds are effectively considered risk free in the sense that they have guaranteed payments and in case of stock when there is a risk component associated with it and also in terms of legal claim, there exists a fundamental difference between stocks and bonds. In case of bonds, you know, it is sort of a binding upon the debtor or the person who has issued the bond in return of receiving money from the creditor they are legally obliged to return the money to you as promised on the maturity date.

However, in case of stocks, this is not necessarily the case and it is a clear upfront to the stock owner that by getting into a situation where they are buying the stock of the firm, they know that while the returns can be good, there are circumstances where their losses might occur and they might lose all their money and so there is no sort of legal binding on the part of the firm that actually issues the stock. It has been observed empirically that because of the inherent nature of the risk that is associated at the stock typically the returns on the stock tend to be in general higher than that of the bond until and unless something goes seriously wrong with the firm and they are facing the possibility of going out of business. This sort of concludes our discussions on the basic security, namely, the bonds and stocks.

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The image shows a digital whiteboard with handwritten notes in blue and red ink. The text is as follows:

- Financial Derivatives → Financial instruments whose payoff depends on the value of another financial variable (eg. price of a stock, price of a bond, exchange rate and so). underlying.
- Not related to physical assets / business opportunities.
- Two Parties Sit Together
- Receive a payment (pointing to the left)
- Make a payment (pointing to the right)
- Payment will depend on the financial variable (underlying).

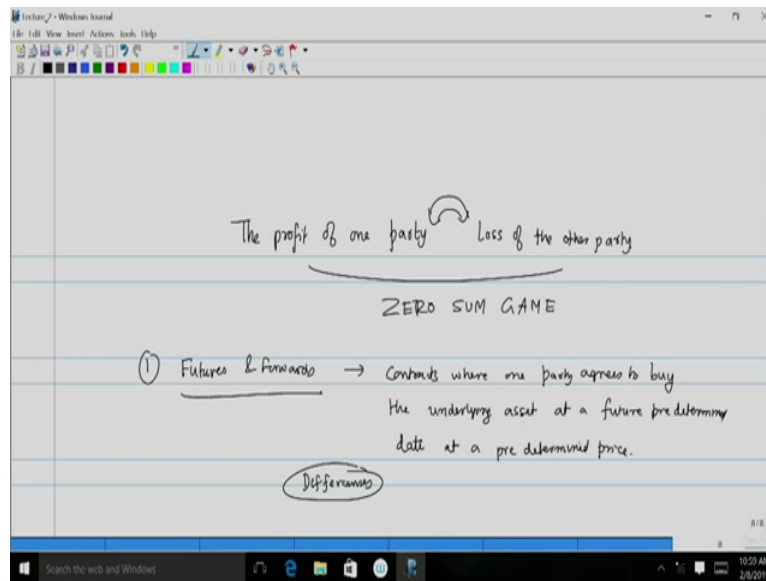
The whiteboard interface includes a toolbar at the top with various drawing tools and a Windows taskbar at the bottom showing the date as 2/8/2019 and time as 10:55 AM.



Now, let us look at the other side of the security, namely, derivatives. For the remainder of the course, derivatives will play a very crucial role and what are known as financial derivatives and I will explain what I mean by derivatives. You know the term derivative what is the connotation and what is basically the interpretation of the financial derivative. So what are financial derivatives? These are financial instruments whose payoff depends on the value of another financial variable, this could be say, the price of a stock, or say, price of a bond, or say, exchange rate. So by this I mean the currency exchange rate and so on. This means that if you invest in a financial derivative, your eventual profit or loss at a future time point will depend unlike in case of bonds or stock. Basically, it depends on what is going to be the price of the bond with the stock. In this case, there is no such direct relation, but rather the profit or loss will be determined by the movement of the values of some underlying asset. So, basically, it means that it is going to be a function of the value of the underlying asset at the time in which the financial derivative is executed. So, in mathematical terms, this would essentially mean that this payoff or the profit at some future time point will be a function of some underlying variable and we will actually discuss this in a lot more detail when we start talking about option pricing.

