Formal Languages and Automata Theory

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Module - 03
Finite Automata
Lecture - 03
NFA ⇔ DFA

So far I have discussed DFA and NFA. So, by introducing some non-deterministic iterations in DFA, we have constructed NFA. Looking at that, that means introducing more flexibility, it looks like that NFA is much more powerful or more general and much more powerful than DFA. That means it looks like that the class of languages accepted b NFA is different from that of the DFA, but actually that is not the case. We are going to show that DFA and NFA are in fact equivalent. So, when you say equivalence between two automatons.

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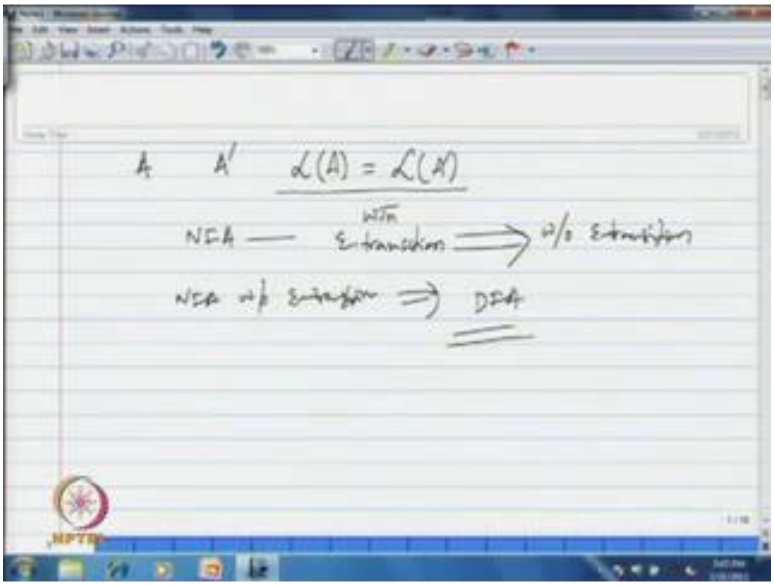

Say automaton A and automaton A dash, we say that A and A dash are equivalent whenever the language accepted by A and the language accept by A dash are the same. In that context we want to prove now that DFA and NFA are equivalent. So, while proving the equivalence, we will see that one direction is quite obvious, that means every DFA can be treated as an NFA, is a special case of NFA, but let us prove the other side that means the converse. Since for every NFA there is an equivalent DFA. So, one will consider an NFA we will first consider an NFA with epsilon transition. An NFA with epsilon transition then what will do, will construct an NFA equivalent NFA without epsilon transition.

Once you do that from that NFA which is equivalent the origin NFA, we will now construct an equivalent DFA. That means there would not be any multiple next state or there would not be any move from a state on input symbol to multiple next states. There has to be a next move or next state from every state on every input symbol. So, NFA without epsilon transition, from there we will construct a DFA. So, we will first start with whatever we are taking example.

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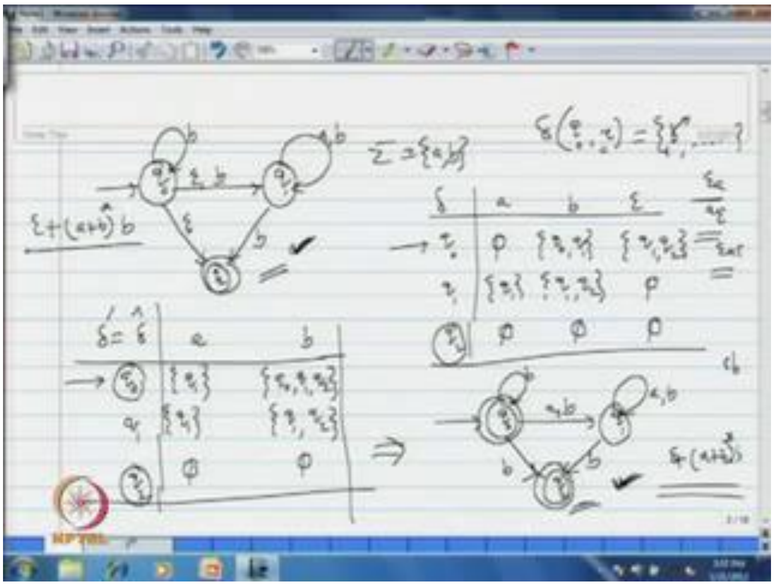

Let us consider this NFA which has… So, 3 states q 0 is the initial state q 1 in the next state, q 0 on a, there is no input symbol. Say the input symbols are only a and b. So, q 0 on b remain the same state or it may also go to the state b, state q 1 on input symbol b and q 0 without taking an input may also go to q 1. That means from q 0 to q 1 the 2 transition on epsilon and in b, q 1 on a and b it may remain on the same state a self-loop on a and b. From q 0 there is a transition to a state q 2 on epsilon and q 2 is also a final state.

Similarly from q 1 there is again a transition on input b to state q 2. That means the transition map. So, it is delta is given by in the 3 states q 0 q 1 q 2 on a b and epsilon. So, q 0 on a it is does not go any. So, it is fine q 0 on b goes to 2 states 1 is q 0 itself and other is q 1. So, the set of states next its basically q 0 and q 1 on input symbol b.

Similarly, q 0 on epsilon it will go to only 1 state sorry 2 states q 1 and q 2. Similarly, q 1 on a goes to single state the single state is q 1 and q 1 on b may remain the same state q 1 or it may go to q 2 and on epsilon it does not have any transition, so it is phi. Similarly, q 2 on a b epsilon there is no any transition. So, it is all fine or a b and epsilon. So, is a transition map for is particular NFA whose epsilon transition. Now, let us say how to remove epsilon transition that means how to get an NFA whose equivalent to this NFA, but there is no epsilon transition.

