Matrix Computation and its applications

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Lecture - 55
Matlab/Octave code for solving SVD

Hello viewers. So, welcome back to the course on Matrix Computation and its Application. So, in the previous lecture, we have discussed the very important theorem about the singular value decomposition of a given matrix. So, today in this lecture we will do some work. We can take the help of MATLAB or Octave and then we will see how we can find out the singular value decomposition of a given matrix.

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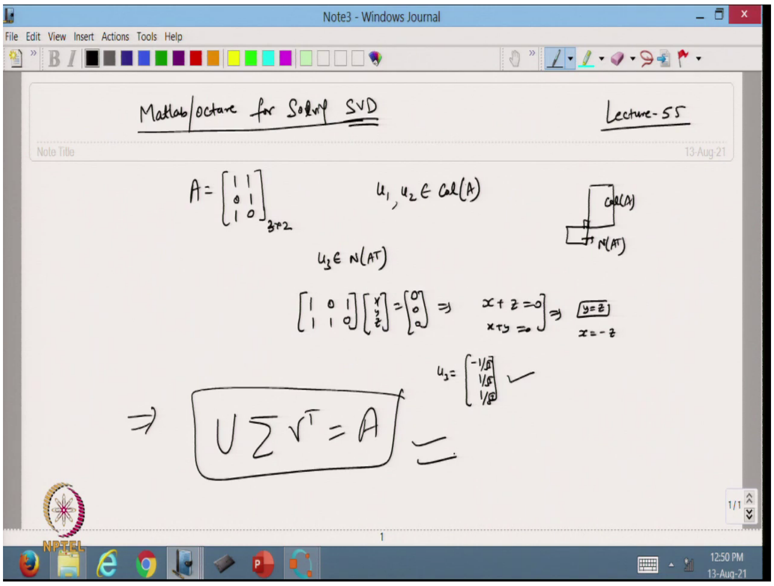

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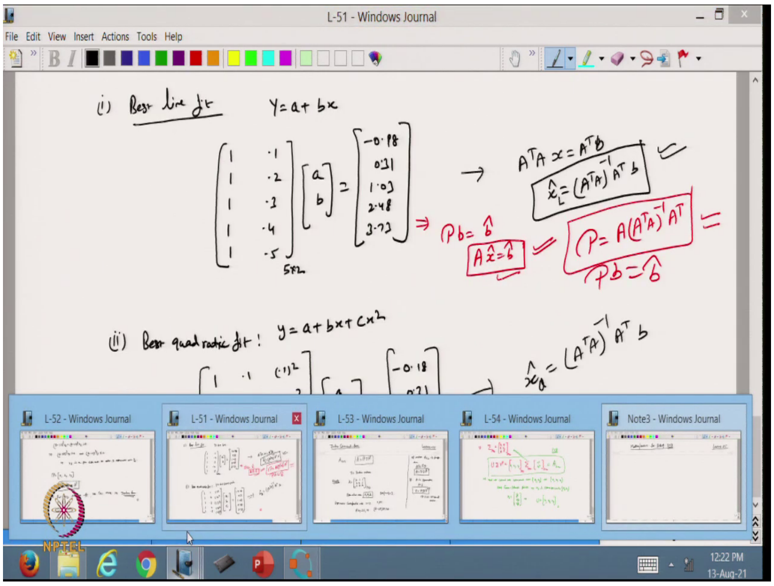

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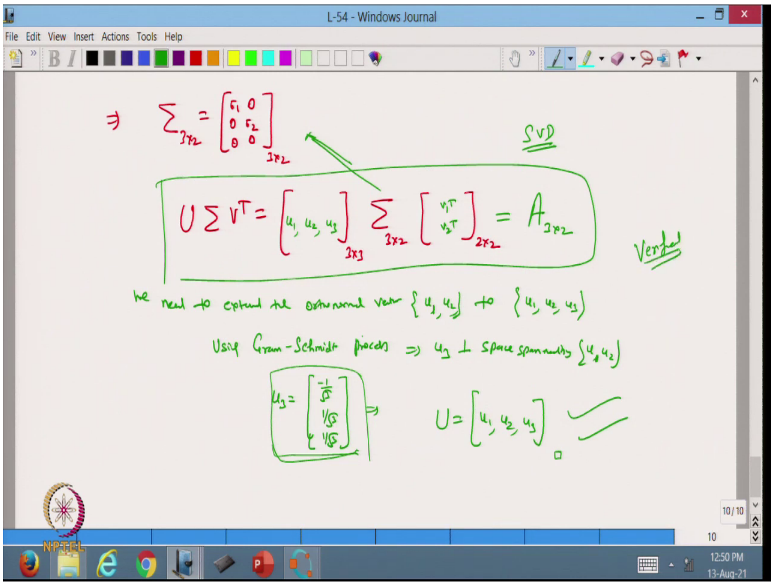

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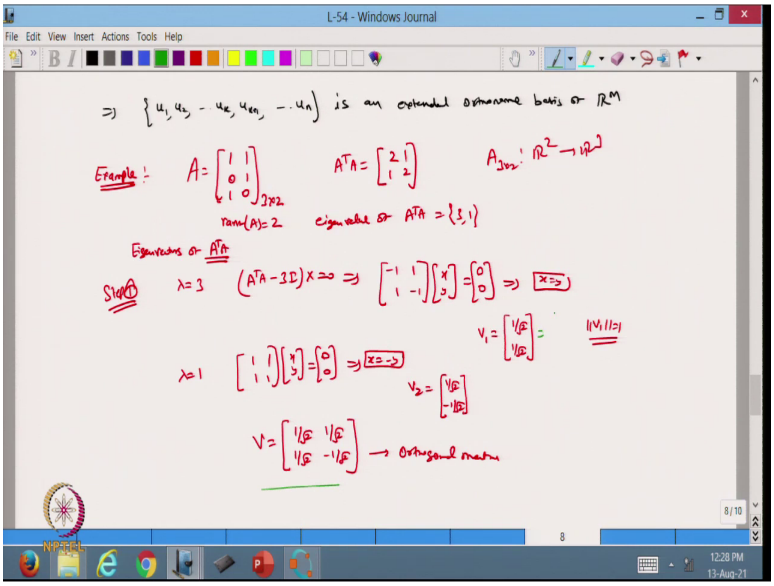

