Matrix Computation and its applications

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Lecture - 54
Singular value decomposition (SVD) theorem

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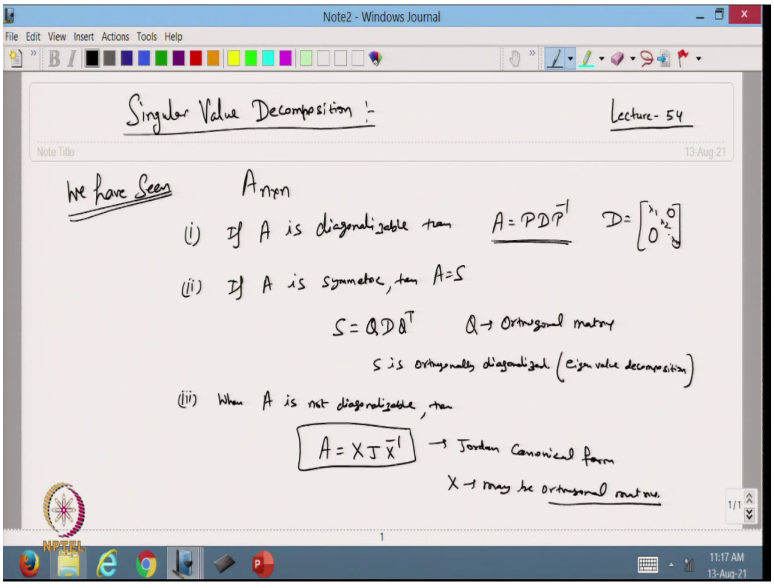

Hello viewers. So, welcome back to the course on Matrix Computation and its application. So, today we are going to start a very new topic that is called the Singular value decomposition for a given matrix A. So, let us do that. So, today we are going to discuss the singular value decomposition.

Now, we have seen that suppose, I have a matrix A that is n* n matrix then, first thing is we have seen that if A is diagonalizable, then I can write A as some $A=PDP^{-1}$; this one I can write. And in this case, my D is a diagonal matrix $\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}$ and P is corresponding to the eigenvectors. So, we have seen that if the matrix A is diagonalizable, then we can write it this way.

Now, the second thing is that if A is symmetric, then I can write A = S; S stands for symmetric just as I am writing. Then, we can write S as some QDQ^T. So, where my Q is an orthogonal matrix. And in that case, we say that this S is orthogonally diagonalized.

So, in this case my Q is orthogonal that is why I change my Q inverse with the Q transpose. So, this form is also called. So, I can write this in the form of eigenvalue decomposition. So, I can write this as an eigenvalue decomposition.

So, then we have seen the third form that when A is not diagonalizable, then we can write A as sum I can write in this form now maybe X and then Jordan form X inverse. So, this one I can write from here, and this is called the Jordan canonical form. And in this case, this X may not necessarily be an orthogonal matrix. So, these are the things we have seen. Now, the next thing comes.

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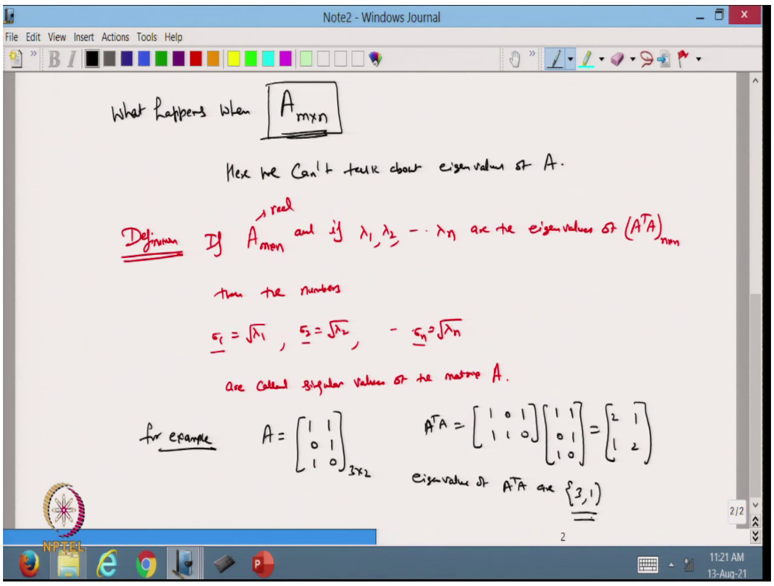

What happens when A is m cross n. So, in that case what we can say about the eigenvalues of the matrix A when so, A is a rectangular matrix. So, here we cannot talk about eigenvalues of

A. So, then we start with the new thing called. So, let us write this one as. So, I will just write the definition.

If A is a m* n matrix . So, I am taking the real matrix and if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of matrix $A^T A$. So, I will take the $A^T A$ matrix. So, this will be of course, n*n matrix. Then the number.

