Matrix Computation and it is applications
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Lecture – 04
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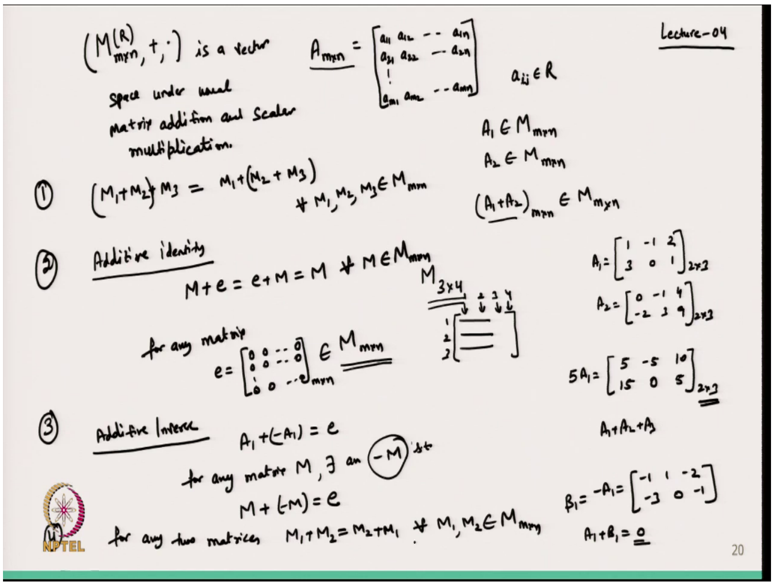

Hello viewers, welcome back to the course on Matrix Computation and its application. So, in the previous lecture, we have discussed a few examples of the vector spaces and today also we are continuing with that one. So, today we are going to discuss another very important example, that is a set of all the matrices of order m x n over the real because we are taking the matrices with real elements, with the usual addition and scalar multiplication.

And, we know that if we take a matrix A of order m x n, we can write this matrix in this form, where all these elements are assuming that is a real number. So, in this case, we know that if I take any matrix A1 and that it also belongs to the set and another matrix A2. Then, we know that we can add A1 and A2 because this is also a matrix of order m x n and that will also belong to this one or maybe I can write this as n. So, this is the way we take the addition of the matrix.

Like, suppose I have a matrix-like, [1, -1, 2; 3, 0, 1] this is a matrix, I just take A1 and I will take matrix A2, the same dimension I am taking. So, it is [0, -1, 4; -2, 3, 9]. So, it is of the same order m x n means, which is 2 x 3. So, it is 2 x 3 and 2 x 3.

So, in this case, we have to take the same matrix of the same order 2 x 3 and 2 x 3 only then we can define this vector addition or the scalar multiplication. So, now just for convenience, I can just take m n, now suppose I take an M of maybe a set of all 3 x 4 matrices.

So, in this case, all the matrices will always have the 3 rows and 4 columns so, 1 2 3 4 and 1 2 3. So, this is always having the 3 rows and the 4 columns. So, this is what we are going to define. So, always having m rows and n columns and this is the vector addition, we are defining the usual addition of the matrices and also I am defining the scalar multiplication.

So, you know that if I take 5A1 then I can write this matrix as [5, -5, 10; 15, 0, 5]. So, this is just the multiplication 5A1 means I am multiplying each element of the matrix by 5 and this is also 2 x 3. So, this is the usual addition and the scalar multiplication we are taking here.

So, the claim is that it is a vector space under usual matrix addition and scalar multiplication. So, now, after defining this addition and the scalar multiplication, we want to check whether it is a vector space or not. So, that is my claim. So, I just define satisfying all the properties. Now, suppose I take the matrix M1, M2, and M3 and add them together, then from here, I can very easily check that this is always equal to M1, M2, and M3.

So, the associative property is well defined in the case of matrices. So, this is true for all M1, M2 belongs to M, that is I am defining M over the set of the matrix m x n matrix. So, this is the 3 elements I have taken and this is true for all. So, the associative property is not the problem, you can also verify from here that A1+A2 and just I take A3, I can add on n matrix.