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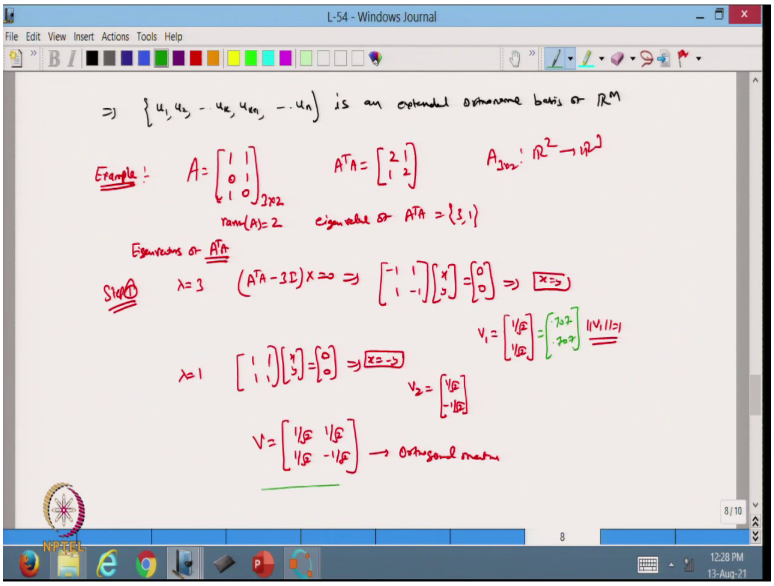

So, in the previous lecture, you see then we are able to take the singular value decomposition for the given matrix and then we have found the value of the matrix V and then we found the value of matrix U and then using this gram Schmidt process, we are able to find U3, but this U3 I can find with the help of the linear algebra what I can do is that, so, I have a matrix. We have started with the matrix $A = [1\ 1\ 0\ 1\ 1\ 0]_{2\times 2}$.

And then we find the value of U1 and U2 that belongs to the column space of A and we also know that the column space of A. So, this is my column space of A and this is my null space of A^T and they are orthogonal to each other. So, I can say that my U3, I can choose from the null space of A^T and that will be perpendicular to this one.

So, this one I can find from, so, now, I can take the A^T. So, A^T will be [1 0 1, 1 1 0] and I am talking from here. So, its value is (x, y, z) and it should be (0, 0, 0) and from here you will see that I will get $x + z = 0$ and $x + y = 0$ and both the things give you $y = z$ and $x = -y$.

Now, from here I can choose my vector U3 as my y and z should be the same and x should be minus of that. So, suppose I take (1 ,1) and it should be -1 and then I have to normalize. So, I will divide by root 3. So, this is where I get the U3 as

$$U3 = \begin{bmatrix} \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

So, that is also we can find out within the gram Schmidt process, but we can do this by this way also. So, now we are able to find my U3. So, let us say that we can verify that whatever we have done is right or wrong.

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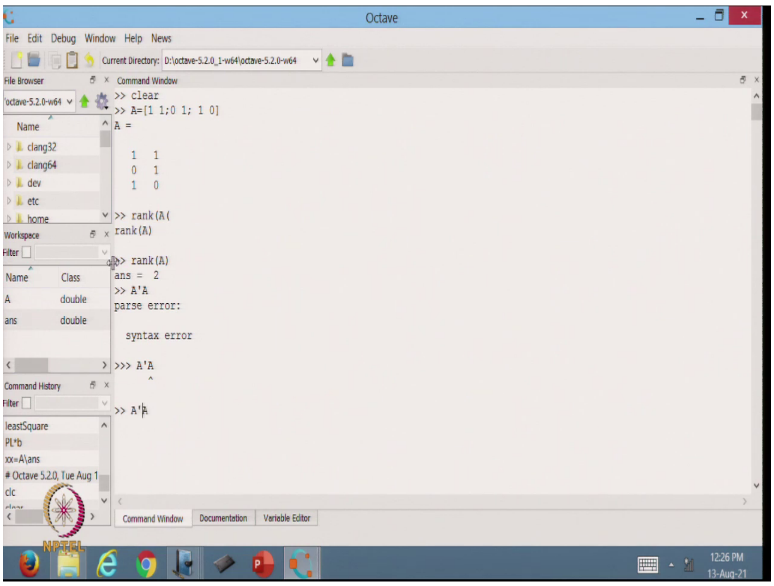

So, let us take the help of Octave. Now in octave I just start with the matrix A. So, this matrix I am writing. So, I am writing here [1 1 ;0 1;1 0]. So, this is my matrix. You can see from here that I can find the rank of this matrix A. So, this rank is 2. Now, from here I can find out the value of A^T A. So, this is my matrix.

