Formal Languages and Automata Theory
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Module - 14
Introduction to Complexity Theory
Lecture - 06
NP-Complete Problems
Part III

So far we have discussed about N P completeness by showing satisfiable problem is N P complete so which enough N P complete problems we have conducted reduction. Then I shown that satisfiability problem is N P complete. Of course, N P satisfiability problem was the first to show that it is N P complete by Stephen Cook. This is a Cook's theorem that we have established in our lectures of course, so far the examples that we have presented to show that this problems are N P complete. They are essentially related to in touring machines and some tailing problem and then we have finally reduced this problem and established satisfiability problem.

So, you know how to establish problem is N P complete by now through this lectures, so in this lecture I will present some more N P complete problems which are of some importance in practice. Now, we will establish them they are N P complete this problems are among the sort of very first that are established to be N P complete. So, as I had mentioned in sometime around 1970-71 Stephen Cook has established that satisfiability problem is N P complete.

Then within one year time cook established about to 21 problems they are mostly the graph theoretic problems they are N P complete. You know graph models are essentially the models for really practical real world problems, so they are essentially sort of transferred to graph theoretic questions and establish those are N P complete. So, in this lecture I cover few more N P complete problems which are of, which are mainly graph theoretic problems and having some importance with some real world problems.

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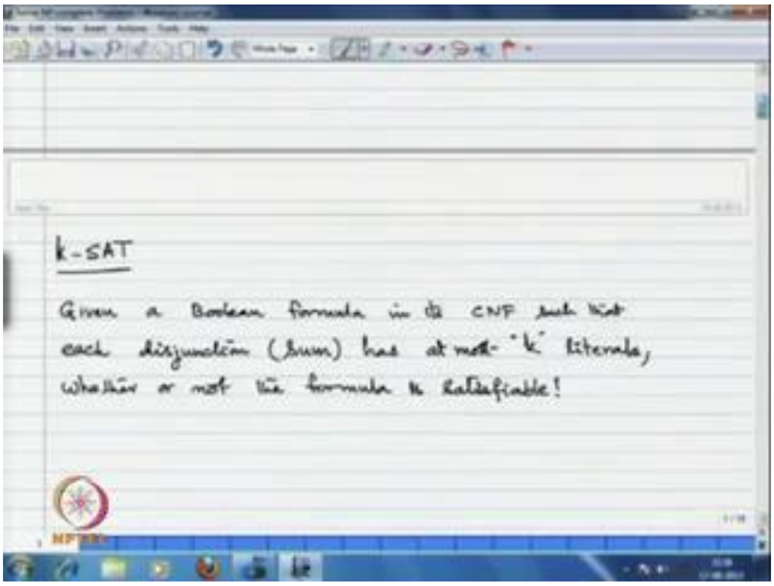

So, in this connection first I will prove that a three set is N P complete of course I had already mentioned what is k satisfiability problem. So, k sat that is given a Boolean formula in C N F in which each disjunction I mean the sum has at most k literals, each sum has most k literals. Now, given such an instance of the problem we have to cross check whether or not the formula is satisfiable, that is what k sat problem is.

Now, I have already mentioned that if you take k equal to 2, these 2 sat is can have polynomial time algorithm right. Now, if k greater than or equal to 3 of course I will establish that 3 sat is N P complete then you can understand for k greater than or equal to 3 these are the hard problems. So, to show 3 sat is N P complete you know by definition we have to establish the 3 sat in N P and 3 sat is N P hard that means 3 sat is as harder as any other N P complete problem. So, that is what we have to look into observing that 3 sat is in N P it is similar to what we have observed that satisfiability problem is in N P, so you can mimic is the similar lines and write that 3 sat is also in N P.

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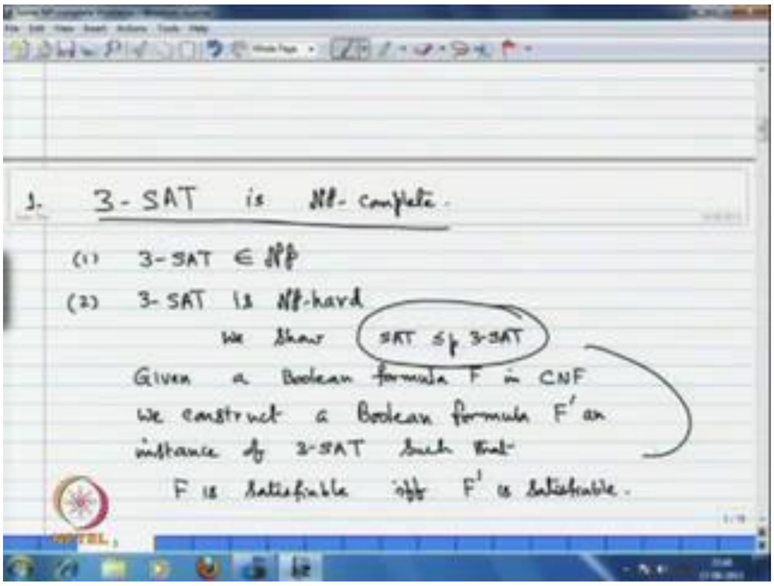

Now, to establish the 3 sat is N P hard what do we do, since we have already established the satisfiability is N P complete we will give a reduction polynomial time reduction to three sat from K sat. So, that means what we have to do given a Boolean formula F in C N F we construct formula F prime and instance of 3 sat such that F is satisfiable if and only if F prime is satisfiable. So, this is what, this is what we have to do the meaning of this particular statement is essentially this is what we have to do, now you take what do we do if you look at this satisfiability problem means tens you will have.

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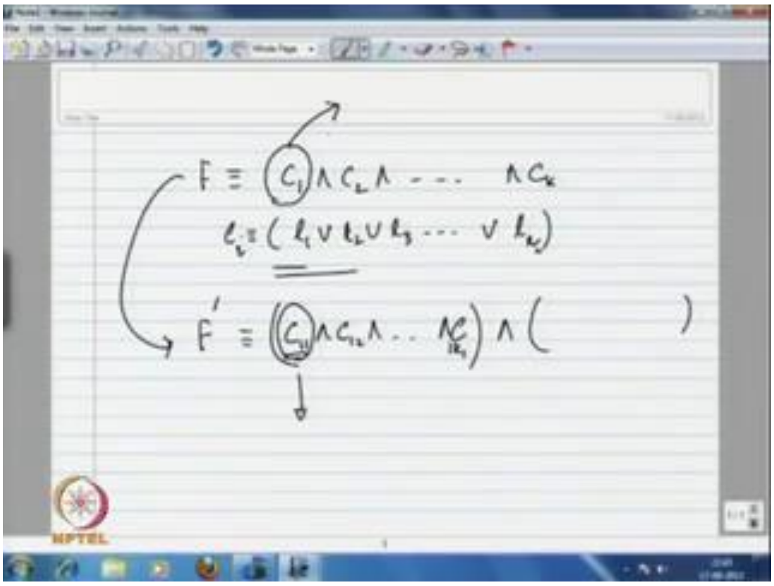

You know a formula is essentially something like you know some certain sums will be there so say C 1 and C 2 and so on, C k for example where each C is a disjunction sum. So, that means is of the form you know some literals say l 1 r, l 2 r, l 3 at maximum you will have three literals because in case of 3 sat.