We next move on to a little bit of discussion on financial derivatives. Derivatives are typically not related to physical assets and/or business opportunities and what exactly the financial derivative is that you have two parties sit together to chalk out the terms and conditions of the financial derivative and so of the two parties, there will be one party which will receive a payment and another party, that is, they will make a payment and this particular payment will depend on the financial variable which is what is known as the underlying. So this financial variable on which this price is dependent, this is, something that is termed as underlying or sometimes basically called the underlying asset.

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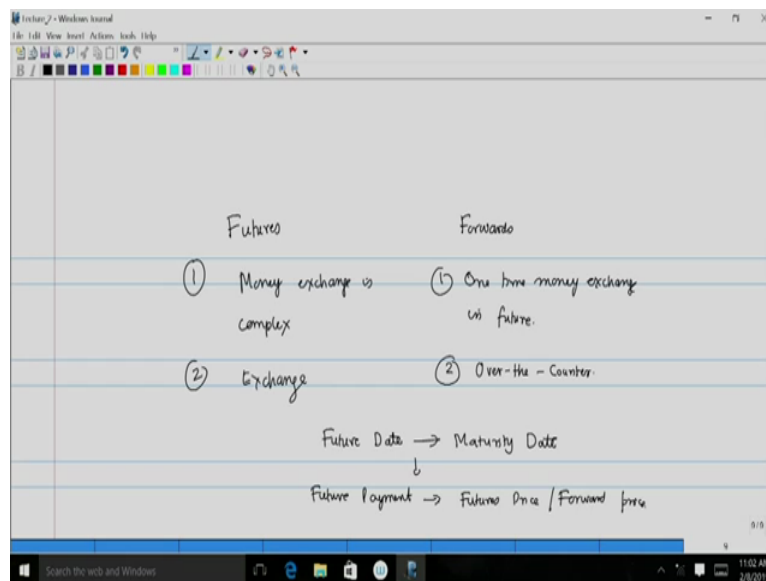
Typically, in case of a financial derivative, the profit of one party basically becomes the loss of the other party and this is what is typically known as a zero-sum game. So, that means then the profit of one party if added to the loss of the other party, it ends up being equal to zero which means that is a zero sum or a zero addition game and this is very typical of financial derivatives and initially the way financial derivatives are priced is that no party is likely to lose money, but eventually as the asset price as the underlying price evolves and a final payoff is made, it ends up being basically a zero-sum game where one party loses and one party wins.

So, in this discussion regarding derivatives, we will essentially look at three different kinds of derivatives, namely, forwards, futures, swaps and finally we will look into options in a lot more detail. In today's discussion, we will just talk about forwards and futures and swaps. So let me start off with talking about futures and forwards. What are futures and forwards? These are contracts where one party agrees to buy the underlying asset, maybe a stock, maybe a bond or you know some other asset, the underlying asset, at a future predetermined date, you know, similar to maturity in case of a bond, at a predetermined price.

And so basically what happens is that there are two parties, say, A and B and they get into this futures and forwards agreement wherein, say, party A tells party B "Okay, I am going to give you this particular product" and party B says "Okay, I will pay you this certain amount of money for the product".

So, here essentially it means that at some present point, you agree upon a supply arrangement where the supply will actually or the transaction will actually take place at a future time point which is predetermined at a price which is also predetermined. So, it is right now that you basically make the deal, but the deal is executed and the money is actually exchanged at some future time point. So, it is something which is a legally binding contract.

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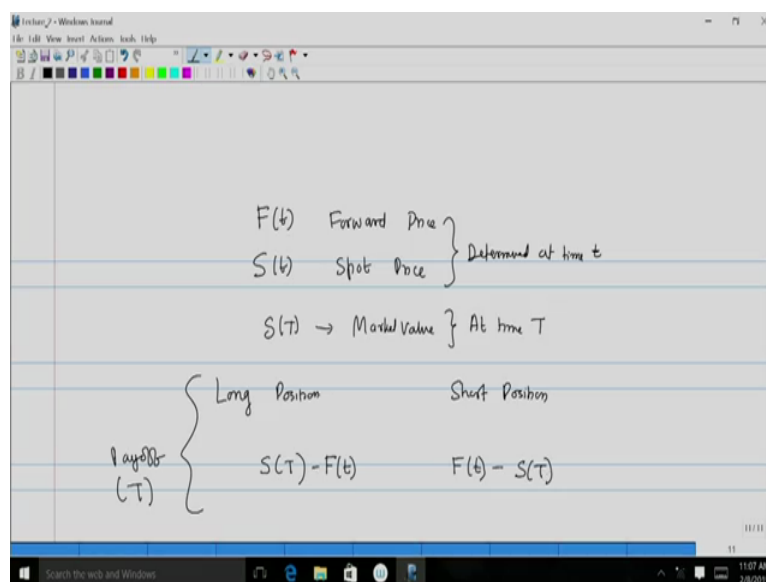
Let me just talk about their differences between these two, is the following. Let us look at what are the differences. So, in case of futures and forwards. In case of futures, it is the money exchange is complex, in case of forwards, there is the only one time money exchange in future. So, by saying that there is complex money exchange, it is basically because when the agreement is signed and suppose that the agreement is to exchange the good for money at a future time point 6 months down the line and so what happens is that depending on the movement of the underlying price at some point, one party or the other is likely to default on this agreement.