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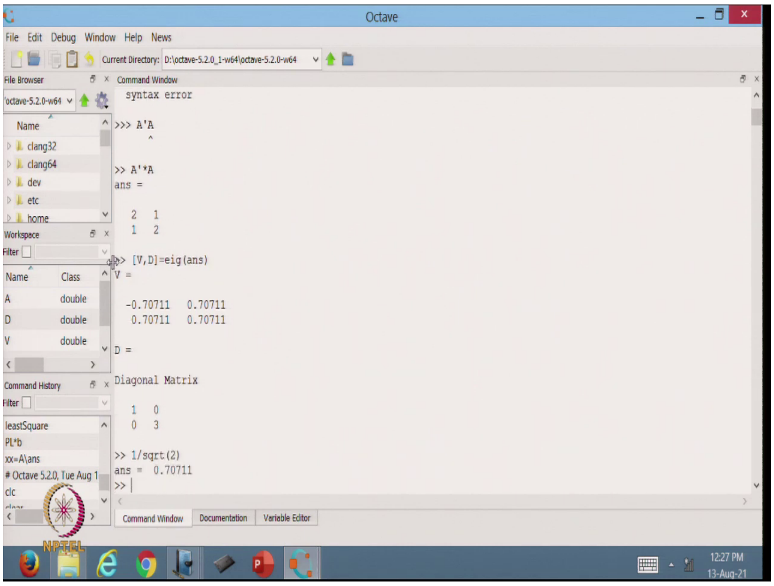

So, I have to write here about star multiplication. So, this is my matrix A^T A. Now I can find the eigenvalues of A^T A and the corresponding eigenvector also. So, I will write 1. So, I will V and I find its diagonal matrix D. So, I let us write this [V,D] =eig(ans). So, whatever the answer I got. So, by using this 1 I can find the eigenvalues and the corresponding matrix made up of eigenvectors.

So, if you see from here, this is my eigenvalues 1 and 3 and this is the corresponding eigenvector and we also found the eigenvector for v1, I got $1/\sqrt{2}$. So, if you see if I find $1/\sqrt{2}$. So, I got 0.701. So, I got this value. It means that the eigenvalue of is 3 and the corresponding eigenvector is

$$V_1 = \left[1/\sqrt{2}\ 1/\sqrt{2}\right] = [0.707\ 0.707\ ]$$

and corresponding to 1 eigenvalue the eigenvector is $\left[-\ 1/\sqrt{2}\ 1/\sqrt{2}\right]$.

So, this one we can take. So, the same thing we got if you see, from here I got this value. So, here v2 we have taken the minus sign here, but in this we got this minus sign and this one. So, that does not matter. So, we can also take the minus sign here ok. So, this way we can find out. So, this is ok because this value is equal to 0.707 that one I can write only, 0.707, 0.707 because it will give you the numerical value only.

So, this is the way we are able to find the value of V and this is the value of D. Now I need to find the value of U.

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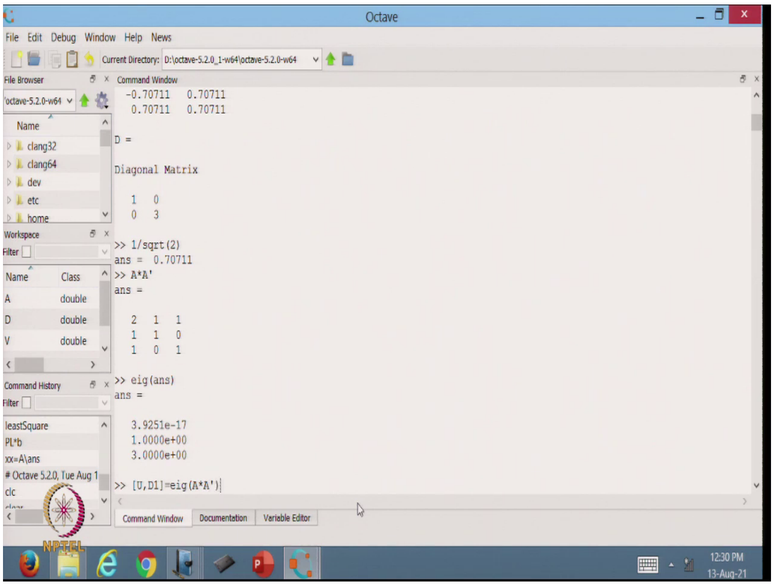

So, I told you that let us see what is going to happen if I take $A^* A^T$. So, this is my matrix 3*3 matrix and if I want to find the eigenvalue of this then you see that its eigen value will be 0 ,1 and 3 because it is of order 3*3. So, eigenvalues for $A^T A$ are 3 ,1. So, in that case we will get the 0 eigenvalue more.

And now I can write down these things and I can write from here. So, now, I can diagonalize the value of this matrix. So, I will write my U here, U and then maybe I can write D1, [U,D1]= eig($A^* A^{'}$).

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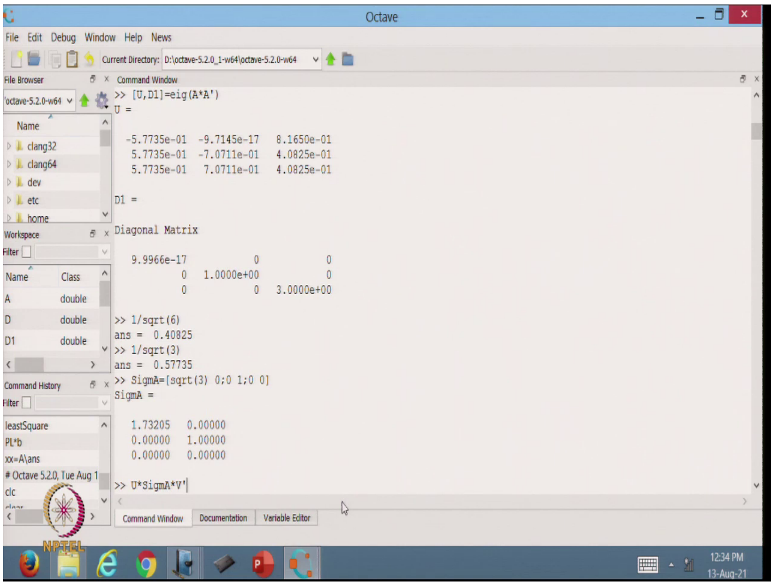

So, these are the values I got. So, now, from here you can see that matrix is this one. So, for the corresponding eigenvalue, 3 eigenvalue I get 3 this vector and this vector is 0.4, 0.4 and 0.8 and if you see from here, I got the value of U is first U I got this value $\begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$

So, if you see from here, I will just check what is the 1 by square root of 6; So, 0.4 so, it means it is 0.4. It is 0.4 and is 2 times 0.4. So, I go to the value of this vector, this value then another one I should get my U2. So, U2 is 0 and this value. So, let us see what is here. So, here also this is point minus 19 minus 17, meaning its value 0 otherwise it is minus 1 by root 2 and this is plus 1 by root 2 corresponding to 1.