So, the main point is that if this NFA, but transition map is delta, we start at q 0 if on input string acts it leads us to some state there be multiple next states. Suppose, P is 1 sub state the multiple next states in such a case. So, these string auxiliary excepted provided from P eventually there is a transition to one of the final states. That means the equivalent NFA all those states will final for which there is an epsilon transition to 1 on d final steps.

So, looking at that what will do is that, from q 0 on epsilon there is epsilon move to q 2 for q 2 is the final state. So, q 0 will be made a final state in the equivalent NFA without epsilon transition, but from q 1 there is no any transition on epsilon to the final set q 2. So, q 1 will not be final step and hence in the equivalent NFA we will have 2 final states q 0 and q 2. Then the transition map say it is delta dash. So, transition map will be nothing, but the external transition function for the given NFA with epsilon transition and here we will have q 0 q 1 and q 2 the same set of states q 2 will be a final states.

So, here q 2 was a final state and here in the equivalent NFA with epsilon transition and here we will have q 0 q 1 and q 2 with same set of states q 2 will be a final states. So, here q 2 has a final state and here an equivalent NFA without ((Refer Time: 08:37)) q 0 will also be a final state. Here q 0 was initial state q 0 still remains to be an initial state, q 0 will be final state, since we have epsilon transition from q 0 to q 2.

Next the transition function for this is defined as suppose on some input symbol a by whenever this N FA without epsilon transition goes to some states, on any input from any state. Then what you need to concentrate is that, we need to concentrate all possible transitions inserting epsilon before and after a, that means we need to look at the strings epsilon a, a epsilon or epsilon a epsilon. So, you need to consider the transition out of every state from for this NFA on the input strings epsilon a, a epsilon and epsilon a epsilon and need to consider that is the set of next states corresponding to q 0 on input symbol a.

So, q 0 on a if you consider there is no move only on a from out of q 0, but taking an epsilon it can go to q 1 and from q 1 taking an a it will remain at q 1. So, q 1 will be for the next steps for this. Next taking epsilon at q 0 it may go to q 2, but from q 2 there is no any condition out of q 2 on a. Therefore, q 2 will not belong to the delta q 0 a. Therefore, q 1 is the only state that will have for delta hat q 0 a. Similarly, q 0 on b, if you consider the string b, so q 0 on b it may taking b it may remain the same state q 0 or taking a single b it may go to q 1 or taking epsilon it may go to q 1.There it may remain the same state q 1 taking b.

That means if consider string epsilon b on that it will again be the single state q 1. So, q 1 is already there or taking epsilon at q 0 it may go to q 1. Then taking this b it may go to q 2. Therefore, q 2 will also be there in the set of next states for delta hat q 0 b. Similarly, if you consider q 0 taking epsilon it may go to q 2, but there is no transition out of q 2 on b.

So therefore, q 2 data will not be included, but q 2 is already included via this parts q 0 q 1 q 0 on epsilon q 1 and q 1 taking b it may go to q 2. So, these are only possibilities that you have q 0 on b. Similarly, q 1 on a it may remain the same state q 1 and there is no epsilon move out of q 1. Therefore, there is the only possible case for delta hat q 1 a. Then for delta hat q 1 b from q 1 taking b it may go to it may remain the same state q 1 and there is no epsilon transition and then q 1 on b it may go to q 2 also. Therefore, the possible next states are q 1 and q 2.

Then q 2 out of q 2 there is no any transition on any input symbol or epsilon a joule. So, there only possible cases that you have considering this. So, just consider the transition diagram corresponding to this automata that we have the diagraph construct it. So, in this case also we have 3 states q 0 is a initial state, then you have q 1 and at q 2, both q 1 and q 2 q 0 and q 2 are final states as well, q 0 on a goes to q 1 and q 0 on b it may remain the same state q 0, it may go to q 1 it may go to b. So, there is state transitions out of q 0 on equal symbol b.

Similarly, from q 1 on input symbol a, if a remain the same state q 1 on a remain the same state and q 1 on b may remain the same state or it may go to q 2. So, the transition diagram for this. So, let us see what are these 2 this NFA and this NFA without epsilon transition this NFA with epsilon transition and you have say that this NFA without epsilon transition. What are the, accept the same language on of. If you see carefully that the NFA which epsilon transition can accept the string epsilon because an epsilon move from the start state to the final state or if you consider by the other part, it may take an epsilon transition may go to q 1.

Then take any strings over a b, because of this loop self-loop on a b and eventually will go to q 2 the final state on input symbol b or before can it can take any numbers of b. This single b and then go to q 1 then take again any strings about a b and eventually that string turned by b expected by this NFA. That means, eventually any string over a b termed by symbol b we accepted by this NFA with epsilon transition. Let us see why that is NFA without epsilon transition can also accept the same string seven language or not.

Since, if the starts at q 0 the final state as well therefore epsilon is automatically exceeded by these NFA and whenever from q 0 it goes to the other final state, then it will go on a single input small b. If you wants to accept if it accept any string, then these string of this from is b star because of the self-loop or otherwise for the other part from q joule it may go to q 1 on taking any a single a or b and then a any numbers of a's and b's because our self-loop again.

So, later terminated by b. So, we also if look carefully is that any string over a b terminated by b is also accepted. Therefore, epsilon plus a b, the language of this and f a as well. That means this NFA is equivalent, this NFA without epsilon transition is equivalent to this NFA with epsilon transition. In fact, how we construct it, this NFA without the epsilon transition and the original NFA with epsilon transition.