So, I take $\sigma_1 = \sqrt{\lambda_1}, \sigma_2 = \sqrt{\lambda_2}, \ldots\ldots, \sigma_k = \sqrt{\lambda_k}$, then the numbers are called singular values of the matrix A. So, A is a rectangular matrix and we call it the σ_1 , σ_2 ,.., σ_n . So, these are called the singular values of the matrix A.

Now, for example, I suppose I take the matrix A. So, let us take matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$. So, this is my 3*2 matrix, then I will take $A^T A$. So, this will be

$$A^T A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

So, I know that this is always a symmetric matrix and its rank will be the same as the rank of matrix A. Now, from here I found that the eigenvalues of A transpose A are. So, these are the eigenvalues 3 and 1. So, I got these eigenvalues 3 and 1. And from here, I can write that I can write $\sigma_1 = \sqrt{3}, \sigma_2 = 1, (\sigma_1 \geq \sigma_2)$. So, these are the eigen these are the singular values of matrix A that is $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$.

So, these are the singular values of the given matrix. Now, in the first we have seen that a matrix can be diagonalizable, it can be orthogonal diagonalizable and then we have seen that it can be converted into the Jordan canonical form.

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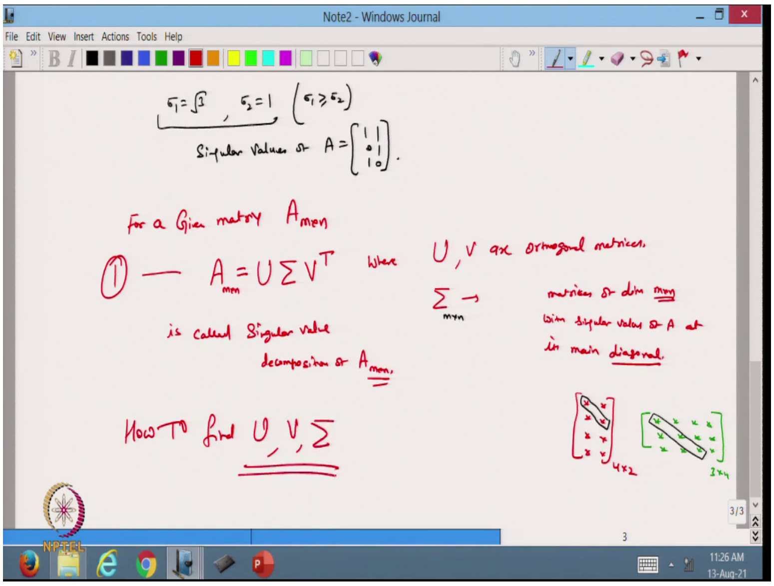

Now, after this one we can write that for a given matrix A; that is m cross n. Now, our condition is that we can write the matrix A decompose this matrix $A = U\sum V^T$.

So, the first time we are seeing that A is decomposed into the matrix which contains three different notations U, sigma and V. In all the previous one, this P and P inverse are just the inverse of P. Here also, Q and Q^T X and X^{-1}, but in this case, we have $U\sum V^T$.

So, we call this that for a given matrix we are able to write this one, then this is called. We decompose this one, where U, V are orthogonal matrices and this is an orthogonal matrix; it is a square matrix and this summation is a diagonal matrix of dimension. So, its dimension will be same as the dimension of A m* n.

So, I can write notes on the diagonal matrix. I can write the matrix of dimension m*n with singular values of A at its main diagonal at its main diagonal. So, suppose I have a matrix like this one and suppose, I have this matrix. So, it is 4 cross 2.

So, the diagonal main diagonal elements will be this one this or maybe, suppose I have another matrix of this form suppose I have a matrix of this form. It is 3 cross 4. So, the main diagonal will be this one. So, this is the main diagram. So, if we have a sigma is the matrix of order m cross n and having the singular values at this as a main diagonal. So, this is the definition of this one.

And now, from here. So, if I take this one and we are able to do this one. So, it is called so, this is called singular value decomposition of matrix A; that is of order m cross n. So, this is what I just write as equation number 1. So, let us see how to find the U, V transpose. So, the next question is how to find U, V and sigma all things we need to find. So, let us see that.

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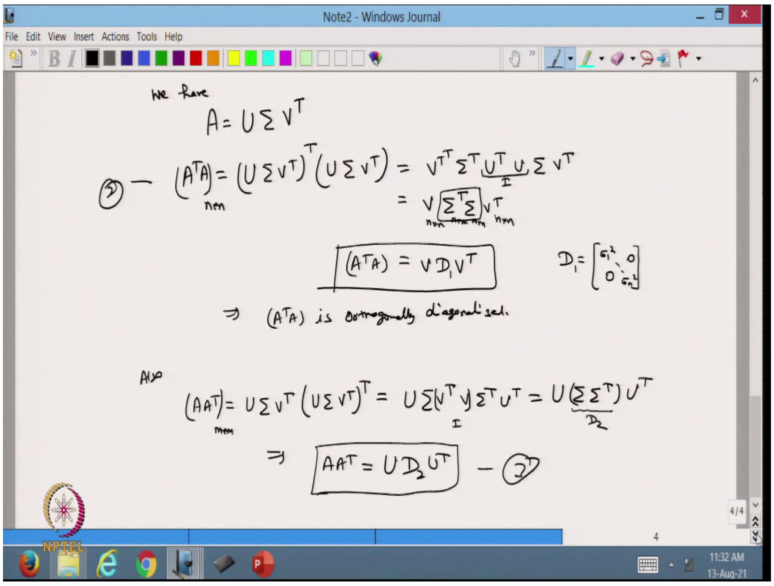