And the last value we have found is $-1/\sqrt{3}$ and if you see from here that is corresponding to 0 eigenvalue and this is my 0.5. So, if I take $1/\sqrt{3}$. So, this is the value 0.57 and I got this value here, 0.57 here. So, in this case I got this value and. So, now, we are sure that whatever the calculation we have done for finding these values of U and V, they are right.

Now, there is one command directly for finding the SVD. Now from here if you see let us check what is going to happen if I multiply U. So, this is the U I got, U^*, now I have to take sigma. So, sigma is as I told you that sigma is the matrix. So, I just write let us define sigma

A. So, that matrix will be. So, $\lambda_1 = 3$. So, I can write from here that it will be equal to $\sqrt{3}$ and then 0.

After that I can write 0 and 1 and it should be of the same dimension. So, I can write from here it should be 0 and then 0. So, this is my sigma A. So, now, if I take U*sigmaA* V' I have taken. So, I just take the transpose and let us see what is going to happen. So, this is so, now, I have to find my A, we got U, this is my V, I got and I got this value, it is 3* 2.

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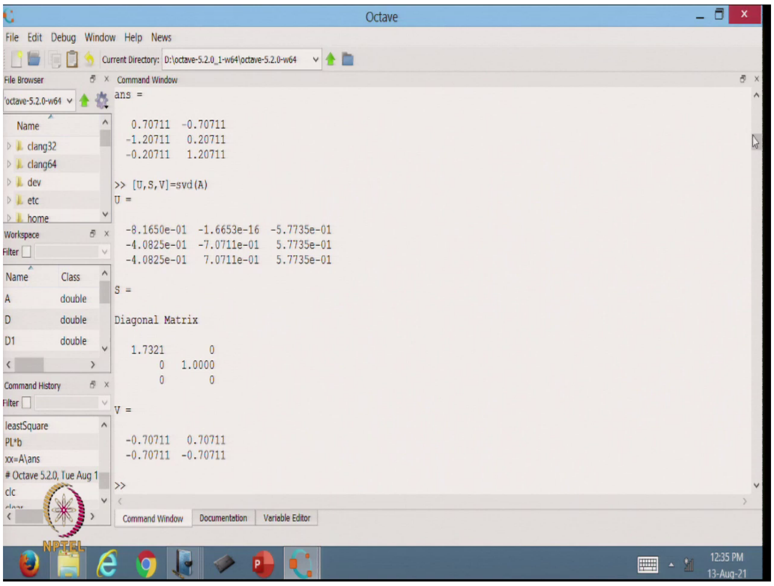

So, let us see if I will write an SVD. So, this is the direct command I can have. So, I can just check by this way. So, I can write my U here then sigma I just by S and then I will write V and I put the SVD of the matrix A. So, here I can directly write. So, now, from here I got my U. So, let us see whether the U is going to be the same or not. So, this is my U and I think I got the same U, only change in signs.

So, this sign actually changes because we take the sign depending upon the penalty and sometimes this change of sign will, when you multiply it, give the different results as we are getting in this, but other things you will see that it is going to be the same. So, U value U is the same as this U= -8,-4,-4 and we have seen that U is 8 ,4 ,4.

So, we have just taken the negative sign. Other things, the diagonal element matrix I already told you will be 1.7 ,1, 0 and V is the same as we have taken.

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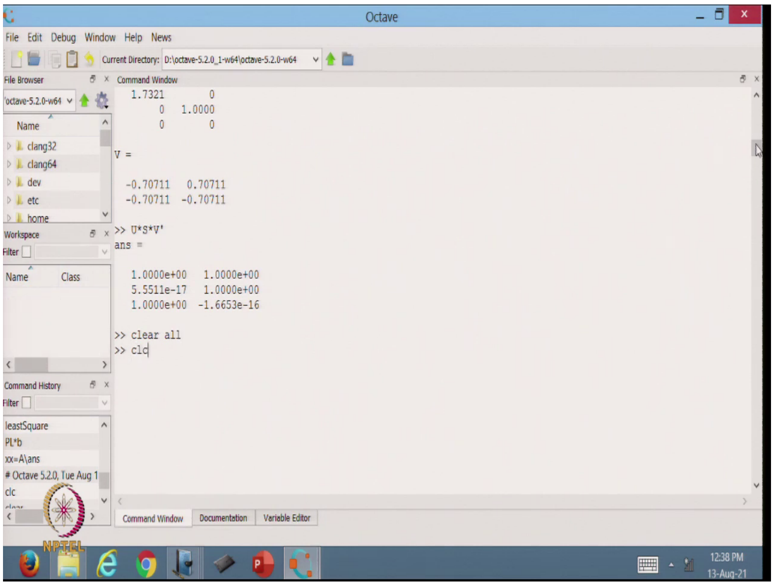

So, let us see that now if we do so, my U and V has changed now. So, my U*S*V' and I get this value. So, this is the same matrix that has got [1 1; 0 1; 1 0]. So, that is the same value we have got. Here I was getting something different because now we have to take care about the sign of the vectors.