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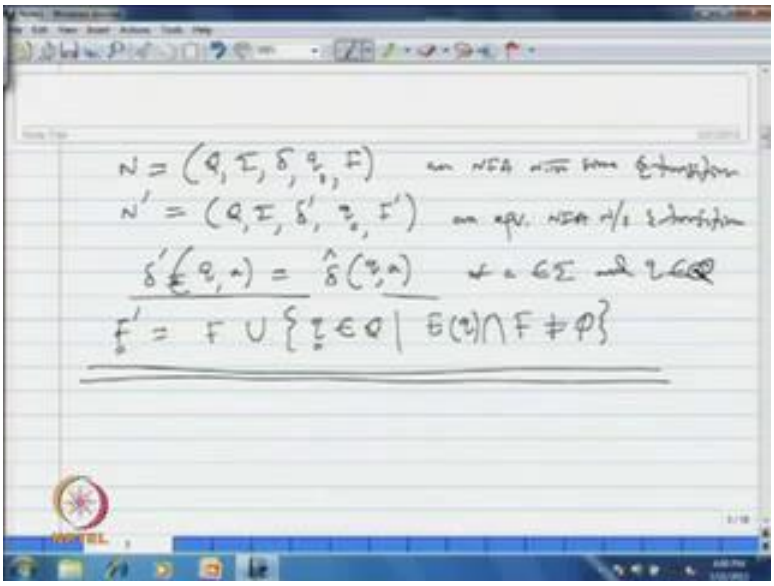

So, it is quite simple that means given an NFA N. So, it is q sigma delta q 0 f, suppose the NFA which some epsilon transition, then we can construct NFA say and does without epsilon transition. Then we will retain the same set of states q what a same input will form it we are going to modify the transition map, which is nothing but the external transition function of the original NFA. Then the set of starts and remain same and set of finite sets will modified to say have this is an equivalent NFA without epsilon transition.

What have done is that, the transition function for this NFA without the function there is delta dash, define like this. So, delta dash q a is basically delta hat q a for all a belonging to sigma and q belonging to capital q set of states. So, for any state q an input symbol a delta raise q a is defined as delta hat q a for the original NFA and f does is nothing but that they have you observe that is nothing but, we have retain the same final states f union any state q in the original NFA.

Such that epsilon proves that means if you can, if you take epsilon moves from any state and the set of next state you see, if it contains any one of final states then we say that that state is also in the final state. That is epsilon closure of q and f if this intersection is not equal to phi, then we consider this q to be included in the set of final states in the modify NFA. So, this is the construction that we have observed and we have seen that take corresponding NFA is equivalent to the original NFA. The original NFA had epsilon transition, but this NFA does not have any epsilon transition. So, this is a constant that we have got, but let us now prove, we have shown it by an example and inattentively it looks like that this constructions works, but let us approve that this construction really works. So, again to give a power proof that these two NFA are equivalent, that means the x are the same languages.

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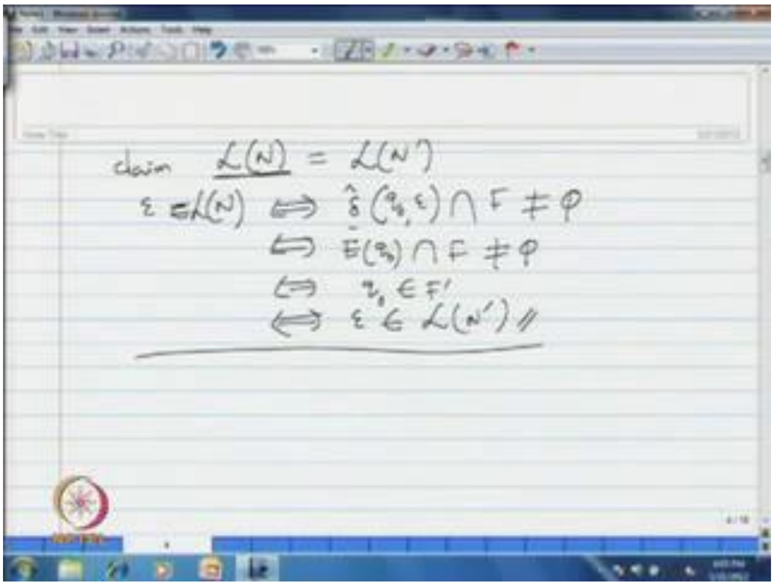

We claim that, the language of the original NFA with epsilon transition is equivalent or identical to the language of the NFA without epsilon transition that we have constructed. So, first we will see that if epsilon belongs to the language of the original NFA which epsilon transition. So, epsilon belongs to L n if and only if, delta hat that means there is transition from q 0 on epsilon from start at q 0 on epsilon. The set of next states that we have on epsilon at least one state that belongs to the set of final state. That means delta hat q 0 epsilon and f has some elements come on that means intersection of the 2 sets not equal to phi. So, there is a condition according to the exceptions when NFA.

So, this is a case if and only if. So, this is nothing but epsilon proves that q 0 delta hat q 0 epsilon is nothing but epsilon q 0 q 0. So, this introduction f is not equal to null set. This is the case, if and only if q 0 belongs to f dash, because this is the way we have defined the set of finite sets. So, f dash is f union all those that is q. Then to the set of states for that epsilon close are q and f the intersection is not equal to phi.

So, accordingly these will this nothing but q 0 must belong to f dash, but what you say is that, since q 0 belongs to f dash epsilon is automatically in the language of NS. So, we have seen that, if epsilon belong to the language of this NFA with epsilon tradition, epsilon mass also belong to the language of this NFA without epsilon transition.