Now, so, it is we have $A = U\sum V^T$. Let us see what will happen if I take $A^T A$. So, A^T means I am just taking the transpose and this is $U\sum V^T$.

$$(A^T A) = \left(U\sum V^T\right)^T \left(U\sum V^T\right) = V^{T^T} \sum{}^T U^T U \sum V^T = V\sum{}^T \sum V^T$$

Now, this is V and if you see from here, it is an orthogonal matrix. So, it will be me. So, from here I can write this sigma V transpose. And I know that this will be of dimension n*n. So, my V will be n* n. So, this is m* n and this will be n* m and this will be again n*n.

I will get only this value. So, this is I will just write that it will be a diagonal matrix D with this value, where D will be in this case $\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \lambda_k & \vdots \\ 0 & 0 & 0 & \ddots \end{bmatrix}$

$$(A^T A) = UDV^T$$

Because, I know that the $A^T A$ is a symmetric matrix and it is positive definite. And we have taken this that this is the eigenvalues and this suppose n*n it has n eigenvalues n singular values so that we have written or if eigenvalues are 0, then to be 0 no problem. So, this will be here.

So, from here, I can say that the $A^T A$ is orthogonally diagonalized. So, and the symmetric matrix will be V. So, from here that this I can write as diagonal form with the matrix $A^T A$. So, now, from here, I can find my value V by taking the eigenvalues of A transpose A and then finding the eigenvectors.

Now, so, from here I can write this one. So, maybe I can write this as 2. Also, let us see what is going to happen about $A^T A$. Now,

$$A^T A = U\sum V^T \left(U\sum V^T\right)^T = U\sum(V^T V)\sum{}^T U = U\sum\sum{}^T U$$

So, its dimension will be m* m, because A is m* n A transpose is n*m. So, it will be m*m. So, it is a new matrix. Maybe, I can write D1 here and then I can just write D2. So, I can write from here that $(A^T A) = UD_2U^T$.

So, I can say from here that U can be obtained from the solving the matrix $A^T A$ and its eigenvector. So, this is the way we are able to find the value of U and V and then, we can find the sigma. So, this is equation number 3 ok. So, let us see how we are going to apply the singular value decomposition. So, in SVD, basically what we are going to do is as we have discussed step 1.

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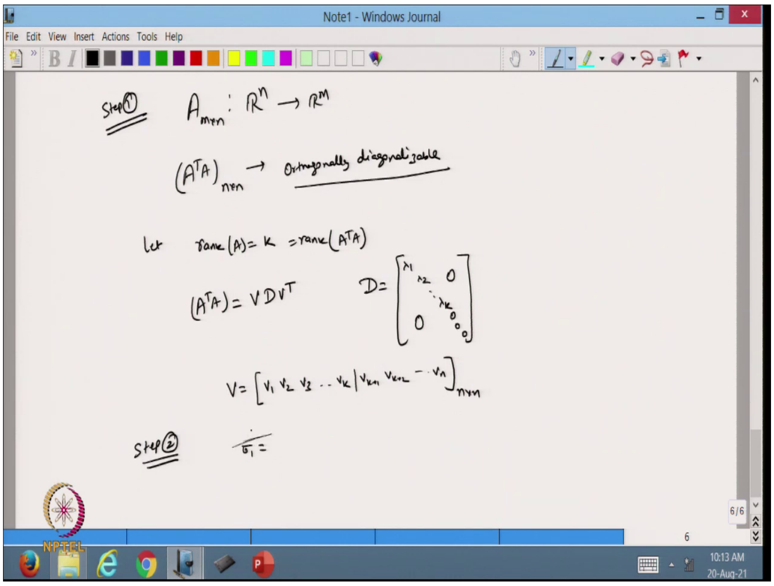

So, it means that first we have a matrix A. So, that matrix A is m cross n and this is from R^n to R^m. Now, I take the matrix $A^T A$. So, this is n*n matrix and I know that this is a symmetric matrix. So, $A^T A$ is orthogonally diagonalizable that I know.

Now, we have considered that the rank. So, I consider that the rank of matrix A is k. And I know that this is equal to the rank of matrix $A^T A$. So, in this case, the matrix I can write this matrix $A^T A$ can be written as a matrix $(A^T A) = VDV^T$.

So, D is the diagonal matrix which contains all the eigenvalues. So, I take the $\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \lambda_k & \vdots \\ 0 & 0 & 0 & \ddots \end{bmatrix}$ and then, it will be going to have 0 eigenvalues, because its rank is K. So, I can have the K non zero eigenvalues and all other eigenvalues will be 0.