In the next one, I have to define the additive identity. So, additive identity means that I need an element such that if I take a matrix M and I need an element e, some element e is there or e plus M then I should get my M back, and this should be true for all M, belonging to m x n. So, in this case, my e is called the additive entity

So, now we know that for any matrix I can define e as a matrix with all the elements 0, i.e. [0 0 0; 0 0 0]. So, all the elements are 0 there and this is of order m x n. Now, from here, we know that this is a zero matrix of order m x n, it belongs to Mmn. And, also if I add any matrix then I am going to get the same matrix with that one. So, this is called my additive identity in the case of the set of all m x n matrices.

3rd one is now I will define additive inverse. So, additive inverse means suppose I have a matrix-like A1, I have taken matrix A1 some matrix I have taken here. Then, I can add minus A1 as we have taken, and then from here, you can see that you will get a zero matrix. So, you will get e, because here A1 is there I can take minus A1 and that will be [-1, 1, -2; -3, 0, -1,] and then this matrix is here. So, I call it B1. So, A1+B1 will be a zero matrix.

So, that will be a zero matrix with all the elements 0. So, in this case, I can say that for any matrix M there exists a -M ok, it is called additive inverse such that, that will be equal to e. And, this is called the additive inverse and this is true for all the matrices of dimension m x n. You take any matrix, I can always define the minus of that matrix, and then we can add this one to the given matrix and you will get the zero matrix. So, this is always satisfying.

The 4th one is additive identity and then I can define that for any two matrices M1+M2 is equal to M2+M1 and this is true for all M1, M2 belongs to this one. So, it is commutative. Now we can define the distributive property.

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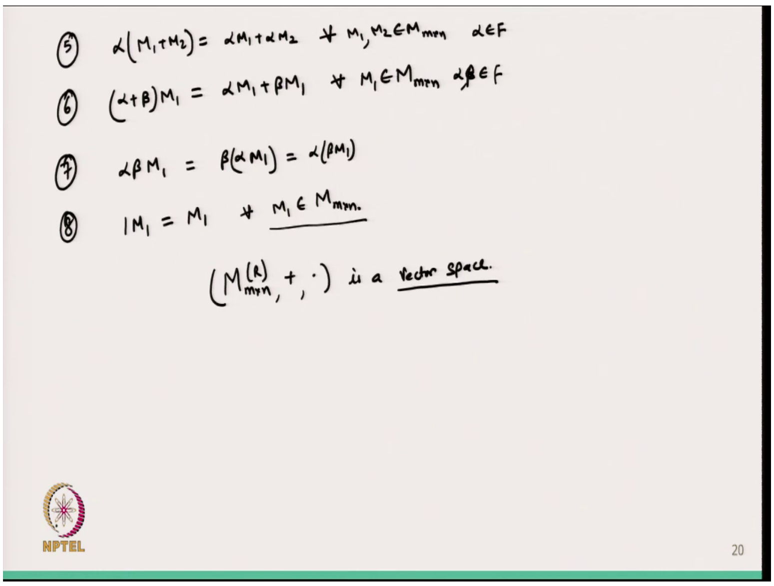

So, the next one is the 5th one. So, for any scalarα if I take M1, M2, 2 matrices then I can show that this is α(M1+M2)=αM1+αM2 and that is true for all M1, M2 belongs to Mmn and α belongs to the field, what is the field, we have taken that is a real number in this case. So, in this one, we can very easily verify the properties of the matrix addition and the scalar multiplication of the fifth one.

Then, the 6th one is that how we can define (α+β)M1. So, this one is also very easy to verify that this will be equal to this one and for all M1 belongs to the set and α, β belongs to field F. 7th is the αβM, I choose any M here for M1. So, this can be written as β(αM1) or I can take α(βM) like this one. So, this is also true for all.

And, the 8th one is 1 if I take from the real number and multiply by some vector M1, then we can show that this is equal to M1, where this is true for all M1. So, it is true for all M1 belongs to the set M of order mn this one. So, now, from here we can say that the set of all the polynomials of order m x n with the usual addition and scalar multiplication, usual addition means the addition we used to take in the matrix and the scalar multiplication is a vector space.

So, and the field I have taken over the real number. So, this is a vector space.

