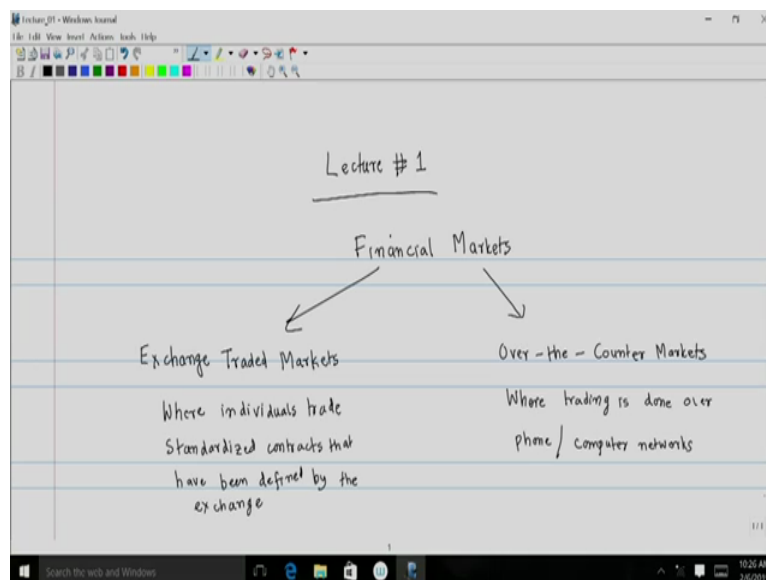


Mathematical Finance
by
Professor N. Selvaraju
and
Professor Siddhartha Pratim Chakrabarty
Department of Mathematics
Indian Institute of Technology Guwahati

Module 1: Introduction to Financial Markets and Instruments
Lecture 1: Introduction to Financial Markets and Bonds

Hello viewers, welcome to the MOOCs course on mathematical finance. In this course, we will look at various financial instruments and address some of the important problems using mathematical tools.

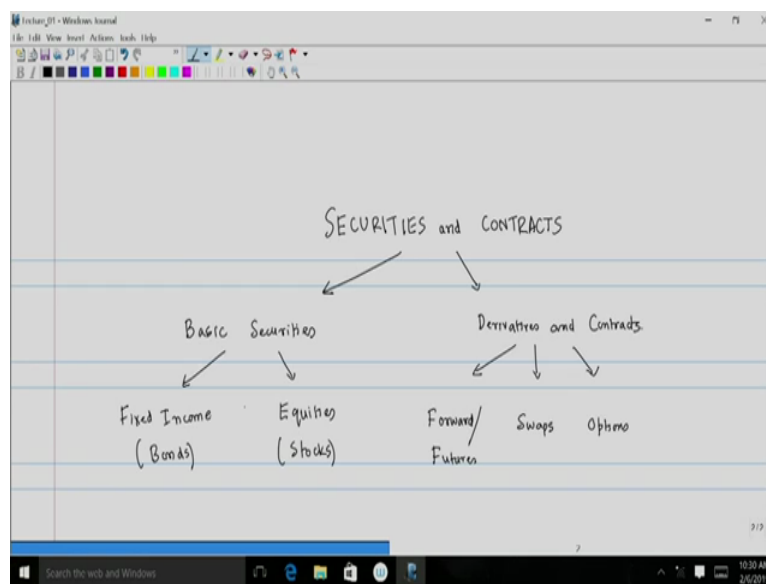
(Refer Slide Time: 0:52)



We begin this lecture, this is, Lecture Number 1, with a discussion on financial markets, financial instruments and in particular, we will look at the instruments of bonds, stocks and financial derivatives, namely, forwards and futures, swaps, and options. So, let us first look at how the financial market actually is set up in most part of the world. The financial markets can broadly be classified into exchange-traded markets and over-the-counter markets. So, let us just go through briefly what each of these two markets actually constitute. What is the exchange-traded markets? An exchange-traded market is where individuals trade standardized contracts that have been defined by the exchange. On the other hand, over-the-counter markets are slightly different in nature, where trading is done over phone and computer networks. So, for the exchange-traded markets what typically happened

traditionally was that they had an open outcry system where traders were physically present on the floor of the exchange in order to carry out the transactions. However, over a period of time it was slowly phased out and today mostly exchange-traded markets involve trading that are done through an electronic system. On the other hand, over-the-counter markets are where trading is done over mostly a computer network and in over-the-counter markets, the trading is done mostly between two large financial institutions and the other possibility is that trading is done between a financial institution and one of its clients. So, to sum it up, exchange-traded markets are mostly open to retail investors, whereas over-the-counter markets are restricted to mostly large financial institutions and their clients. So, in order to protect retail customers, typically exchange traded markets have far greater regulation as compared to over-the-counter markets, which have more flexibility in terms of deciding on the terms and conditions of the financial contract that is executed between the parties.