Now, in case of satisfiability problem here you can have any number of literals that there is no resection for satisfiability problem. So, say l 1 r, l 2 r, l 3 and so on r say some l r you can have each C i and l r i, I will put, so you may have something like this each clause will be of the form.

Now, what do we concentrate if we can write each clause an equivalent formula an equivalent formula which fits in 3 sat. Then we can do it for every C i and thus this F prime whatever that we are constructing that an equivalent formula which is having in each clause. So, say for example C 1 is now somehow divided into say C 1 1 and C 1 2 and so on say some C 1 k 1, C 1 k 1, so this is corresponding to C 1 and similarly for C 2 and for C 3.

So, what we are going to do each sum C 1, now when we are writing this conjunction of disjunction of literals each C i j. Here, we will maintain that it will have utmost 3 literals by maintain this we will construct the formula F prime an equivalent formula equivalent formula. So, we mean whenever this is satisfiable this is satisfiable if and only if this is satisfiable likewise we will construct the truth assignment you can have if it, if it satisfies this and this satisfies here and vice a versa.

So, that is what essentially we are targeting to, so that means it is sufficient to concentrate in one disjunction one sum if I do you can do it for the other sums and establish the things. Now, each disjunction in F with more than k literal with more than 3 literals say for example if you take one sum disjunction which is having say for example x 1 r, x 2 and so on r x k. So, if you have like that this will be replaced with by the following sum what do we do we will take this formula C prime corresponding to that.

So, corresponding to this C 1 whatever that we are writing here if it is having more than 3 literals then only I do this if say for example C 2 has one only one literal. So, no problem if it has 2 literals no problem, because it is anyway fitting to the instances of 3 sat. So, if it is having more than 3 literals then only we do this process whatever I am now mentioning, so that is what essentially C 1 prime is for example.

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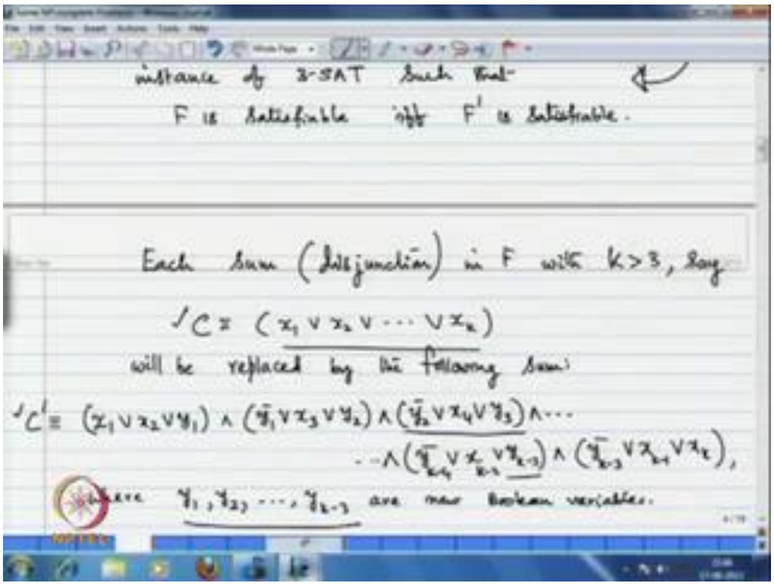

So, what is the formula here I am writing you just carefully see this, so this disjunction C will be replaced with this C prime in the formula F what is this if we have this literals say x 1 r, x 2 and so on. So, x k do not just think that these are variables this I could have written maybe you know some l 1 r, l 2 likewise because this can be you know complement of the Boolean variable. So, whatever that on the consideration so consider a sum x 1 r x 2 and so on x k what we are doing this x 1 r x 2 r by 1.

Now, what is y 1 we are going to choose now some k minus 3, new Boolean variables and construct a sum in the following. So, x 1 r x 2 or y 1 and y 1 bar, because these are Boolean variables this is negation of this y 1 and r x 3 whatever is the third component. Here, in this in C 1 what you have that you put it here r y 2, so what I am doing here y 1 bar r, y 3 r, y 2 I will explain you.

So, and then this is y 2 bar r, y 4, y 3 then if you look at here i have four then I have 2 here, so if when I go to this x k minus 2. Then I will have the here thus if you continue this way y k minus four bar and here y k minus 3. So, these are the new variables that we have chosen and this y k minus 3 bar I mean complement of that negation r x k minus one r x k, so this is what is the formula constructed. Now, let me explain you the theme like if you look at any of this sum, here say if you look at y 1 bar r x 3 r y 2 y 1 complement r x 3 r y 2 if you look at this is equivalent to.

Now, this is same thing as y one implies x 3 r y 2 right what is the meaning how do we look at the truth assignment for this if y 1 is true then x 3 is truer y 2 should be true you know. So, that is what is the formula equivalence here, so the theme here is if y 1 is true then one of these should be true right, so with that we have constructed this formula.

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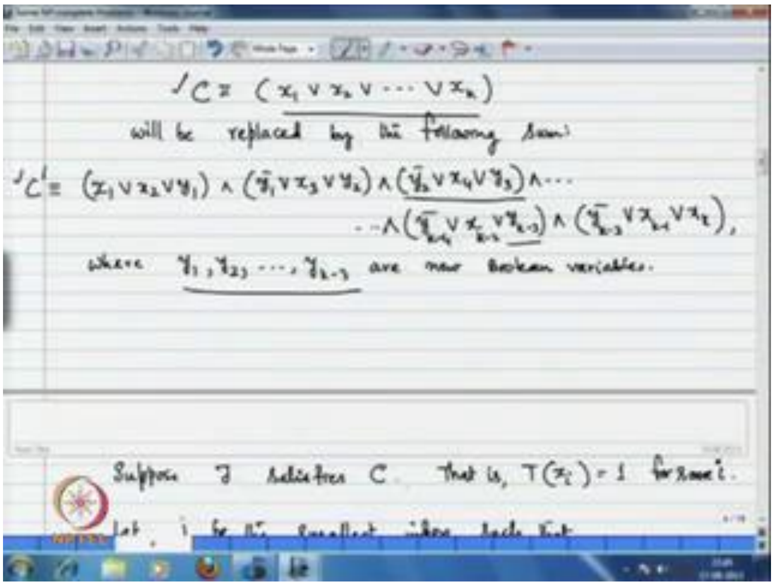

Now, we can see that this C satisfiable if and only if C prime is satisfiable that is what we will we will look at. Now, the point is one side you can see that we are going to observe that C is satisfiable if and only if C prime is satisfiable. So, that means if you have a truth assignment which satisfies C I will construct truth assignment that satisfies C prime and vice a versa. Now, that is what we have to do now assume you have a truth assignment t that satisfies C; that means you look at this sum a truth assignment satisfying this sum.

But, we mean when at least one of these x I will be satisfied by the truth assignment right so that is t x I equal to 1 for sum I. Now, what I will ask you to do here let j be the smallest index such that t x j equal to 1, so that means here in this x 1 r x 2 and so on x k what we have looked into here. So, at x j we have truth value 1 and before that all r receiving truth value 0 that is how we have now what do we do. So, we will extend this truth assignment t to all these variables this all these Boolean variables x 1 x 2 x k and the new variables y 1 y 2 and so on y k minus 3.