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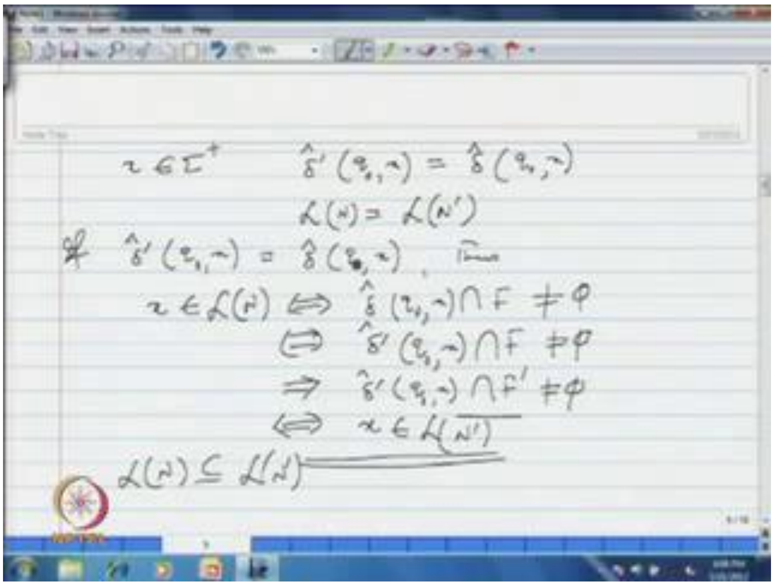

Now, for any string around epsilon belong to sigma. That means for any string x belong to sigma plus. We proved that delta dash hat q 0 x equal to delta hat q 0 x. So, that this even says that L of n equal to a of n dash this because if delta dash hat q 0 x. So, if delta dash hat q 0 x equal to delta hat q 0 x, then x belongs to one of n x belongs to one of n if and only if delta hat q 0 x intersects on f not equal to phi that is by definition, our acceptance over the string x by the original automaton original NFA. So, if and only if delta dash hat q 0 x intersection f not equal to phi because that is how we have defined the transition function of the new NFA delta dash. Now, what we know is that, this f dash contains. So, f is a subset of f dash, so f dash may contain some element which not there in f, therefore if this is the scenario. So, delta dash hat q 0 x intersection f not equal to phi.

Then this implies that delta dash hat q 0 x intersection f dash not equal to phi, but the converse may not be true. That means if delta dash hat q 0 x phi f dash intersection f dash not equal to phi, then it may not be the case that delta dash hat q 0 x intersection f not equal to phi, because f is a subset of f dash. If this is the case then it implies that x belongs to L of n dash according to the definition.

If x belongs L of n dash then delta dash hat q 0 x intersection f dash must on equal to f d differentiate. That means at least one state thus common to f dash and the set of next is from q 0 delta dash hat q 0 x. So, that is by definition, therefore, it says, it is clear that L of n is a subset of L of n dash. So, it is quite clear that L of n is a subset of L of n dash.

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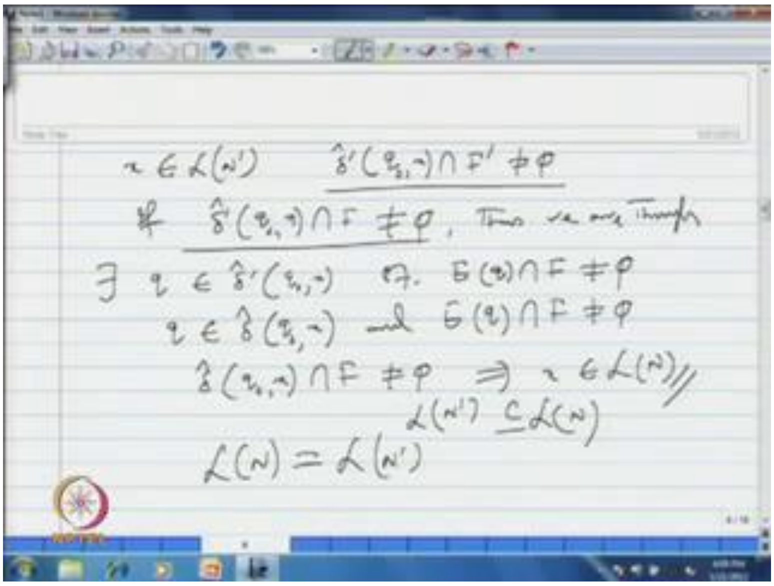

Conversely suppose that x belong to L of n dash. That means delta dash hat q 0 x intersection f dash not equal to phi. So, according to definition of acceptance the string x by this NFA n dash delta dash hat i q 0 x intersection f dash must on be equal to phi. So, in this case if we can you show that delta dash q 0 x intersection f, delta dash hat q 0 x in an f is also not equal to phi. So, that was the case for f dash and we can show that this is n f also not equal to phi then we are true.

Otherwise, then we are true because that will also belong to x must will also belong to L of N. Otherwise there exist some set q that belongs to delta dash hat q 0 x such that there exist q belong to this set of n x dash for this transition on string x. Such that epsilon closer up q intersection f not equal to phi. This implies that q belongs to delta hat q 0 x and e q intersection f not equal to phi which in turn implies that delta dash q 0 x intersection f not equal to phi. That is x belongs to L of n therefore, L of n dash is a subset of L of n. So, considering the previous result that L of n is a subset of L of n dash and L of n dash is a subset of Lf n. We now conclude that L of n is equal to L of n dash.

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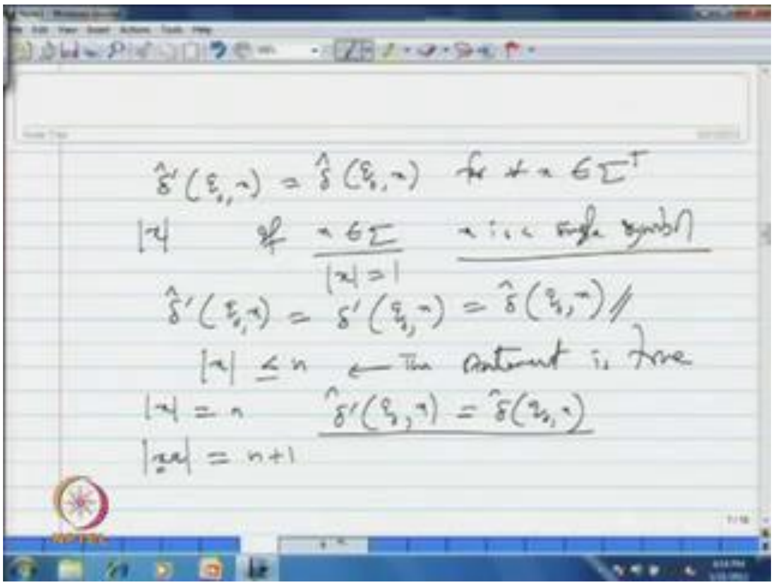

So, it is enough to prove that delta dash hat q 0 x equal to delta hat q 0 x for all x belonging to sigma plus that means for any string over sigma or an epsilon. So, we prove this by induction on the length of the string x, does prove it by induction on the length for string x. So, if x belongs to sigma that means if x is a single symbol. That means length of a x equal to 1 that is what we have said length of x equal to 1. That means x has single symbol, that means x belongs to the input of all it.