(Refer Slide Time: 4:45)

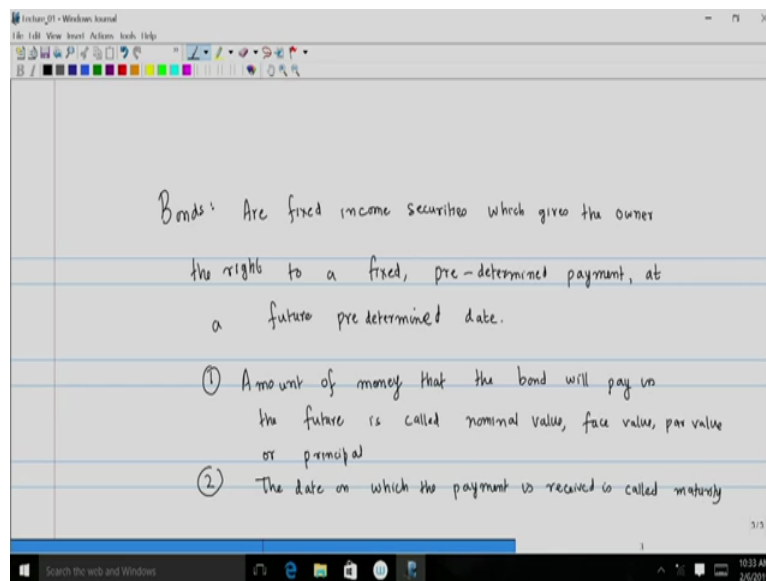


Now, let us look at a little bit about securities and contracts. So, I will just look at a very broad overview of securities and contracts that are currently existent in most financial markets. We will start off with securities and contracts. The most basic classification of securities and contracts involve basic securities and derivatives and contracts. Now, basic securities can broadly be classified into what will eventually be referred to as the risk free kind of securities and what is risky securities. So, in particular, the basic securities that we will consider in this course are the risk free or fixed income securities, the most common example of which is bonds and we will go through this definition and the details on bonds

later in the lecture and the second is securities or equities (which are risky in nature) and we will consider stocks when you are actually talking about risky securities.

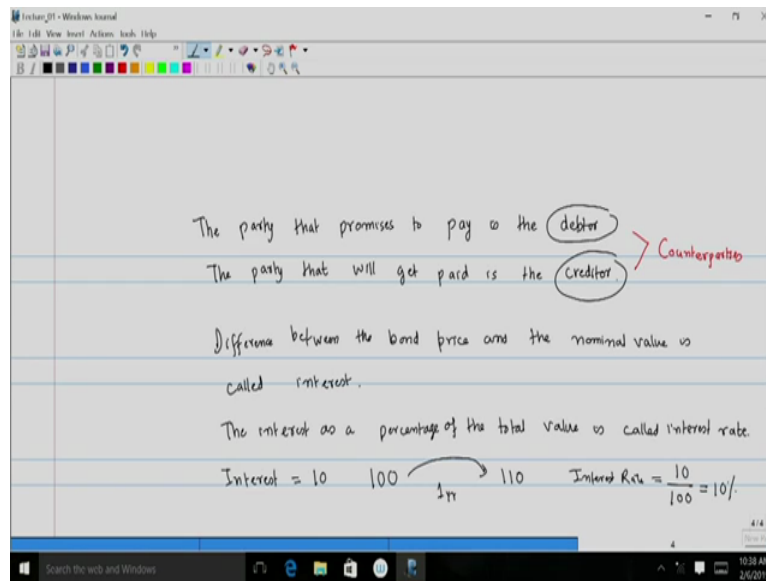
Now, coming to classification of derivatives and contracts, in this course, we will essentially look at three different kinds of contracts. First of all, we will look at what is known as forward/futures. Secondly, we will look at swaps and finally, options and as far as options are concerned, we will go through the various properties of option as well as pricing of option in a fair amount of detail as we move along the course.