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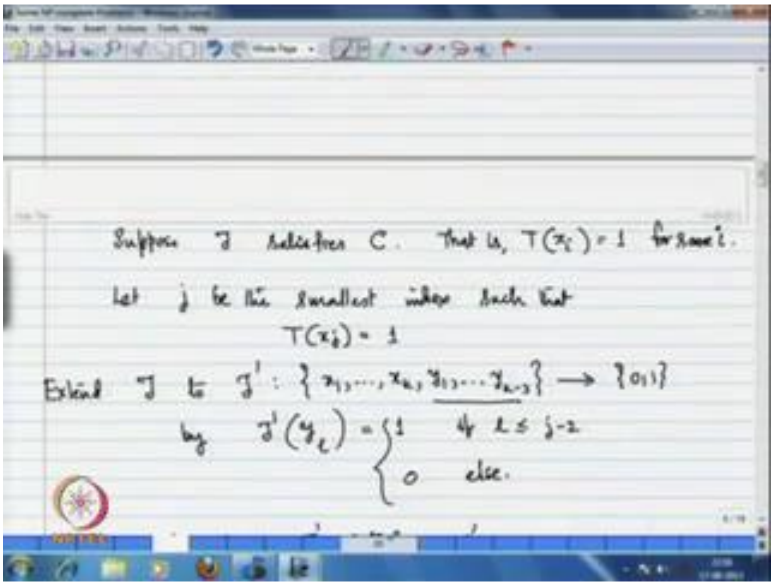

We give truth assignment this way because this is an extension, so that whatever the truth assignment t that gives for all x i that will be the same. Now, we have to tell for other variables this y 1, y, 2 y k minus 3, so what are what are we going assign here is this t prime at y l. So, if you take any variable here in among y i for all l with less than or equal to j minus 2 we give truth value 1 otherwise we give 0, so now you can cross check that this t prime sat satisfies that c prime how it is?

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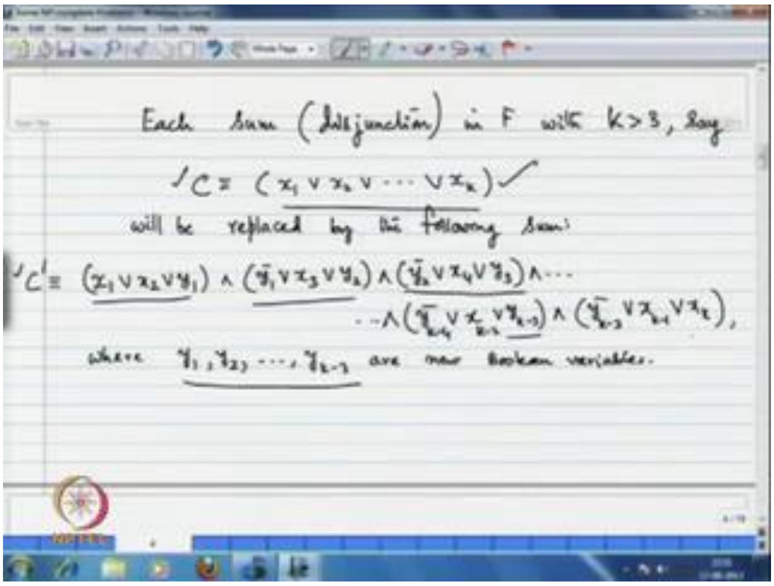

Now, look at the formula because what I have to say that this t prime satisfies each of these sums, this sum, this sum and so on all these things. Now, we know that till x j all the way all these literals x i they are receiving truth value 0, but now from the definition of t prime you see till l dash than equal to j minus 2. So, we have, we have assigned truth value one for this y also that means in the, if x j is somewhere here in between.

Now, y 1 has received truth value one therefore this sum will be satisfied by t prime, similarly y 2 is receiving truth value 1, so this will also be satisfied and so on. Now, wherever we have say for example x j here, so that has been satisfied that has that has been satisfied by the truth assignment t and therefore t prime. Now, thereafter, now you look at, thereafter you look at what is here thereafter we are assigning this y i truth value 0 for this j minus 2.

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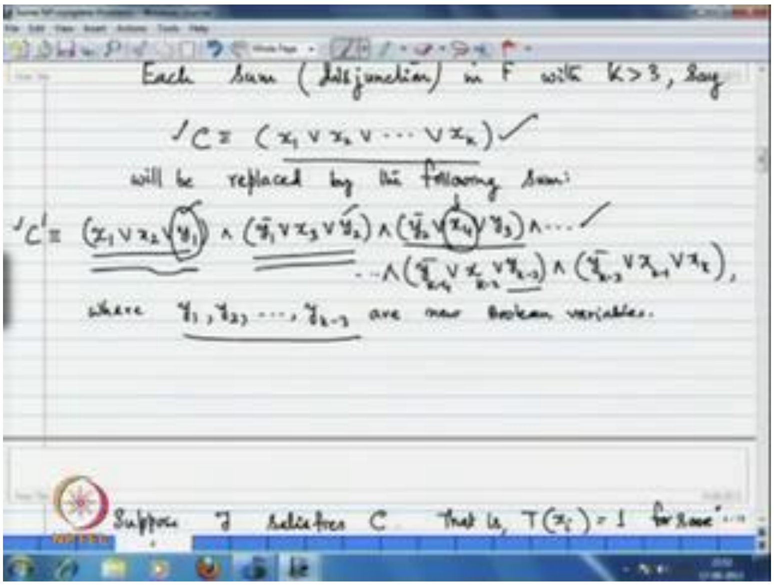

So, whatever here if I have truth value one that is the least, here of course till this point we have assigned truth value 2 for all these y i. So, you can concentrate on these things these variables and see this sums are satisfied now thereafter for all the variables y. So, we are assigning truth value 0 for y i, now you concentrate on the first one, now you concentrate on the first one. Now this candidate we have assigned for y k minus y k minus 4 for example we have truth value assigned truth value 0, so its complement we will receive truth value 1 similarly here.

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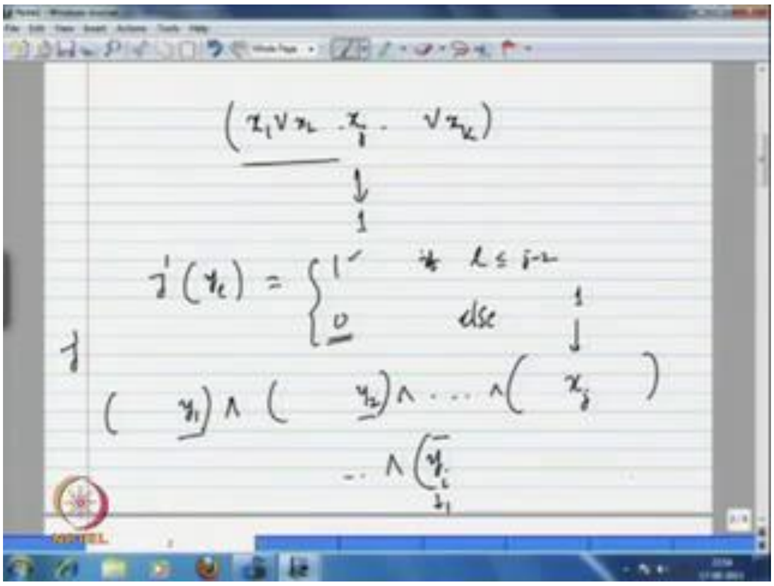

So, the truth assignment t prime we have assigned this way for all y l truth value 1, if this l is less than or equal to j minus 2 and elsewhere 0 I mean. Thereafter, that means when you are looking at the first one you have y 1, here this will assign truth value 1 and when I have y 2. Here, this will assign truth value 1 and so on when I have got this x j which is having truth value 1.

