Mathematical Finance

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Module 4: Fundamental of Derivatives
Lecture 1: No Arbitrage Principle and Pricing of Forward Contracts

Hello viewers, welcome to this class on Mathematical Finance. We will now begin a new module which will give you a gentle introduction to a little more mathematical aspect of financial derivatives and in particular, we will talk about our first no arbitrage principle and we will look at how this particular no arbitrage principle is actually widely applicable, when it comes to pricing a wide range of financial derivatives. And in particular, we will look at determining the price of a forward contract, we will look at future's contract and finally, we will look at some of the properties of options.

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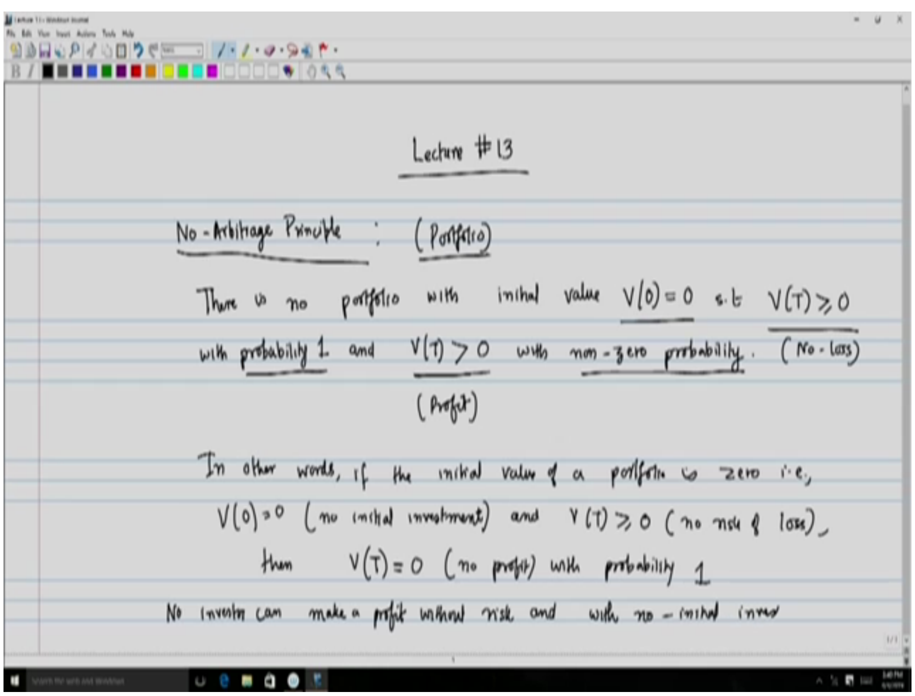

Let us begin this lecture with the no arbitrage principle. So what does the no arbitrage principle say? So we will discuss in the context of (say) a portfolio and it says the following that there is no portfolio with initial value $V(0) = 0$ that means no initial investment. Such that at the final time T the value $V(T) \geq 0$ has probability 1 and $V(T) > 0$ with non-zero probability.

So, what is saying basically that you cannot have a portfolio where you can make a riskless profit. That is, you start off with an initial amount of 0 and you surely either will remain at 0 or will make a profit and there are always the possibility that you will actually make a profit. So this is what I meant by that $V(0) = 0$ and finally either $V(T) \geq 0$, it is probability 1.

So you are certain not make any loss and that is a nonzero probability that $V(T) > 0$, so that means there is always a nonzero probability that you actually make a profit and this means no loss when I have greater than or equal to 0.

So in other words, if the initial value of a portfolio is 0, that is, $V(0) = 0$, which means that no initial investment has been made and $V(T) \geq 0$, that means no risk of loss then you cannot have $V(T) > 0$, this means that you must then have $V(T) = 0$, that is no profit with probability 1.

So from an economic point of view this means that, no investor can make a profit without risk and with no initial investment. Alright, so now that we have made this particular statement that you cannot make a riskless profit, were now going to apply this in case of forward contracts, recall that a forward contract was binding agreement for both the sides wherein one of the parties agrees to sell an underlying asset to another party at a future date for a price that is predetermined at the time of getting into the forward agreement.

And at that point we had asked the question, as to what is going to be the fair price at which this particular asset is going to be sold at a future time point but which has to be specified at the initial time point and we are going to make use of this no-arbitrage principle in order to ascertain this particular price under several conditions.

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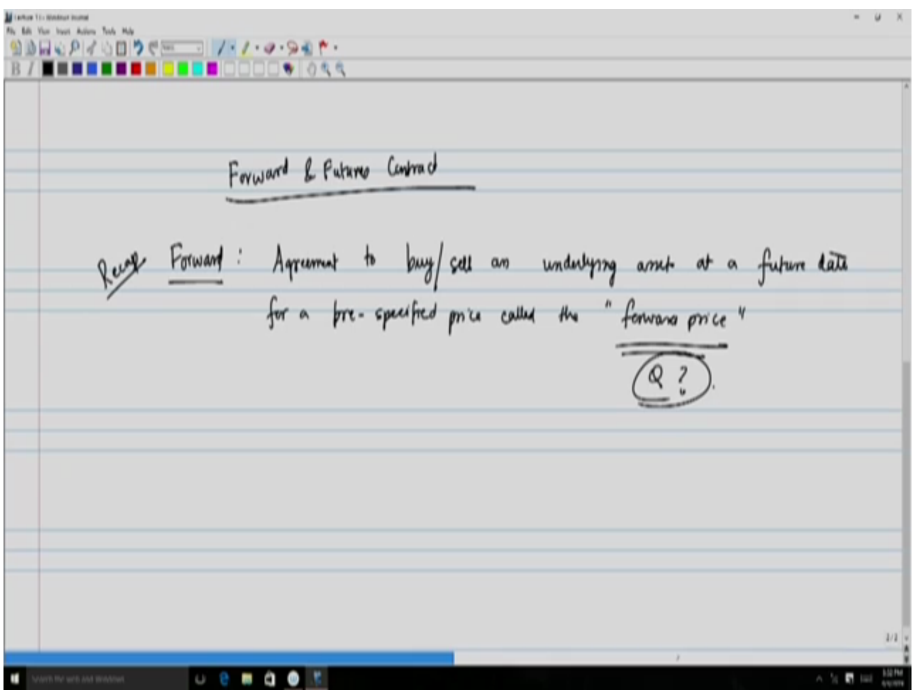

So we begin by talking about forward and futures contract. So we do a recap, so this is just a recap of what you have discussed previously that a forward is an agreement to buy or sell and underlying asset at a future date for a pre-specified price called the forward price, alright. And this is something we are actually going to determine while making use of the no-arbitrage principle.

So, in order to determine the, what is going to be the forward price? We begin by introducing certain terminologies that are applicable and that is customary in case of derivatives.

So, first of all will start talking about long and short position. So what is a short forward position? A short forward position is basically the party is said to have a short for position because they agree, so it is a party which agrees to sell the underlying asset and consequently the long forward position is going to be the party which is obliged to buy the asset. And remember that this is basically binding on both the parties.

Now once you get into an agreement say at initial time period $T = 0$ and the expiration or the time point

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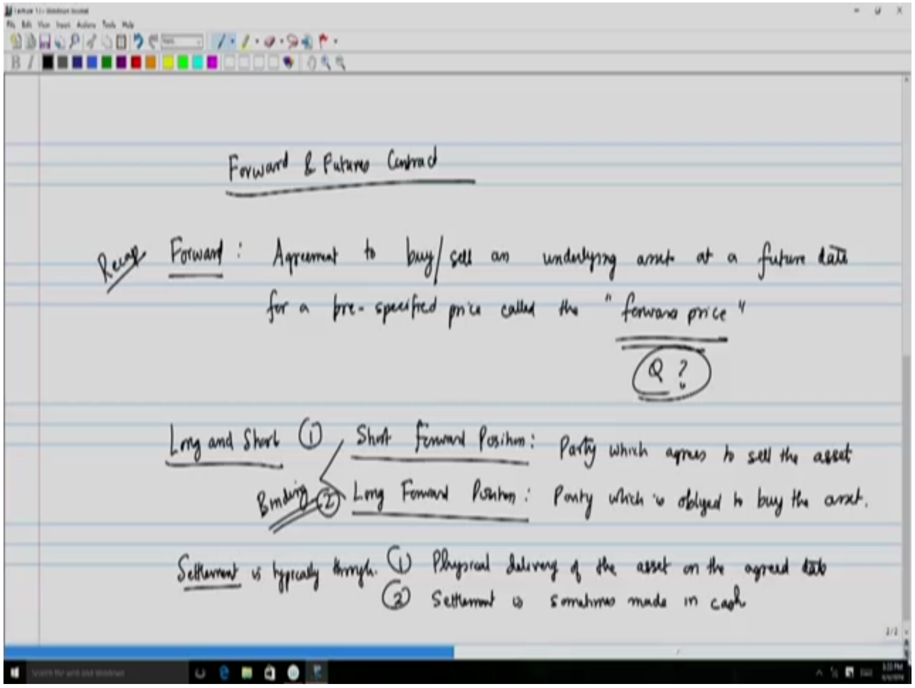

at which the transaction actually takes place in terms of the purchase of the underlying asset, the settlement at that particular time point of expiration T can happen and one of the two ways.

So the settlement is typically through one physical delivery of the asset on the agreed date that means the party with the short position will actually physically deliver the asset to the party with a long position and then receive the payment which is basically the forward price or alternatively the settlement is sometimes made in cash.

So, by this I mean the following that suppose one of the ways you can do it of course is that you just pay the agreed price and you just buy the underlying asset. The alternative is that you settle this in cash where, for example that the price that is agreed upon is actually lower than the current asset price at time T. So in that case the party with the short position instead of actually handing over the particular asset just pays the difference between the forward price and the current price of the asset to the party with the long position and this is what is known as basically the cash settlement.

So it can also happen the other way around when that the party with a long position might actually have to pay the party with the short position in order to make that cash settlement.

So this is it intricately related to the concept of payoffs. So let us start the pricing aspect of it, so accordingly we let the time at which the forward contract is exchanged that means is agreed upon to be $t = 0$ with the delivery time being $t = T$. Further we let the forward price that both the parties agreed upon to be (say) $F(0, T)$. So $F(0, T)$ indicates that this is the agreed upon price, agreed at time 0 for eventual execution at time T.

Now, let $S(t)$ be the market price or the spot price the underlying asset at time t, so basically this means that $S(0)$ is going to be the price at the initial time and $S(T)$ which is a random variable is going to be the price at the final time. Recall that this is the same notation that we had used in case of stocks. So at time $t = 0$, when this agreement is made no payment is made.

So there is no exchange of other assets or any payment and at time $t = T$, the amount $F(0, T)$, which is agreed upon forward price is paid for the particular asset.

Alright, so as I have already said that you agree for a price see the time $t = 0$ the asset price is $S(0)$ and at time T the prevailing market asset price is going to be $S(T)$ but the amount actually paid is going to be $F(0, T)$. So that means there are two prices that are actually relevant at the final time point T.

