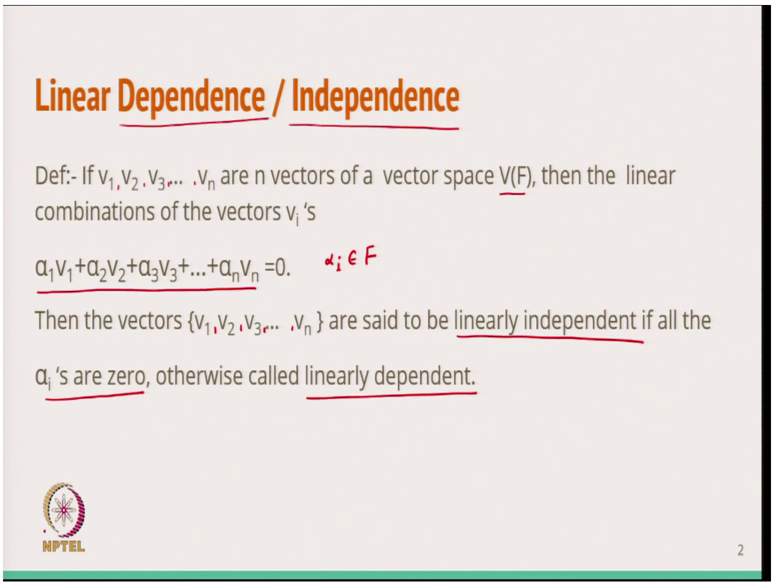
Matrix Computation and its applications

Dr. Vivek Aggarwal
Prof. Mani Mehra
Department of Mathematics
Indian Institute of Technology, Delhi

Lecture - 09
Linearly independent and dependent vectors

Hello viewers welcome back to the course on Matrix Computation and its Application. So, we are continue with the linear algebra today we are going to discuss about the Linearly dependent and independence vectors and then we will discuss the basis and the dimension of a vector space. So, let us start with that.

(Refer Slide Time: 00:43)



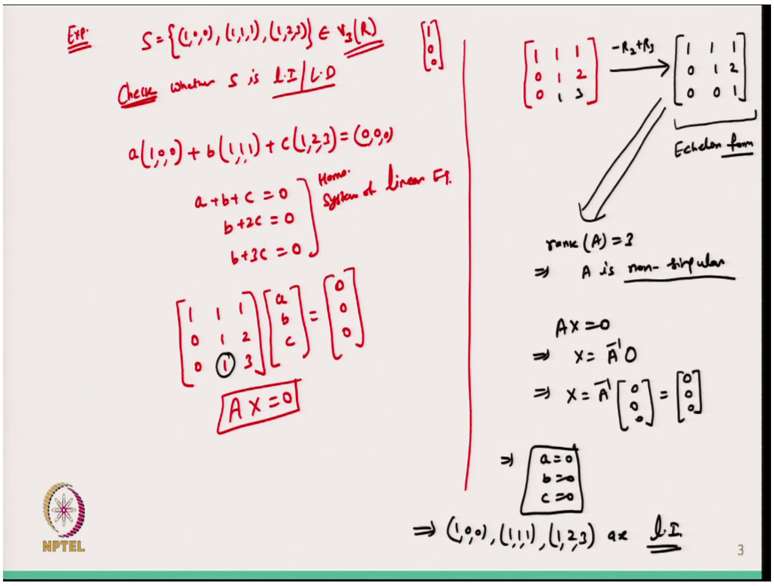
So, today we are starting with the definition of linearly dependence and independence. So, suppose I have a n number of vectors and that vector are coming from a vector space V F. So, I am choosing v 1 v 2 v 3 and v n are the n vectors of a vector space V with the define over the field F then the linear combinations of the vectors.

So, this is a linear combination I am taking alpha 1 v 1 plus alpha 2 v 2 plus alpha 3 v 3 up to this is equal to 0, where this alpha i's belongs to the field F. So, this linear combination we are putting equal to 0 and why we are doing this one? Because, we know that 0 lies in the vector space and all these vectors are coming from this vector space. So, 0 is the additive 0 their additive identity. So, that we are putting equal to 0.