(Refer Slide Time: 6:40)



Let us now first begin with a basic security, and in particular, we will first begin with bonds. What are bonds? Bonds are fixed income securities which gives the owner the right to a fixed, pre-determined payment, at a future predetermined date. I want to make two observations here. First of all, the amount of money that the bond will pay in the future is called nominal value, face value, par value or sometimes it is called the principal. Secondly, the date on which the payment is received is called maturity. So, this means that in the definition of bond the predetermined payment that I have mentioned that is what is called as the nominal value or sometimes face value or par value or simply the principal and the future predetermined date at which this payment is actually made that is what is known as the maturity of the bond.

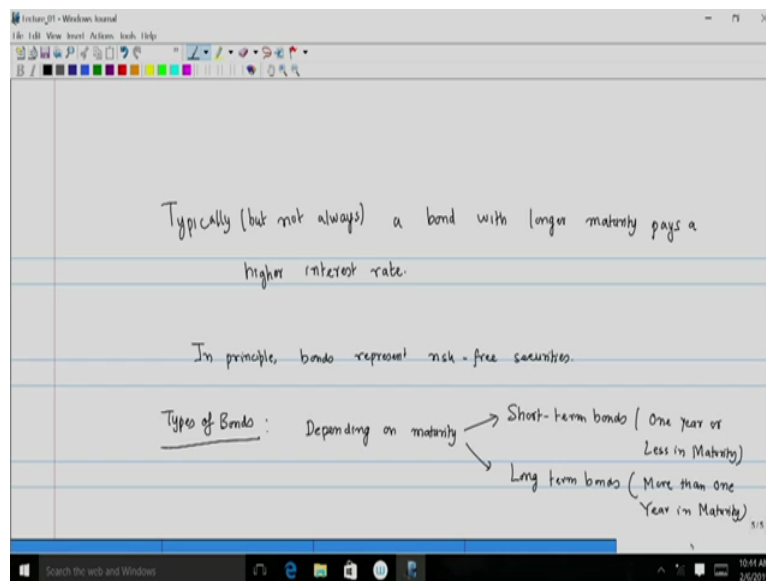
(Refer Slide Time: 8:41)



Now, let us talk about the two parties that are involved in the bond. The party that promises to pay is the debtor and the party that will get paid is the creditor. So, a simple example of the bond in our domestic market would be the fixed deposit. Suppose we go to a bank and then we get a fixed deposit. So, in that case, we pay a certain amount of money to the bank and the bank gives us the fixed deposit receipt with a promise to pay a certain amount of principle at a future date or the maturity. So, in this case, since the bank is promising to pay you a certain amount of money in the future, they are called as the debtor because in a sense they are in debt to you and because you are purchasing the bond or the fixed deposit by making a payment to the bank or essentially you are extending a credit to the bank. That is why you will be regarded as the creditor. So, creditor is the party that pays money to the debtor at the initial time and that at the maturity they receive the promised payment from the debtor. In this case, both the debtor and the creditor are known as counterparties. This means for the debtor, the creditor is the counterparty and for the creditor, the debtor is the counterparty. Now, the difference between the bond price the creditor pays to the debtor and the nominal value, is called the interest. So, essentially what it means is that when as a creditor you are purchasing a bond, you are paying a certain amount of money and in return, you will receive the promised amount at maturity. So, the difference between these two is the interest on that particular bond. This means that I can write this as difference between the bond price and the nominal value, which is called interest. For example, you are investing an amount of 100 rupees and at the end of one year the bank pays you back 110 rupees. Then the interest for this one year period is going to be 10 rupees. Now, another way of looking at the interest is

the following that the interest as a percentage of the total value total value is called interest rate. Now, coming back to the example, if I invest an amount of 100 rupees today and at the end of one year I get 110, then what is the interest going to be. As I pointed out, the interest is going to be $110 - 100 = 10$ and the interest rate will be the interest as a percentage of the initial value, so that is $10/100 = 10\%$.