So which means that we need to consider two cases? So let us take the first case where if your $F(0, T) <$

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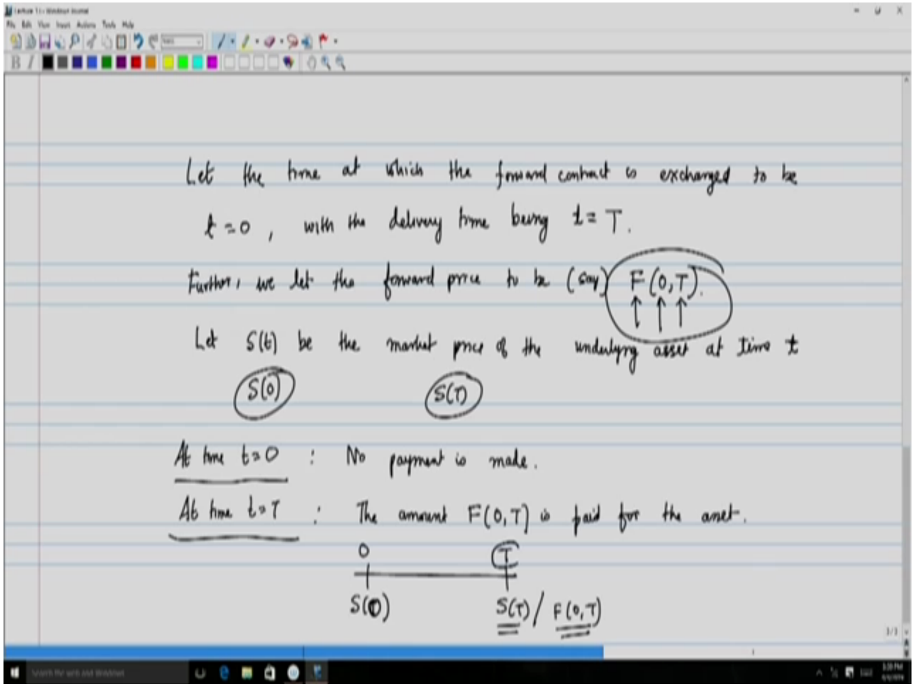

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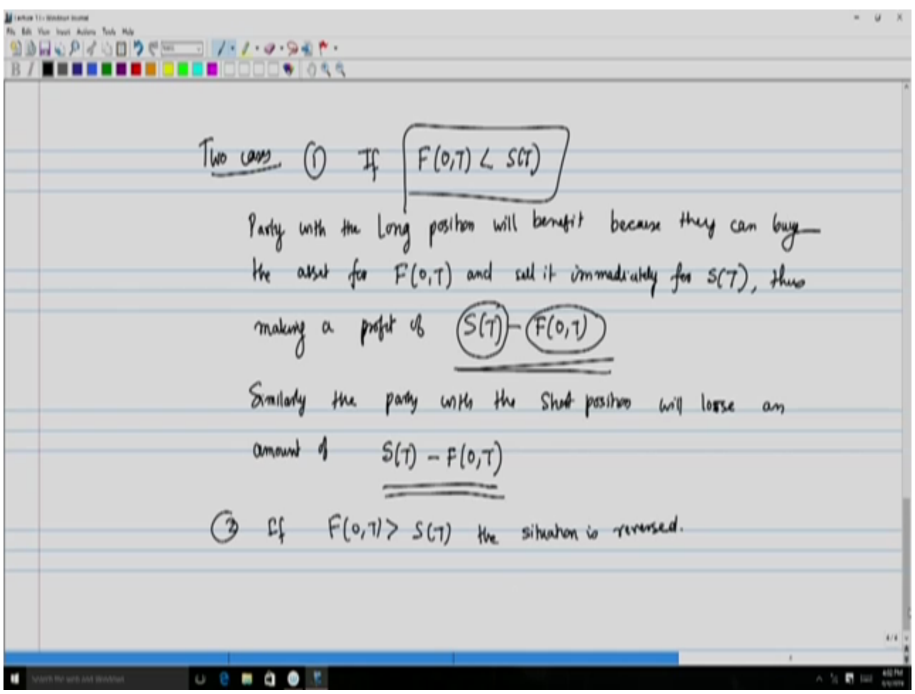

$S(T)$ that means the forward price is less than the prevailing stock price than what happens? Then the party with the long position will benefit, why will the benefit? Because they can buy the asset for the pre-agreed amount of $F(0,T)$ and sell it immediately for the prevailing market price of $S(T)$, thus making a profit of $S(T) - F(0,T)$ that means they have paid $F(0,T)$ and they are receiving $S(T)$ and this is obviously because of this particular assumption this is going to be a positive quantity.

And so, similarly the party with the short position will lose an amount of $S(T) - F(0,T)$ and the reason is that they are being compelled to sell the asset for a lower price F of 0,T whereas had they have not got into the forward contract they could have sold it for a higher price of $S(T)$, so that means they are losing out on the difference of these two amount. So whatever is the gain for the first party that is the party with a long position becomes the loss for the party with the short position showing that this is basically a zero sum game.

Now likewise if $F(0,T) > S(T)$ the situation is reversed and in this case the party with the short position will again and the party with the long position is basically going to lose.

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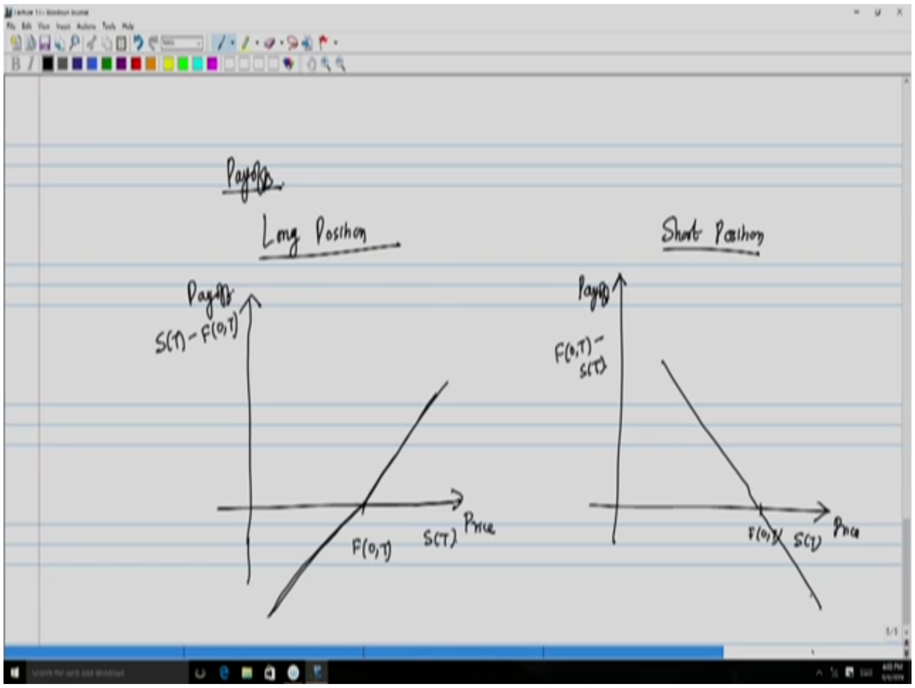

So what are going to be the Pay-offs? Alright. So let us look at a graphical representation of this. So first consider the party with the long position and then consider the party with the short position. We take a two-dimensional post to this, so on the x-axis I will have the price that is my S of T and on the y-axis I will have the payoffs which in case of long position is $S(t) - F(0,T)$ and in case of the short position this is $F(0,T) - S(T)$ and this is going to be the agreed upon price $F(0,T)$, then the payoff will look something like this, okay.

So as you are on the right of $F(0,T)$ that means $S(T) > F(0,T)$ there is a positive payoff otherwise there is a negative payoff and likewise in case of the short position the payoff will look something like this, okay.

So we now make an observation and observation is the following that if the contract is initiated at time $t < T$ instead of $t = 0$, then we will denote the forward price by, so F is a forward price, t is the expiration and the first entry here which denotes the time at which the contract is initiated this is going to be small t with the payoffs, obviously being denoted or rather is actually is $S(T) - F(t,T)$ and $F(t,T) - S(T)$ for the long and the short position respectively.

So now we come to the critical question of what is a forward price. Alright, so in order to address this question, what we do is that, we consider an underlying asset such as a stock which pays no dividends, remember dividends was basically a share of profit that goes to the shareholders, so it is not dividends.

Further let r denote the risk-free rate and this risk-free rate is used using continuous compounding which is assumed to be unchanged or constant during the time window of 0 to T that means during the duration of the of the forward contract or during the lifetime of the forward contract.

So then we have following theorem. So it states the following, for a stock which pays no dividends, the forward price is $F(0,T) = S(0)e^{rT}$, so this means that if you are getting into a forward contract than both the parties must recognize that if the underlying asset is a stock which does not pay any dividend then

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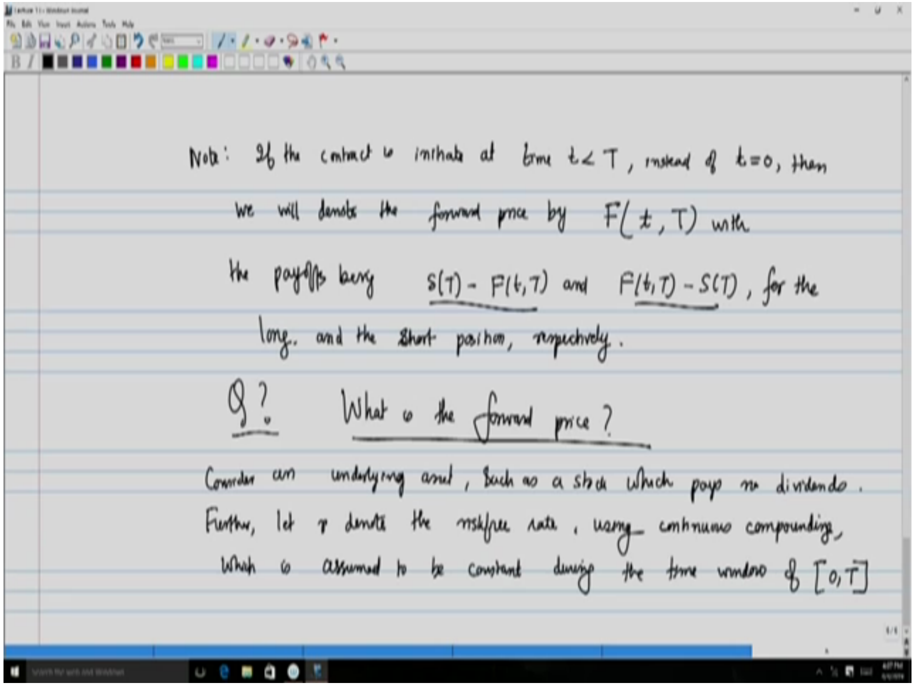

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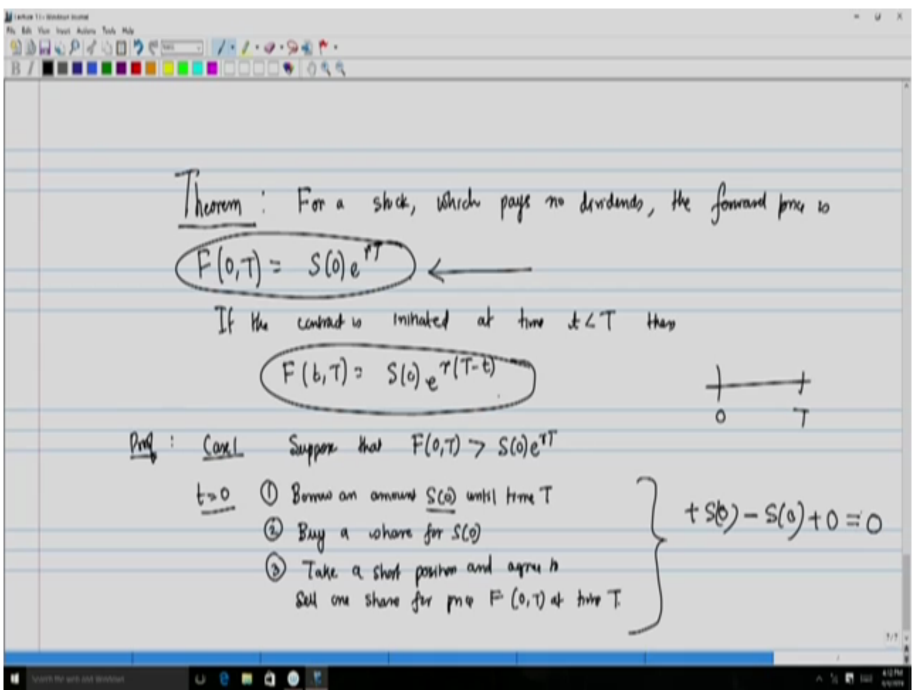

the price that is agreed upon for a stock at a future time T when the interest rate or the risk-free rate is r is continuously compounded and remains unchanged during the lifetime of the contract than the price of the forward contract will just be the current price of the underlying asset $S(0)$ multiplied by the growth factor of e^{rT} as if it would have been invested in a risk-free security with initial investment of the $S(0)$.