So, sometimes if you do that one, you get some different values of the solution and that is what is happening in this case, but otherwise everything is completely right except for some change in sign. So, these things we can do now we can apply some other examples and see how we can do the SVD for other matrices. Now let us take another matrix. So, I will just write everything. So, it will clear.

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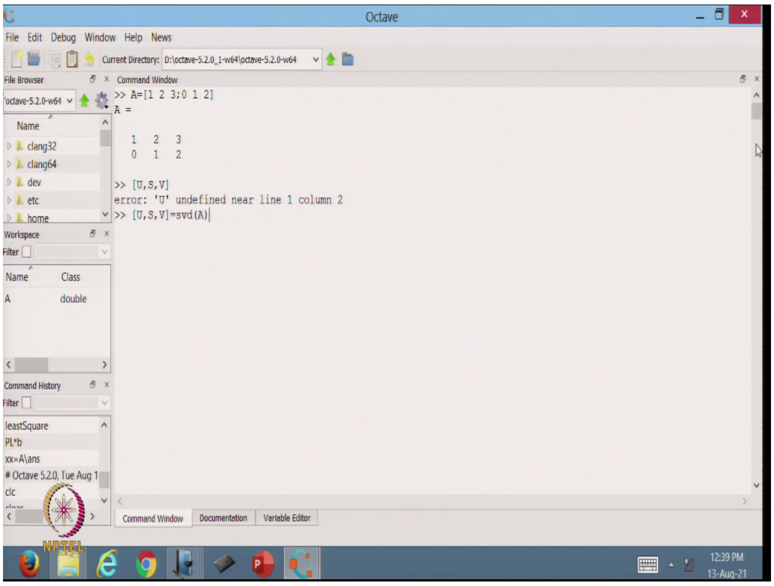

Now, I can define some different matrices. Suppose I take the matrix A and I just take it. Maybe I just take matrices [1 ,2, 3; 0, 1 ,2]. So, let us take this matrix. It is a 2* 3 matrix. Now I want to find out what is going to happen in this case if I take the S. So, just I take U and then I will write S and then I will take V and I will write is equal to SVD of the matrix A.

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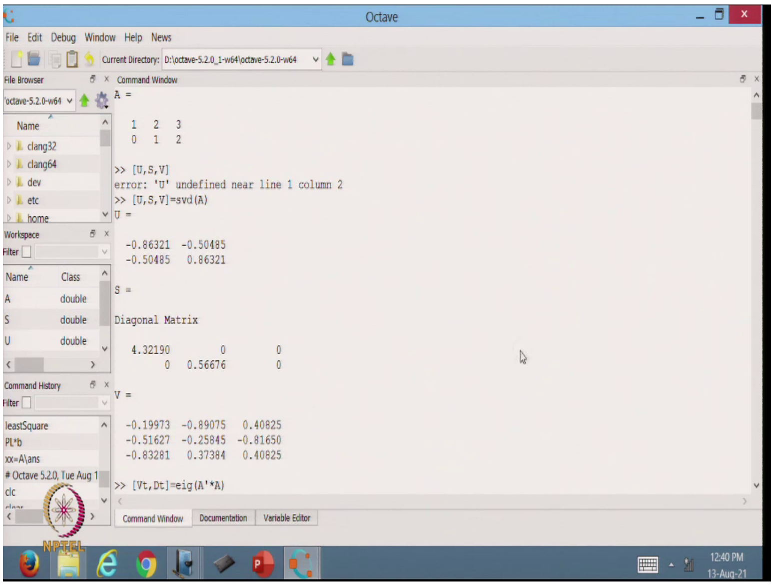

So, this is my matrix now my U is 2* 2 ,V is 3*3 and its eigenvalues, you will see from here then I just write A, I just find let us see what is going to happen. So, I just write A. Maybe I will write corresponding to a transpose A. So, I write VT. VT means V corresponding transpose and DT. I just write that one and this one equal to I write eigenvalues of A’*A. So, let us see what is going to happen here.

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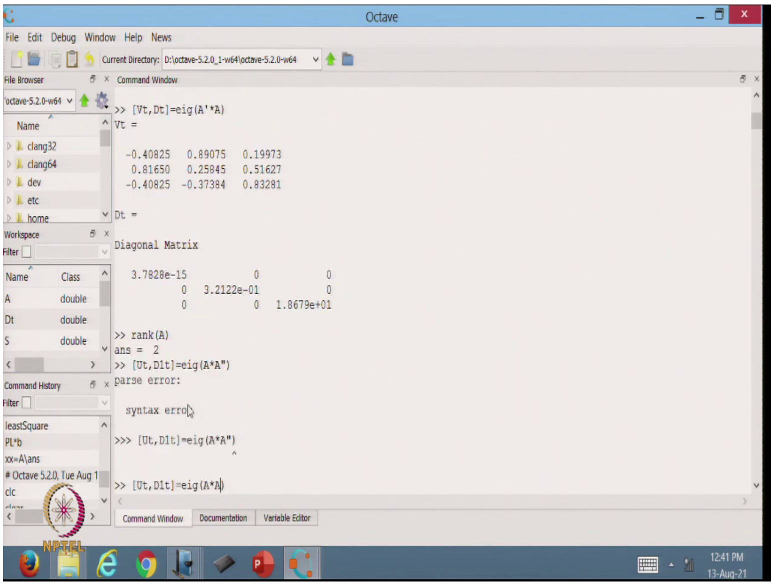

So, this is my A’ and that is 3*3 matrix of course, it will be 3*3 and this is the 3 eigenvalues. So, it is 1.8 10 raised to power 1. So, 1.18 and this is point 3 and this is 0. So, 1 eigenvalue is 0 because the rank is 2, if you see the matrix A, its rank is 2. So, one eigenvalue has to be 0 in this case as it was 0 in the previous case for U. So, one eigenvalue should be 0 and this is the value we got. Now, the same thing I can check for U. So, let us see what I get for U and maybe I can write as D 1 and this is A, A transpose oh I get this value this one.