So, this is my diagonal matrix. And I can take my V as a matrix that is corresponding to the eigenvectors corresponding to $\lambda_1 V_1, \lambda_2 V_2, \ldots\ldots, \lambda_k V_k$ and then, corresponding to 0 eigenvalues will take $V_{k+1}, V_{k+2}, \ldots., V_n$. So, this is my matrix V and this matrix will be n*n. So, that we already know, because this matrix A^T A is n*n matrix. So, this is what we are going to do in step 1. So, it is basically step 1.

Now, after doing this one. In step 2 what I am going to do is I will choose the non-zero eigenvalues and I will find out sigma 1. So, first I will choose these eigenvalues and I will put them in the order such that. So, I am putting them in the order and now, I am considering that.

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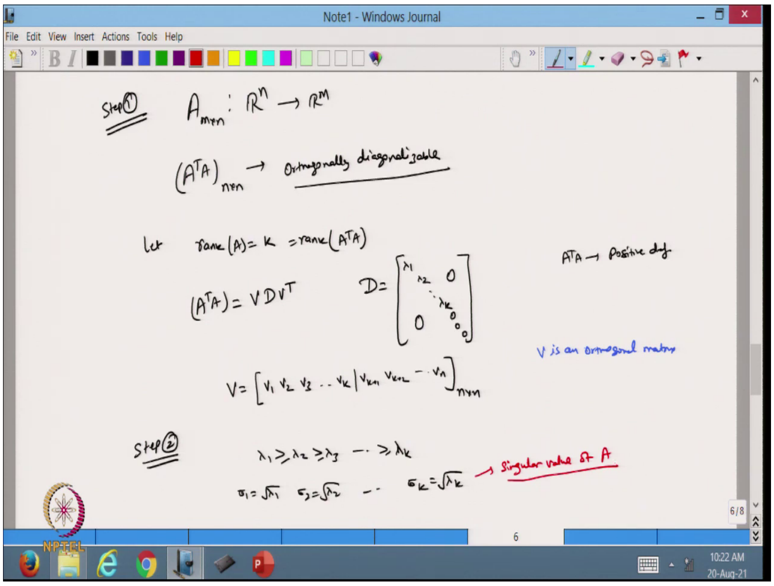

So, let I am considering that my $\lambda_1 \geq \lambda_2 \geq .... \geq \lambda_k$. So, this is I am taking the non-zero eigenvalues and I am putting this in the order. So, now from here, I am. And I know that this A^T A is positive definite. So, I can take its square root. So, now, I am considering sigma 1 I will take $\sigma_1 = \sqrt{\lambda_1}, \sigma_2 = \sqrt{\lambda_2}, ..., \sigma_k = \sqrt{\lambda_k}$.

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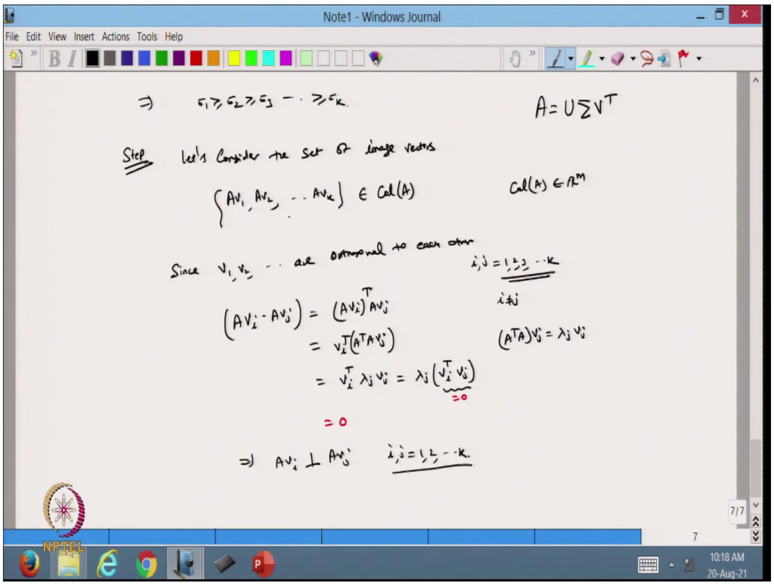

And now, from here, I can say that my singular values $\sigma_1 \geq \sigma_2 \geq \ldots\ldots \geq \sigma_k$. So, this is the k non zero eigen singular values of the matrix A. So, this is what I will put in step 2.

Now, after doing this one. Now, I need to find the value of the U, because in our singular value decomposition, I am going to do $A = U\sum V^T$. So, V we have already found. So, now we are going to find the U. So, let us take this one. So, these are my non zero singular values.

So, now, let us consider the set of image vectors. So, I just consider $\{Av_1 , Av_2 ,\ldots,Av_k\}$. So, I am taking the k image of the k vectors v_1, v_2 , v_3 and v_1, v_2 , v_3 is coming from here. So, that we know now. So, if you see this then, then this belongs to the column space of A and you know that the column space of A will belong to R m, because this is a linear transformation from R^n to R^m.