So now let us look at another observation out of this. So it says that if the contract is initiated at time t instead of $t = 0$, then obviously we need to modify this as $F(0,T)$, $S(0)$ and e^r and instead of T, we just take into account the remaining period of the contract that is $T - t$. So what I will do is, we will do the proof for this one and sort of conclude in an analogous way that this is also true.

So while you are doing the proof we will make use of a no arbitrage principle and how we are going to make use of the no arbitrage principle is the following. That we are targeting to prove that this is the result and we will assume that this does not hold which gives rise to two cases. Either $F(0,T) > S(0)e^{rT}$ or it is less then $S(0)e^{rT}$ and in breach of this case is one by one we will see that if we assume that one of them is larger than the other it always leads to an arbitrage opportunity which cannot happen as a result of the no arbitrage principle and hence the only other possibility is that both of them are actually equal because you have ruled out the possibility that one of them is greater than the other.

So accordingly we will first look at say this case when we suppose that the forward price $F(0,T) > S(0)e^{rT}$, then let us see what happens at time $t = 0$. Remember we are basically looking at this time window $[0,T]$, so what is going to be a strategy at time $T = 0$. At time $T = 0$ what we would do is that, we will borrow an amount $S(0)$ until time T.

Secondly, we use this amount $S(0)$ to buy a share for $S(0)$ and then we take a short position and agree to sell one share for price $F(0,T)$ at time T. So what have we done here? So while I am borrowing an amount of $S(0)$, so that means I receive an amount of $S(0)$, then I used the money to buy a share, so this is going to be $-S(0)$ and you get into the forward contract while you basically don't need to make any investment, so the net loss or gain here is going to 0. So that means your overall investment is basically going to be equal to 0.

So this means that in the power lines of the no arbitrage principle your $V(0) = 0$. So now what you will do is that, we will endeavor to prove that actually this strategy lets to $V(T) > 0$, which immediately violates the no arbitrage principle.

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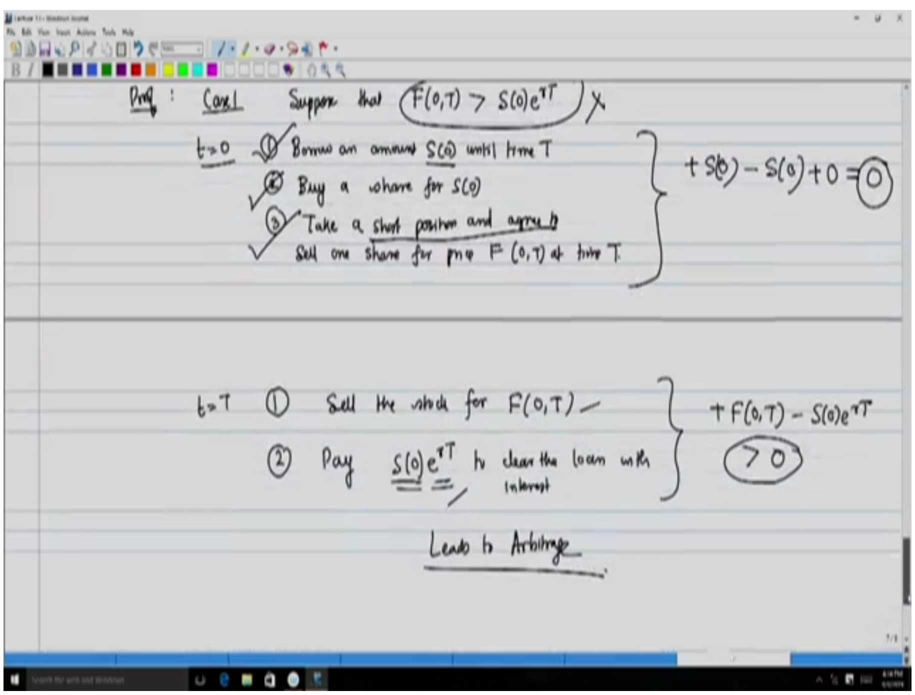

So accordingly now what is going to happen at time $t = T$? So at time $t = T$, what would be the following? You have taken a short position to sell the stock, so obviously you will have to then sell the stock because it is a binding contract, so we sell the stock for $F(0,T)$, so this obligation is taken care of and we have gotten rid of the stock that we have and the other obligation that we have is that we have borrowed an amount $S(0)$.

So accordingly what we have to do is that, we will then pay an amount of $S(0)e^{rT}$ to clear the loan with of course interest, right? This was the loan amount and this is basically the interest amount, so this creates all the position taken at time $t = 0$. So that means what? When you sell the stock for $F(0,T)$ you receive an amount of $F(0,T)$ and when you pay an amount, this amount $S(0)e^{rT}$, so you subtract an amount of $S(0)e^{rT}$.

And this, by this assumption here this is going to be strictly greater than 0, so this means that you start off with an initial amount of 0, but you end up surely making the profit which is positive and therefore this leads to arbitrage. So this means that this assumption has been proved to be incorrect.

Alright, so this is one side of the inequality, let us look at the second case where $F(0,T) < S(0)e^{rT}$, so this is going to be my case 2.

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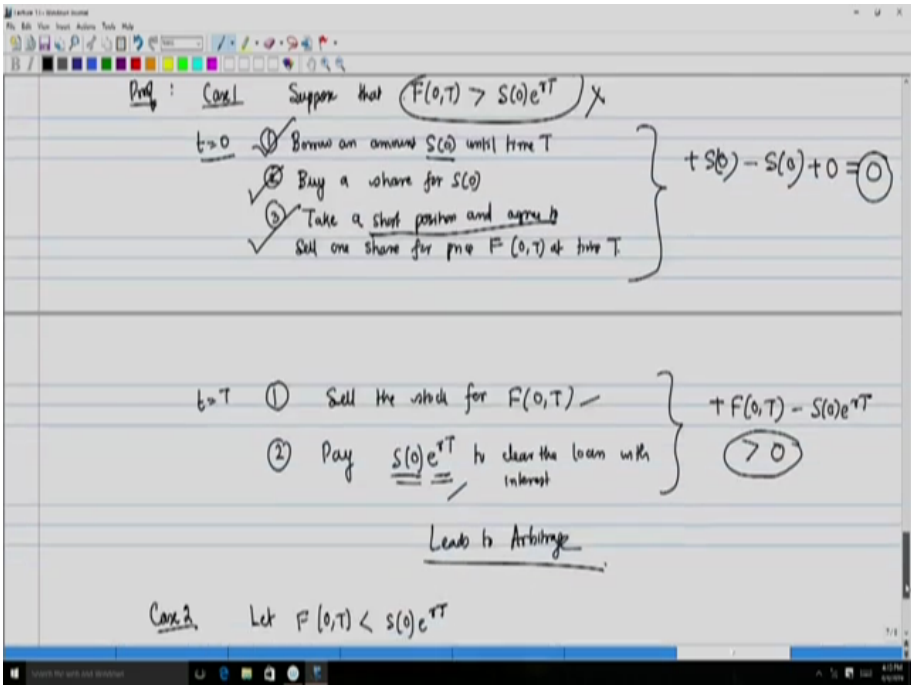

So in this case we let $F(0,T) < S(0)e^{rT}$, right?

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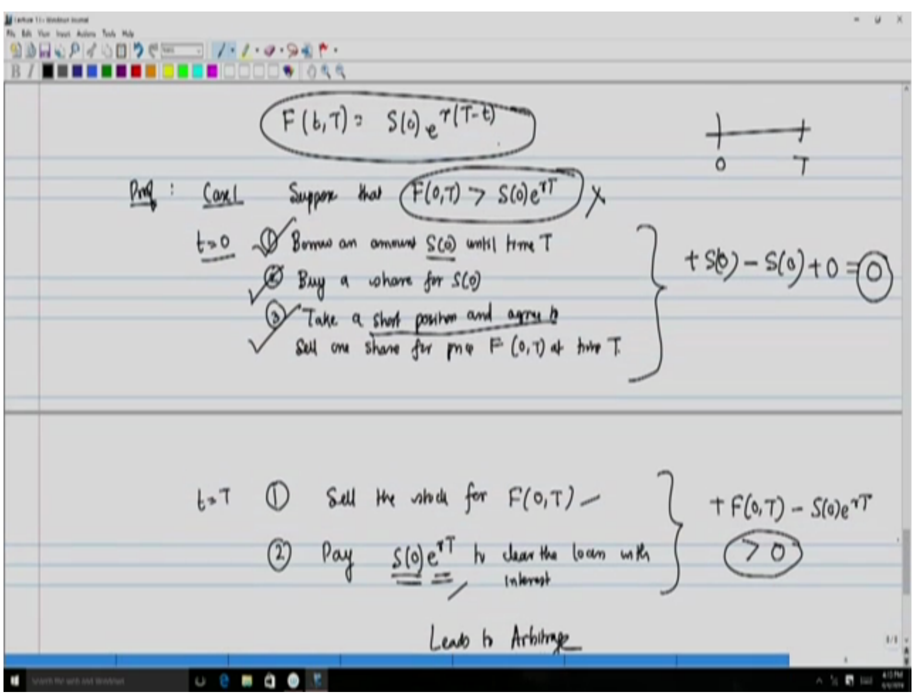

Just the compliment of this.