Then the vectors I call it v 1 v 2 v n. So, these are the set of vectors are set to be linearly independent if all the f i alpha i's are zero, otherwise they are called linearly dependent. So, this is the definition of the linearly dependence or independence or the vectors. So, this one we can do.

Now, so, how we can check this one? So, let us do this one. So, let us start with the some example.

(Refer Slide Time: 02:35)



So, just take one example. Suppose I take a set S this is made up of suppose I take three vectors 1 0 0, 1 1 and 1 2 3. So, these are the three vectors I am taking that belongs to v 3. So, this is the v 3 the vector space I am taking. So, I know that this is the vector.

So, generally we also represent the vector like this 1 0 0. So, basically this is the vector I am representing here. So, this is the column vector. So, now, I take three vectors out of v 3 and we want to check whether s is linearly independent or linearly dependent; s means the set of vectors they are linearly independent or dependent.

So, for this one what I am going to do is I just take the linear combination. So, I will take maybe I will define by a 1, 0, 0 and this v 3 I know that is defined over the a real line. So, field is real b 1, 1, 1, c 1, 2, 3 and that I am taking 0. So, 0 means this is the 0 vector belongs to v 3.

Now, these all these vectors. So, what I will going to do is that, I am going to take the compound vice and then putting equal to 0. So, from here I will get first component if I choose. So, it will be a plus b plus c that is going to be 0. Then, I will choose the next component.

So, in this case b plus 2 c that is also coming equal to 0. And then I am taking here b plus 3 c that is equal to 0. So, in this case I get this system. So, it is a system of linear equation it is a homogeneous basically homogeneous system linear equation and I can write this equation in this form. So, I will get 1, 0, 0; 1, 1, 1 and 1, 2, 3.

So, you can see that I have written this first vector as a column vector here in the matrix second is here and the third one is here and the scalar I can write as a a b c and then it become 0 0 0. So, it is you can write as a A x is equal to 0. So, it is a homogeneous system of linear equation.

Now, I want to solve this one. So, we know that how we can solve this. So, I what I can do, I can just convert this into the echelon form and then I by using the gauss elimination. So, what I want to do that? Now what I do is that I will reduce this 1 into the upper triangular matrix.

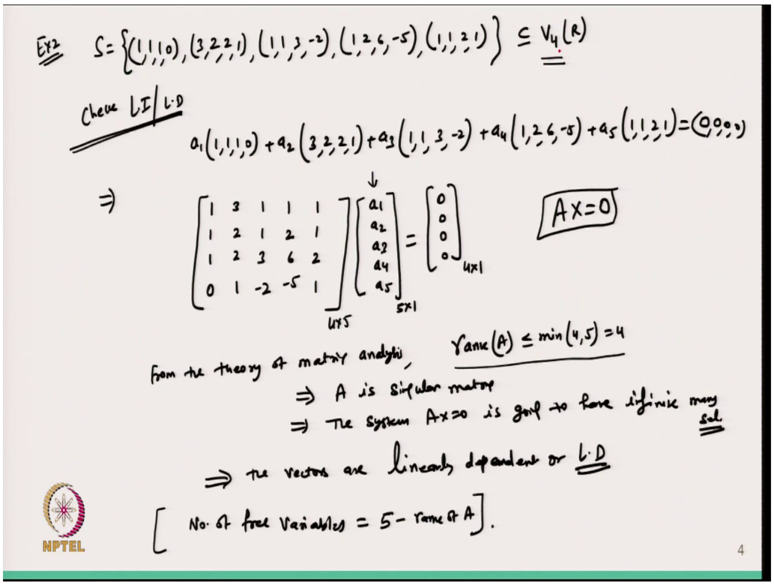
So, I will get 1 0 0. So, 1 1 1 2 now I have to make this element and this element. So, basically I just want to make this element 0. So, and this one I can do just multiplying the. So, I can here I can write first I write 1, 3 then I apply minus of R 2 plus R 3 and from here you can see that I will get 1 1 1 0 1 2 and it is 0 and this is minus 1 and this is 0. So, minus 2 it is 1 ok.