So, this will be satisfied by that t prime in which and thereafter whatever the clauses whatever the sums that we have these clauses you just concentrate on the first one because we have assigned truth value 0 for all other cases. Here, I have the candidate that you if you look at this y i prime, so thereafter this first component now you concentrate on and this y i.

Since, we are giving truth assignment 0 with respect to t prime, so this will receive truth value one and therefore all these sums in this formula will be receiving truth value 1 with respect to this t prime this truth assignment. Hence, the C prime is satisfiable, now converge assume you have a truth assignment that satisfies this formula let me write this formula once again.

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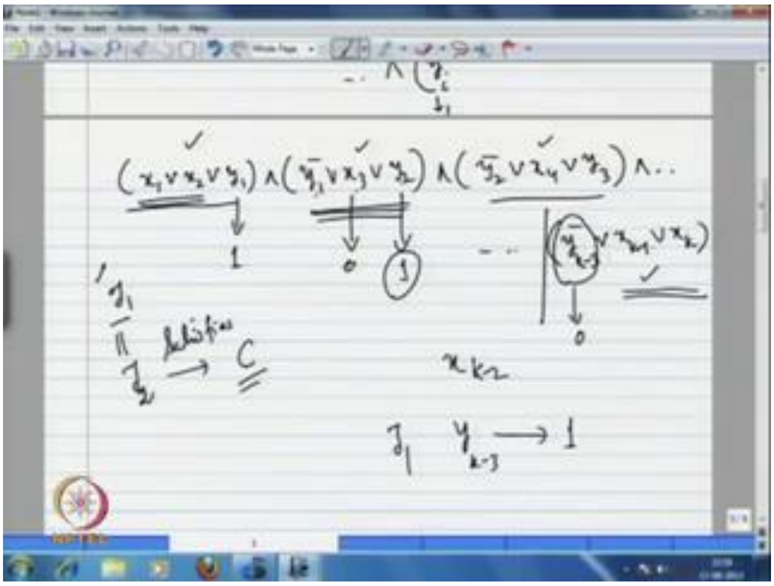

So, the formula is this x 1 r x 2, y 1 y 1 bar, x 3 y 2 and y 2 bar x 4 y 3 and so on. And of course at the end we have this y k minus 3 prime y k minus 3 complement r x k minus 1 r x k this is what we have. Now, if you have the truth assignment that is satisfied this that means everyone of this is getting satisfied by the truth assignment. So, let me properly write that truth assignment say t 1 what is the meaning of this t 2 satisfies this and all these sums will be satisfied by this.

Now, what I have to say this C is satisfied by some truth assignment that is what we have to look into, so here it is very easy. Now, whatever this t 2, let me call which satisfies that C, how we look at, now look at this t 1, which is this is satisfying this. So, for example if this t 1 is giving truth value to y 1 1 only, but not for x 1 and x 2 if anyone of these x 1 and x 2 is getting truth value 1 by this t 1. So, we are true because the same t 1 will work to satisfy C because in C we have x 1 r x 2 and so on x k. So, if x 1 r x 2 if anyone of them is getting truth value one with respect to this t 1 then we are true of not this y 1 should get truth value 1.

Now, when y 1 is getting truth value one then y one bar will receive truth value 0, but if you look at this sum if when y 1 is true this one is true or this one is true. Since, y 1 getting 1, this will receive truth value 0 when it is receiving truth value 0, since t 1 satisfies this sum also one of these y x 3 or y 2 1 of them should get truth value 1. Now, if x 3 is getting truth value 1 then we are true if we are not getting truth value one for this. So, that means if you are getting 0 here this should get truth value one then this will be satisfied by t 1 correct.

Now, let us continue this way suppose if none of these x I are receiving truth value 1 till this point that means till x k minus 2. Now, you let us look at that means this y k minus 3 must be receiving truth value 1 through this t 1 correct, therefore this candidate should received truth value 0 because y k minus 3 is receiving truth value 1. Therefore, it is complement must be 0 and now whatever variables left over here literals this x k minus 1 x k. So, one of them should received truth value one then only this clause will be satisfied this is a very simple argument.

Therefore, what is our t 2, this same t 1 will work for us, so if you have a truth assignment tone which satisfies the formula c prime; that is this formula everyone of its sum should be satisfied by that t 1. Now, the same t one will serve our purpose, because if anyone of these x i is satisfied before you know in anyone of these clauses, then we are true if not what is going to happen if not the first one then second one in x 3. So, we have x 3 if it is satisfied then fine if x 3 is not satisfied by that t 1 i mean is not receiving truth value 1 then it has to be receive 0 which forces us to get truth value 1 for this y 2.

So, you can argue this and continue till you know x k minus 2 if you are not getting truth value 0 1 with respect to the assignment t 2. Then what will happen is either x k minus or x k should receive truth value 1 with respect to this truth assignment t 1, and hence what are the truth assignments. So, which satisfies C prime will also satisfy C, hence I conclude that C is satisfiable if and only if C prime is satisfiable.

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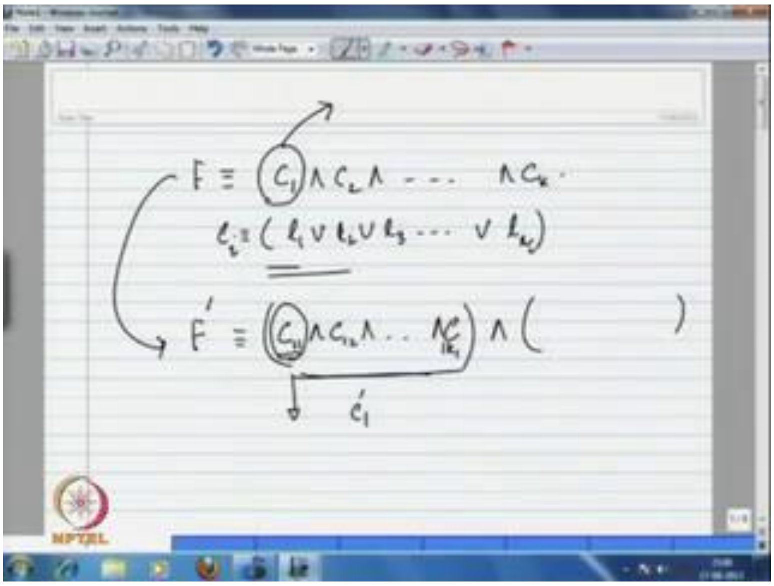

Now, as I had explained in the beginning you take any formula, now each of this sum we will replace it by you know, if it is having more than 3 literals we will replace it by this kind of constructive sum. Now, construct this F prime which is instance of 3 sat and we can observe that F prime is satisfiable if and only if F is satisfiable.