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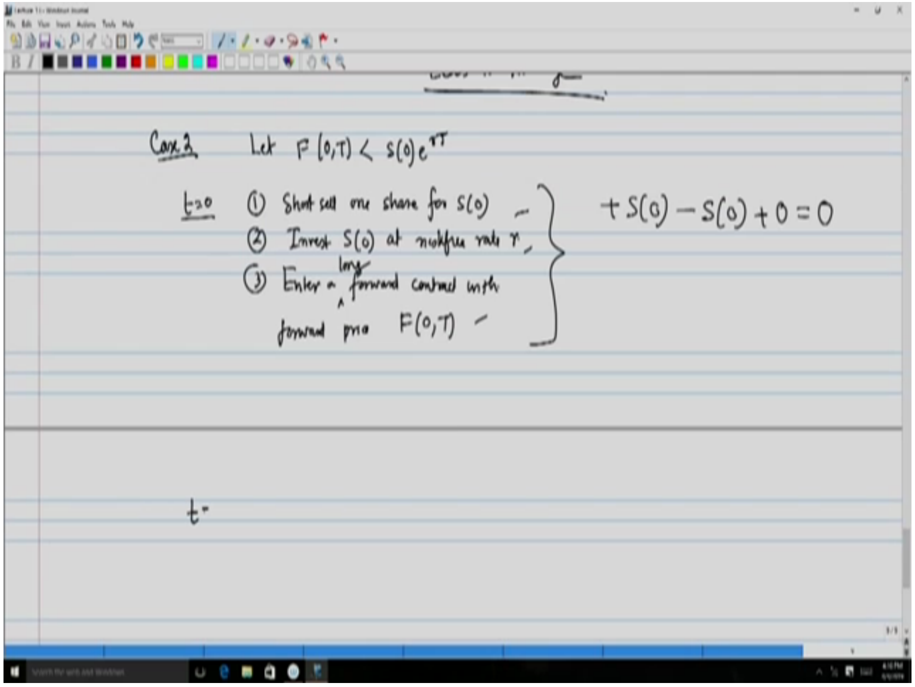

So what you will do? What is going to be my strategy at time $t = 0$? The first strategy would be we will short sell that means borrow somebody as a stock, one share for S0 once you receive this amount of $S(0)$ what you will do is that, we will invest $S(0)$ at risk-free rate r and third we will enter as long forward contract with forward price $F(0, T)$.

So when you short sell one stock you receive an amount of $S(0)$ when you invest $S(0)$ at risk-free rate that means you spend amount of $S(0)$ and because you have entered along forward contract there is no investment, so initial value of this position is going to be equal to 0.

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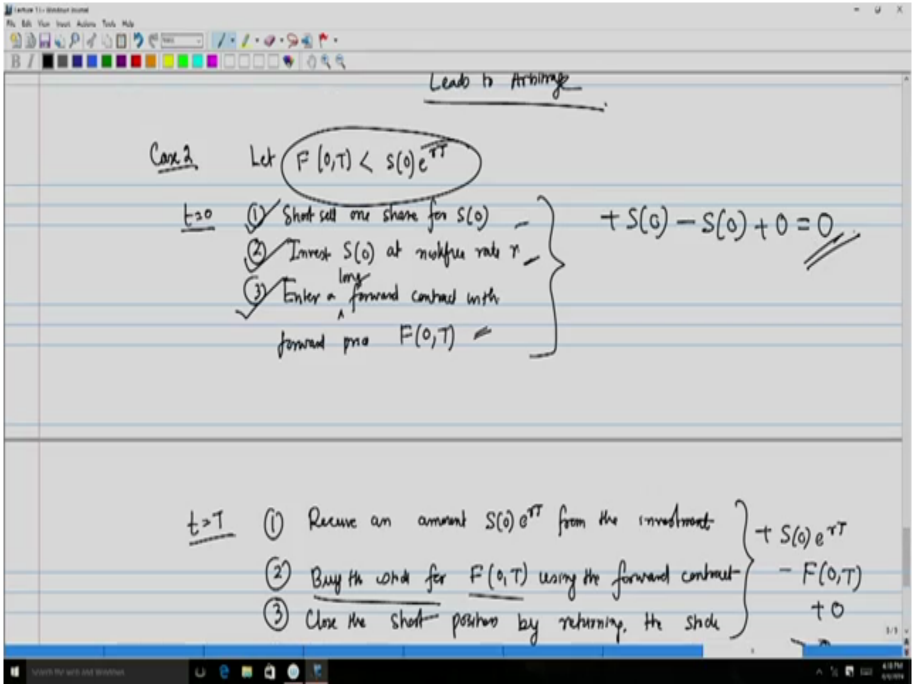

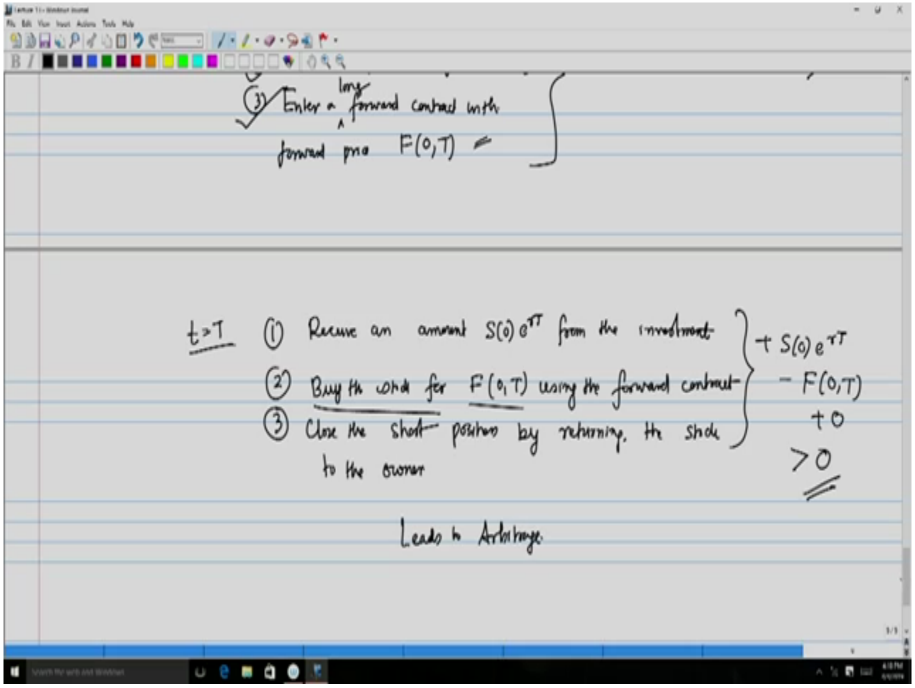

Now, let us see what happens at time t equal to T, at $t = T$, from your investment of $S(0)$ at rate r, so you will receive an amount $S(0)e^{rT}$ from the investment.

Now, so this has taken care of this particular position that you have received the amount. Now, you had gone into a long followed contract, so we have to close this position. So accordingly you buy the stock for what amount $F(0, T)$ using the Forward Contract and finally now that you own a stock you are now in a position to close this outstanding position of the short selling that you had done.

So, that means you can close the short position by a returning the stock that you have purchased by returning the stock to the owner. So, this means that what is the gain or loss, so you have received an amount of $S(0)e^{rT}$, you have bought a stock for $F(0)$, so an amount of $F(0, T)$ goes out and plus you have closed the short position so there is no transaction.

And this by assumption by this assumption the $V(T)$ or the final value is going to be strictly greater than 0. So this means that an initial investment of 0 leads you to a positive profit and so accordingly leads to arbitrage.

So this means that both this condition as well as this condition they both of them they lead to an arbitrage situation so they cannot actually hold.

So therefore only the third possibility that is $F(0, T) = S(0)e^{rT}$ this holds. Okay, so let me just try to motivate actually how I came up with this particular argument.

See in this particular argument, what was my hint? My hint was that that my $F(0, T) > S(0)e^{rT}$. So this gave me the hint that at time T, I should be receiving $F(0, T)$ amount of money and I should actually be spreading $S(0)e^{rT}$. Now, how do I spend $S(0)e^{rT}$? The way I spent is that that means I have to invest an amount of $S(0)$ at time $t = 0$, which will grow to $S(0)e^{rT}$.

So that is the reason why I borrowed an amount of $S(0)$. Now, when I borrowed amount of $S(0)$, what I must supposed to do with it. So, I decide that, okay I will buy it the shear. So it takes care of two things, one is that then I have ensured that yes this amount of $S(0)$ will grow to $S(0)e^{rT}$ and now that I have used the money to purchase the shear that means I have the shear to basically fulfill my obligation by taking a short position in the stock.

Likewise for the second case when I see that $F(0, T) < S(0)e^{rT}$, this gives me the suggestion that $F(0, T)$ is the amount that I should spend which means that I should be purchasing a stock at time T for $F(0, T)$. So, what this means is the following that in order to do that and this is a larger quantity and that means I will receive an amount of $S(0)e^{rT}$.

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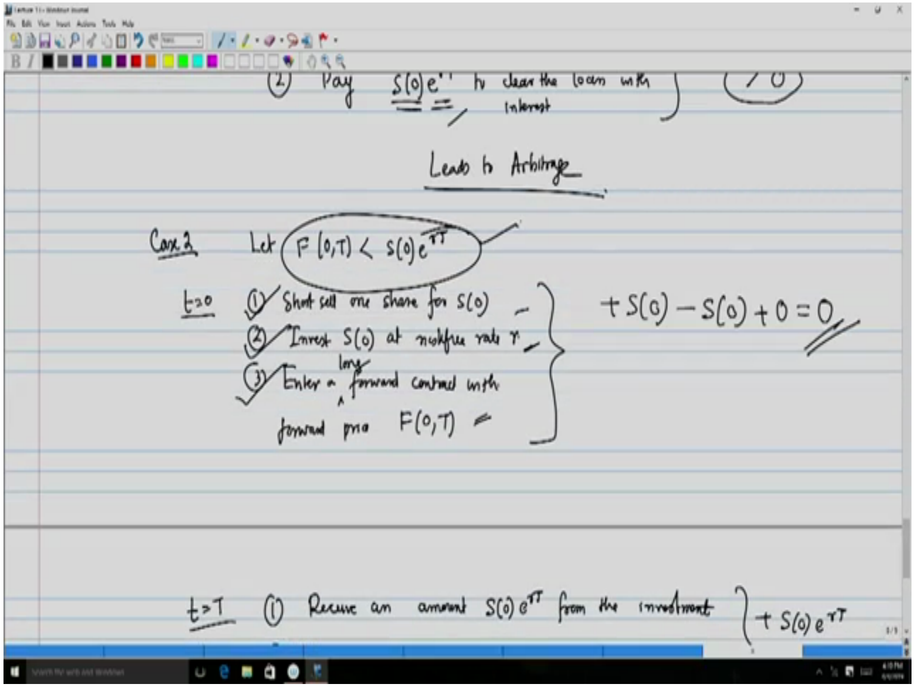

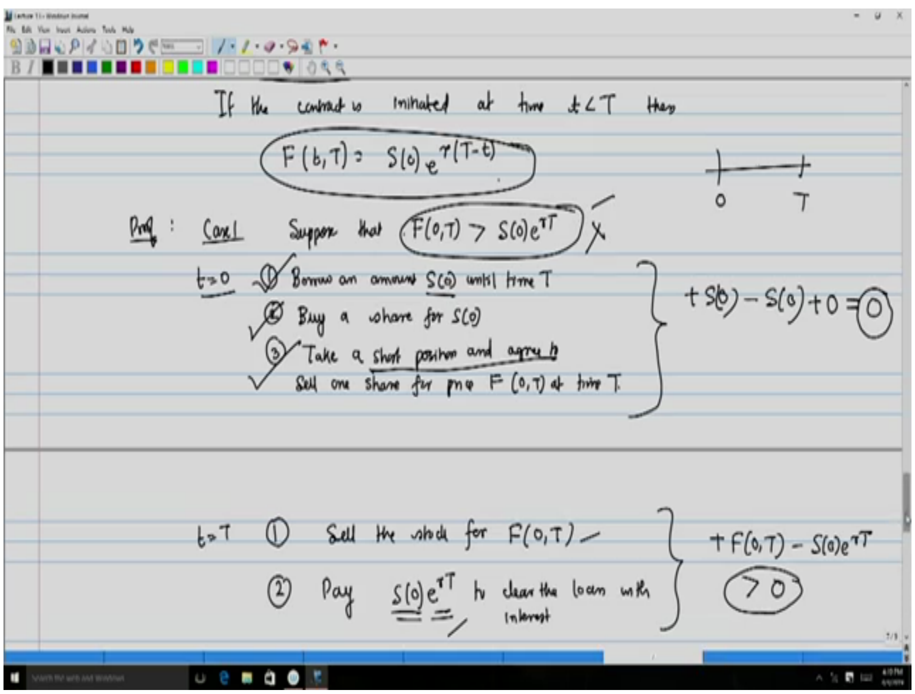

Now, how do I actually get an amount of $(0)e^{rT}$ at time T? That means I will have to make an investment of $S(0)$ at time $t = 0$. How do I raise the money for $S(0)$? That is why I go ahead and do a short selling and raise the money $S(0)$ and also this will automatically cover my situation that I will eventually purchase a stock for $F(0, T)$ and so that I am not wondering as to what I should be doing with this particular stock that I have purchased, by short selling I have also taken care of that that I am just not getting held up with the particular stock because I am anyway going to sell it for F or rather buy it for $F(0, T)$ to cover my short position.