(Refer Slide Time: 12:41)



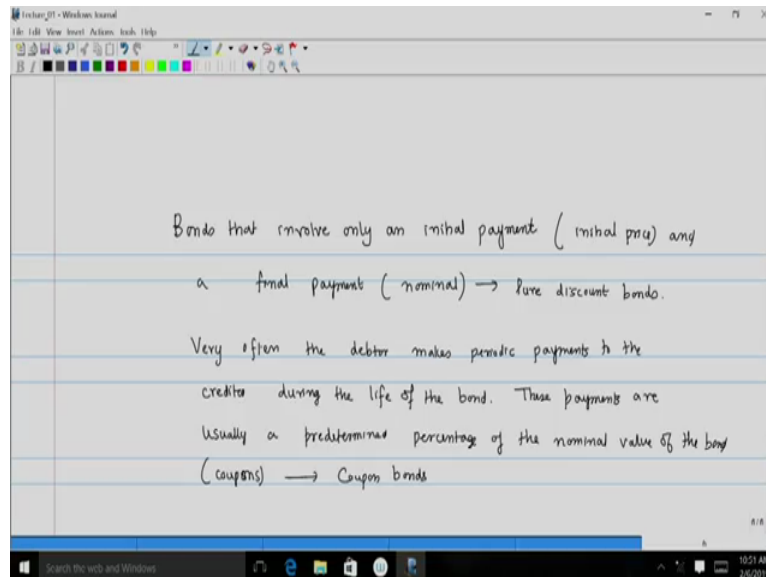
One interesting feature of bonds or fixed deposits is that, typically (but not always), a bond with longer maturity pays a higher interest rate. This means that if you hold a 1 year bond, then whatever interest rate that you are getting and in comparison, if you hold a 5 year bond, then the interest rate in that case will be higher than the interest rate that you get for 1 year and this is something that is typically true in case of bond, but not necessarily always.

Now, let us talk about the term risk-free in the context of bonds. In principle, bonds represent risk-free securities. So, what I mean by risk-free securities is the following, that when you are making an investment in a bond, then at the time of investment you know apriori the exact amount of money that you are going to receive at maturity and this is in contrast and we will see this later in case of stocks where your initial investment of money does not predict eventually the amount of return that you are actually going to get. So, for all practical purposes, given the nature of no uncertainty in terms of the amount that you are going to receive at maturity they are considered as risk-free securities.

Next, we will look at types of bonds. We will look at the different classifications of bonds depending on various different criteria or parameters. First of all, we will look at the

classification of bonds depending on their maturity. Depending on maturity, that means the period for which you hold the bond, you can classify them as short-term bonds which are one year or less in maturity and long-term bonds which are more than one year in maturity.

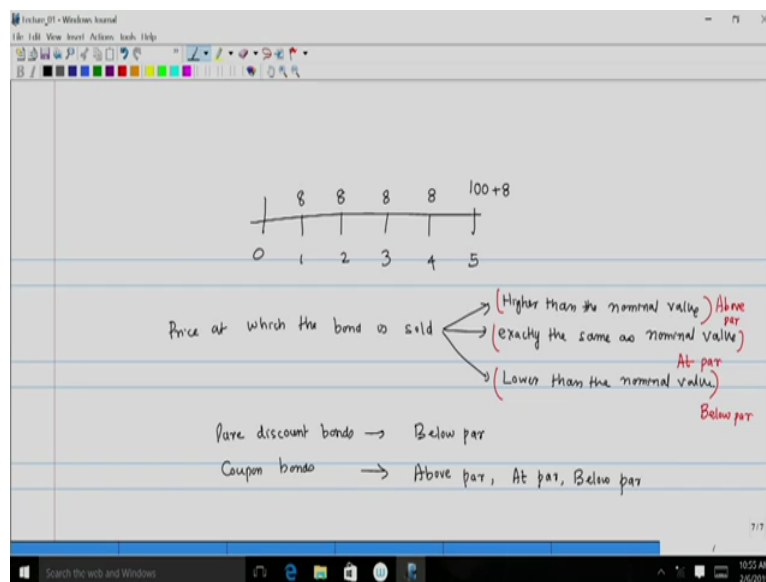
(Refer Slide Time: 15:56)