Accordingly, something which is called a margin account which is like a security deposit account, where both the parties put a certain amount of money and periodically there is, typically, on a day to day basis, money is moved from one security account or security deposit account to the other security deposit account and the money flows from the account of the party which might default in future or which has a chance of defaulting in the future because the asset price has moved down, is transferred to the account of the other party which is less likely to default and this continuous exchange of money takes place in this manner until the final expiration.

And why do these two different kinds of structures exist? It exists because in case of futures, these are more exchange-traded and that is the reason why they have to put more safeguards into place because futures are something where no net worth individuals might invest. So, in order to protect such investors, exchange have put this mechanism of a security deposit account, but in case of forwards, this is mostly over-the-counter agreements between large financial institutions or a financial institution and one of the clients.

Now, this particular future date on which the exchange of money and the product takes place this is what is known as the maturity date, just like we had the maturity date in case of bonds. So, now at a future date, we basically make a payment and so this future payment that you have, depending on whether this is a forwards or a future, this is what is known as the futures price or the forward price, as the case might be.

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Now, these are futures and forwards price and basically the regular market price. So this is in contrast to regular market price which is sometimes known as the spot price. Spot price is the price at which you will purchase the asset. Suppose I go to the market and I sign an agreement so that I will buy a certain product in a future time, then it becomes a futures price or a forwards price, but if I decide to buy the product immediately right then and there on the spot, then that particular price is what is known as the spot price, because it is the price that is being quoted on the spot for immediate purchase.

Again, coming back to the long and short position just as we had defined in case of bonds and stock, in case of forwards/futures, the long position is basically the party which buys the asset

and the party with the short position is the one which sells the asset. So, basically the party with the long position ends up paying the predetermined price, what is we called as the futures or forwards price to the party with the short position.

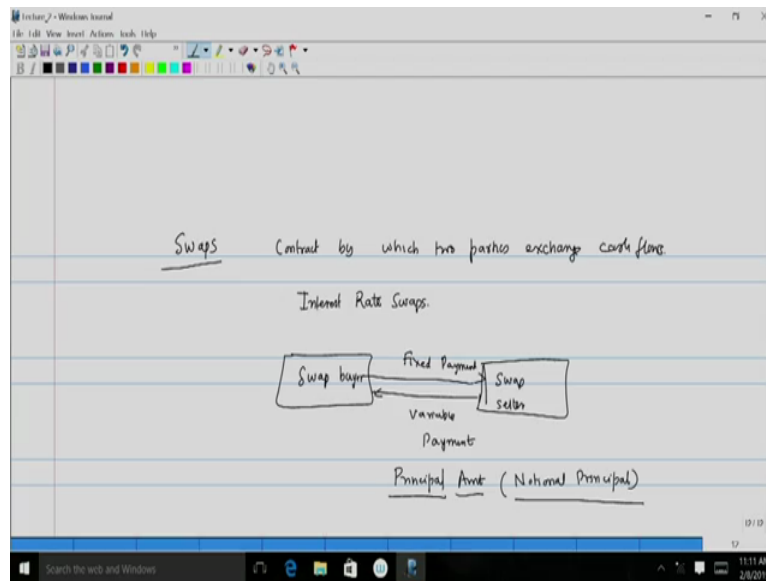
Now, let us talk a little bit in terms of variables for that what we will do is that we will introduce the notation. Suppose that the current time is  $t$  and  $T$  is the maturity, then  $F(t)$  is the forward price and  $S(t)$  is the spot price. Basically,  $S(t)$  essentially means that  $S(t)$  is the price at which you can purchase the underlying asset, say, the stock at the current time and  $F(t)$  is the price that you agree upon, but you do not make the purchase now, but you make the purchase at time  $T$ . These are the two prices that are determined at time  $t$  and  $S(T)$  is basically the market value which we do know at time  $T$ , which is the maturity.