Okay, so the similar argument you can just take $t = 0$, you change this to $t = T$ and you essentially this will give you a result $F(t, T) = S(t)e^{r(T-t)}$.

So just a little correction here, this should be t.

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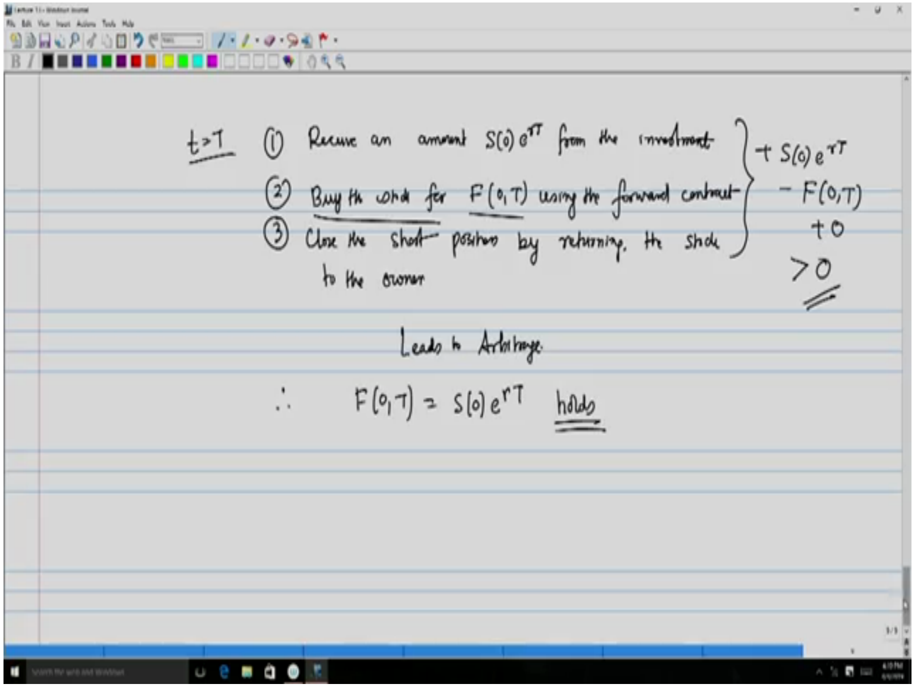

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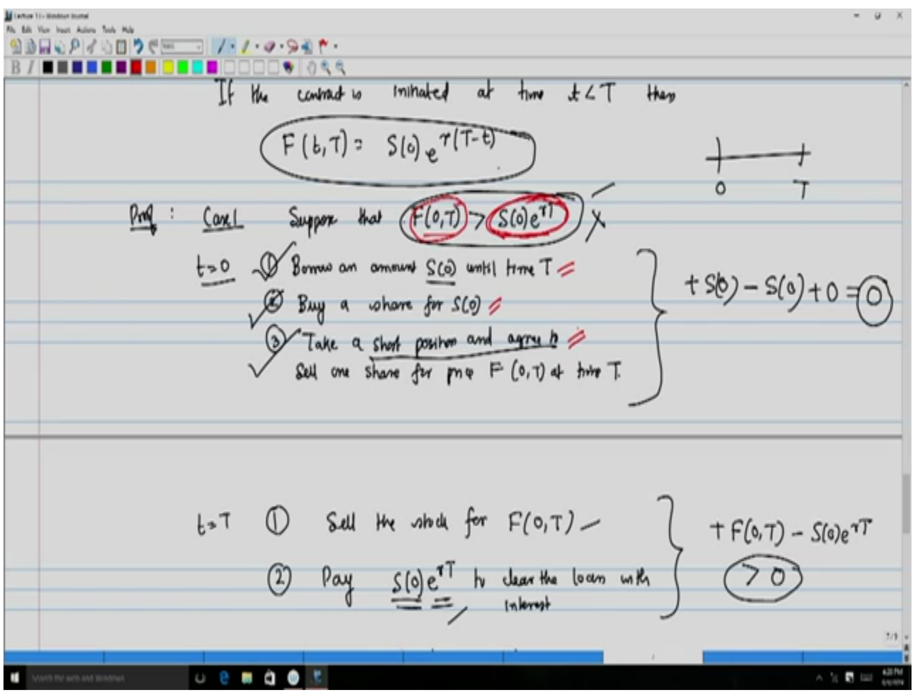

Okay, so next we will look at a case when actually dividends are included. So let us now look at the scenario when you can actually including dividends.

So with dividend as state the following theorem, so it states the following that the forwards price of a stock paying dividend let me call this dividend to be a small division at time t which is different from the initial or the final time, so strictly lies between 0 and T is given by $F(0,T) = [S(0) - e^{-rT}div]e^{rT}$.

Notice that I mean when the dividend was not there, that is a dividend equal to 0, then this term vanishes and you basically recover the previous result of $F(0,T) = S(0)e^{rT}$. So the proof actually goes on similar lines. So accordingly, we will look at two cases. So, first of all, let us look at case 1. In case 1, I will assume

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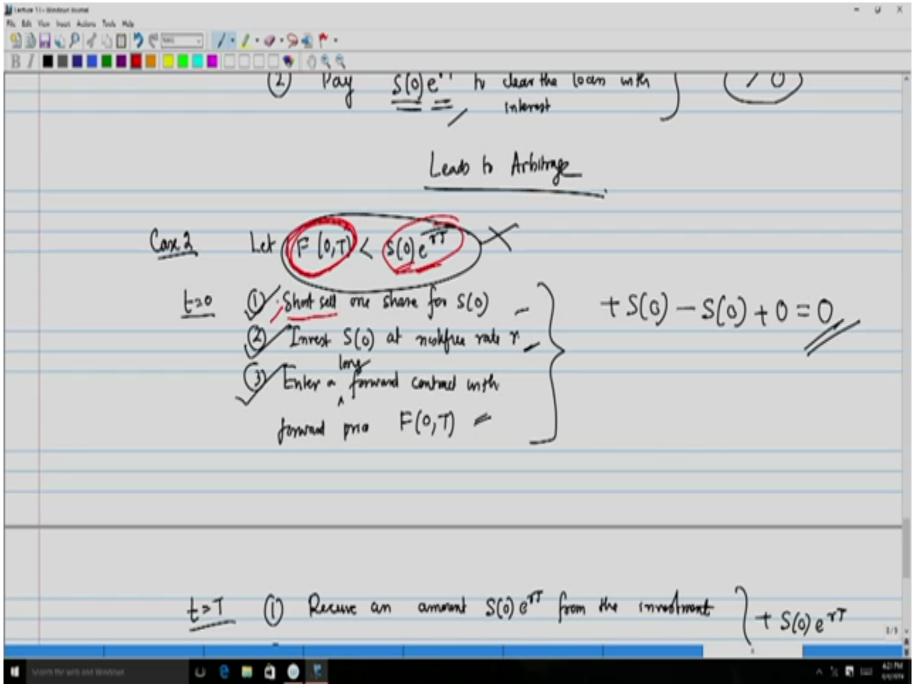

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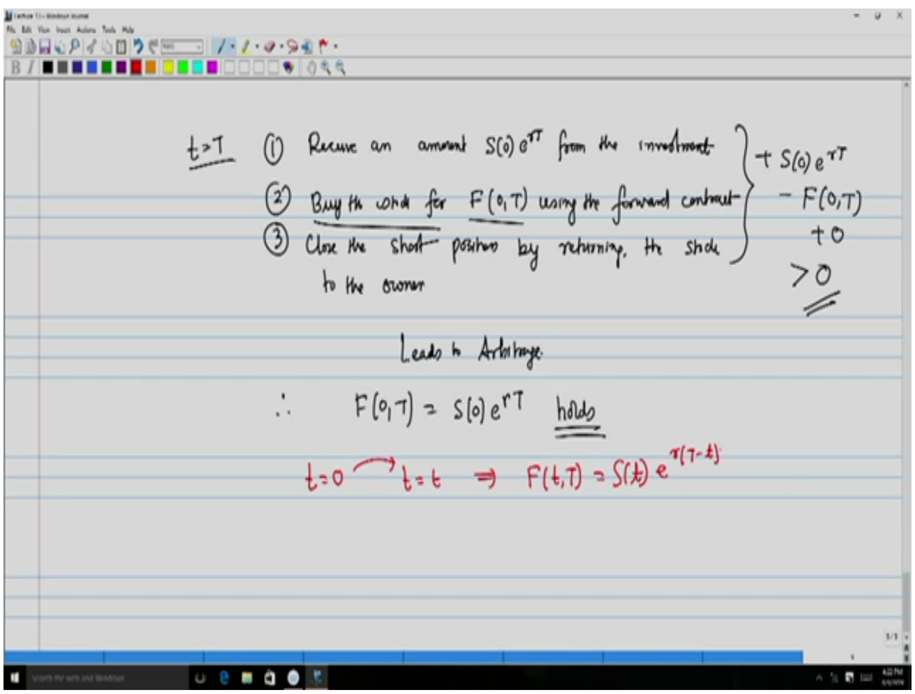

that $F(0,T)$ that means this quantity is greater than this quantity, so $F(0,T) > [S(0) - e^{-rT}div]e^{rT}$.

So, what is going to be my strategy? At time $t = 0$, I will go with the following strategy, that in order to get to be this quantity minus this quantity to be positive I need to receive this amount of money at time T and spend this amount of money at time T.

So accordingly, if I receive $F(0,T)$, this means that I will sell the underlying asset for $F(0,T)$ which means that I will get into a short position of a forward contract on the underlying dividend paying asset. So this means that the strategy would be enter into a short forward contract with forward price $F(0,T)$.

Now, when I get into a short forward contract, it means that I have taken the obligation to sell the asset.