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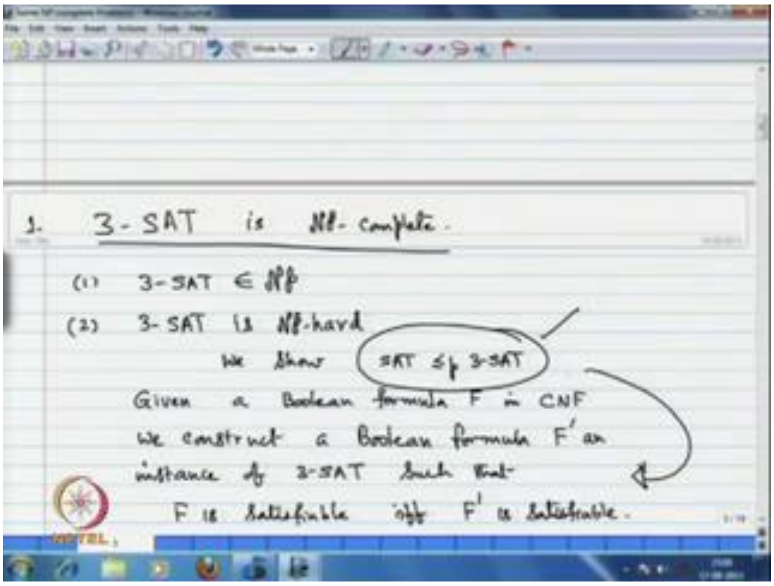

Thus, the conclusion is we have reduced 3 three sat to sorry satisfiability problem to 3 sat, now the question is whether this reduction is possible in polynomial time because you look at this is less than or equal to p. So, what did it indicates less than or equal to is the reduction and p indicates that should work in polynomial time.

Now, you tell me whether this works in polynomial time what is the construction that we are doing here what are the input that you have if you have this. So, you know each sum you consider because total input if you look at the formula the entire formula you have C 1 and C 2 and so on. So, C k these are the disjunction, these are the sums that we have and now with respect to the length of r the literals which are appearing in C 1, now you look at if I have this x 1 or x 2 and so on, x k is a sum k say a number given to you.

Now, what we are constructing here we have included with respect to k few more new variables which are not appearing anywhere in F. So, this have to consider and construct this fixed formula because this formula is a fixed one with respect to that k. Hence, this construction with respect to the input parameter k we do not have, you know anything bigger than just the multiple of k. Therefore, you can easily this task within polynomial time the constructing the formula in polynomial time.

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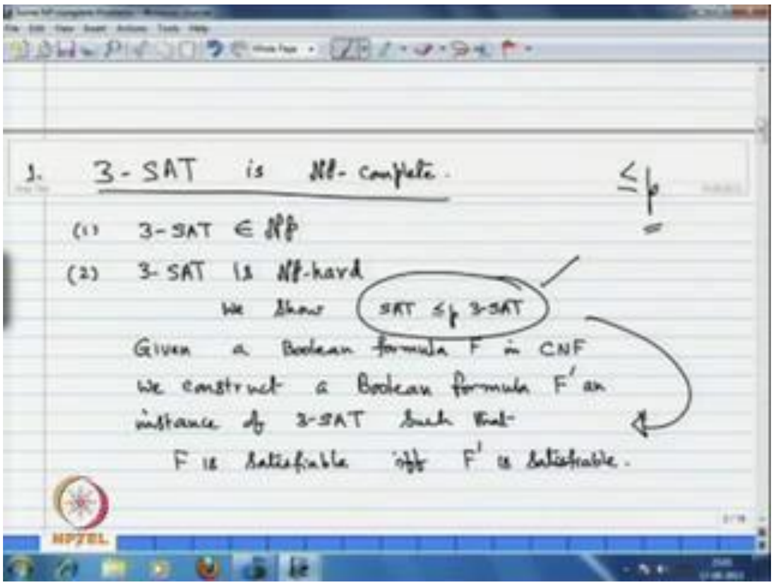

Hence, this reduction is say polynomial time reduction therefore 3 sat is N P hard to conclude that 3 sat is N P complete. Now, then what I will do, I will present few more graph theoretic problems as I have mentioned which are N P complete for the purpose. So, I make use this 3 sat establish also because just we have established the 3 sat is N P complete I can use 3 sat I mean I can use 3 sat to reduce to the problems. So, that we are targeting to other than to satisfiability or other problems which we have established so far.

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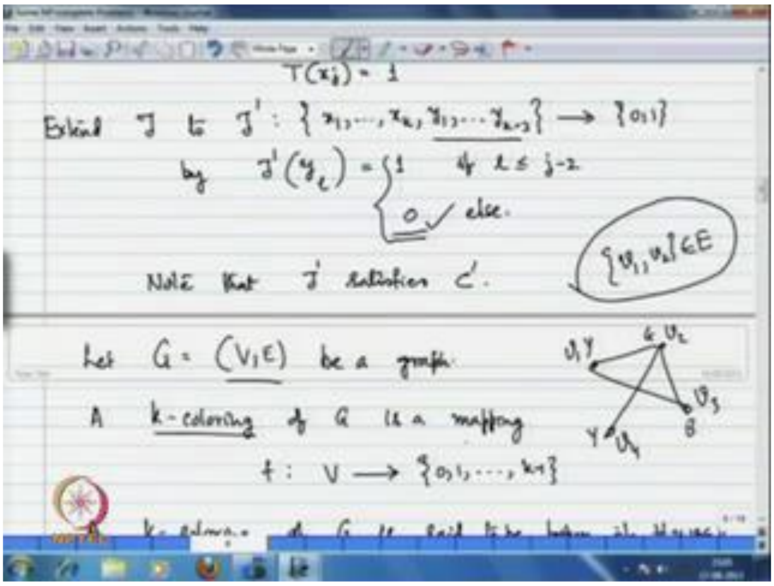

So, let me present this problem coloring problem in graphs, let G equal to V a pair be a graph I hope you are aware of this definition, because V sat of what it is. So, this first component of this pair E sat of edges I am writing graph this is undirected graph that means I am not going to if you are concentrating an example.

So, I am not going to put any directions of this undirected I mean something like this here each edge say V is vertices say let me give label say V 1, V 2 ,V 3, V 4 for example. Now, E you know adjust we have put here between V 1 and V 2, V 2 V 3, V 1 V 3, V 2 V 4, so since this is undirected graph I make I may right this one V 2 edge by another pair V 1 V 2, V 1 V 2. So, this is an element of a alright, similarly V 2 V 3 another pair it is an element of a V 1 V 3 it is an element of a. So, here essentially E the kernel t of E in this example I have 1, 2, 3, 4 edges, so another pairs here and vertex set of course it has 4 vertices.

So, V kernel is also 4 here anyway, so let me that is what is I mean a graph, here let us consider a graph G. Now, in a graph a k coloring of G this is a mapping this is a function F from the vertex set to you know some k elements set I might choose 0, 1 and so on k minus 1 essentially coloring I mean. Here, what do we do for each vertex, we associate a number that means we label we give a color maybe you can take this set of colors maybe red blue green that way.

So, just to say that these colors are now named as 1, 2, 3 and so on k minus 1, say some k minus 1 colors, some k colors, so 0, 1 and 2 and so on k minus 1. So, essentially coloring a graph we mean you just give some color to each of its vertex for example this I give red for example this. Now, I give green this maybe some blue some yellow whatever and you can give maybe a yellow to this as well whatever it is nearly just a function from vertex set to a set of colors. Here, we are choosing colors 0 to k minus 1 for some k colors instead of naming red, green, blue, whatever.

Now, we say k coloring is a proper coloring we said it is a proper coloring when whenever you know you see take an edge the colors which are given to you know this vertices the x n vertices should be different such a coloring. So, we called it as proper coloring, so a k coloring of G is said to be proper any take any two vertices whenever there is an edge.