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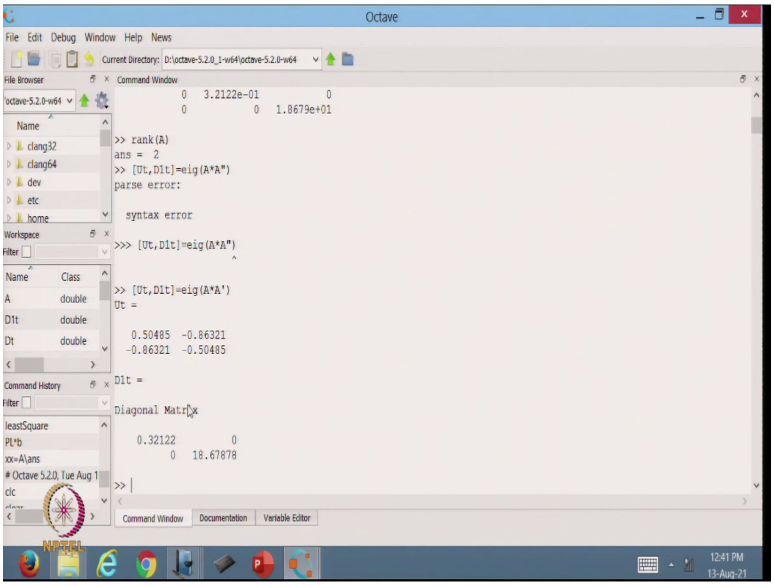

So, it will be 2* 2 and the eigen value will be the same 0.32 and 18 and this is my corresponding matrix. So, now in this case also if you see everything is written differently yeah. So, in the previous case if you see the U and V.

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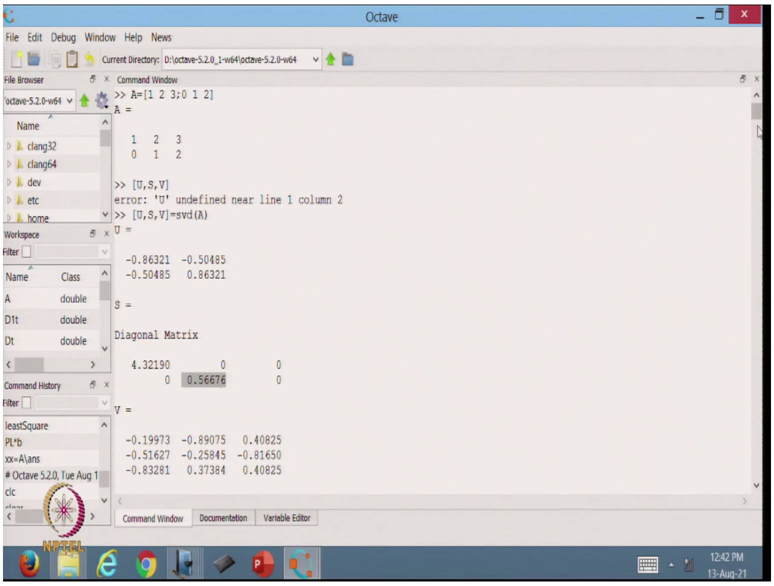

Now, if you see from here, the U we have written here in this fashion and the eigenvalues are given to us with the ordering that it is a maximum order then the smallest one, but if you see from here taking A transpose A. In this case the eigenvalues are written in the, that this is a 0 then it is 0.32 and this one. So, that is why in the previous case when we multiplied, it was changing the order.

Similar case is here. It is 0.3 and 0.18. So, that is why we have to keep in the same order the highest power and the lowest power. So, that is why if you see in the previous example, we were not able to get the same eigenvalue or same matrix, but here we are able to do that.

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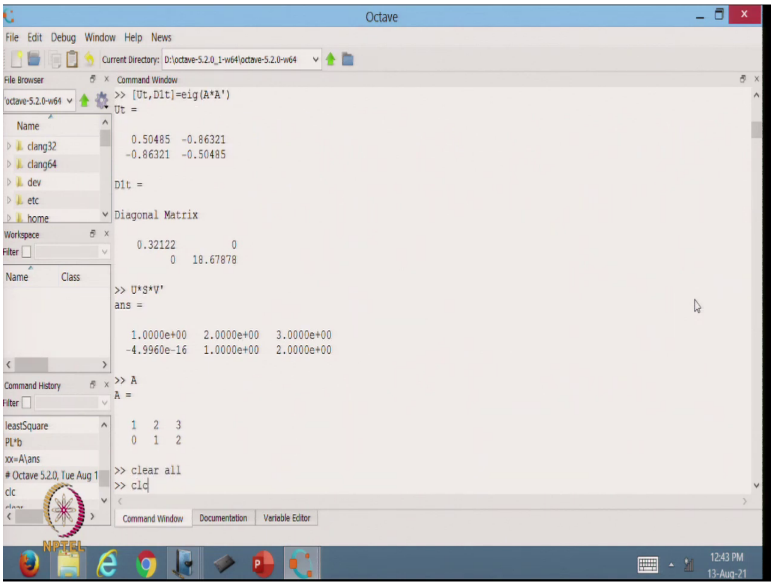

So, now from here you can see that now if I take my U multiply by S and then multiply by V transpose, I get this matrix that is [1 2 3;0 1 2]. So, this is the matrix I have started with, this is. So, we go to the same matrix using this value. Now, let us see what went wrong in the previous example. So, let us see that one because now we are able to see why it was not coming.