Also, since v_1, v_2 all are orthogonal to each other that we already know. Now, let us see what is going to happen if I take the dot product of Av_i with Av_j ; let us see the dot product of this. And I am taking this i, j= 1, 2, 3,… k. So, I am taking from here the image sets. I am taking the dot product.

Now, let us see I can write this as

$$(Av_i.Av_j) = |Av_i|^T \ Av_j$$
$$= v_i^T A^T Av_j = v_i^T \lambda_j Av_j = \lambda_j (v_i^T v_j) = 0$$

And from this I can say that Av_i is perpendicular to Av_j and this is I am taking i ,j =1, 2 ,…, k. So, this is all we are considering about the column space.

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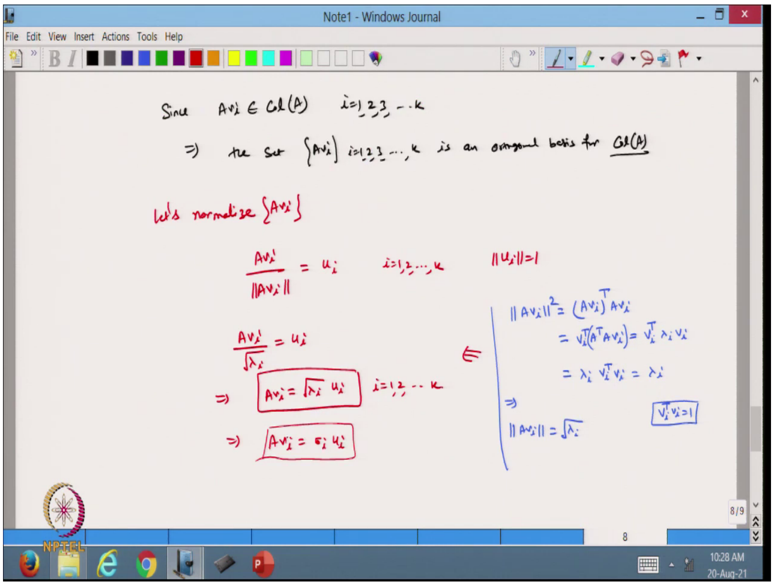

Now, from here, we can say that since all Av_i belong to column space of A where i =1, 2, 3 ,…., k then, from here, and I can say that these are. So, the set Av_i for i =1, 2, 3,…, k is an orthogonal basis for column space of A. So, this is the orthogonal basis for the column space of A; that is we are able to understand from here. Now, this is the orthogonal basis. So, let us say that let us normalize A v i all this one for this i' s.

So, for this one what I do I $\frac{Av_i}{\|Av_i\|} = u_i$ because this belongs to my column space of A. So, I call it u_i, where i =1 ,2,…., k and u_i is unit vector, because I am dividing this one by. Now, from here, let us see what is going to happen in this case. So, what is the. So, this one I am going to do here.

$$\|Av_i\|^2 = (Av_i)^T Av_i = v_i^T A^T Av_i = v_i^T \lambda_i v_i = \lambda_i (v_i^T v_i) = \lambda_i$$

Now, v_i is the orthonormal vector, because we have seen that this matrix is orthogonal, because I know that the matrix V is an orthogonal matrix and each of v_i is orthonormal sets. So, from here, I can say that this will be equal to lambda i.

Now, from here, I can write like this one. So, from here, I can say that the norm $\sqrt{\lambda_i} = Av_i$. So, now, from this I can write this as

$$\frac{Av_i}{\sqrt{\lambda_i}} = u_i$$

$$Av_i = \sqrt{\lambda_i} u_i, i = 1, 2, \ldots, k$$

Now, this thing is from here, and I can write even as this $Av_i = \sigma_i v_i$, if you want to write in the term of singular values. So, I can write this one as, because I know that these are the singular values of A. It is a non-zero singular value, because all 0's are also there. So, we are just writing the non-zero singular values. We are concentrating on that one. So, I get this value $Av_i = \sigma_i v_i$.

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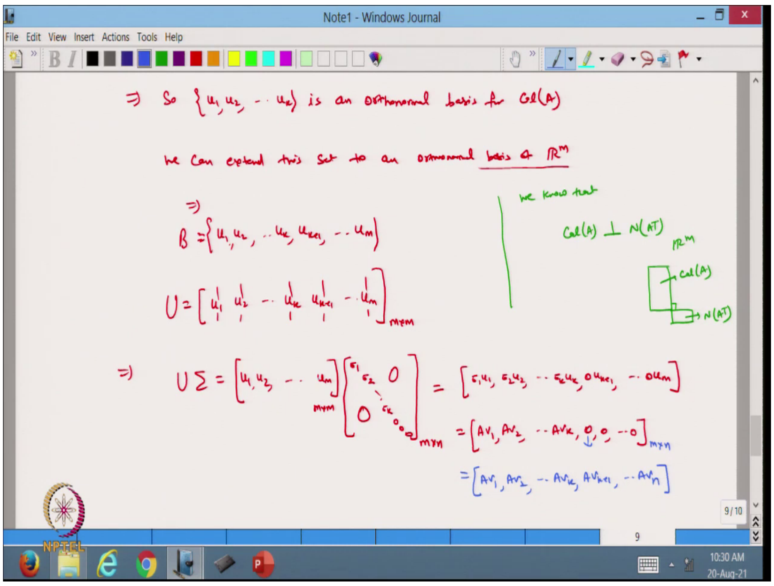