But, we require the colors given to the vertices should be different whatever the in this example I had written this is anyway proper coloring for example if I choose here also a yellow, now you see this is at a proper coloring. Now, a graph is said to be a graph is a k colorable, it is k colorable if it has a proper k coloring, so using some k colors you will be able to give a proper coloring to the graph in we say graph is k colorable.

Now, regarding this problem, so what is the problem, here you will be given a graph and given some k colors and you will be asked whether this is k colorable. So, I give you a graph of 10 vertices for example and I give you some 3 colors and ask you whether there is a possibility of having some coloring using these 3 colors. So, the coloring should be a proper coloring you can always have a coloring because this is simply assignment of colors you just keep on. But, what we are looking at what we are looking at you can have a proper coloring proper coloring we mean no 2 existence vertices should have the same color, so that is what is the problem.

Now, of course the problem we will look at is a descent problem, here because this I had already mentioned when I am talking about satisfiability we are not worried whether you give me what is called a solution for this. So, that means if you have in this context, if you have a, such a k coloring what is the coloring I am not worried about we are worried about the counter descent problem. Now, we mean if you are given a graph and some k colors whether it is colorable or not you have to say yes or no. So, this is a problem that we are looking into first N P complete problems the descent problems.

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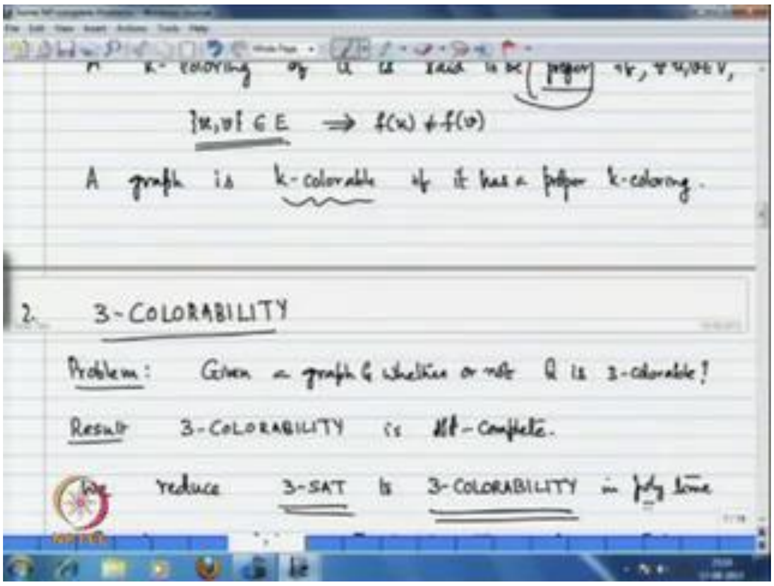

In this connection this 3 colorability is the problem given a graph G, whether or not G is 3 colorable and we will establish that this 3 colorability problem is N P complete. So, once again to establish this is N P complete we have to observe that this is a N P problem as well as it is N P hard to observe that this is N P problem. So, what do we require, you are given a coloring you have to only cross check whether it is proper coloring or not.

So, that is very easy you just take each edge and cross check the vertices end of that edge whether they are having the same color or not. But, if they having same color somewhere for some edge you simply say that this is not a proper coloring otherwise you report yes this is a proper coloring. So, what is a time you require you have to just traverse through all the edges and see what is, what the color is given to the end of those edges. So, that is also it is very quick that you can give an algorithm that works in a polynomial time with respect to the input, here the input is graph you have vertices edges.

So, you look at you have to simply just with respect to the input edges that you it will work in polynomial times, so it is not a problem for you to check whether it is in N P. Now, what do we do we reduce this three sat problem which we have established as N P complete to this 3 colorability in polynomial time of course. So, what I have to do I given an instance f of 3 sat, so I will construct the instance of a 2 colorability that means a graph.

So, given a Boolean formula in 3 sat that means in each of which has utmost three literals right, what do we do, we will construct a graph and such that whenever this formula is satisfiable we have the corresponding graph is 3 colorable and vice a versa. So, this is what we have to establish, now what I will do instead of considering I mean you take an arbitrary instance of 3 sat we can always. Now, say in 3 sat I will always construct the formula in which each sum has exactly three literals because you know if I have only one literal like I can always extent.

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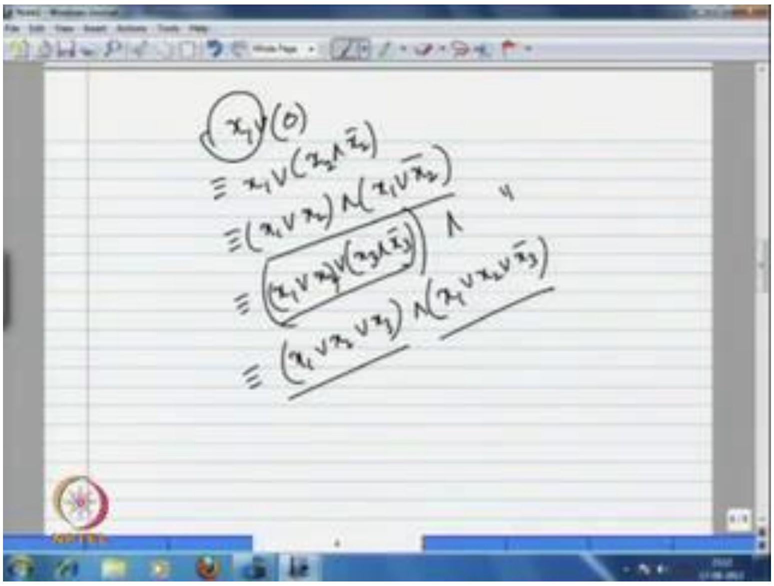

So, say for example I have only x 1, what I will do, I will just adjoin a few more, few more literals it is easy for you, because this r you can put with 0. So, for example this is equivalent to x 1 r say another variable you bring into the picture x 2 and x 2 bar, this is. Now, if you expand this x 1 r x 2 and x 1 r x 2 bar, so when I have a clause with only one literal, I can now write, I can make it equivalent formula in which I can bring two literals each.

Similarly, if you want one more literal to bring it into the picture what do you do here x 1 r x 2 and now for example r x 3 and x 3 complement this is same thing with this and this candidate also, what do I get here. Similarly, this x 1 r x 2 r x 3 and let me put a bracket here, x 1 r x 2 r x 3 complement, you see you can easily extent a given formula in which you can have you know exactly 3 literals.

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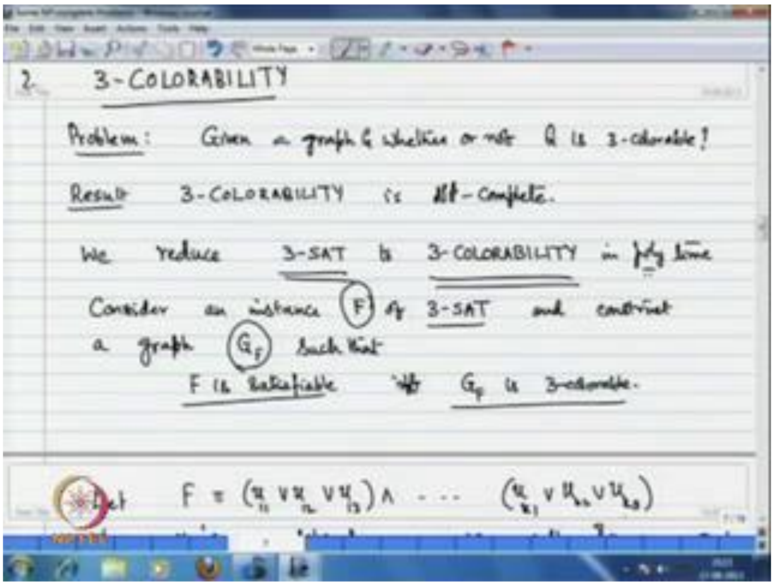

So, what I will now assume a better clause of generality that means whatever the formula that I am considering an instance of 3 sat. So, I assume that each of its sum has exactly 3 literals, so by assuming that what I will do, I will constructs graph corresponding to G F such that f is satisfiable if and only if this G F is 3 colorable.