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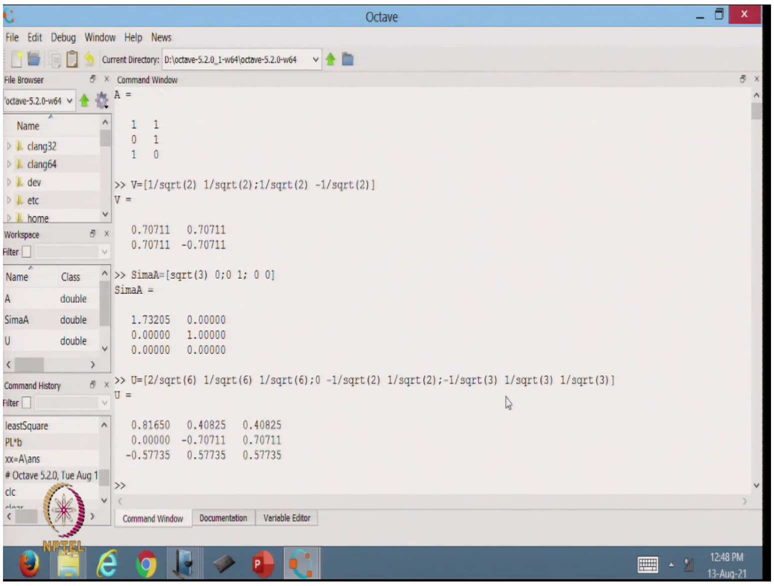

So, let us take A as the same as the previous one. It was [1 1 ; 0 1; 1 0]. Then we found our value of U that, so, I will take this here. So, we go to the value of first, I will write down the value of V1 and V2. So, V1 and V2 is this one. I will write like this one.

So, let us now take V =[1/sqrt(2) 1/sqrt(2); 1/sqrt(2) -1/sqrt(2)] .

So, I can write control V and minus sign control V. So, this matrix I got is my V. So, this is a V we have taken. Now I will write down my sigma. So, sigma we have taken sima(A). So, that is equal to what I can write. So, it was [sqrt(3) 0; 0 1; 0 0] because it should be of order 3*2. This is my sigma and U we have taken.

So, my U was, now we should write U =[2/sqrt(6) 1/sqrt(6) 1/sqrt(6); 0 -1/sqrt(2) 1/sqrt(2); -1/sqrt(3) 1/sqrt(3) 1/sqrt(3)].

This is my U. Now we have to write it as a column vector.

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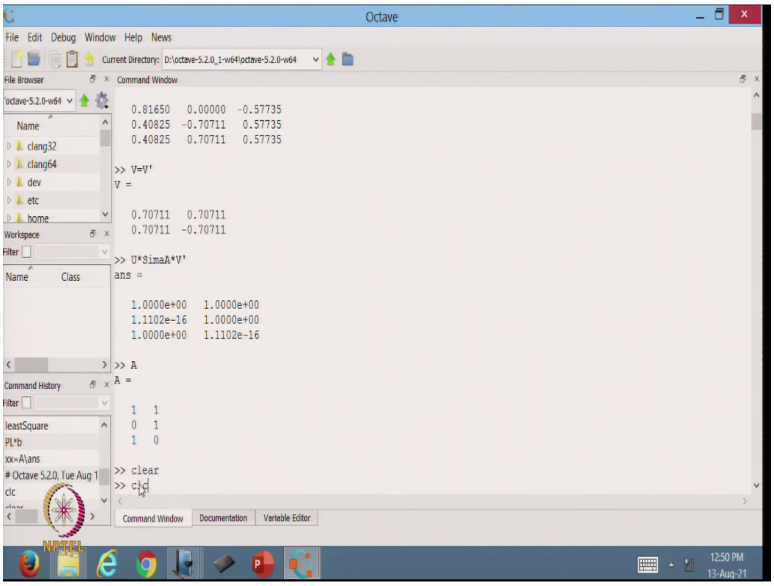

So, I think we should write U as U’. Now it is ok 0 minus this yeah. So, that should be the U. So, I think we also have to write. So, V should be also V’ this one .

So, now we can take the multiplication U star and then I will write sigma. I am just right here in this and then I take a star and then V transpose. Now we get this value. So, this matrix is the same as our matrix A =[1 1; 0 1; 1 0]. So, if you see now it is coming the same.

So, everything is ok in the previous example, whatever the example we have done in this case. So, it is verified now that we are getting this value. So, from here we are able to see that my U sigma V’ is giving my A. So, whatever we have done is completely right.

So, this way we can take any matrix. We have taken 2*3. We have taken a 3 *3 matrix. Maybe I can take some other matrix. Maybe I can take a very big matrix that is I just take [0 1 2 3 5] and then. So, this is my first row.

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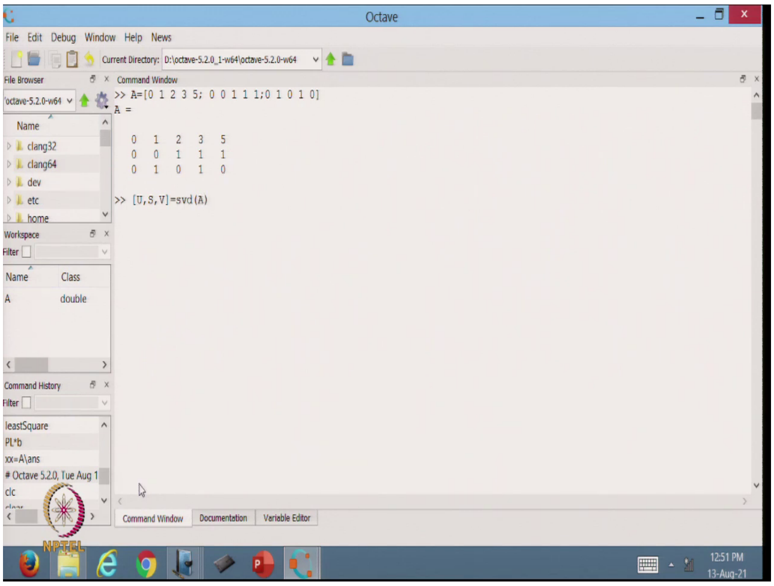