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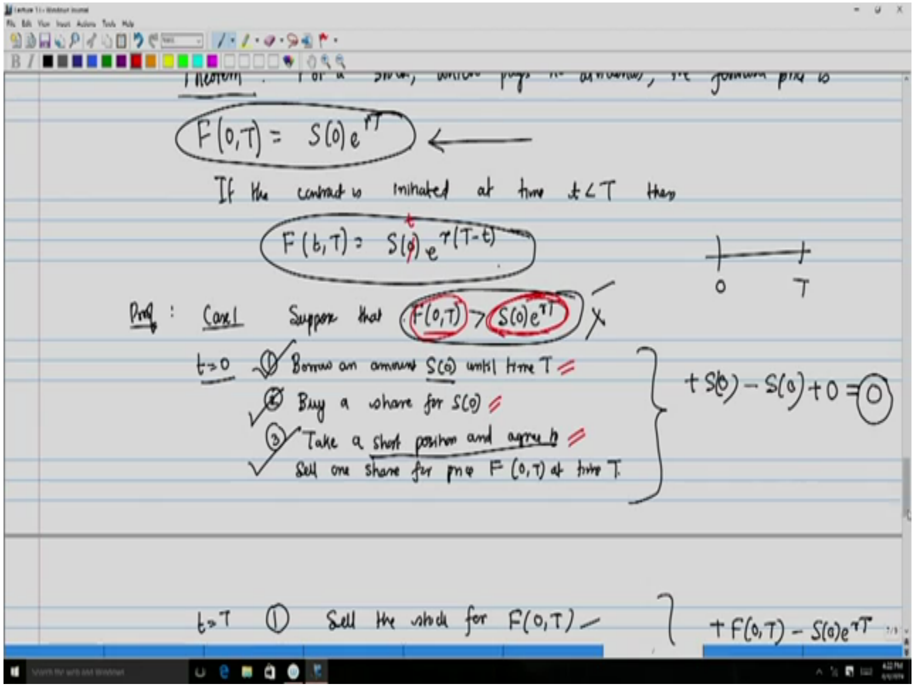

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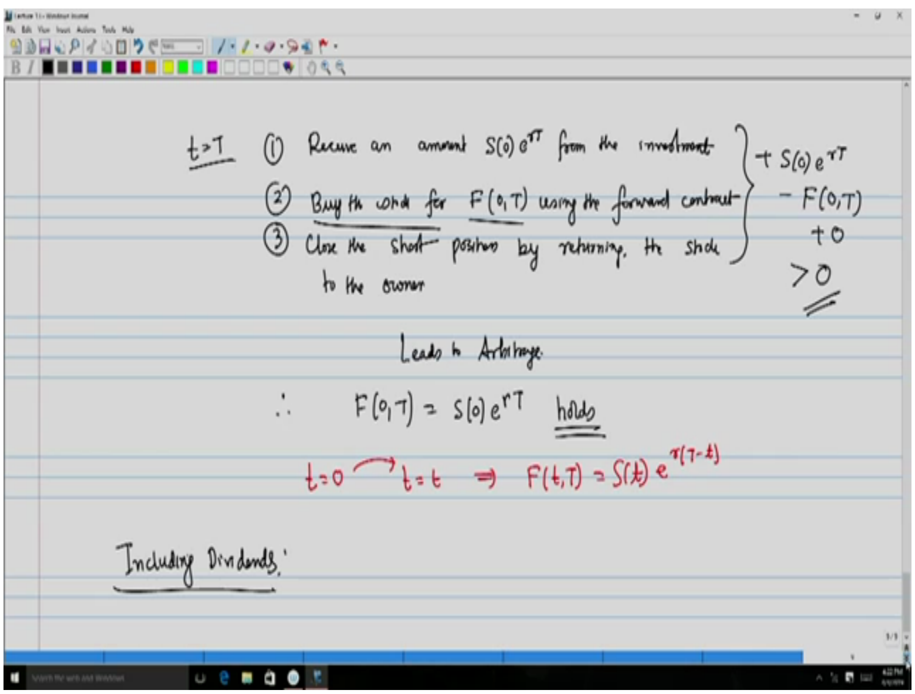

So accordingly, I need to buy an asset. So, how do I get the money to buy the asset? I will, accordingly in order to get the money to buy the asset I will borrow an amount of $S(0)$ at interest rate r for time T.

And then use this money $S(0)$ to buy one share price $S(0)$. So, what is going to be the net money transfer? When you enter the short forward contract it does not cost you anything, when you borrow an amount of $S(0)$, so that means it is $S(0)$, and when you use the money to buy one shear that means you have spent an amount of $S(0)$, so that means your net investment is basically going to be equal to 0.

So, then I basically need to just show that this particular investment at time T will end up (give me) giving me a positive value. However, this case is slightly different from the previous case that you have to

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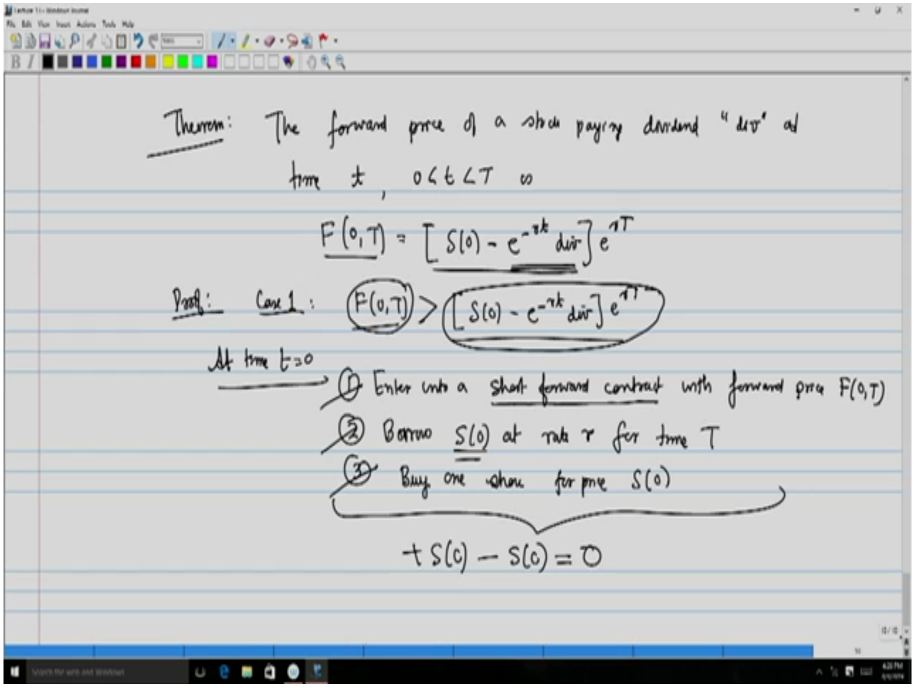

account for the dividend that is being paid at time t.

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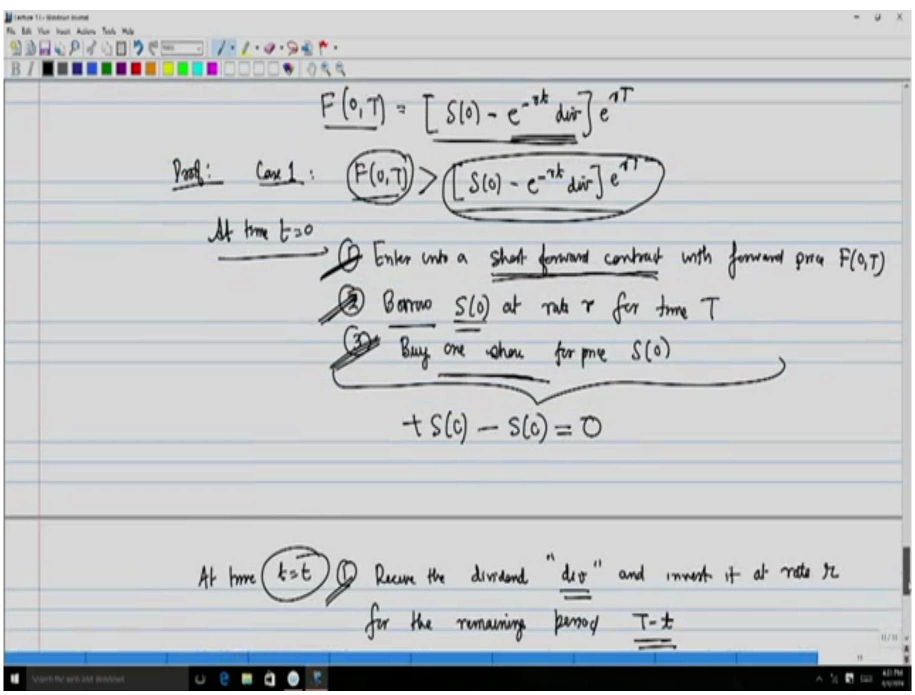
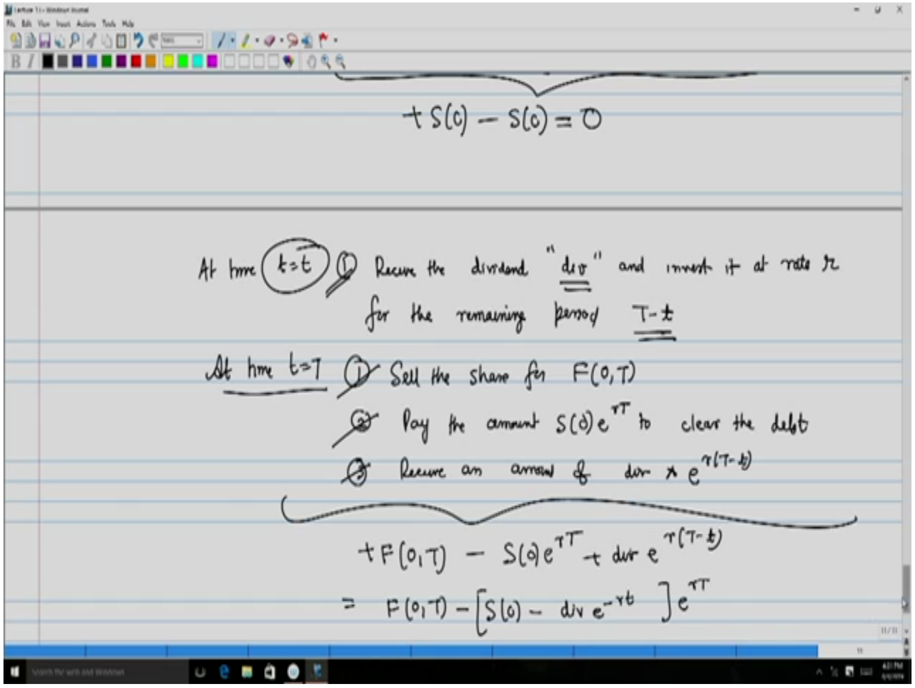


So at time t equal to t, you receive a dividend, right? Because you actually own the shear. So what do you do is that you receive the dividend, the dividend "div" and invest it at rate small r for the remaining period. What is the remaining period? Because you have done this at time t, so the remaining period is $T - t$.

Now, you arrive at the final time. At time t is equal to T, what happens? First of all, you have already purchased the shear and you had got into a short forward contract. So, you basically close the forward position. So, then you sell the shear for $F(0, T)$. So that means that you have gotten rid of your shear and



you have closed your short position in the forward contract.

Next, you had borrowed an amount of $S(0)$ at interest rate for, at interest rate r for time T. So that means you have to pay the amount of $S(0)e^{rT}$ to clear the debt. And finally, so this position is also taken care of. The only position that you have not closed is that you had remember you had received an amount of div.