And then it became the upper triangular matrix or the echelon form. So, echelon form now I can write this one as. So, from here this matrix you can check that the rank of this matrix is 3, it means A is non singular and if it is a non-singular matrix then I can take the inverse of that one.

So, A x equal to 0 can be written as x is equal to A inverse 0 ok and from here I can write my x as A inverse and that is 0 0 0 and we know that if we multiply anything matrix with the 0. So, it is going to be 0 0 0. So, from here you can say that my a 0, b 0 and c 0. So, all these three scalars are coming 0. So, from here I can say that this vector 1, 0, 0; 1, 1, 1 and 1, 2, 3 are linearly independent.

So, this way we can define that these vectors are linearly independent. Now what will happen going to happen if I have another type of vectors.

(Refer Slide Time: 09:12)



So, this is a just simple vector I have taken. Now I take another example. So, in this case suppose I have a set vectors 1, 1, 1, 0; 3, 2, 2, 1; 1, 1, 3 minus 2 and 1, 2, 6 minus 5; 1, 1, 2, 1. So, this is the 5 vectors I we are taking and you know it is a. So, this is a subset of v 4 define on the real line; why it is v 4? Because, each vector has 4 component and define on the real line so this is belongs to v 4. Now, the question is that whether check linearly independent or dependent.

So, the same way I can just take the linear combination. So, I will just take a 1 the first plus a 2 3, 2, 2, 1 plus a 3 plus a 4 and a 5. So, I am taking 5 element from here and putting that a linear combination and as we have done in the previous example the same way we can find here. So, from here I can write this in the matrix form.

So, I can write here 1 1 1 0 and 3 2 2 1, 1 1 3 minus 2, 1 2 6 minus 5, 1 1 2 1. So, this is the 5 vectors we are taken and this is a 1 a 2 a 3 a 4 a 5 and that is coming equal to 0. So, you can see that it is a 4 cross 5 matrix, it is 5 cross 1 and it is 4 cross 1. So, this is become A x is equal to 0 homogeneous system.

Now, in this case we want to check that whether these vectors are linearly independent or dependent. Now from the theory of matrix analysis we know that this is the matrix. So, I can 4 cross 5 matrix. So, in this case one thing I know that the rank of the matrix A is always less than equal to minimum of 4 and 5. So, that is 4.

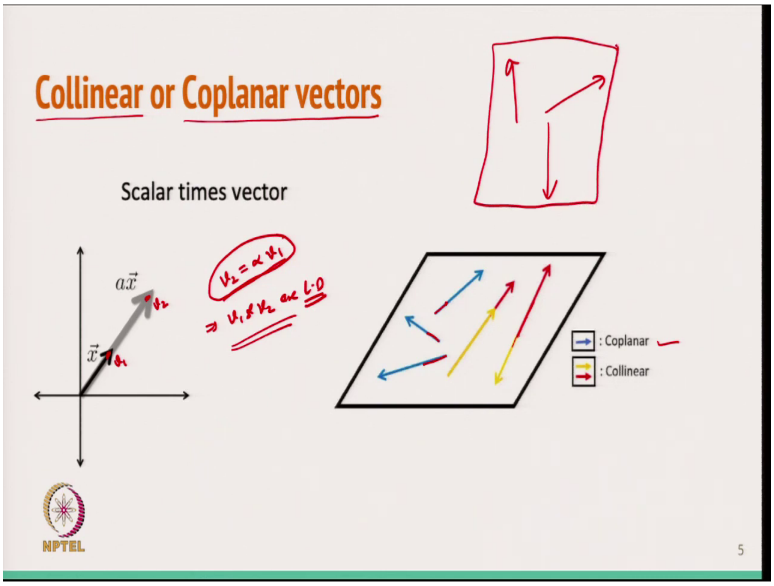
So, in this case the rank of the matrix A is always less than equal to 4 because this is 5; 5 cross 1 and the component of each vector is 4. So, from here the rank of A is less than equal to 4. So, from here one thing is to that A is singular matrix and from here I can say that the system A x equal to 0 is going to have is going to have infinite many solution.