Then by definition of delta dash, delta dash hat q 0 x equal to delta dash q 0 x which nothing but delta hat q 0 x. So, for any string of length 1, it is true and there is a base case. Now, we assume that this result is true for all strings of length less than or equal to N. That means for n length of string x less than or equal to N say the second 1 is true.

There is a inductive hypothesis, that means for string x of length n delta dash q 0 x equal to delta dash hat q 0 x will delta dash delta hat q 0 x. Now, consider the string x a for a j single symbol length of the string is n class 1. Since, length of x is n and by inductive hypothesis did the case is this true does the second is true.

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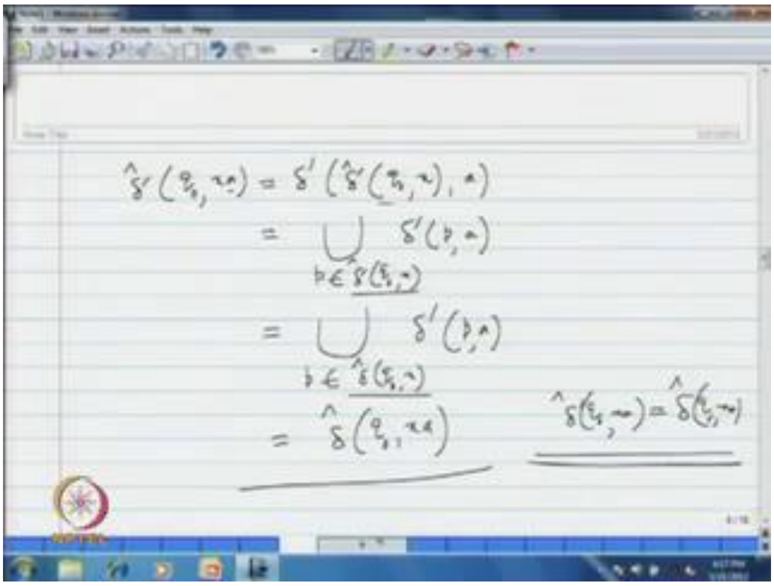

Now, delta dash at q 0 acts a equal to. So, this can be written as, first we apply delta dash hat q 0 on the string x. Then what about the state of that we get apply here, the transition function delta dash. So, this how we define the epsilon transition function, but this is nothing but since this delta dash at q 0 x is state of states. For every subset we apply this delta dash A and hence we leave it take the union of all those. That means union of all those for every P belonging to delta dash hat q 0 x.

If we apply delta dash, this delta dash on for every P on symbol for all a input symbol for the input symbol a, that we have over here. So, by definition. So, just this is the case by definition. Now, according to in that is inductive hypothesis this delta dash hat q 0 x can be written as b belonging to delta hat q 0 x because the length of x is equal to n and according to inductive hypothesis delta dash hat q 0 x nothing but delta hat q 0 x.

So, this is delta P n again. According to definition this is nothing but delta hat q 0 x pi by the definition of epsilon transition function delta. Hence by induction I have shown that delta dash hat q 0 x a, is nothing but delta hat q 0 x n. Hence this completes the proof that means the language exceed by the original NFA N with sum epsilon transition can be replace is q a identical to the languages exceed by the NFA, that we construct that we does not have any epsilon transition. Next, once we have constructed this NFA from the original NFA we have epsilon transition. This epsilon transition does not have any, this NFA does have any epsilon transition.

Now, we will construct from this NFA who is does not have any epsilon transition may have multiple next it is on input symbol on some input symbol from out of any state, we will construct a DFA.

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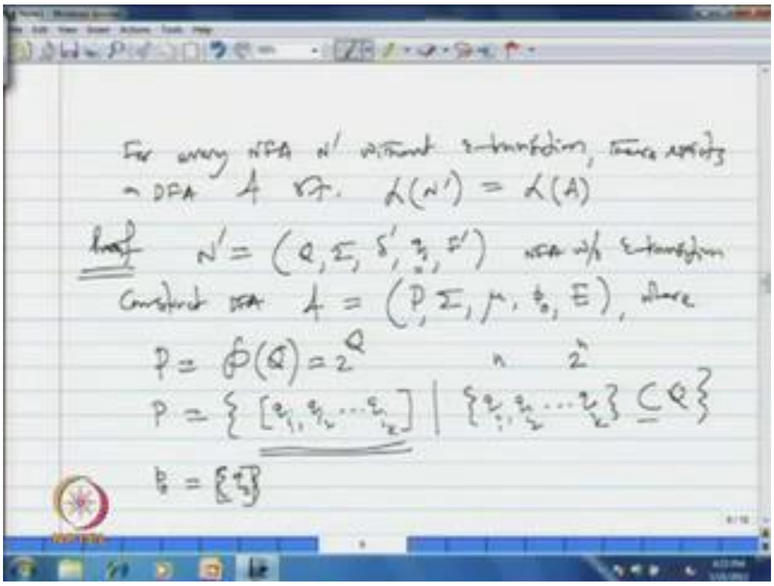

That means what show is that, for every NFA say N dash without epsilon transition, but it might have some other transitions like transition multiple excess on some input symbol or no transition out of any state on some input symbol then there exists a DFA. Say a, solves that the language of this two are same. Now, we will give a proof for this. Just consider that the original NFA without epsilon transition as set of states q sigma identical for that delta dash is the transition function q 0 is the start state and every case is the state of final steps. So, this is the NFA without any without epsilon transition.