Then let us look at what is going to be the payoff. The payoff for the long position and the short position. What is going to be the payoff? For the long position, this is the person who buys the asset. What price do they buy the asset? So, the payoff will happen at time  $T$ . The person who buys the underlying asset or the stock, they pay the agreed price of the forward. What was the agreed price? The agreed price was  $F(t)$  and since they are making a payment. So, we will put it as  $-F(t)$  and then if the person sells the asset immediately in the market, then the price that they get is  $S(T)$ .

Similarly, the person with a short position what they do is that they sell the underlying asset or the stock to the party with the long position. Since there is  $-F(t)$  which is the amount that is paid by the long position. Short position party they gain an amount of  $F(t)$  and suppose that they had to purchase the stock for  $S(t)$  at that time, so the difference is  $F(t) - t$ . So, one of them is obviously going to be a positive quantity. If it is zero, then it is fine, nobody wins or nobody gains. If  $S(T) - F(t) > 0$ , then the party with the long position gains and the party with the short position loses and vice-versa and if you see that both the terms that we have here  $S(t) - F(t)$  and  $F(t) - S(T)$ , both of them add up to zero and this is what I was referring to earlier as the zero-sum game.

Now, what is the reason for buying the future is that basically people want to hedge, that people are concerned that the price of the underlying asset that they want to buy might fluctuate heavily in the future. So, essentially through a forward and futures agreement, they lock in the price that they have to actually pay at a future time point.

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Now, we move on to another kind of derivatives which are known as the swaps. So, what is the swap? A swap essentially is a contract by which two parties exchange cash flows. One simple example of where a swap is used, that suppose you decide to go and take a home loan or a housing loan, in which case, you have to play a floating interest rate. By this, I mean that the interest rate if it is today, say, 12% it might change at a future time point and you do not want that to happen. So, you basically go to another entity and sign a swap by which you tell the other party, say, I call this party to be B “Okay, I will give you a fixed interest rate, say, 14% and in return you basically give me whatever the floating interest rate is so that I can go and return it to the bank from where I had borrowed”. This is what is known as interest rate swap and it is one of the most common form of swaps. Basically, it is a contract by which two parties exchange cash flows at some future time point, so graphically we can put this, so that means that me who is taking the housing loan and wants to go ahead and only pay the fixed interest loan, I am what is known as a swap buyer and the party which agrees to pay, take the responsibility of the variable interest rate or the floating interest rate in exchange of a fixed rate that I give to them, they are what are known as the swap seller. So, swap buyer will pay the swap seller a fixed rate and the swap seller will receive a fixed rate or a fixed payment and the swap seller will give the swap buyer the variable payment. Another of the typical areas where swaps are actually used are in basically foreign exchange to take care of foreign exchange fluctuations and the principal amount, so I am talking here about a loan so the principal amount or the loan that is borrowed that is not actually exchanged, but this is what is known as, it is only used for the purpose of calculation and this is what is known as the notional principal.

Again, the swaps can also be traded just like a bonds can be sold and bought. So, likewise swaps can also be sold and also there is also the need of valuation of swaps and another feature of swaps is that even though we are exchanging a floating and a fixed rate of interest, it does not necessarily mean that the swap seller will agree to take care of all possible floating interest rates or variable, so there are limits. For example, if currently you have a 12% interest rate and you agreed to exchange this floating rate with the fixed 14% but at some point the floating interest rate might go up to 20% and at which point the swaps seller or the party which is agreed to take care of your variable payments that party will no longer do so and this is what is known as fixing caps, so basically there is an upper cap on this.

This is so much we have to discuss for today. In the next class, we will talk in detail about the last of the three kinds of derivatives, namely, options that we have mentioned earlier and look at some of the basic properties of options and discuss in some amount of detail about the purpose which options serve. Thank you for watching.