So, there will be no unique solution it will be infinite many solution in this case that we need to find if we want to show that how it is going to happen, but one thing is true that it is going to have a infinite many solutions and from here state we can say that that that the system the vectors are linearly dependent or L D. So, without checking we can just say that it is going to be linearly dependent.

Because suppose it we has a rank 4 then still it has a rank 4 and we have a matrix 4 cross 5. So, it is going to be a singular matrix and then we if we solve this one then we can check that they are going to be linearly dependent because the number of variables are 5 or the free variables. So, in this case you can also check the number of free variables that will be equal to total number of variable 5 minus whatever the rank of A.

So, suppose your rank of A is coming 4, then still one will be the free variable and that free variable we have to choose and then we can have a infinite many solutions ok. So, from here you can one thing you can just keep in mind that you have a v 4 and suppose you take a 5 vectors or 6 vector out of v 4, then it is always going to be linearly dependent. So, these things we can conclude from this example and later on we will prove also this.

(Refer Slide Time: 15:44)



Now after this one I want to discuss one more term is that what is the meaning of collinear or coplanar vectors. Because, just now from there we can check whether it is going to be linearly independent or dependent. So, like this one.

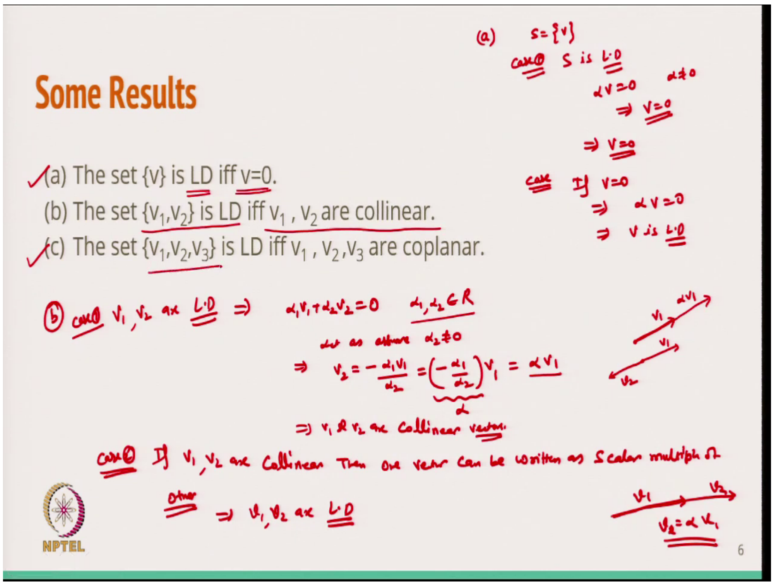
So, in this case it is a just a vector multiplied by some scalar. So, it is a type of collinear vectors suppose I have some vector x is there and some another vector y is there ok and from here suppose it is my v 1, it is v 2 and let my v 2 is some scalar multiple of v 1. So, it means both are coming on the same line.

So, from here you can say that these vectors v 1 and v 2 because from here it can be parallel also because we know that, but in the parallel vector also we just shift that one and then it became the type of collinear here. So, if the two vectors are moving or coming on the same line in this case, then they are linearly dependent ok.

So, I am just putting on the same vector. So, the so, collinear vectors are always linearly dependent another type of vector is coplanar. Coplanar means if the I take the vectors like this vector. So, this vector this one, this one, this one these are lying on the same plane. So, they are called the coplanar vectors and this is the collinear vectors, one vector is in this direction another is coming in this direction.

So, this is a collinear vector it is a of opposite direction, but here it is coming in the same direction. So, they are the collinear vectors. So, now, we talk about 2 vectors or 3 vectors, then we want to check whether they are going to be a collinear vector or the coplanar vector.

(Refer Slide Time: 17:57)



So, this is some results that we can discuss that the set v is linearly dependent if and only if v is 0. So, in this case I just discussed the first part a 1, you have a set that contains only one vector ok. So, in this case it says that. So, I can take the case 1 as is LD linearly dependent set of vectors ok. It means if it is a linearly dependent set of vectors then I can have some alpha v that is equal to 0 for alpha is not equal to 0. So, if alpha is not equal to 0 and v in alpha into v is equal to 0, then it is going to happen only when v is 0.