Now, from here, I can say that. So, the set $\{u_1, u_2, \ldots, u_k\}$ is an orthonormal basis for column space of A. Now, what we want. We can extend this set to an orthonormal basis of R^m, because this belongs to R^m. And these things we can do with the help of the Gram Schmidt process.

Or we can because in this case, we also know that the column space of A is perpendicular and orthogonal to the null space of A transpose that we know, because if you see we have the picture of four subspaces. So, in the R^m, we have this is the column space of A and this is the null space of A transpose and they are perpendicular to each other.

So, now, I know that the basis of this column space is also an orthogonal orthonormal set. It is the basis of this one and I want to extend this one. It means I need the all the vector which are orthonormal to all these vectors $\{u_1,u_2,...,u_k\}$. So, I can choose the vector from the null space of A transpose and that is also orthonormal and then we are done. So, we can extend this set of orthogonal basis of R^m to make my A orthonormal basis of R^m. So, we can extend $\{u_1,u_2,...,u_k\}$ and then, I can have $B=\{u_1,u_2,...,u_k,u_{k+1},...,u_m\}$

$$U=\begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_1 & u_2 & \cdots & u_k & u_{k+1} & \cdots & u_m \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

And this matrix will be of m*m, because all these u_i belong to R^m and m in number. So, this is.

So, now, from here I can write that I want to see what my U sigma is. $U\sum=[u_1u_2......u_m]\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \lambda_k & \vdots \\ 0 & 0 & 0 & \ddots \end{bmatrix}$ and then I can put the remaining 0 eigenvalues here and I know that this is of order m* m and this is of order m *n.

$$\begin{aligned} &=[\sigma_1u_1,\sigma_2u_2,.....,\sigma_ku_k,0u_{k+1},.....,0u_m] \\ &=[Av_1,Av_2,.....,Av_k,0,...0] \\ &=[Av_1,Av_2,.....,Av_k,Av_{k+1},.....,Av_m] \end{aligned}$$

So, everything depends upon that in this case, which is greater whether m is greater or n is greater. So, it will go up to, because we know that our matrix V is of n cross n.

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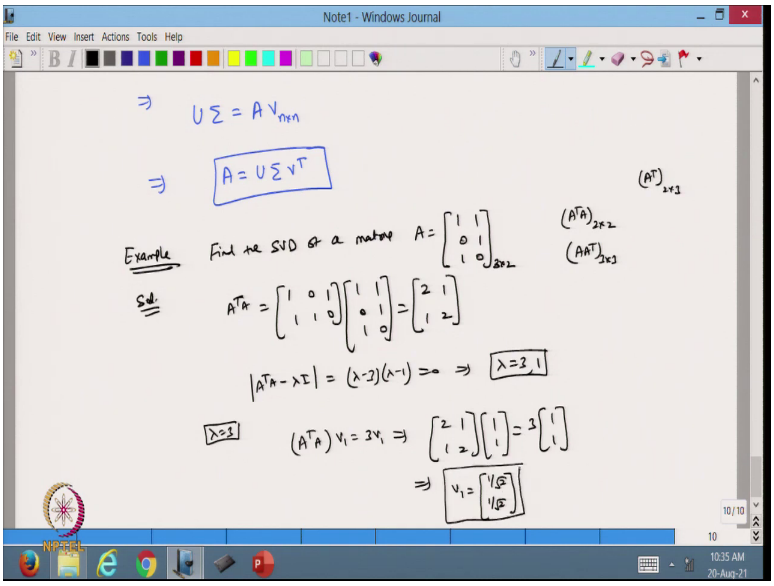

So, now, from here, I can write my

$$U\sum = AV_{n\times n}$$

So, I will get this matrix here and and matrix with A from here I can get these values, because this will be of order m* n. So, m is the number of rows, n is the number of this one. So, I will get up to n here. So, this will up to n if you do this multiplication. So, if you see from here, it is going up to n.