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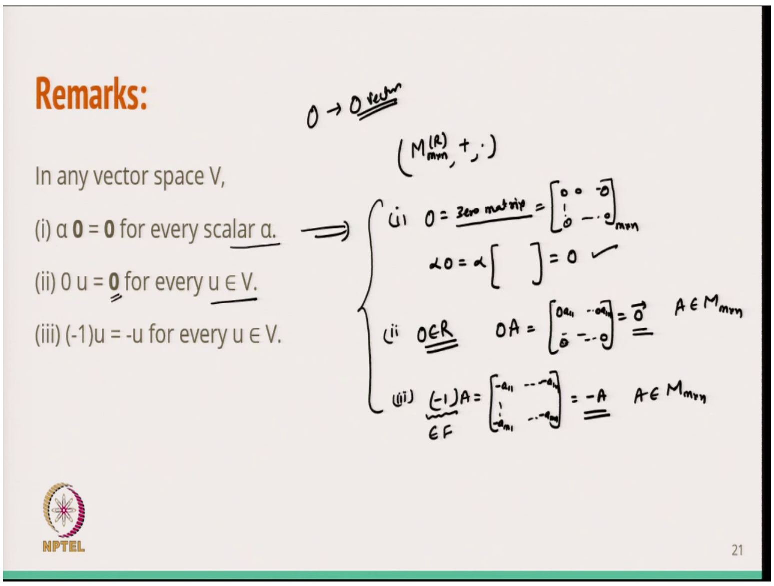

Now, we want to define some properties that belong to the vector space. So, this is just the three properties we have to keep in mind that if I take any α and multiply to the 0 element. So, this 0 means the 0 element in the vector space ok. So, this 0 means 0 vector. So, if I take any scalar multiplied by 0 vector, then I will get the 0 vector again for all scalars alpha.

And, suppose I have a 0 element from the field that is the scalar multiplied by any u, then I will get the 0 vector here. So, we are taking a field as a real number. So, in that real number, we know the 0 is there. So, if I take u.0 and take u, then it will be the 0 element of the vector space. And, -1.u will be equal to -u for every u belongs to the V. So, these three properties we have to keep in mind.

For example, just now we have taken the matrices of order m x n under addition and multiplication over the real line. So, in this case, if I take any scalar and I know that 0 element is a zero matrix. So, it is a zero matrix. So, I know that the zero matrix is this one [0 0 0 ;0 0 0...] all are 0. So, m x n is a zero matrix.

So, if I take any α into 0 then definitely I am taking α and then multiplying this matrix and that will be again the zero matrix. So, this is true, always true in the case of vector spaces. So, this is the first property we can verify. The second one is if I take the 0 element.

So, 0 belongs to the field R. And, if I take any matrix from the M_{mxn} then we know that 0 multiplied by the matrix A, then I will multiply all the elements with the 0, 0.a11,..., 0.a1n something like this. So, all the coefficients will be 0. So, this will become the zero matrix. So, I just write 0 and this 0 is coming from the field that is a scalar and I am writing this vector because it is a zero matrix.

So, that is true for all elements A belongs to where A belongs to M_{mxn} . So, this is the second one. And, the third one is -1.A. Definitely, I am going to multiply each element with the minus sign, -1.am1,..., -1.amn and that is equal to -A. So, this is also true for all elements A belongs to this one and this -1 is coming from the field, belongs to my field.

So, if this is there then it is always satisfying. So, if it is a vector space then this is always true because sometimes we need to define the inverse. So, this way we can define the inverse or the 0 element we want to find out. So, this is the way we can find out the 0 element. So, these three properties are always there if we have a vector space.

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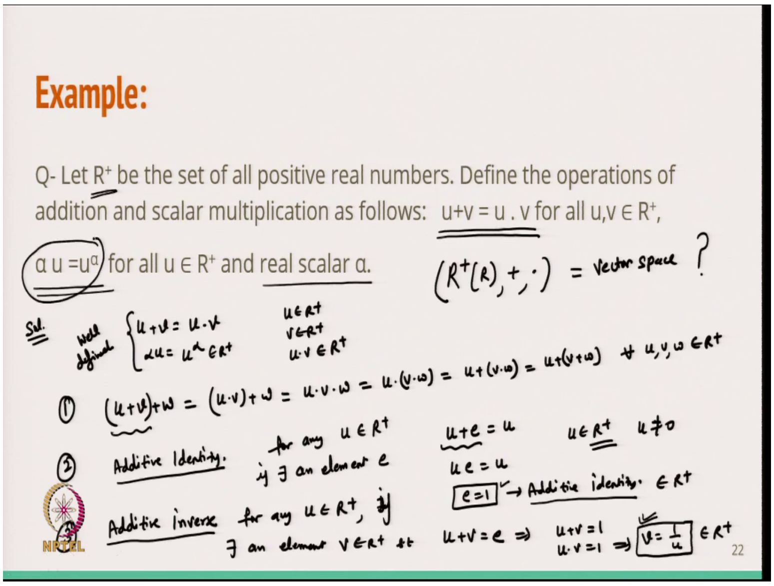