So, either alpha is should be 0 or v should be 0. So, in this case I am taking the alpha is need not be 0. So, in that case v will be 0. So, from here if it is LD, then it shows that v is equal to 0 zero element and the case 2 is that if my v is 0 then I take any alpha multiply by v that is always going to 0 ok.

So, from here I can say that it is independent of alpha whatever the alpha I take this value is going to be 0. So, in this case I can say that v is LD. So, just one single element is there and if it is a LD only possibility is there when v is equal to 0. So, this is ok the second 1 is that the set of vectors is LD if and only if they are collinear.

So, this b part I can discuss here. Now it is given that that v 1 and v 2 are LD. So, I just take the case 1 and I assume that that the set v 1 and v 2 are LD. Now from here if it is LD then I can write that alpha 1, v 1 plus alpha 2 v 2 is going to be 0 for alpha 1 and alpha 2 belongs to the real line and it can be any number need not be 0. So, in this case that is true.

So, just for the convenience I just assume let us assume that alpha 2 is not 0 because I know that both are never going to be 0 only in one case it can be 0 both the case is 0 v 1 plus 0 v 2 can be equal to 0 no problem, but there if either of them is not zero then is also it going to be 0 that is the meaning of linear dependent.

So, let us assume that alpha 2 is not equal to 0. So, from here I can write this one as. So, I can write v 2 is minus alpha 1, v 1 divided by alpha 2. So, this one I can write as a alpha 1 alpha 2 v 1. So, this is my v 1. So, from here and this is just a scalar. So, it is a new scalar I just call it alpha. So, I can write is a alpha v 1. So, from here I can say that v 2 is some scalar multiple of v 1.

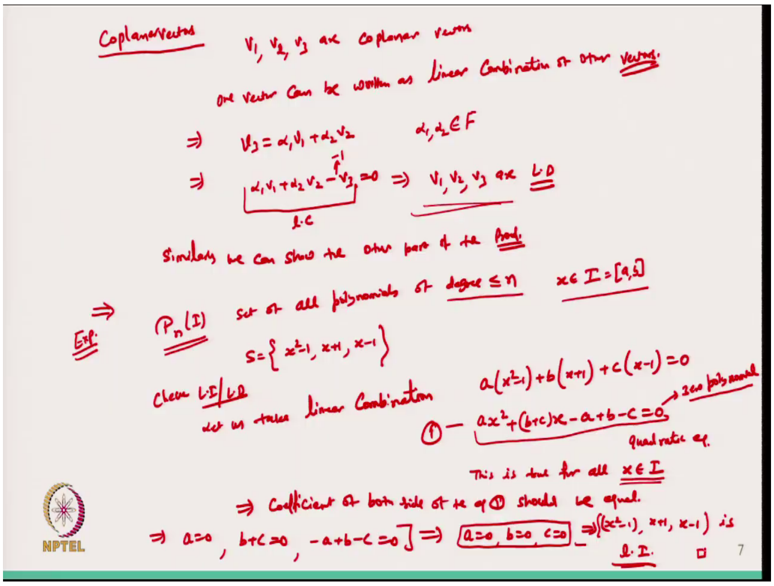
So, if v 1 is there, then v 2 is just the scalar multiple of this one. So, in that case my vectors either it can be suppose, this is my v 1 ok and suppose this is the point now my v 2 can be this it is alpha v 1 or suppose this is my v 1, then I can have an nactive of that one. So, it is my v 2.

So, this is the form of collinear vectors I am taking here so obviously, from here I can say that v 1 and v 2 are collinear vectors and then I just take case 2 that if v 1 and v 2 are collinear then one can be written as scalar multiple of other then one vector can be written as scalar multiple of other.

Because if it is a one vector is this one, another vector it is v 1 and another vector is suppose v 2. So, definitely I can write my v 2 as some scalar multiple of v 1. So, this one I can write. So, from here you can say that and then doing the same process from here I can say that v 1 and v 2 are linearly dependent.