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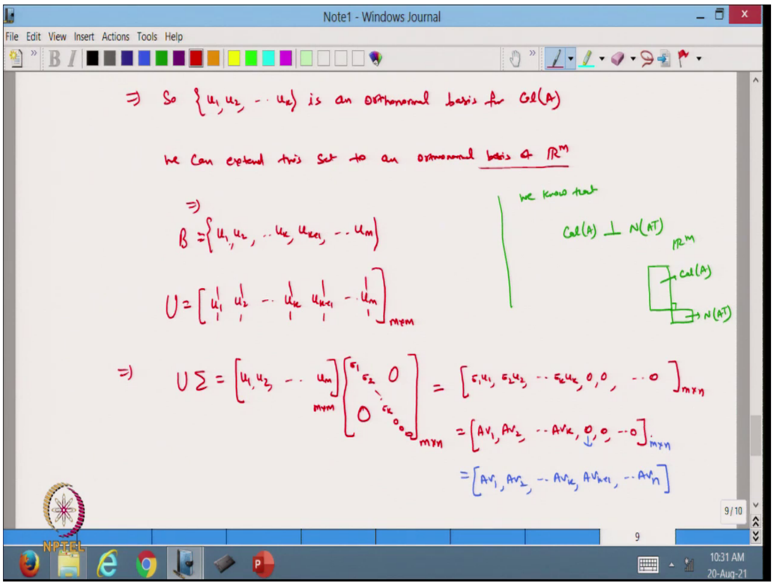

I think I should take this as equal to n. So, I can go up to 0. So, maybe I can just write here. I think instead of this writing I should write like here. So, I should write it like 0 0 0. So, and this is of the dimension m* n.

So, that we are writing, because from here, I get the matrix basically of order m* n. So, after doing this one, I will get this matrix A and this is V n* n matrix. So, from here I can write that after doing this one I can write my matrix A; that is $A = U\sum V^T$ and that is my SVD. So, this way we are able to find all the elements of the matrix V and all the elements of the matrix U.

So, now, after doing this one, we can take a quick example, then how we can find the SVD of a given matrix. So, let us take the example find the SVD of a matrix A. So, this matrix we are taking $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and this is 3* 2 matrix.

So, in this case if you see now, my $A^T A$ will be 2* 2 and my $A^T A$ is going to have 3*3, because A transpose is 2*3. So, it will be 2 cross 2's and. Now, we will go with the first step. So, let us take the solution.

So, first I will find out $A^T A$

$$A^T A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$| A^T A - \lambda I | = (\lambda - 3)(\lambda - 1) = 0$$
$$\lambda = 3,1$$

So, these are the eigenvalues I am getting.

Now, corresponding to the eigenvalues I am, I want to find the eigenvectors. So, I have my vector. So, corresponding to $\lambda = 3$. My $A^T A$ if you see, then I can write from here, I want to write it as

$$(A^T A)v_1 = 3v_1 => \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

And from here now, I want to make these vectors orthonormal also. So, this is a vector and I want to normalize this vector. So, I just take $v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$. So, I have normalized this vector and I have taken the first eigenvector as this one. So, after doing this one.

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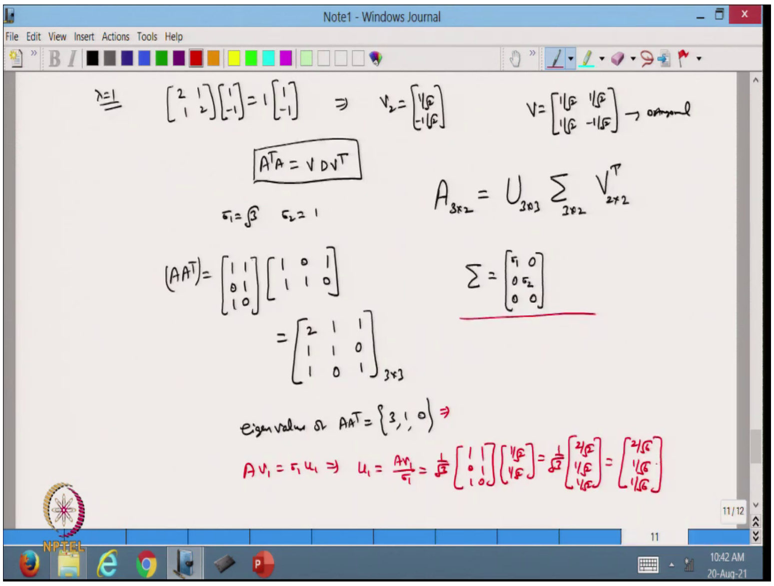

Now, for corresponding $\lambda = 1$ I will go the same way. So, I have

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

Now, based on this one, I can show my that my $A^T A$ can be written as. So, now, my $V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$. So, this is my matrix and this is my orthogonal matrix. So, I can write that $A = VDV^T$. So, this is what we are able to get.

Now, I will find out the value of u i's. So, now, from here my $\sigma_1 = \sqrt{3}, \sigma_2 = 1$. And if you see that my A is 3* 2. So, my U should be 3* 3, sigma should be 3* 2 and V transpose is 2* 2.So, this one we want. So, we got this value $\sum = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix}$. So, this is my sigma basically. Now, from here, I need to find the U.

So, if you see from this $AA^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

So, it is a symmetric matrix. So, I know that and this is 3* 3.