So, how do we do that, let F be this formula say u 1 1, u 1 2 and so on u 1 3 you know these are the 3 literals as I had just mentioned in this sum. Now, I assume that I have that k sums in the formula F where each u i, here whatever I have mentioned these are literals or the variables. So, say x 1 x 2 x n these are the variables that I have x 1 x 2 x n, now how do I construct a graph, what I will do to each sum of F.

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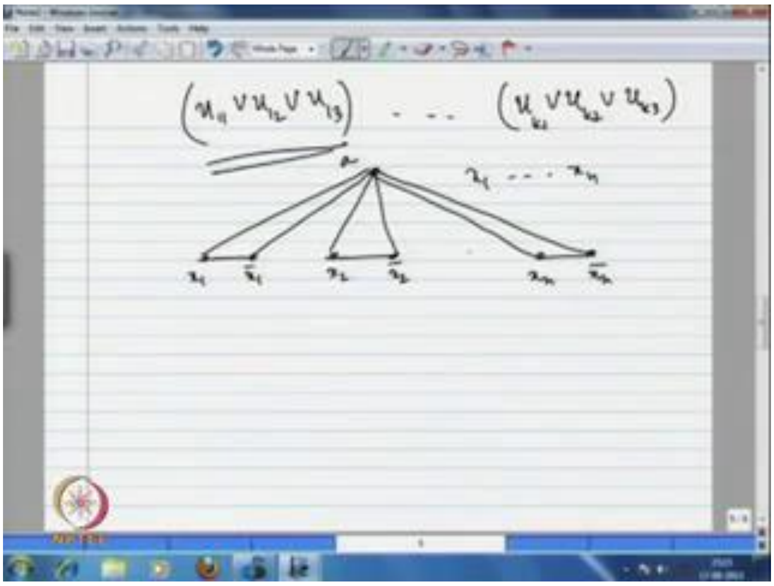

So, say what do I have u 1 1 r, u 1 2 r, u 1 3 this is the first sum and so on u k 1 or u k 2 or u k 3 this is the second sum. Now, when we have like this or variables x 1, x 2, x n what I am asking you corresponding to each of these variables you consider 2 nodes like this x 1, x 1 prime x 2, x 2 bar complement and so on. So, x n, x n bar likewise you consider and take a new node, let me call it as a, and make a triangle. Here, for all these literals make it like this and then corresponding to each of this sum I will now tell you what we have to do.

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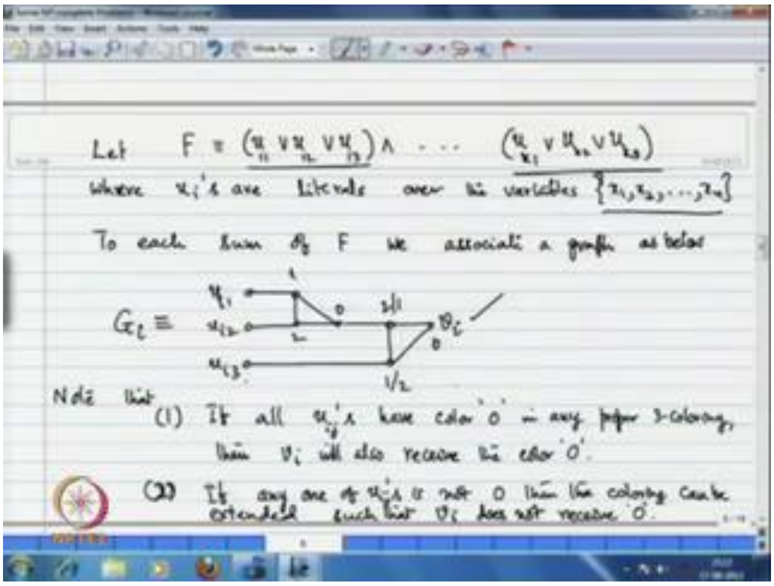

Now, you consider a graph of following type you look at, so these many vertices we will be choosing let me call it as G i, so u 1 1, u 1 2, u. For example for G 1 u 1 1, u 1 2, u 1 3, so in general for each of this sum you construct a graph like this.

Now, what is the important feature of this graph if you observe, let me for example give you know 0 colors to all these 3 nodes what is given to happen. Since, I am giving 0 to this can receive 1 or 2 among the three colors, since I am giving 0, here 0, here to give a proper coloring of this here. So, for example if I am giving 0, I have say I can give 1 here since I am giving 1 here and this is 0 here, I should get 2 only here since I am having this 1 and 2 here. So, I should give 0 here there is no alternative because these two are given 0, since I am getting 0 here if I am giving 1 here I can give 2, I am giving 2 here, but I can give here 1, so that means let me write 2 or 1, one of them only.

So, in this triangle, now it will receive 0, so if these all these three nodes if we are giving 0 colors you see it will force us to give 0 color to vertex V i there is no alternative. So, that is what I am making, here point, here if all u i j have 0 color then any proper 3 coloring any proper 3 coloring will give you know V i color 0. Moreover, if anyone of them is relax with to you know have 0, so that means if one of them is none 0 you can always have some proper coloring 3 coloring. So, that this V i can also receive a non zero color among the three 0, 1, 2, so that can graph G i is this.

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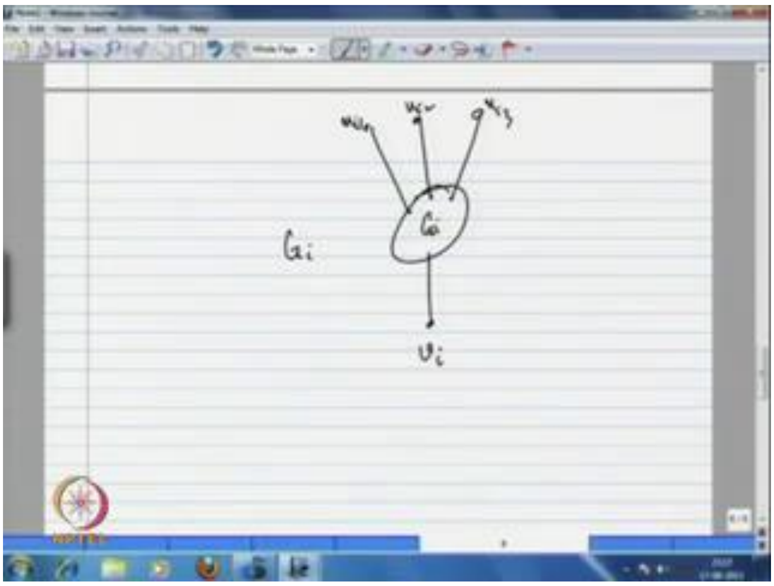

So what I would suggest you this G i how it is this G i it has say something like 3 inputting, so and say V i is here. So, this is what is u i 1, u i 2, u i 3 let me look at G i graph like this there are several vertex vertices inside this, I can only look. But, I will only look at these 3 vertices and this, so what I will do I take for each of this sum a copy of this and put it here.