So, this is the same case we can have now the third one is the set of vectors is linearly dependent if and only if v 1, v 2, v 3 are coplanar. So, this is a coplanar. So, coplanar means like this one. So, all the three vectors are coming in one plane like I take one vector here, then another vector here, then another vector here. So, both are coming in the same plane. So, that is the called the coplanar vectors.

(Refer Slide Time: 24:58)



So, in the coplanar vectors. So, we know that from the definition of coplanar vectors, now we know that. So, I am taking v 2, v 3, v 1, v 2, v 3. So, it is given that v 1, v 2 and v 3 are coplanar vectors it means. So, this is possible if one vector can be written as linear combination of other vectors because that is only possibility only then we can say that this is coplanar vector.

So, implies that let we take v 3 is written as some alpha 1 v 1 plus some alpha 2 v 2 ok for alpha 1, alpha 2 belongs to the scalar field whatever we are taking and from here I can add this as alpha 1, v 1 plus alpha 2, v 2 minus v 3 equal to 0. And from here I can say that this is a linear combination and putting equal to 0, but here the coefficient is always minus 1. So, from here I can say that v 1, v 2, v 3 are linearly dependent.

Because, here in this case the coefficient is always 1 minus 1 basically. So, I can choose any value of alpha 1, alpha 2, it can be 0 0 also, but in this case it is always coming minus 1. So, from here I can say that they are linearly dependent. So, if it the coplanar vector then it can be it is a linear dependent and the similarly we can show the other part that in that case we are assume that this v 1, v 2, v 3 are linearly dependent and from there I can go back like this one and that will show that one vector is a linear combination of the other remaining two.

So, it will be coplanar vector. So, in this case the I can say that this vector will be linearly dependent and with the linearly dependent then the vector would be coplanar. So, this way we can check now. So, after discussing these things I can also discuss in some other vector spaces that how we can check the linearly dependence or independence of the vectors.

So, suppose. So, let us take one another example suppose, I take set of all polynomials P n I ok. So, this is all the set of all polynomials of degree less than equal to n ok. So, I am choosing this one. And I is some interval ok. So, from here I just take one set of polynomials that is x square minus 1, x plus 1, x minus 1. So, this is the my set I am taking here. Now x belongs to I. I is some interval you can just take some interval a, b.

So, this one this interval I can take. Now, I just want to check whether this are linearly independent or dependence. So, check linearly independence or dependence of the set of s now in this case also. So, let us take linear combination. So, I take the linear combination. So, let us take a x square minus 1 plus b x plus 1 plus c x minus 1 equal to 0. So, this is my linear combination of the vectors coming from set of polynomials.

And then from here I can write I just can separate the power of x square plus. So, the x I can. So, it can be written as b plus c x. So, the this is I am just separating the x and then the constant. So, constant I can write minus a plus b minus c equal to 0 ok. So, this is my quadratic. So, in this case it is a my quadratic equation and this 0 is a 0 polynomial because I am choosing that from this vector space. So, I know that this is a 0 polynomial.

Now, this is true. So, this is true for all x belongs to the interval I and if it is true for all the x belongs to into I. So, that it implies that that the coefficient of both side of the of the equation should be equal because this is going to be true for all x. So, this is possible if I compare the coefficients of x square and x and the constant.

So, from here I can say now from here I can say that my a will be 0, b plus c will be 0 and minus a plus b minus c that will be equal to 0. So, if I taken from here that this is my a 0 I just taken. So, if I put a equal to 0 here, then b minus b plus c and b minus c both are coming 0.

So, from here you just it is very easy to check that in this case my a will be 0, b will be 0 and c will be 0.

So, all these value will be 0. So, from here we can say that that the set of x minus 1, x plus 1 and x minus 1. So, this is the set of vectors is linearly independent. It means, that this polynomial or the quadratic I am taking here they are linearly independent to each other by solving this one and this is the value of the scalars.

So, let we let me stop here. So, today we have started with the definition of linearly independence or dependence and then we also discussed about the collinear vectors and coplanar vectors and we have studied some example also. So, in the next class we will continue with this one. Thanks for watching.

Thanks very much.