Next, we will talk about another classification of bonds, namely, bonds that involve only an initial payment which we call as the initial price or simply the price and a final payment which is known as the nominal or face value. So this means that you make an initial payment required to purchase the bond and there is no payment that you receive during the entire period and finally at maturity you get back your promised sum. So, at the initial time point, you pay the bank and get the fixed deposit and at the final time point, they basically or in the maturity the bank pays you back the promised amount of money. Now, these kind of bonds are known as pure discount bonds, and the reason why they are called the pure discount is that the initial payment that you make to purchase the bond can be obtained by simply taking the final payment of the bond and discounting it by a factor which takes into account the interest rate of the bond. However, there are other kinds of bonds, where there is one initial payment but there are multiple payments that happen in between the initial payment and the maturity. Accordingly, on the other hand, very often the debtor (remember the debtor is the bank in the context of our discussion) that makes periodic payments to the creditor during the life of the bond and these bonds or these payments for such bonds are usually a predetermined percentage of the nominal value of the bond. And these periodic payments are known as coupons and these are called as coupon bonds. So, this means that in case of a pure discount bond, there are exactly two payments, one that is paid by the creditor to the debtor at

the initial time point and at the final time point the debtor pays back the original amount that was paid plus the interest to the creditor. However, in case of coupon bond what happens is that the creditor makes an initial payment to the debtor. In our case, we pay a single amount to the bank at the initial time point, but the payment that we received from the debtor or the bank is not just a single payment that happens at maturity, but also they are made periodically time to time to the creditor and these periodic payments are known as coupons and that is the reason why such bonds are referred to as the coupon bonds. Typically the coupon payments are dependent or specified as a percentage of the nominal value. It means that if the nominal value is 100 (say) and I say that the rate of coupons is 8 percent, that means that every period we will basically get an amount of 8 assuming that the bond is something where the coupons are actually have been paid annually. Specifically, for example, if you have a 5-year bond and it is a coupon bond where the nominal value is 100 and the coupons are paid at 8 percent, so this means that at the end of the first, second, third and fourth year you will get an amount of 8 for each of the years and finally, at the maturity you will get the nominal value of 100 and also the last coupon of 8. So, this means that you will get payments of 8, 8, 8, 8 and 108 at the end of the first, second, third, fourth and fifth year, respectively.

(Refer Slide Time: 21:32)

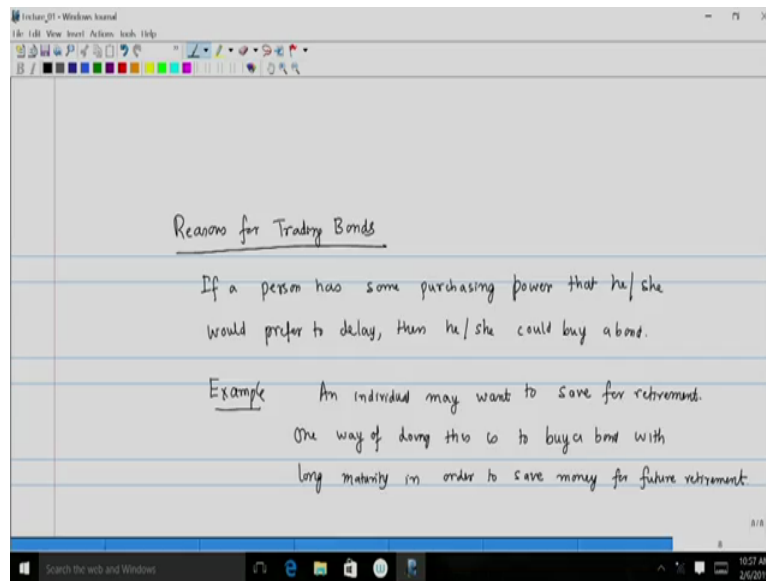


Next, we move on to just an interpretation of coupon bonds. You can treat coupon bonds as something that is like a collection of pure discount bonds. It means that each of the payments of 8, 8, 8, 8 and 108 can be considered as a basket or a collection of 5 different pure discount bonds with the nominal values being 8, 8, 8, 8 and 108. So, basically, this is what it is going to look like, you have zero, first year, second year, third year, fourth year, fifth year. So, in

case of pure discount bond, you will simply get an amount of 100 at maturity, but in case of a coupon bond in the example you considered, you will get 100 plus 8, 8, 8, 8 and 8.