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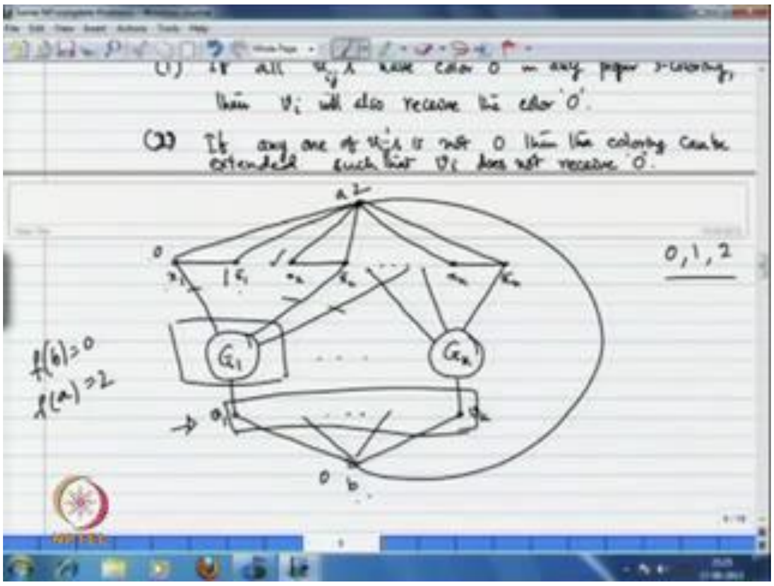

So, G 1 and so on, I have k sums, so G k whatever these vertices what are these vertices this is either x i or some x i bar whatever it is you connect it to that. So, if it is say for example x 2 the first one you connect it to x 2 and say for example this x i 2, sorry u i 2 is x 1 bar you connect it to that. Now, say for example something else you connect it to like this, so similarly this G k whatever these literals that you have that you connect it to that. But, whatever the connections you have here I have V 1, say in between some V i and this is V k, this is what we have.

So, what is the in this construction what we have done first we have made these triangles and what are the graph that we have constructed G i you put it here and look at these literals wherever those literals are appearing in this list you connect to them. Then I am not worried about this G i, I have already mentioned this graph is here in between, now then these V i we will now connect to a node say some b. So, you make it like this and moreover make this a edges into b where is the construct clear, once again all these variables you have x 1, x 2, x n corresponding to which you consider 2 n vertices limit.

So, the labels x 1 x 1 bar, x 2 x 2 bar, x n x n bar, so that you know I can quickly see that this is having a straightforward correlation with these variables. Now, if you consider any sum you see corresponding to which I want to consider graph G i in which these let me call them as input nodes and this is an output nodes sort of these are the you know what are the sum that you are considering. So, you have exactly 3 literals there these literals wherever they are appearing in this list they have to appear here you connect to them and this V i.

So, you connect to your vertex b and make these vertices a and b are also a descent I hope this construction is clear to you that is what is exactly I want to look at. Now, we will argue this the formula F is satisfiable if and only if this graph is 3 colorable how do we how do we argue that, now you cross check carefully suppose we have a truth assignment which satisfies that formula. So, that means what is the meaning of that this in this sum one of them will receive truth value 1, one of them will receive truth value 1 that means for this graph among these 3 connections.

But, you see at least one of them will have truth value one that means a non zero color, so I will give a color here using 0, 1, 2 these three colors. But, whatever the truth assignment that you are giving to this you first give them straightaway this x 1 x 1 bar, if x 1 is receiving truth value 1 you give it to that. Then automatically x 1 bar will receive truth value 0, so you make that color, similarly for x 2 whatever the truth value it is having you just give it to that. Now, its complement will receive the opposite truth value that means either 0 or 1 will be you know, so if it is 0 x two bar will be one and so on.

Now, you cross check since this is formula is satisfiable each of this literal at least one of these literal in each sum you will have truth value 1. So, that means among these 3 connections you will have truth value I mean the color 1, here once you have color 1, I can always make it a proper coloring such that this V i will receive non zero color. But, V i will receive non zero color when it is receiving non zero color, now what I will do, I will give this I can always give 0 color to this because these are all these V i will receive none zero colors.

So, I can put 0 here and then, here since x i and x i complement they are having 0, 1 as colors I will give color 2 here. Now, you see this graph I can have a proper three coloring once again the truth values whatever that you are having 2 x i, x i you just give them as color and automatically their compliments will receive the opposite truth value that means the corresponding color.

So, that since each clause at least each sum, since it is satisfied by this truth assignment at least one of the literal will receive truth value 1 once we have having that. So, once we are having that this each V i can receive truth value and a non zero when I am having this truth value, I mean not truth value.

Now, you can always give a proper 3 coloring such that each V i can receive a non 0 color, so since we are having a non zero value, here I can now take then as colors and I can easily give coloring 0 to b. Since, anyway these two are having 0 and 1, I will give color 2 to a, so that you see this graph can be given as a proper 3 coloring and then suppose you have a proper three coloring of this. So, suppose we have a proper 3 coloring of this converge part, I wanted to give a truth assignment which satisfies the formula F, how do we do that assume you have some proper three coloring F to this graph

Now, what do, what do I suggest you in this in this graph whatever the proper coloring that you are considering you make a small permutation. So, that this F of b is receiving 0 and F of a is receiving two among the colors 0, 1, 2 why I mean how it is possible you just give a permutation whatever the colors. Now, we have this as a proper coloring you just relabeled them if I am receiving say for example z equal to 0 just you called. But, wherever 0 is there you called them as 2 just a permutation between 0, 1, 2 you give that will be that will still be a proper coloring fine.

So, what I am asking you just re relabeled it and give the coloring color 0 to b and color 2 to a first you do that, since it is a proper three coloring what is going to happen. Since, a is receiving truth value, sorry color 2 this x 1 and x 1 bar should receive among the truth value colors 0 and 1 because it is a triangle. So, in this triangle to have a proper coloring here if it is receiving truth value 2, either this should be 0 and 1 or this will be 1 or 0. So, in all these triangles we will have 0, 1, 2 colors in all these triangles we will have 0, 1, 2 colors, now you see let us look at b, b is given color 0.

Therefore, none of these b i can get color 0, none of these can get 0, now in b proper coloring you can argue that. But, since this is getting a proper coloring which is having truth value a non zero and in this proper coloring each G i here in this proper coloring each G i anyone of these connections at least one of these connections, you will have color 1.

Thus, each of these sums here in F can have truth value one of these literals can have truth value 1, each color that we are assigning to. So, that can give a truth assignment which satisfies the formula F, and therefore F is satisfiable, so likewise we can construct a truth assignment which satisfies F.

Thus, we can conclude that this 3 satisfiability is can be reduced to 3 colorability, now polynomial time again you can argue very quickly because you look at the literals we have the size of this. Now, this graph construction with respect to each of this can work in constant time with respect to the input parameter a multiple of the input parameter. So, this works in a polynomial time, and hence 3 satisfiability can be reduced to 3 colorable, 3 colorability, so that 3 colorability is also an N P complete problem.