So finally, you receive an amount of which is what you had invested into remember that you had invested amount of div for the remaining period $T-t$ and we had assumed that the interest rate r remains unchanged during the period so it is dividend into $e^{r(T-t)}$.

So, then what is going to be your net gain or loss? So, here you sold the shear for $F(0,T)$, so you receive $F(0,T)$, then you pay an amount of $S(0)e^{rT}$, so I have minus $S(0)e^{rT}$ and then you receive an amount of div into $e^{r(T-t)}$. And if you look carefully what was my assumption? My assumption, so this can be rewritten as $F(0,T) - [S(0) - dive^{-rt}]e^{rT}$. All right?

We had, we factor out e^{rT} and we have $S(0) - div\ e^{-rT}$, alright. So, that means if you go back to this assumption here, that $F(0,T) > [S(0) - e^{-rt}div]e^{rt}$, so it turns out that this particular assumption leads to the final value to be greater than 0 and this basically this arbitrage. So, this means that our inner end assumption for case 1, this is not correct.

Now, let us look at case 2. For case 2, we assume that $F(0,T) < [S(0) - e^{-rt}\ div]e^{rt}$. Remember that this is just the opposite of the previous assumption. Earlier $F(0,T)$ was the larger and now we assume that this is basically going to be the smaller. So, what happens, at time $T=0$, let us say, what should be the strategy?

Now, remember that here eventually I want that this term should be bigger than this term, so this means that $F(0,T)$ is the amount that actually I spent. So, when I spend in amount of $F(0,T)$, this means that I have purchased the underlying stock for an amount of $F(0,T)$. So, which gives me the suggestion that this means at time $T=0$, I would have gotten into a long position of the forward contract and agreed to buy the underlying asset for an amount of $F(0,T)$.

So, this means that at time $T=0$, we will enter into a long forward contract with forward price $F(0,T)$. Okay. So, now when I attain to a long forward contract this means that eventually I know that I will pay an amount of $F(0,T)$ for a share and at time $t=T$, I will have the ownership of a share. So, this means I will have a disposable share at time T, so that means that I can now safely take up a position where I actually short sell a share.

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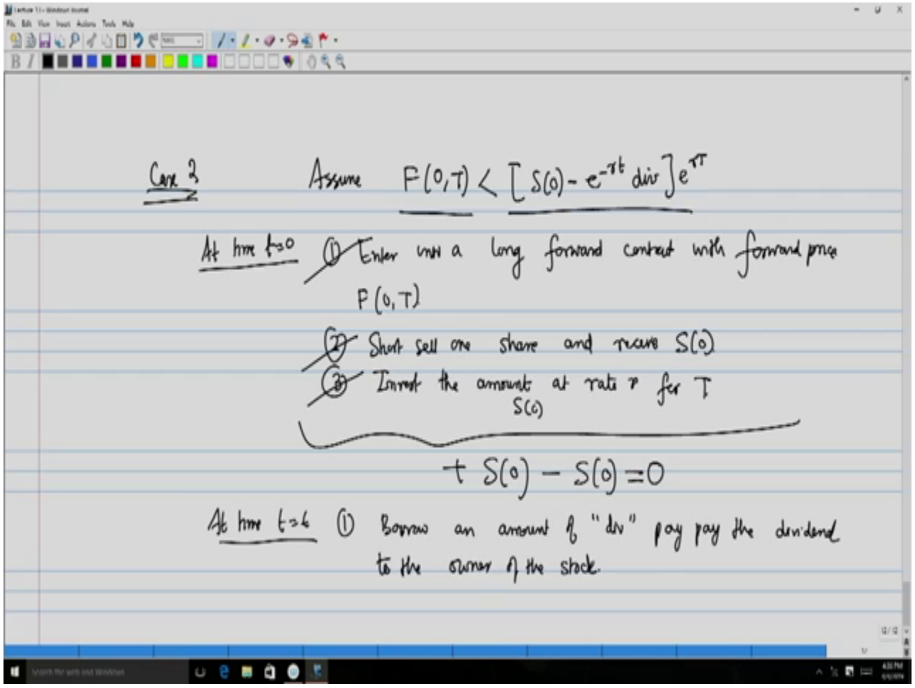

Accordingly this will give me the hint that I will short sell one share and receive $S(0)$ and I will then invest the amount $S(0)$ obtained by short selling at rate r for time T. So, what is going to be the net amount of money here? So, you are nothing, no cost is involved in getting into the long forward contract. When you short sell you receive an amount of plus $S(0)$ and then you invest the amount at risk free rate r, so an amount of $S(0)$ goes out, so your initial value is going to be $V(0)$.

Now, please remember that at time $T = t$, when the company pays the dividend that means that dividend is actually due to the owner of the stock, which means that since you have borrowed the stock from the owner and shorts, have got into a short selling position, this means that it is your obligation then to pay the owner of the stock, the dividend, remember that you have actually borrowed the stock and you have sold it, so you are not getting a dividend from the company because you are no longer in the ownership of that stock because you have sold the stock for $S(0)$ and have invested that at an interest rate r.

But since, you had borrowed from the owner of the stock, your obligation still remains that you have to pay the dividend back to the owner, so the only choice that you have that is basically you have to borrow the dividend amount of "div" and pay the dividend to the owner of the stock.

Now, let us see what happens at time $t = T$. So, at time $t = T$, first of all you you have a forward position, you had entered a long forward position so you close this so you buy a share for $F(0, T)$. Secondly, you close out the short position in the stock, okay. So, by short position, I mean that you have short sold the asset, so once you have used the long position in the forward contract to get the asset you simply return the asset or the stock to the owner.

And remember that you have invested an amount of $S(0)$ at rate r so this means that you will get this amount back, so you will receive an amount $S(0)e^{rt}$ with the interest. And finally at time t you had borrowed an amount of dividend for the remaining period so that means that you have to now pay an amount of dividend into $e^{r(T-t)}$ to clear the loan amount with interests.

So, this means that when you bought the share of $F(0, T)$ so you have spent an amount of $F(0, T)$ when you close the position then there is no transaction actually taking place, you have received an amount of $S(0)e^{rt}$ and you pay an amount of $-div\ e^{r(T-t)}$. And this can be rewritten as $e^{rT}S(0) - div\ e^{rt} - F(0, T)$.

Remember that we had, the assumption was that $F(0, T) < [S(0) - e^{-rt}div]e^{rT}$.

So, this means that since $F(0)$ is less than the quantity so this is going to greater than 0. So, again you see that you had made an initial investment of 0 and it has resulted into a riskless profit, and this means

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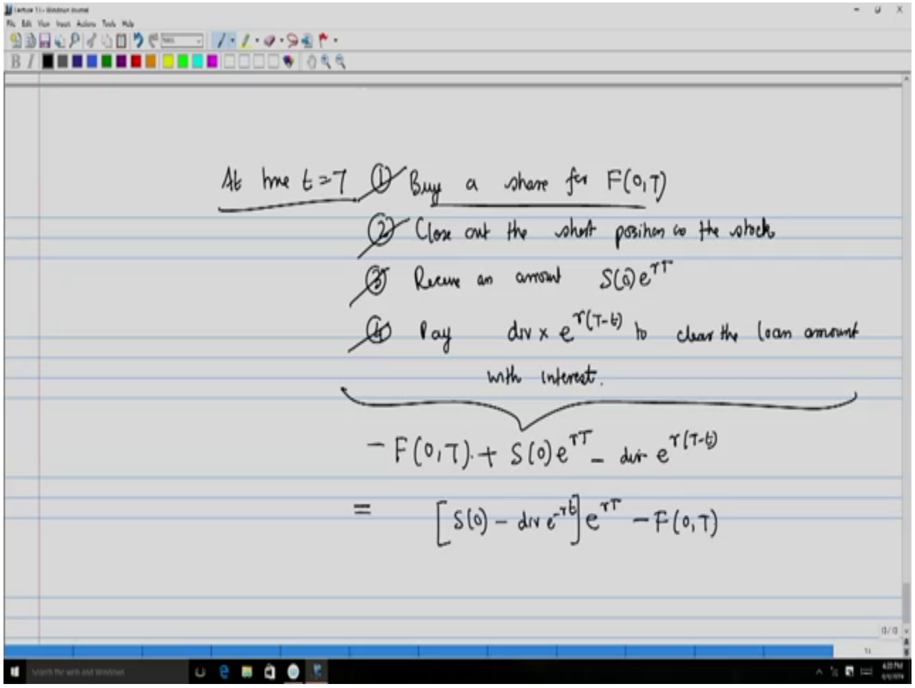

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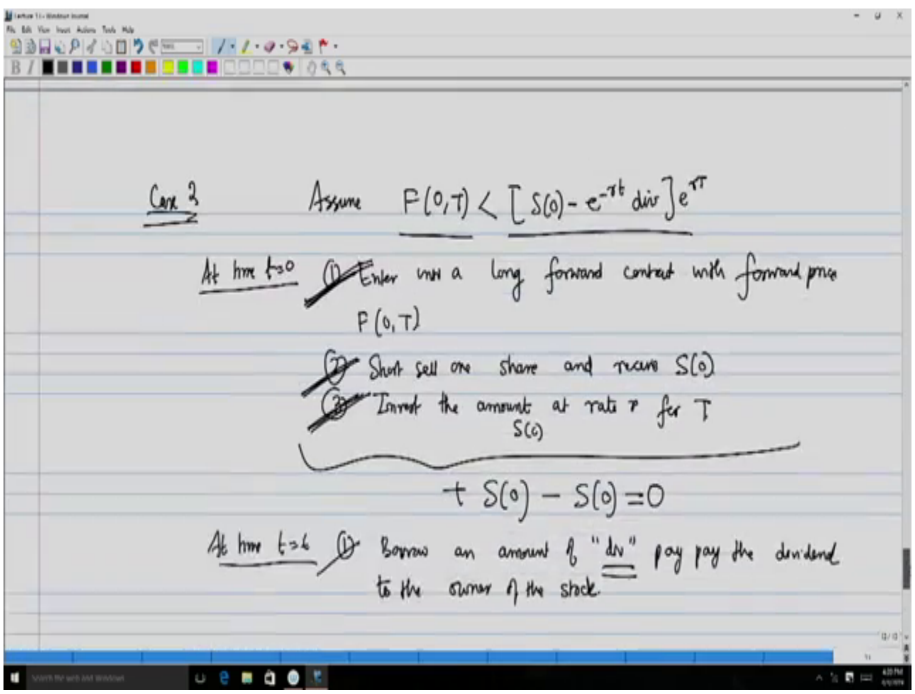

there is an arbitrage opportunity.

And so our assumption, in the second case also does not hold true.

So, having ruled out this possibility therefore you end up getting $F(0,T) = [S(0) - e^{-rt}div]e^{rT}$.

So, I will just state one more proof, one more theorem without proof, so this is in case of continuous dividends. So, the forward price of a stock paying dividends continuously is given by $F(0,T) = [S(0) - e^{rt}\ div]e^{rT}$, where the dividend is or the continuous dividend, is paid at rate $r\ div$.