Now, if you find out its eigenvalue. So, you will find out the eigenvalues of AA^T. So, this is going to have the eigenvalue {3 ,1,0}, because if you see, then it is a 3* 3 and the rank of A transpose is 2. So, it is going to have 1, 0 eigenvalue. So, it is a singular matrix and is going to have 0 eigenvalue. So, this is what we got from here. Now, I want to find the value of u_1 ,u_2. So, either you can solve this one and find out the eigenvectors or we have the way we discuss.

So, what we need to do is now. So, I want to find the eigenvectors. So, I will start from here and then, I will try to find what is mine. So, I know that $Av_1 = \sigma_1 u_1$. So, from here, my u 1 I can find that

$$u_1 = \frac{Av_1}{\sigma_1}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 2/\sqrt{2} \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

So, this is my u_1 and it is a vector with magnitude one; that we can see from here. So, I am able to get this value.

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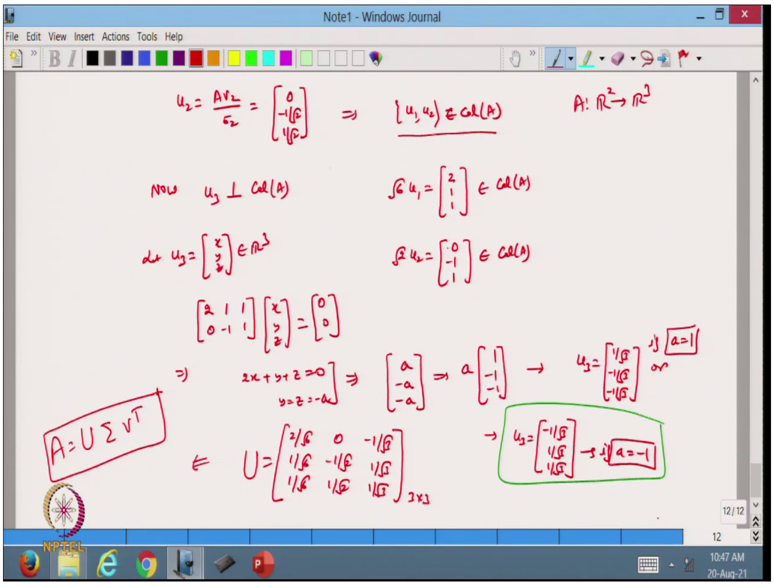

Now, corresponding to I will get my u_2 as

$$u_2 = \frac{Av_2}{\sigma_2} = \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

because I just substitute the value here, for v_2 and I get this value.

So, now, we are able to find the 2 from here. So, and you can see that this $\{u_1,u_2\}$ that it makes belongs to the column space of A and they make the column space ok. So, these are the column spaces of matrix A. Now, I want to extend this one. So, that is my question, because my U is 3* 3. So, I am able to find these two eigenvectors or maybe these vectors u_1,u_2 based on the condition that Av is equal to sigma 1 u_1 and Av_2 is equal to sigma 2 u_2.

Now, I can take my u_3 that should be perpendicular belongs to should be perpendicular to the column space of A; only then, it will be going to make the orthogonal basis for R^3, because our matrix is moving from it is 3* 2. So, it is from going from R^2 to R^3.

So, now so, perpendicular to column space. Now, what we do is that just for the calculation we can make life simpler I can take my u_1. So,

$$\sqrt{6}u_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \in col(A)$$

$$\sqrt{2}u_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \in col(A)$$

And this belongs to column space of A and also belongs to column space of A. So, now, what I do let my $u_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in R^3$. Now, it should be perpendicular to both, because these are the basis for the column space of A that we have seen. So, it means that the dot product. So, I just put this as $\begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ And from here, you can see that this is going to have a basis 2. So, now, its rank is basically 2.

So, now, from here you can see that I can have my $2x + y + z = 0, y = z = -a$. So, from these two, I can have the vector. So, y and z should be the same in this case and 2 x is equal to this one.

So, maybe I can just take this vector as I just take this as

$$\begin{bmatrix} a \\ -a \\ -a \end{bmatrix} => a\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$u_3 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix} or \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

So, I can choose any one.

You can use both the u_3 if you see. So, I can say this or this; this is equal to this, if a=1 and this is equal to this if a = -1; everything depends on this one. So, they and but this and these are orthogonal to each other that should be there. And if you see that taking the dot product. So, this will be orthogonal to each other. So, this way we are able to find the value of U.

And now, I can write my matrix $U = \begin{bmatrix} 2/\sqrt{6} & 0 & -1/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$ So, I have taken this value.

So, maybe you can say that this value we have taken corresponds to just wanting to have a smaller number of negative signs. So, that is why we have taken this one. So, my this is U that is 3* 3. And from here, we can verify that my $A = U\sum V^T$ This may be verified in the next lecture. So, we will stop here.

So, in today's lecture, we have discussed a very important theorem that is called the singular value decomposition of a given matrix. And then, we have seen that how we can generate the value of U and V that is the matrix involved in the singular value decomposition for the corresponding to the given matrix A. So, in the next lecture, we will show that how we can compute this singular value decomposition you, taking the help of MATLAB or Octave. So, I hope that you have enjoyed this lecture.

Thanks for watching.