We again look at the classification of bond from the point of view of the nominal value. The price at which the bond is sold now, it can either be exactly the same as nominal value, the second alternative is that the price is higher than the nominal value and the price is lower than the nominal value. So what do you mean by price? Suppose that I have purchased a 1-year bond or a fixed deposit whose maturity value or the nominal value is 110. If the interest rate is 10%, then I know that the amount of money that I have to pay initially is going to be 100, because on that amount of 100 interest rate of 10% would be applied giving you the nominal value of 110. This means that we must understand the distinction between the price and the nominal value. Price is the amount that you pay to the bank or the debtor at the initial time point and nominal value, as I mentioned before, it is the amount that you receive at maturity. Now, if it turns out that this price 100 (in case of an example), if this is higher than the nominal value, then we have a term which is known as we say that the bond is selling above par, if it is lower than the nominal value, then we say that the bond is selling below par and if it is exactly the same as the nominal value, then we say that this is at par. So this means that it is obvious that in case of a pure discount bond which you sometimes refer as the zero coupon bond because it does not pay any coupons in between, I will pay a lower amount of money initially and receive a higher amount of money at maturity. So, that means the price of the bond in case of a pure discount bond is always going to be less than the nominal value. This means that pure discount bonds is always will be sold below par. However, in case of coupon bonds, all the three possibilities can actually happen. This means that in case of coupon bonds, we can have the price to be above par or the bond might be trading above par, at par and also below par.

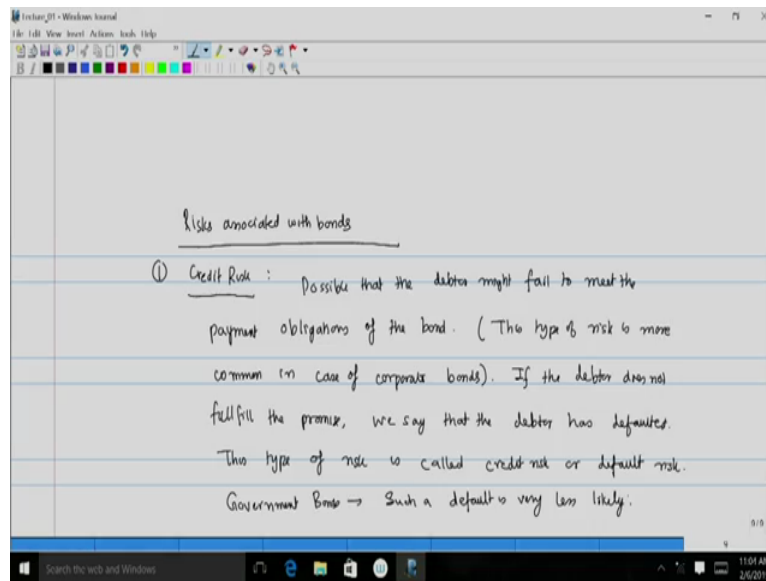
(Refer Slide Time: 25:34)



Now, let us talk about briefly about the reasons why one would actually trade in bonds. I mean what is my incentive as an investor in order to trade in a bond or equivalently buying a fixed deposit or putting my money in a fixed deposit in a bank. What are the reasons for trading bonds? Now, if a person has some purchasing power that he or she would prefer to delay, then he or she could buy a bond. Let me just illustrate this particular statement (what I mean by delay that purchasing power for a future time point) through a little example that is extremely practical in the context of our day to day activities. One example for this is that an individual may want to save up money for retirement. One way of doing this is to buy a bond with long maturity in order to save money for future retirement. The reason why we basically get into these bonds is that we invest a certain amount of money that is not needed at this point of time, probably classify, as a disposable income and so we put it in the bank and basically it accrues interest over a period of time and that is at future time point, so we essentially get a larger sum of money back.

Now, here we have so far made two observations regarding the bonds, one is that they are risk-free assets. This means that there is no risk associated with it, as the name would suggest. Secondly, we look at it as an investment that basically provides some sort of security in the future. However, in the context of these two points or these two paradigms of risk-free and having a future return, I would at this point like to make a few observations regarding the risks that are associated with bonds.