Now, will construct the DFA which contains. So, P state of steps the input, then how it is same say transition function is suppose mu, then will have say P 0 is the start state and e is the set of phi states, where the set of states P is nothing but the power set of q that is every subset of q will be considered as a state in the DFA. We also written as sometimes 2 power q. So, it means that if the original NFA has n states the equivalent DFA may have 2 power n states total into maximal.

What of states and set of states P can be written as q i 1 q i 2, q i k within square bracket. So, this is step such that this q i 1 q i 2 q i k be a subset of the set of steps of the NFA. So, we indicate a state in the DFA within square bracket for a subset of the original set of states in the NFA. The start state P 0 is nothing but the original start state is considered as subset contain the single element the original state only NFA. Then that mean this is the starts from DFA, let us make it clearer. So, this indicates that the subset containing q 0, is basically consider as the start state of the DFA.

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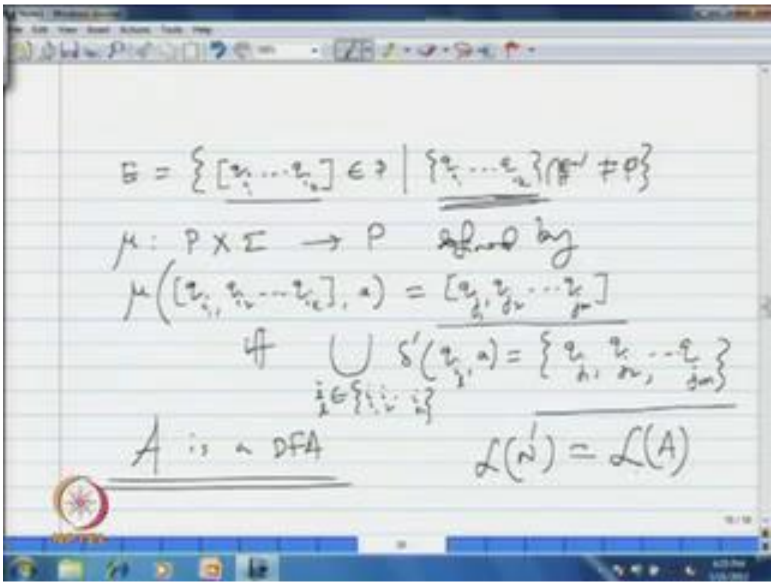

Then the state of final states e is nothing but say, q i 1 up to say q i k. So, this is considered it belong to P is considered as the set of finite sets and a DFA, provided q i 1 up to say q i k this is a final state. This intersection f f dash not equal to phi. That means this contains, this subset contains one only finite states in original NFA, then this state corresponding to that subset be considered as a final state in the DFA.

This transition function mu which is from the set of states P, then sigma to P is defined by defined by mu of say q i 1 q i 2 up to say q i k. This mu on this state either set of states or original NFA on input symbol a is nothing but say q j 1 q j 2 up to some say q j N, if and only if we have the transition in the NFA the delta dash q i L on a say it is q j 1 q j 2 up to say q j m for every L i L belonging to the set of states every. I will going to i 1 i 2 up to say i k and it talk the union of all those. Once when we need we get set q j 1 q j 2 q j n and such a case we provided provide this proof mu q i 1 q i 2 up to q i k on a it will go to q j 1 q j 2 q j N.

So, clearly the automaton that we have define by this construction is a DFA because it satisfies all the properties to being, to be a DFA. Now, what I understood is that L n dash is equal to L of A. To prove this what we need to prove is that.

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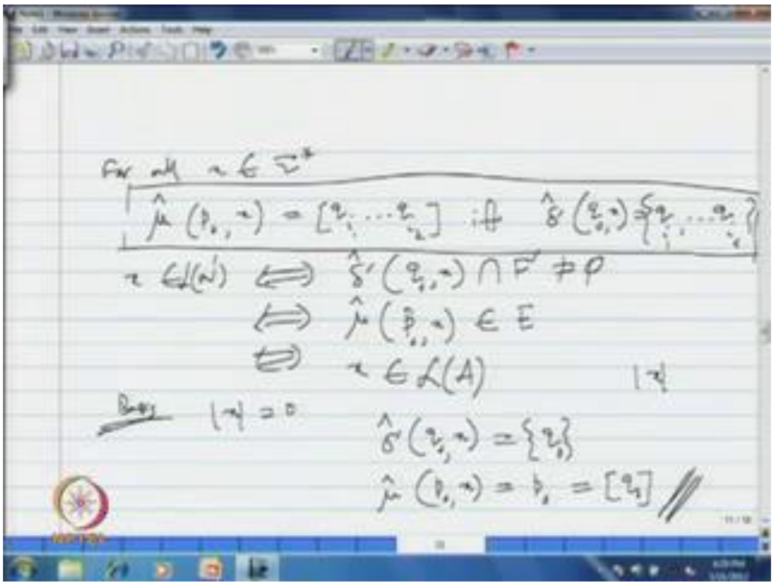

For all string x belongs to sigma star mu P 0, where P 0 is a star state mu P 0 x for mu dash extended transition function for this DFA. So, mu dash P 0 x equal to q i 1 up to say q i k if and only if delta dash hat q 0 x equal to q i 1 q i 2 up to say q i k.

If we can show this, then we can prove that L of n equal to L of n dash equal to L of A. This surprises the result because x belongs to L of n dash if and only if delta dash hat q 0 x intersection f dash not equal to phi, according to the definition of acceptance with the string by this NFA. This is the case if and only if according to the construction of this DFA from this NFA. We have seen that mu hat P 0 x belongs to e if and only if x belongs to L of A.