Now, after doing this usual addition and scalar multiplication, I want to give you one example that is completely different from what we have done till now. So, let us take this example. Let I take the R^+ set of all the positive real numbers. Define the operations of addition and scalar multiplication as follows. So, this is my vector addition I am defining for all u and v belong to R^+.

And, scalar multiplication I am defining α.u is equal to u^α. See, here I am defining u+v which is a vector addition. So, that I am defining that is equal to u.v for all u and v belong to R^+. And, this one I am defining α.u is equal to u^α for all u belongs to R^+ and a real scalar α. It means I am defining this one; this is a set of vectors defining over the whole real line with this addition and with this scalar multiplication.

And, I want to check if it is a vector space or not. So, this one I want to define R^+, you know all the positive real numbers. So, let us try to find out. So, the first property we are defining and is well defined. So, no problem, because u+v is equal to u.v and is well defined, u is coming from R^+, v is coming from R^+. So, definitely u.v, that is also coming from R^+. It cannot be 0 or negative.

So, it is definitely if you take any real number, positive real number, multiply two positive real numbers, that will definitely belong to the real number, positive real number. So, this and α.u is u^{α}. So, now, if I take any positive real number and take the power any power α, where α is any real number, then this also belongs to R^+ always ok. So, we are taking this one and this is well defined. So, it is a well-defined binary operation, addition, and scalar multiplication.

Now, we want to check whether it is a vector space or not. So, let us satisfy the first property that is (u+v)+w and this one I take like this one. Now, (u+v)+w, so, I am applying the first vector addition here and then the whole. So, u+v will be (u.v)+w, and then from here it will be u.v and then I am applying w.

Now, it just belongs to R plus. So, it is just a positive real number. So, from here I can write this as u.v.w, and then it will be u+(v.w), and then it becomes u+(v+w), ok. So, from here I can say that this is true for all u, v, w belongs to R^+. So, the property is satisfied, so it is associative. So, this associative property is defined in this case.

The second one is that I am just taking the additive identity. So, in this case for any u belongs to R^+, if there exists an element e such that u+e should be u, ok. So, for any u that belongs to this if there exists an element e, such that this property is satisfying then we call it e as an additive identity.

So, now, in this case, we have to find out. Now, see u+e is u.e by the addition and that should be equal to u. Now, u belongs to a positive real number right, e also going to be a positive real number if it exists there. Then from here and u is, of course, u is not equal to 0, because I am taking all the positive real numbers. So, from here we can say that e is equal to 1 because u.e is equal to u and that is always positive real numbers, then from here, e is 1.

So, from here I can say that this is my additive identity. So, till now we have seen the additive identity is a 0 element, but the first time we are able to see that in this case, the additive identity is 1. And, 1 belongs to the R^+. So, it means that additive identity exists in the given space, that is R^+.

The third one is the additive inverse. So, for the additive inverse for any u belongs to the set, if there exists an element, we call it v belongs to R^+ such that, u+v is equal to e, then we call v as an additive inverse. So, now, from here we know that u+v=e=1 and u+v is u.v, which is equal to 1. So, from here now you know that neither u nor v is 0 because it is R^+. So, from here I can take my v is 1/u and this is the additive inverse of u.

So, for any u, I can find the additive inverse 1/u and that also belongs to R^+, because if you take any positive number and take its inverse 1/u, that also belongs to R^+. So, in this case, this is my additive identity or additive inverse sorry in this case. So, now, from here we are able to see that it can also be an additive inverse instead of a minus of that number that we have seen in the previous example.