(Refer Slide Time: 28:00)



I start and specify the risks associated with bonds. So, here even though we are saying that the bonds are actually risk-free, there are some various sort of obvious risks that are present, some of which are result of the market forces and also it could be a result of certain exigency that requires you to essentially liquidate your bond at some intermediate time prior to the maturity.

We first talk about something which is known as a credit risk. What do I mean by credit risk? Recall that we had identified the person who purchases the bond as the creditor and the extent the credit to the debtor. They basically give the money which can be viewed as some sort of a loan to the debtor and this means that there is always a possibility that the debtor (in this case, the bank or a corporation, in case you are buying a corporate bond) could end up not fulfilling their obligation which you would not expect, because bonds are supposed to be assured returns and you do not expect any default on the part of the entity, the financial institution or a corporation which has issued the bond. But nevertheless from the practical point of view, the credit risk does exist in case of bonds.

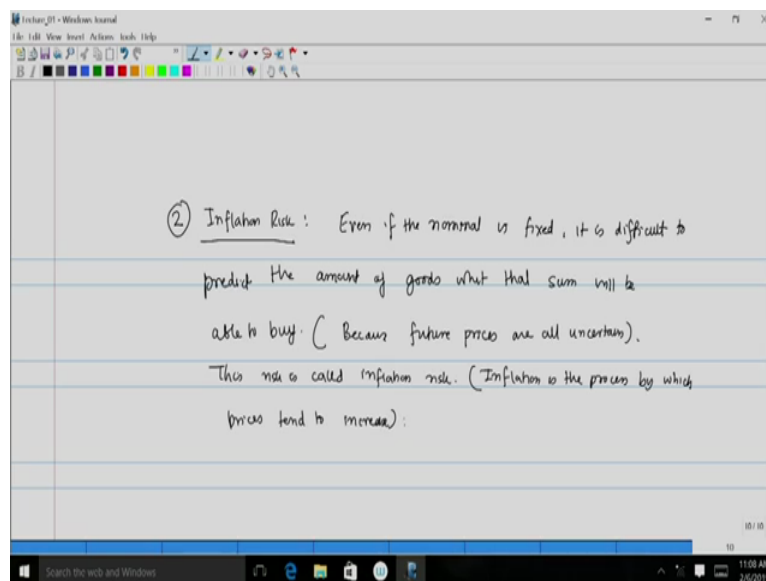
Let us specify this little more that it is possible that the debtor might fail to meet the payment obligations of the bond and just one little observation that this type of risk is more common in case of corporate bonds. Corporate bonds are bonds that are essentially issued by companies in order to raise money. The companies typically can raise money through bonds for investors who are less willing to take risk or through equities something (that we will discuss subsequently) so as compared to government bonds or bonds issued by, say, well-established

banks. The chances of default are much more in case of bonds that are actually issued by companies.

Continuing from here, that in this situation, where basically the debtor fails to meet the requirement, or in other words if the debtor does not fulfil the promise, we say that the debtor has defaulted and this type of risk is called credit risk or default risk. Now, as I have just already observed that it is more likely that the corporate bond or the bonds issued by the companies are more likely to default and one of the reasons being that the company raising money through bonds might not have the successful business venture for which the bond was issued and the consequent result of which is that the buyer of the bond loses money.

However, in case of government bonds, typically, such a default is very less likely and one of the primary reasons why I am saying that this default is very less likely is because governments can always print currency and meet the obligations as a part of the bonds that they have actually issued.

(Refer Slide Time: 32:57)