We finally come to one last point that I want to make and that is what is the value of a forward contract, right? And I state this as a theorem, it says that, for any t such that $0 \leq t \leq T$, the time t value of a long

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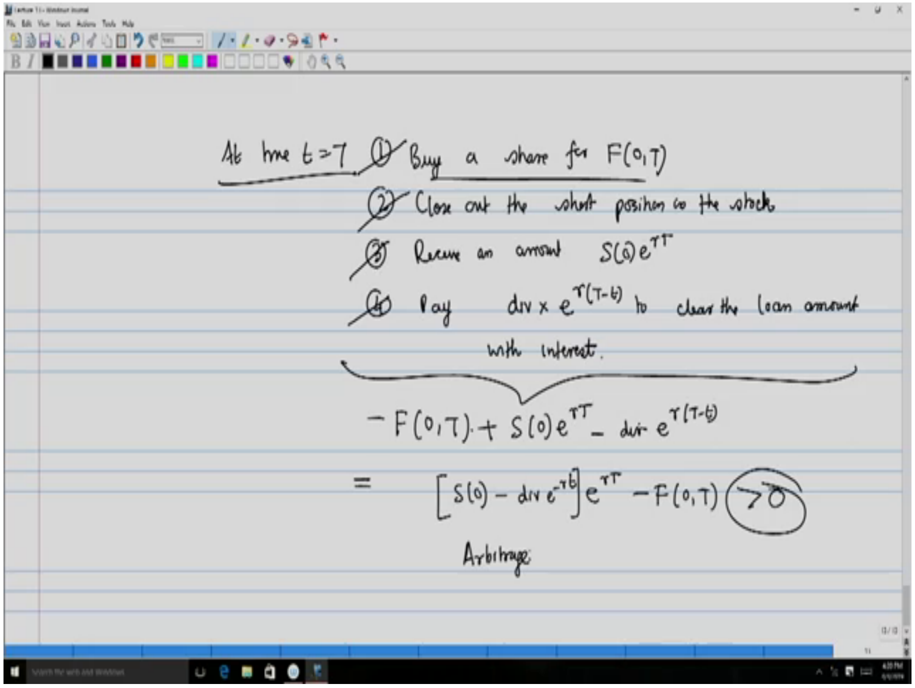

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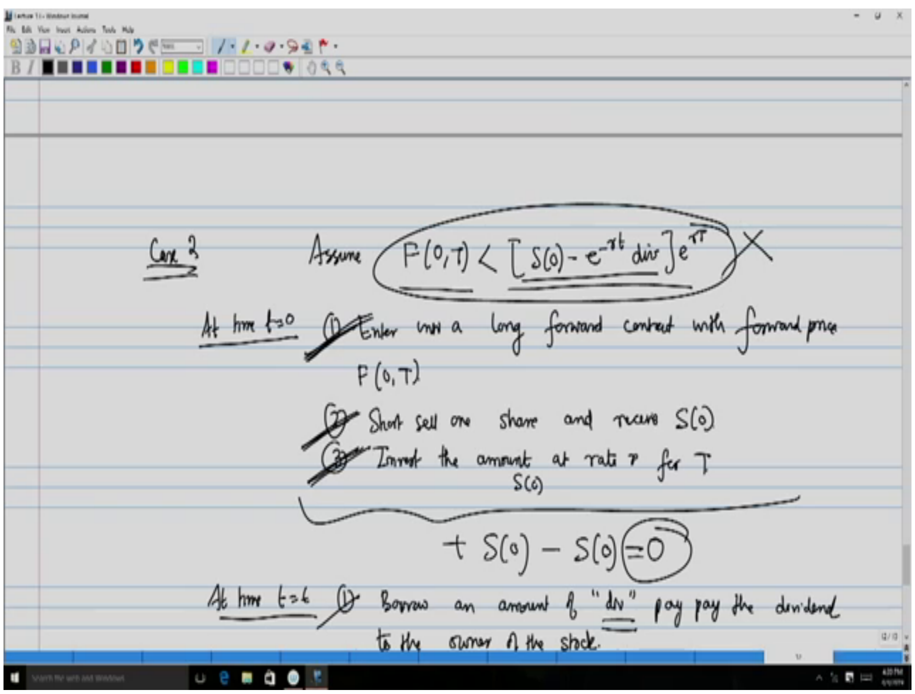

forward contract with forward price $F(0,T)$ is given by $V(T) = [F(t,T) - F(0,T)]e^{-r(t-T)}$.

If you observe carefully what you do is that $F(0,T)$ is the price that you would actually pay for the underlying asset if you enter into the forward agreement at time $T = 0$. Similarly, $F(t,T)$ is the forward price that you will pay at time T if you get into the forward agreement at time t instead of 0.

That means the value of the at time $T = F(t,T)$ and $F(0,T)$ and so its value at discounted back to time t is obtained by multiplying which is the factor of $e^{-r(T-t)}$.

So, in particular if you observe that if you put $t = 0$ here, you will see that $V(0) = F(0,T) - F(0,T)e^{-rT}$, which is 0. And this is synonymous or consistent with the fact that the value of the forward

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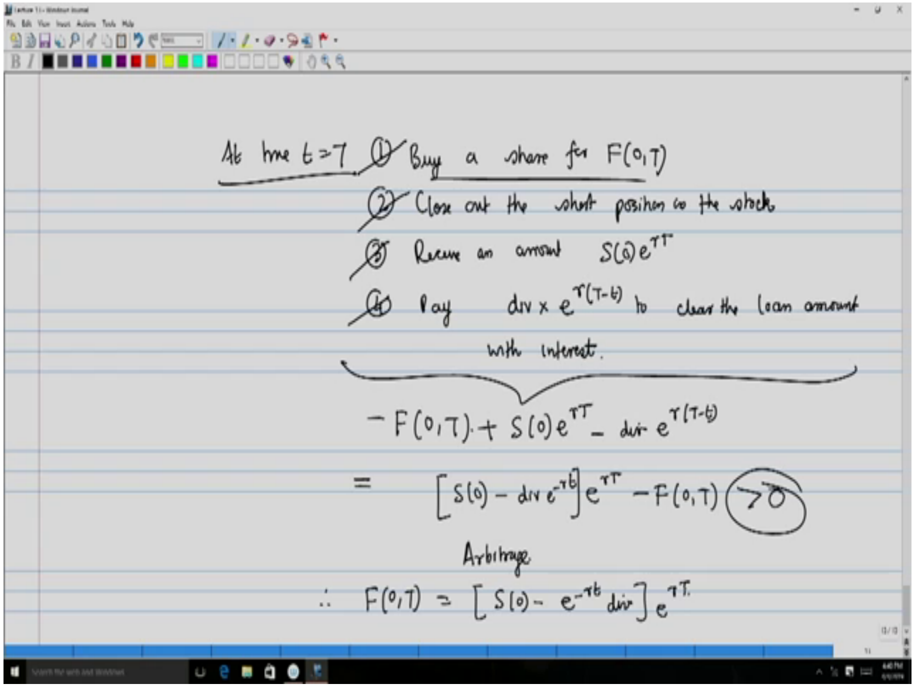

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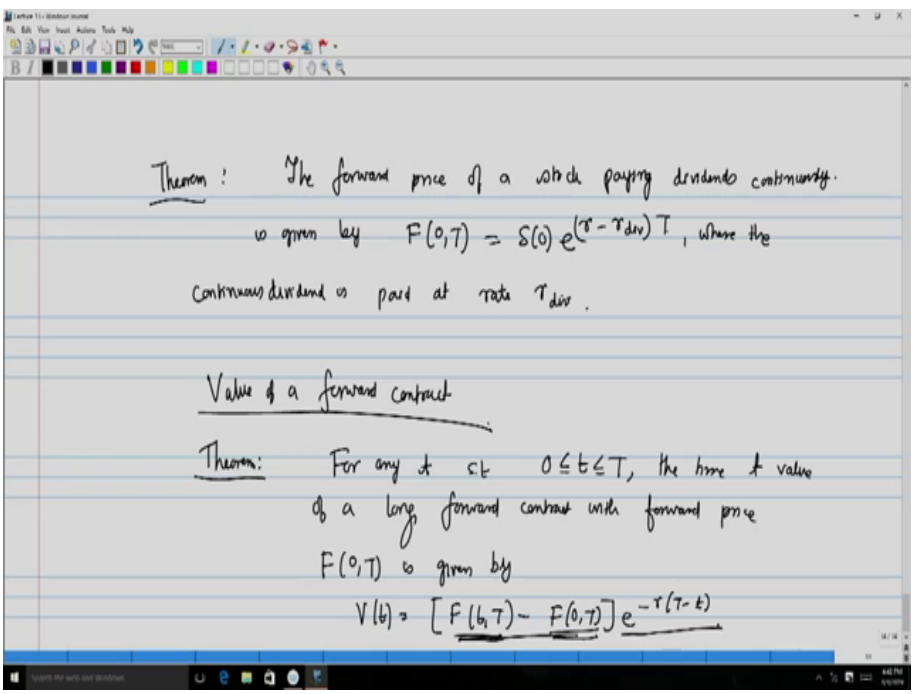

contract initially is basically going to be equal to 0.

So, this concludes our basic discussions on how one can make use of the no-arbitrage principle in order to determine the price of a forward contract and in particular we looked at the derivation of two cases, one when the underlying asset was the a stock without which does not pay dividends and secondly the price of the forward contract when the underlying pays a single dividend at a particular intermediate time point and we stated a theorem when the dividend instead of being paid at one intermediately time point is actually paid out in a continuous fashion and we concluded by discussing about what is the value of a forward contract.

So, in the next class, we will talk more about futures which you would recall is about, is similarly nature

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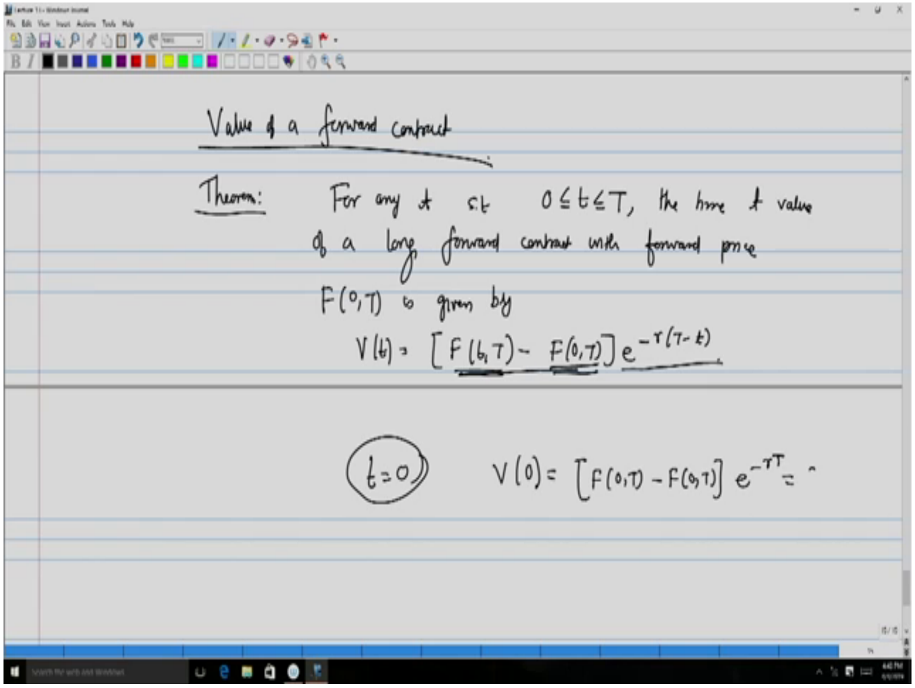

to forward except that they are traded on exchanges and that is the reason why there is a continuous inflow and outflow of money through the margin account and then we will move on to discussing basic properties of option both European and American. Thank you for watching!