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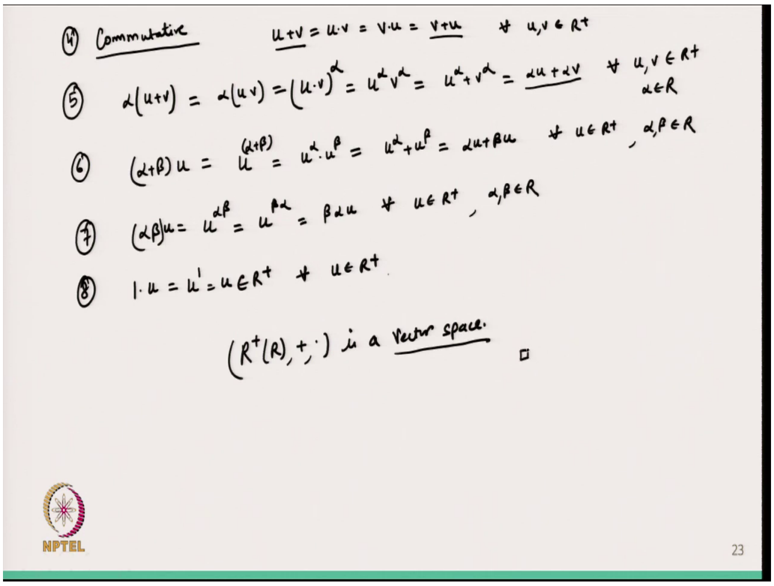

So, after doing this one the 4th property is commutative. So, commutative now u+v, I am taking. So, this is equal to the u.v that is there and this is a just multiplication of two positive real numbers. So, I can write this as v.u also. And, this one I can write v+u by the definition and this is true for all u and v belong to the set R^+ so, satisfying. So, this property is satisfying. So, it is commutative under the defined addition. So, now, it satisfies all the properties.

So, for the 5th property, I just want to see the distributive property, so, α(u+v). Now, I want to take the help of scalar multiplication and vector addition. So, let us see what is going to happen in this case. Now, α is a scalar and this is a vector. So, I can write this α and u +v, I know that this is u.v. And, now if α is multiplying the to the vector, then if you see from here, then this is the scalar multiplication.

So, I have to take the scalar multiplication here $(u.v)^{\alpha}$. And, now this is equal to so this is just the multiplication of two positive real numbers raised to power α, then I know that from the algebra that I can write it is $u^{\alpha}.v^{\alpha}$. And, then from here, I can write this as$u^{\alpha}+ v^{\alpha}$, right. So, this one I can write from here, and now I can write from here α.u+α.v.

So, this one is true and from here this is what I got from here. So, this is from here I can say that this is true for all u and v belongs to R^{+} and α belongs to R. So, it is a distributive property satisfying. The 6th one is again the distributive property. So, (α + β).u and I can write this as by the scalar multiplication it becomes $u^{(\alpha+\beta)}$. And, this is again $u^{\alpha}.v^{\beta}$.

And, again I can write this as $u^{\alpha}+ v^{\beta}$, by this vector addition and this becomes$\alpha u+ \beta v$. So, this is true for all u belongs to R^{α} and α, β belongs to real number R. So, this is satisfying. The 7th one is (αβ).u and this one I can represent, that this will become $u^{\alpha\beta}$ and it can be written as $u^{\beta\alpha}$ no problem, we can write this also.

And, from here you can see that I can write this as a (βα).u. Just we can write this as so, this is also true for all u belongs to R^{+} and α, β belongs to R. And, the 8th one is now my 8th property 1.u. Now, 1 is a real number that is coming from the field into u. So, by the scalar multiplication u it will become u.1, because it is just a scalar. And u^{1} is always u for any positive real number. So, this belongs to R^{+} and this is true for all this one.

So, this is true for all u belongs to R^{+}. Now, this property is also satisfied. So, from here we can say that all the properties are satisfying, and then R^{+} defined on the field R with given addition, and scalar multiplication is a vector space. So, we will stop here. So, today we have discussed two more examples based on vector spaces; the first one is the set of all the matrices of order m x n and the other example was that in which we are defining the vector addition and scalar multiplication in a different way.

So, in the next lecture, we will continue with this one, and thanks for watching.

Thanks very much.