And maybe I can take the second row as [0 0 1 1 1]second row and in the third row I can take [0 1 0 1 0]. So, this matrix I have taken. So, it is a 3*5 matrix. So, this matrix we have taken now I can find out is SVD directly. So, this is U, S, V. I can write it down as SVD of the matrix capital A and this is the corresponding SVD.

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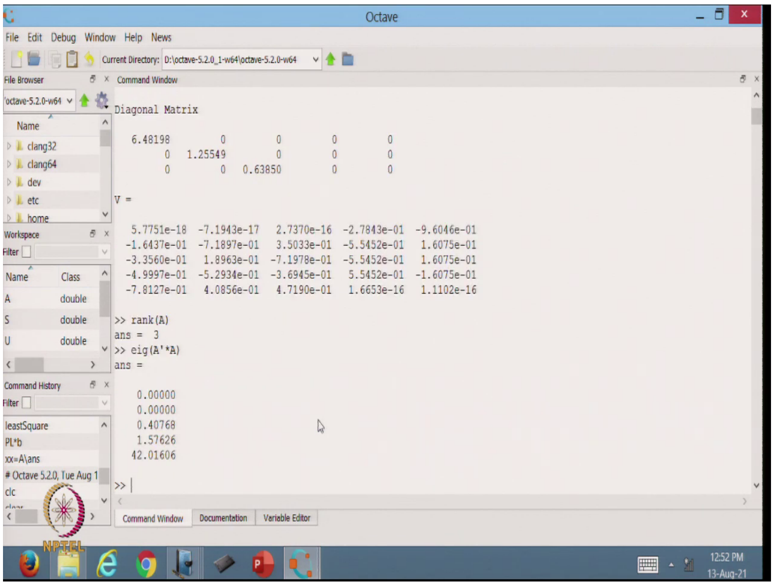

Now, you can see that my U is 3*3 and the diagonal matrix is now 3* 5. So, it was 3* 5 so, 3*5. So, this is my corresponding singular values, the largest one and then the next and the next and this is my corresponding V.

So, in this case A'A is a 5 *5 matrix and it is going to have 5 eigenvalues. So, 2 eigenvalues are 0 because in this case if you see the rank of A is 3 and if I want to find the eigenvalues of A'*A. So, it will be[0 ;0 ;0.41 ;0.5 ;42]. So, you can see from here. So, it is a 5 eigenvalue and 2 eigen values would be 0. So, the remaining eigenvalue. So, we have seen that this will be the 3 eigenvectors that are non 0 and 2 eigenvectors will be 0 and then from here we can find the eigenvalues.

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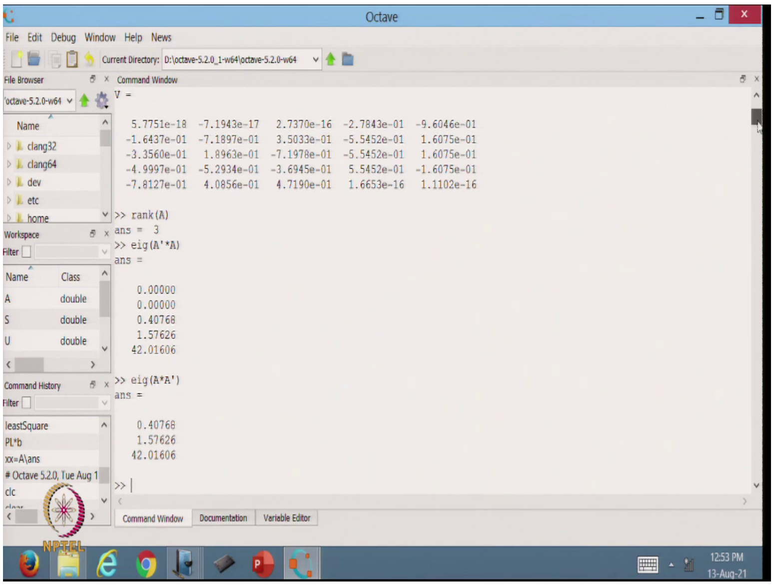

Similarly, I can find the eigenvalues of A star A transpose star and this eigenvalue is the same except the 0,0. So, there is no 0,0 eigenvalue there. And then we can find the value of corresponding eigenvectors and then if you see then this is my U, S and V directly I can take with the help of inbuilt function in the Matlab or Octave and that gives me the SVD and you can also verify by solving on with the pen and paper and then you can do the multiplication in this case.

So, you will also be able to get the same result. So, this is the way actually we can find out the SVD for various values of various matrices with the help of octave. So, now we will stop here. So, in the today's lecture we have just taken the help of MATLAB or Octave to verify that whatever the calculation we have done to find out the singular value decomposition is whether right or wrong and we found that the whatever the multiplication whatever the calculation we have done is quite right and then we have compared it with the inbuilt function that is SVD function and we found that both the cases we are getting the same result.

So, you can just it is just the introduction that how we can manipulate the with the help of octave and you can play with this 1 and you can find out the SVD or the eigenvalues of different type of matrices you can take and then you can verify the results for a given matrix to find the SVD. So, I hope that you enjoyed this lecture.

Thanks for watching.