Now, it proved the statement that for all x belonging to sigma star, we have that delta dash hat P 0 x equal to q i 1 up to q i k if and only if delta dash hat q 0 x equal to the set q i 1 up to q i k. Now, the discussion is that we can again produce a result on the length of the string x. So, the basic is that the length of string is 0. In such a case delta dash hat q 0 x equal to q 0 also mu dash P 0 x equal to P 0 this is nothing but q 0.

So, this is proved or does taken holds for length for strings of length 0. This will also be the case for strings of length equal to 1; that means for every symbol belonging to the input of alphabet. It is very clear from the definition, that you have given the transition function mu. It holds for strings of length of 1 as well.

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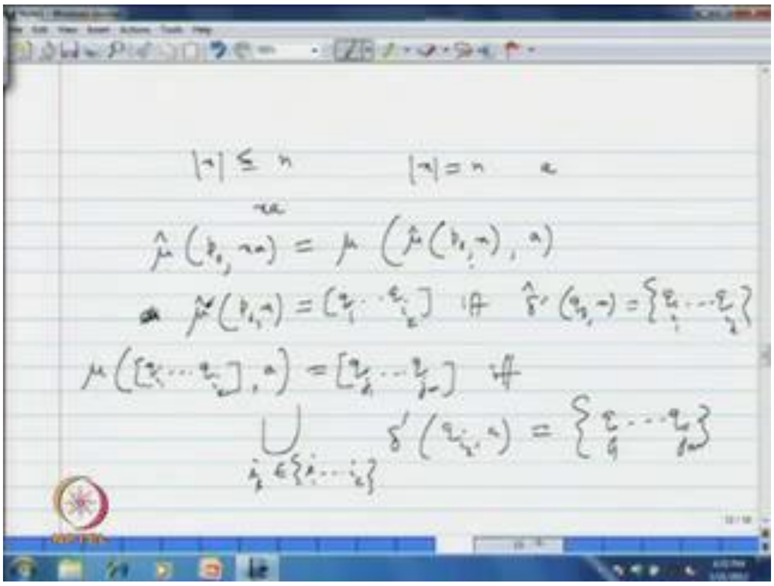

Now, assume that it is true for all strings of length n or less, for all string n of length for all strings of length n or less say this is to be true this true. Now, consider say length of string x equal to n and the single symbol a. Now, if you consider string x a, but you will have delta hat P 0 x a, can be written as mu of mu hat P 0 x then a. So, for supplied is a transition function on x and then whatever set of states we get there we will apply the transition function mu on each of that.

Now, by in that you hypothesis we have founded mu hat mu hat P 0 x is nothing but q 1 up to say q i 1 up to say q i k if and only if delta dash hat q 0 x equal to q i 1 say q i k. So, this is why in that hypothesis because the length for string axis equal to n. Now, by definition of mu, we know that mu q i 1 up to say q i k on a is nothing but q j 1 up to say q j n. If and only if union of i L belonging to i 1 up to say i k delta dash qi L a equal to q j 1 up to say q j n.

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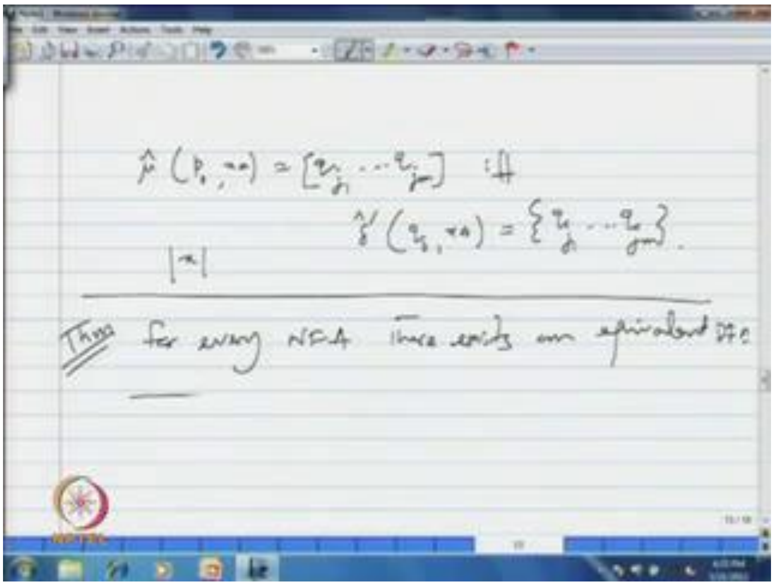

Thus, mu hat P 0 x a equal to q j 1 up to say q j m, if and only if delta dash hat q 0 x a equal to q j 1 up to say q j n . Therefore, by induction on the length and the string x the statement that we have already said is true. Hence theorem follows. Now, from the above two theorem, that is first one is that for an NFA, if NFA with some epsilon transition then we can have an equivalent to NFA without a transition. Then if we have an NFA without f n transition then will have there exists DFA was equivalent to this NFA. Therefore, for every NFA there exists an equivalent DFA. So, these are theorem that we have proved from the above. Now, let us construct an example or given example. So, how to construct an equivalent DFA from any given NFA without epsilon transition.