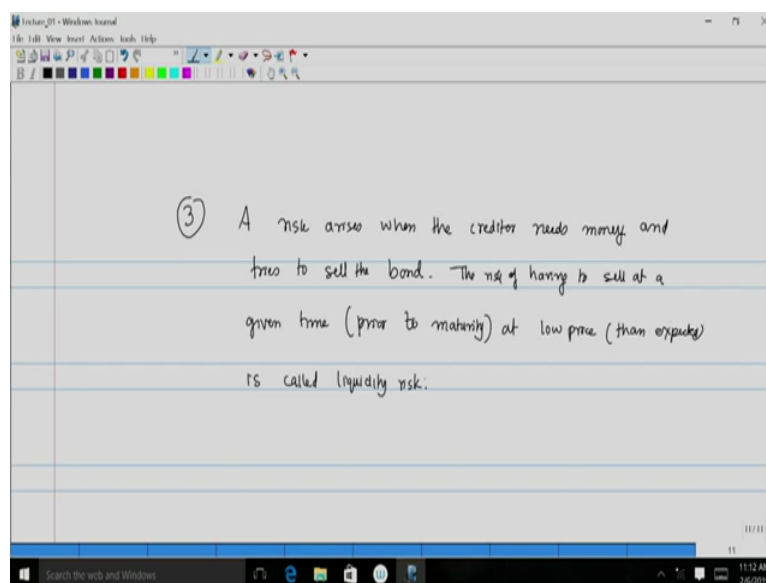


All right, now we move on to a second kind of risk that is associated with bonds and this is what is known as the inflation risk. So, we are all familiar with the type inflation. I mean inflation is something that affects everybody. It is basically the rise in the prices of commodities thereby making it difficult in terms of meeting our daily requirements.

In this context, we like to make the observation that even if the nominal is fixed, it is difficult to predict the amount of goods which that sum will be able to buy and the reason for this is

that because future prices are all uncertain. For example, suppose today you purchase a bond for an amount of 100 and you know that this amount of 100 is enough to purchase a certain good or a commodity. Further, you make an investment of 100 for 1 year with the interest rate of 10 percent and at the end of 1 year, you receive an amount of 110. However, the same good or commodity that would cost 100 today might cost 120 at the end of one year, but your investment of 100 today gives you only 110 at the end of 1 year. This means that the amount of 100 which would have been enough for you to purchase the commodity today, as a result of the investment in the bond it grows up to an amount of 110. But you find that you are no longer in a position to just utilize that money to purchase the same commodity and there is a shortfall, the shortfall is by an amount of 10 and this is an illustration of how the pricing and the bonds are vulnerable to inflation despite being risk-free from the point of view of fixed pre-determined return that you actually can get at the maturity. Then this risk is called inflation risk and you recall that inflation is the process by which prices tend to increase.

(Refer Slide Time: 36:06)



Finally, I come to the last of the three types of risks that are facing a bond. The third kind of risk is known as a liquidity risk. What do you mean by liquidity risk? A risk arises when the creditor, that means the person who has purchased the bond needs money and tries to sell the bond. For example, today I purchase a bond for an amount of 100 and the bond is a 1-year pure discount bond and the interest rate is 10% as before, so I am assured to get an amount of 110 at the end of 1 year.

However, at the end of 6 months if a necessity arises where I need the money, then I might decide to sell the bond at that point of time. Now what I would typically expect is that when I

invested an amount of 100 for a period of 1 year, I was guaranteed an interest of 10 for the whole year. This means for half the year or for 6 months the interest that I should get is 5. So, when I try to sell off my bond at the end of 6 months, the price that I expect to get is 105. However, between now and 6 months, there is a possibility that the interest rate might change and you might end up getting an amount that is less than 105 and that is the scenario where you end up getting a value because of this premature the selling of the bond, you end up getting an amount of money that is lower than what you had expected and this has happened, because the risk-free aspect of the bond is no longer in place because the risk-free aspect of the bond applies only if you hold on to the bond till maturity. Consequently, in this case, you are compelled to sell off the bond for a lower amount of money than what you would have anticipated when you had invested at the bond at the initial time point, namely, your expectation was 105, but you ended up actually receiving lesser amount of money as a result of the interest rate change that has happened between during this period of the first 6 months of the bond. This kind of risk that is what, so that means the risk of having to sell at a given time prior to maturity at price than expected is called liquidity risk. Liquidity essentially means that your ability to sell off the asset. Since, this kind of loss that you had to incur is a result of selling off your bond prematurely, that is the reason why this kind of risk that is associated with bonds is known as the liquidity risk.

But having said so, for the remainder of the course, we will essentially look at bonds as a risk-free investment, for practical purposes, simply for the reason that there is no uncertainty in terms of the final amount that you actually are able to get. So, this concludes our discussion on bonds. In the next class, we will talk more about stocks and we will introduce financial derivatives with a particular emphasis on forwards and futures as well as swaps. Thank you for watching.