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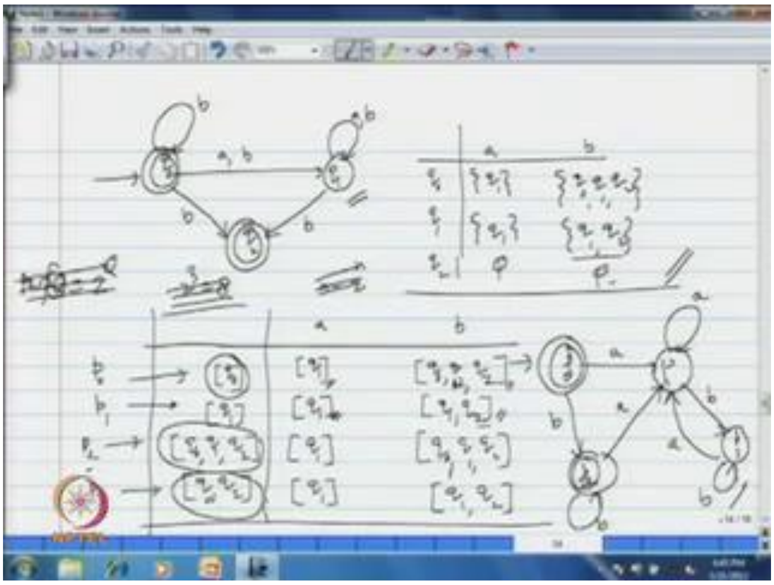

So, let us start with the same example that we have already constructed. Let us redraw the equivalent NFA that we have drawn from the original NFA, which I have for we have q 0 as the star state as well as the finite state on input b their self-loop on input a and b it goes to q 1, q 1 on a b, it remains in the same state and q 2 is a final state again q 0 on b will goes to q 2 and q 1 on b and tell it goes to q 2. So, this was the equal NFA without f n transition for a given example.

So, the transition map for this was simply q 0 q 1 q 2 this on a and b goes to q 1 and q 0 q 1 q 2 q 1 on a goes to q 1 again and on b it goes q 1 and q 2 and q 2 on a and b it goes to phi. So, is there transition f for is given NFA without epsilon transition. So, let us construct the equivalent DFA for this NFA.

So, according to our construction we have seen that, since there are 3 states in this NFA there may be 2 power q that means the set of states that we have P which is equal to the power set of q. There may be 2 power q states that means they may be total 2 power 3 eight states, but all those eight steps are given an n step NFA the equivalent DFA even though then we possible to about n states. All those states may not be accessible from the start state or all the final states may not be accessible from the remaining states of the DFA. Therefore, in the construction what will do, we will start with only the accessible steps and then we proceed in such a way that only the states which are accessible will be here in the DFA.

So, in that construction we will start like this. Let us see construction transition map for this DFA. There two symbols a and b, so according to the definition q 0 is the only start state that we have. This is the start state and q 0 on a, it goes to q 1. So, only state for it goes to. So, the subset containing q 1 is a state. So, we start q 0 because the start state because is a only excisable state initially in the NFA or in the DFA equivalent DFA.

So, on a it goes to q 1. So, the subset containing q 1 is the state, where that if you goes on input a and this state is accessible. Similarly, on b q 0 on b may remain in the state q 0 or it may go to q 2 or it may go to it may go to q 1 or it may go to q 2. So, there are three possibilities. So, the subset containing q 0 q 1 q 2 is the other state does accessible from the start state q 0. Now, we will start constructing a transition or complete transition for all those states which are accessible, that means which are introduced newly in a table. So, q 1 is the new state introduce in a table.

So, we start with q 1 on a, the subset containing q 1. So, q 1 on a, it may remain the same state q 1 and there is no the possibility. So, this is the only possible state which is accessible from q 1 and q 1 on b it may remain on q 1 or it may go to q 2. So, these are only 2 states for q 1 goes on b therefore, this state contains the 2 states q 1 and q 2. So, initially we introduced this q 1 is a new state and then q 0 q 1 q 2 the subset containing q 0 q 1 q 2. Therefore, introduce this construct a transition function for q 0 q 1 q 2 the subset containing q 0 q 1 q 2.

So, we will consider the transition from this state on a, which is nothing but the union of the first column q 0 on or this NFA q 0 on a goes to q 1, q 1 on a goes to q 1 and q 2 on a goes to phi and we take union of our all those. So, that is the only possibility that q 1 is the possibility. Similarly, q 0 q 1 q 2 individually on b goes to q 0 goes to q 0 q 1 q 2 q 1 goes to q 1 q 2 and q 2 goes to phi, taking the union we will get q 0 q 1 q 2. Next, q 1 is already done in this second line second row and q 1 q 2 is the only error state that we have.

So, far with new for which we have not constructed the tradition map, traditional function. So, q 1 q 2, q 1 may go to only 1 step this is q 1. Similarly, q 1 q 2 on b may go to q 1 and q 2. So, this is the only subset that we have correspond to this. We have found it no more states, new states are introduced in the table. Therefore, we did not compute further or process further. That means we have 4 states only q 0 q 1 q 0 q 1 q 2 and q 1 q 2.

So, in this case q 0 is a star state and we construct the final state by considering, if any subset contains a final state of the previous NFA. For example, say this q 0 this state starts at q will be a final state, because original NFA contains q 0 is the start state and final state.

Similarly, this q 0 q 1 q 2 both q 0 q 2 is a final state and q 1 q 2 these must also be a final state because q 2 is a final state, but q 1 is not a final state because in the original NFA q 1 was not a final state. So, this is the equivalent DFA that we have got from the given NFA without any epsilon transition. So, if we can become we can our rename or re-label the states for example, this state we can call it as P 0, this may be called as P 1 this may called as P 2 and this may called as P 3.

Then we can construct the transition function corresponding to this transition map, which is the DFA equivalent DFA for the given NFA. So, let us quickly draw it. So, P 1 is the start state as well as final state. P 1 on a goes to P 2 and P 1 on b goes to P 3, which is also a final state then P 2 on a remains on P 2, because P 2 on a remains on, this is P 0 this is P 1 and this is P 2.

So, P 2 q 1 on a remains the same state and P 1 on a remains in the same state q 1 and on b it goes to q 1 q 2, but q 1 q 2 is a new state q 1 q 2 does P 3. On b it goes to that state and then P 2 on a goes to q 1 and P 2 on b remains was in same state P 2 q 0 1 q 2, that means q 0 q 1 q 2 on b, it remains the same state. Finally, P 3 on a goes to state q 1 that means it is P 1 and P 3 on b remains the same state if q 1 q 2. So, this is the equivalent DFA for the given NFA. We have proved that this DFA is equivalent to this NFA, which are multiple next states on some input symbol.